

Diagnosis, Fault-tolerant and Robust Control for  
a Torsional Control System

Mandatory assignment in DTU course 34746

Part A

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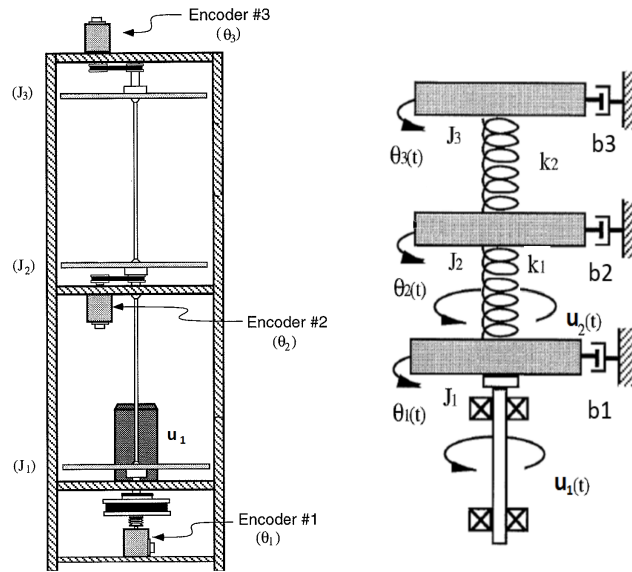
Table 1: Revision history			
version	date	description	changes
1.a	05.02.2024	new	all pages new

# Assignment part A - 2024

## Introduction

This exercise presents a model of the ECP M205a system. It is a three-mass system that comprises three identical disks, connected in parallel via a flexible low-damped shaft. The system is equipped with three incremental position encoders (one for each disk) and two actuators, i.e. a motor applying torques to the bottom disk and one more for the middle disk.

The control goal is to ensure that either of the disks tracks a specified motion profile. The system considered is sketched in Figure 1. The variables are explained in Table 2.



The normal behaviours of the systems are described by the following con-

Table 2: **List of variables**

variable	unit	description
$\theta_1$	rad	angular position of bottom disk
$\omega_1$	$\text{rads}^{-1}$	angular velocity of bottom disk
$\theta_2$	rad	angular position of middle disk
$\omega_2$	$\text{rads}^{-1}$	angular velocity of middle disk
$\theta_3$	rad	angular position of top disk
$\omega_3$	$\text{rads}^{-1}$	angular velocity of top disk
$u_1$	Nm	torque command for the bottom disk
$u_2$	Nm	torque command for the middle disk
$y_1$	rad	measured angular position of bottom disk
$y_2$	rad	measured angular position of middle disk
$y_3$	rad	measured angular position of top disk

straints:

$$\begin{aligned}
c_1 : 0 &= \dot{\theta}_1 - \omega_1 \\
c_2 : 0 &= J_1 \dot{\omega}_1 - u_1 + b_1 \omega_1 + k_1 (\theta_1 - \theta_2) + d \\
c_3 : 0 &= \dot{\theta}_2 - \omega_2 \\
c_4 : 0 &= J_2 \dot{\omega}_2 - u_2 + b_2 \omega_2 + k_1 (\theta_2 - \theta_1) + k_2 (\theta_2 - \theta_3) \\
c_5 : 0 &= \dot{\theta}_3 - \omega_3 \\
c_6 : 0 &= J_3 \dot{\omega}_3 + b_3 \omega_3 + k_2 (\theta_3 - \theta_2) \\
d_7 : 0 &= \dot{\theta}_1 - \frac{d\theta_1}{dt} \\
d_8 : 0 &= \dot{\omega}_1 - \frac{d\omega_1}{dt} \\
d_9 : 0 &= \dot{\theta}_2 - \frac{d\theta_2}{dt} \\
d_{10} : 0 &= \dot{\omega}_2 - \frac{d\omega_2}{dt} \\
d_{11} : 0 &= \dot{\theta}_3 - \frac{d\theta_3}{dt} \\
d_{12} : 0 &= \dot{\omega}_3 - \frac{d\omega_3}{dt} \\
m_{13} : 0 &= y_1 - \theta_1 \\
m_{14} : 0 &= y_2 - \theta_2 \\
m_{15} : 0 &= y_3 - \theta_3
\end{aligned}$$

where  $d \triangleq T_C(\omega_1)$  is a function of the bottom disk angular velocity and represents the *unknown* Coulomb friction torque on that disk. The control input saturates at 2 Nm, i.e.  $u_1 \in [-2, 2]$  Nm. The parameters in the forgoing con-

straints are listed in Table 3.

