Proof

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To prove that Elias Omega code is prefix-free from any other code word generated by Elias Omega code, consider two integers n and m, where $n \neq m$, $n < m$ and $n, m \in \mathbb{R}^+$.

Using a proof by contradiction, assume there does exist two positive integers where the code word of n is a prefix of the code word of m. Therefore, the length of n must be $\leq$ to the length of m and the contents of n are a prefix of m.

The result of n being less than m will result in two different cases when comparing the two outputs of their Elias Omega code.

Firstly, the length of the binary representation of n and m can be the same. In this case, the length components of each Elias Omega encoding would be equal and contain the same bits. However, the binary representation of the number will be different. Thus, n can not be a prefix of m.

In the other case, the binary representation of n is shorter than that of m. This will result in the bits of the two length components to not be equal as they represent different subsequent lengths. Or, on the other hand, the total length of the length components of n to be less than that of m. Since the leading 1 of the each length component is changed to a 0, this will create a missmatch with the leading 1 of the binary representation of n.

- For example:

$$0\ 00\ 100$$

$$0\ 00\ 000\ 10000$$

- The 4th bit represents this case.

A contradiction is met in both cases. Therefore, by contradiction the Elias Omega code is prefix-free on any two positive integers where $n \neq m \in \mathbb{R}^+$.