KENNETH JOSEPH ARROW

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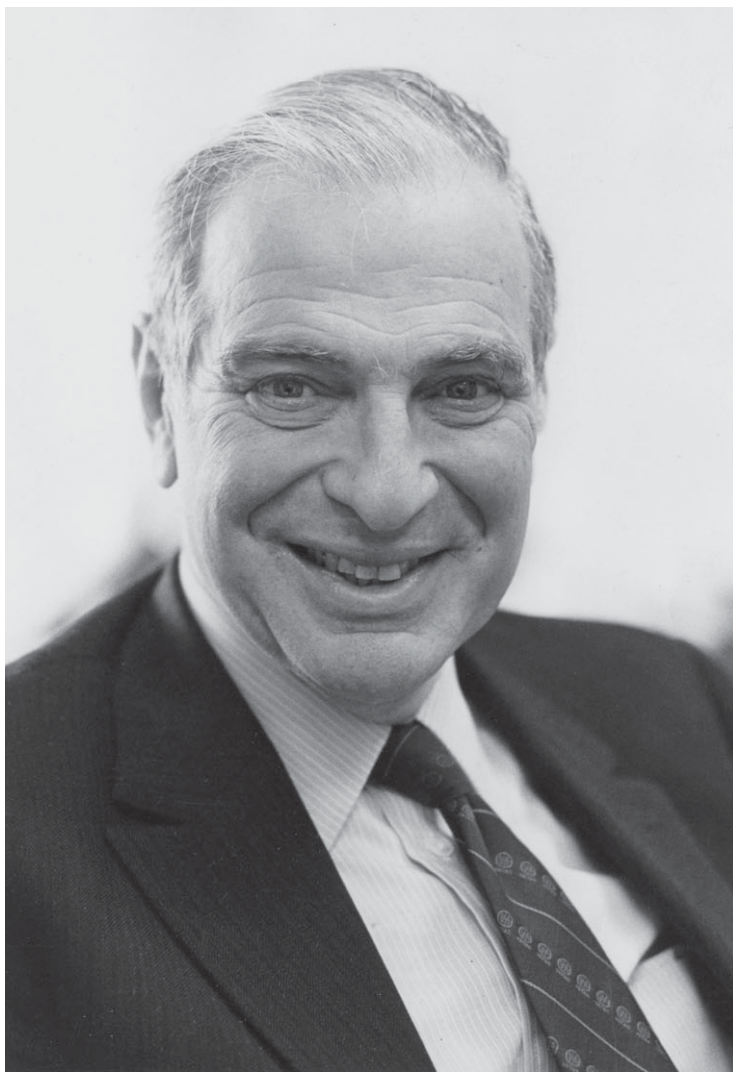
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Elected ForMemRS 2006

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Kenneth Arrow was a mathematical economist and political scientist who made many ground-breaking contributions to the theory of economics and social values. His great mathematical ability led him to introduce new approaches to theoretical economics and in particular to a series of fundamental theorems in the discipline. These included the Arrow Impossibility Theorem, the two fundamental theorems of welfare economics and the existence of a competitive equilibrium. For these and many other contributions he was awarded the 1972 Nobel Prize in Economics shared with Sir John Hicks. He took a particular interest in computation and computability in economics. He was active and very productive as a researcher for over seven decades and was renowned as a generous and inspiring teacher and colleague.

FAMILY BACKGROUND AND EARLY YEARS

Kenneth Joseph Arrow was born on 23 August 1921, in New York City, NY, the son of Harry and Lillian (née Greenberg) Arrow who had married on Sunday, 21 November 1920, in New York. Kenneth's parents came to the US at a very young age as Romanian Jewish first-generation immigrants and were brought up on the lower east side of New York (Feiwel 1987a). Harry came from the town of Podu Iloaiei, in Iaşi county, and Lillian, from the same county's 'capital', and second largest town in Romania, Iaşi, in the Polish part of the region of Moldavia.

Harry was gifted intellectually and educated at Townsend Harris Hall, as it was then. Harry's parents had considerable difficulties in financing his travel to school but their value system, adopted by their son, made it imperative that he was educated at the best possible

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institution. He graduated from the high school and then attended the New York University Business School for his undergraduate education, and after the First World War successfully established himself in the banking industry. The onset of the Great Depression, initiated by the Wall Street crash of October 1929, brought an end to the immediate prosperity of the banking industry and resulted in hardship of typical severity for the Arrow family's previous comfortable circumstances.

Harry and Lillian Arrow also had two younger daughters, Anita and Ellen. Anita (or Anette) married Robert Summers, brother of Paul Samuelson, and became the mother of three sons, one of whom is the distinguished economist, Larry Summers. Anita is distinguished in her own right: a professor emerita of the Wharton School; inductee into the Hunter College New York hall of fame; and recipient of an honorary doctorate of humane letters from the same institution in 1995.