Table 3: List of parameters			
symbol	value	unit	description
$J_1$	0.0025	$\text{kgm}^2$	Bottom disk moment of inertia
$J_2$	0.0018	$\text{kgm}^2$	Middle disk moment of inertia
$J_3$	0.0018	$\text{kgm}^2$	Top disk moment of inertia
$k_1$	2.7	$\text{Nmrad}^{-1}$	Stiffness of the bottom shaft
$k_2$	2.6	$\text{Nmrad}^{-1}$	Stiffness of the middle shaft
$b_1$	0.0029	$\text{Nmsrad}^{-1}$	Damping/friction on the bottom disk
$b_2$	0.0002	$\text{Nmsrad}^{-1}$	Damping/friction on the middle disk
$b_3$	0.00015	$\text{Nmsrad}^{-1}$	Damping/friction on the top disk

A Simulink model and a parameter initialisation file have been uploaded to DTU Learn in the *File share/Matlab and Simulink files/Mandatory Assignment Part A* folder for your convenience.

## Question 1

Make a structural analysis:

1. Determine a complete matching on the unknown variables.
2. Find the parity relations in symbolic form.
3. Investigate other properties you find relevant from a structural analysis.
4. Reformulate the parity relations to an analytic form, as functions only of known variables and the system parameters.

## Question 2

1. Design a set of proper residual generators, i.e. residual generators that are stable and are causal.
2. Discretise your residuals with sampling period  $T_s = 4$  ms.
3. Implement your residual generators using a Simulink model of the system and demonstrate by simulation (in open loop) that your residuals are insensitive to changes of control inputs.

## Question 3 (Experimental work)

A single sensor additive fault afflicted the measurements.

1. Test the implemented residual generators on the ECP M502a torsional system. Demonstrate their fault-detection properties.
2. Which of the three sensors is afflicted by the fault? Justify your answer.
3. Are the residuals insensitive to input changes? Comment on the results.

**Hint:** In order to load the recorded inputs and outputs in Simulink, you can set them in a timeseries structure (Matlab command `timeseries()`) and use the “From Workspace” block in Simulink.

## Question 4

Faults are possible on any of the sensors or at the actuator.

1. Model these as additive faults and write the system in the standard form

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}_x\mathbf{d} + \mathbf{F}_x\mathbf{f} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{E}_y\mathbf{d} + \mathbf{F}_y\mathbf{f}\end{aligned}$$

where the matrices need be determined and the vector  $\mathbf{f}$  contains all actuator and sensor faults.

2. Determine the transfer function  $H_{rf}(s)$  from faults to residuals in your LTI design.
3. Investigate strong and weak detectability of the faults.
4. What would change in terms of fault detectability if the Coulomb friction function  $T_C(\omega_1)$  were known?

**Hints:** For this task it is useful to find a basis for the left nullspace by using the Matlab symbolic toolbox in defining the various transfer functions (e.g. `syms s; G = 1/(s + 1)`). You may find the following functions useful:

- `simplify` - simplifies symbolic expression,
- `expand` - expands symbolic expression,
- `numden` - extracts numerator and denominator of symbolic fraction,
- `sym2poly` - converts symbolic polynomial to numeric,
- `minreal` - gives a minimal realization of a transfer function,
- `zpk` - expresses a transfer function as a zero-pole-gain product.

## Practical notes

- The deadline for the report is **Monday March 11, 2024, at 23:55 hours.**
- Please note that max. 20 pages are allowed for part A, excluding front matter. Any pages in excess of 20 (excluding front matter) will be discarded.
- Layout must be reasonable: A4 paper, 20mm margins and font size 11pt.
- The report need be delivered in electronic form, as a .pdf file, via the DTULearn assignment system. Only pdf files are accepted.
- Group-work is encouraged (max 2 persons in one group).
- Please do **write your name(s) and student number(s) at the front page and as running heading on each page of your report.** In addition, do not forget page numbers.