

Diagnosis, Fault-tolerant and Robust Control for
a Torsional Control System

Mandatory assignment in DTU course 34746

Part A

Dimitrios Papageorgiou

February, 2024

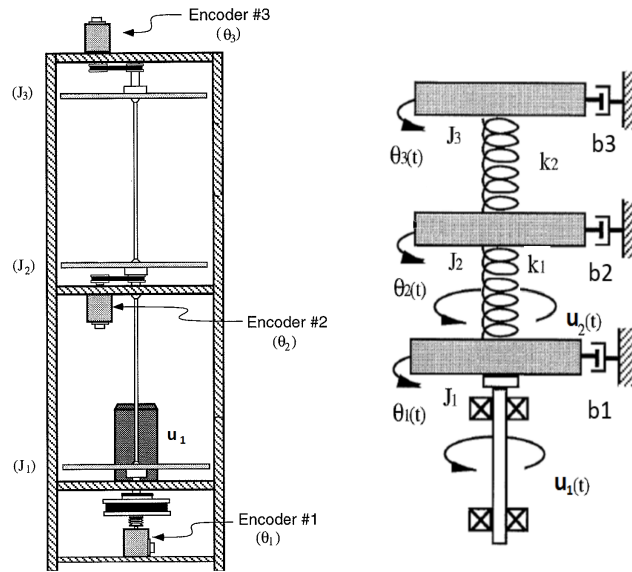
Table 1: Revision history			
version	date	description	changes
1.a	05.02.2024	new	all pages new

Assignment part A - 2024

Introduction

This exercise presents a model of the ECP M205a system. It is a three-mass system that comprises three identical disks, connected in parallel via a flexible low-damped shaft. The system is equipped with three incremental position encoders (one for each disk) and two actuators, i.e. a motor applying torques to the bottom disk and one more for the middle disk.

The control goal is to ensure that either of the disks tracks a specified motion profile. The system considered is sketched in Figure 1. The variables are explained in Table 2.



The normal behaviours of the systems are described by the following con-

Table 2: **List of variables**

variable	unit	description
θ_1	rad	angular position of bottom disk
ω_1	rads^{-1}	angular velocity of bottom disk
θ_2	rad	angular position of middle disk
ω_2	rads^{-1}	angular velocity of middle disk
θ_3	rad	angular position of top disk
ω_3	rads^{-1}	angular velocity of top disk
u_1	Nm	torque command for the bottom disk
u_2	Nm	torque command for the middle disk
y_1	rad	measured angular position of bottom disk
y_2	rad	measured angular position of middle disk
y_3	rad	measured angular position of top disk

straints:

$$\begin{aligned}
c_1 : 0 &= \dot{\theta}_1 - \omega_1 \\
c_2 : 0 &= J_1 \dot{\omega}_1 - u_1 + b_1 \omega_1 + k_1 (\theta_1 - \theta_2) + d \\
c_3 : 0 &= \dot{\theta}_2 - \omega_2 \\
c_4 : 0 &= J_2 \dot{\omega}_2 - u_2 + b_2 \omega_2 + k_1 (\theta_2 - \theta_1) + k_2 (\theta_2 - \theta_3) \\
c_5 : 0 &= \dot{\theta}_3 - \omega_3 \\
c_6 : 0 &= J_3 \dot{\omega}_3 + b_3 \omega_3 + k_2 (\theta_3 - \theta_2) \\
d_7 : 0 &= \dot{\theta}_1 - \frac{d\theta_1}{dt} \\
d_8 : 0 &= \dot{\omega}_1 - \frac{d\omega_1}{dt} \\
d_9 : 0 &= \dot{\theta}_2 - \frac{d\theta_2}{dt} \\
d_{10} : 0 &= \dot{\omega}_2 - \frac{d\omega_2}{dt} \\
d_{11} : 0 &= \dot{\theta}_3 - \frac{d\theta_3}{dt} \\
d_{12} : 0 &= \dot{\omega}_3 - \frac{d\omega_3}{dt} \\
m_{13} : 0 &= y_1 - \theta_1 \\
m_{14} : 0 &= y_2 - \theta_2 \\
m_{15} : 0 &= y_3 - \theta_3
\end{aligned}$$

where $d \triangleq T_C(\omega_1)$ is a function of the bottom disk angular velocity and represents the *unknown* Coulomb friction torque on that disk. The control input saturates at 2 Nm, i.e. $u_1 \in [-2, 2]$ Nm. The parameters in the forgoing con-

straints are listed in Table 3.

Table 3: List of parameters			
symbol	value	unit	description
J_1	0.0025	kgm^2	Bottom disk moment of inertia
J_2	0.0018	kgm^2	Middle disk moment of inertia
J_3	0.0018	kgm^2	Top disk moment of inertia
k_1	2.7	Nmrad^{-1}	Stiffness of the bottom shaft
k_2	2.6	Nmrad^{-1}	Stiffness of the middle shaft
b_1	0.0029	Nmsrad^{-1}	Damping/friction on the bottom disk
b_2	0.0002	Nmsrad^{-1}	Damping/friction on the middle disk
b_3	0.00015	Nmsrad^{-1}	Damping/friction on the top disk

A Simulink model and a parameter initialisation file have been uploaded to DTU Learn in the *File share/Matlab and Simulink files/Mandatory Assignment Part A* folder for your convenience.

Question 1

Make a structural analysis:

1. Determine a complete matching on the unknown variables.
2. Find the parity relations in symbolic form.
3. Investigate other properties you find relevant from a structural analysis.
4. Reformulate the parity relations to an analytic form, as functions only of known variables and the system parameters.

Practical notes

- The deadline for the report is **Monday March 11, 2024, at 23:55 hours**.
- Please note that max. 20 pages are allowed for part A, excluding front matter. Any pages in excess of 20 (excluding front matter) will be discarded.
- Layout must be reasonable: A4 paper, 20mm margins and font size 11pt.
- The report need be delivered in electronic form, as a .pdf file, via the DTU Learn assignment system. Only pdf files are accepted.
- Group-work is encouraged (max 2 persons in one group).

- Please do **write your name(s) and student number(s) at the front page and as running heading on each page of your report.** In addition, do not forget page numbers.