Kenneth described with considerable frankness his early 'family circumstance' (Feiwel 1987b):

> I did not come from a scholarly family, although my father, who actually came from extreme poverty, got a degree from New York University Business College, which was then very unusual. The house had a lot of books; all sorts of books, there were encyclopedias, novels … My parents were newly rich and they acquired a lot of these matched sets of books that are not so fashionable today but were quite in style then. But they were good books. My parents adapted well to their new affluence. In fact, they became well off rather early and then lost everything. So my first ten years of life were spent in considerable affluence, my next ten years in very considerable poverty. But the result was that there was capital built up in the earlier period. So, from my point of view, it was better that than the reverse. And, of course, among the things we had left were the books. My parents were people of taste; what they did buy they bought well—the furniture was good quality, the books were good quality.
>
> Of course, though my father would have preferred if I had become a businessman, being a scholar was high in his value system. So there certainly was no opposition. They sort of worried that I read too much; they thought I should go out and play football …

He attended Townsend Harris High School, like his father, in New York City. Mayor Fiorello La Guardia ordered the school closed in 1942, stating that it was a 'non-essential educational unit', despite its high standing in the USA [USA] as a whole and its very high status in New York itself. (It reopened in 1984, first in temporary buildings on Parsons Boulevard and, then, in 1995, permanently to the campus buildings at Queen's College, both in New York.)

Although Kenneth's attendance at Townsend Harris High School was marked by 'ten years of very considerable poverty' because his parents 'lost everything' in the harsh years of the depression, nonetheless the Arrows were still able 'to retain a … servant, Mary Hermann' in their house, which they 'rented for $200 per month' (Read 2012). At that time, they lived at 1 Darley Road, in Great Neck, Long Island, New York, and the Townsend Harris High School was located at Amsterdam Avenue and 136th Street on the upper west side campus of the City College of New York (CCNY) (Schweber 1994). Kenneth is reputed to have had to walk a considerable distance back and forth from home to school during these years of hardship.

In 1947, Kenneth married Selma Schweitzer, who was then a graduate student at the University of Chicago where she held the Sarah Frances Hutchinson Fellowship. Selma later re-trained and practised as a psychotherapist. They had two sons, David Michael born in 1962 and Andrew Seth in 1965 (figure 1). Selma died in 2015.

Figure 1. Kenneth with his family. Clockwise from top left: David, Andrew and Selma. Photograph provided by David Arrow. (Online version in colour.)

University, war, research and consultancy years

As an undergraduate, Kenneth was educated at the CCNY, where free tuition gave his generation a tremendous start in the pursuance of an academic career. Graduation from the Townsend Harris High School 'automatically guaranteed entrance to CCNY' (Schweber 1994). Because of the family's poverty due to the Great Depression, Kenneth had no choice but to accept this opportunity (Wright 2017). CCNY was also fondly known as the 'Harvard of the Proletariat' or the 'Poor Man's Harvard', as well as 'Harvard-on-the-Hudson'.

In 1940 Kenneth obtained a Bachelor's degree in social science, with a major in mathematics, from CCNY. In Feiwel (1987a), he says that he 'was at City College which had a fairly good faculty and an extremely good student body'. That student body was to include three future Nobel Prize winners: Julian Schwinger (physics, 1965) and Herbert Hauptman (chemistry, 1985), who belonged to the immediately previous cohort at CCNY, and Robert Aumann, who won the Nobel economics prize in 2005, 33 years after Kenneth's award in 1972.

Kenneth's above observation in Feiwel (1987a) is stated in slightly stronger terms in Schweber (1994), together with the point that the 'city colleges could boast of their impressive faculties. At CCNY, . . . Emil Post was a professor of mathematics'. Post was a renowned mathematician and logician, and a member of the faculty of CCNY from 1935 until his death in 1954 (Davis 1994).

In early January 1987, Paul Samuelson rang me at UCLA and, among other things, asked me what I was working on at that time. I replied that I was concentrating on aspects of computability in economics, and outlined, at Samuelson's request, what I meant by it.

Figure 2. Two 'Impossibilists' conversing: Amartya Sen and Kenneth, Washington DC, June 2016. Source unknown. 'My work [in choice theory] is still being referred to. [Amartya] Sen certainly does when developing his own work' (Kenneth, cited in Feiwel 1987a). (Online version in colour.)

Samuelson listened patiently, thought a little and responded: 'You should speak to Ken Arrow; he was a pupil of Emil Post at CCNY, during his university undergraduate years.' Computation and computability were recurrent themes in various of Kenneth's writings—published articles, letters to one of his past pupils and also his act of generosity to me personally.

The other eminent mathematician and mathematical logician who strongly influenced Kenneth during their years together at CCNY was Alfred Tarski (Feiwel 1987a, 1987b). This is particularly important for the papers published by Kenneth in 1950 and 1951, which used— and made familiar to economists—axiomatic analysis, the notions of orderings and relations over finite or infinite sets of alternatives. Amartya Sen, more than any other economist, made imaginative use of partial orderings in social choice contexts (figure 2). The best description of the pervasive influence of Tarski's *Introduction to logic* (1941) on Kenneth, is Suppes (2005): 'Ken's work is written in the modern axiomatic style of pure mathematics, now dominant in large parts of mathematical economics.' However, this observation is qualified: 'I am not suggesting that Ken learned about the axiomatic method only from Tarski.' A few pages earlier, Suppes points out: 'Ken has said to me [Suppes] he probably first learned about ordering relations from a course in algebra using Birkhoff and MacLanes' well-known text (1941), as well as from Tarski.' However, Kenneth's nephew, Lawrence Summers, in his email reaction to the first draft of my *Nature* obituary of Kenneth, told me: 'I was pretty close to Kenneth and never heard him mention Tarski.'

Kenneth studied at Columbia University, New York, for his Master's degree in mathematics, which he obtained in June 1941 with Harold Hotelling, who, although a mathematical statistician, was in the department of economics. His PhD at Columbia, with the economist Albert Gaylord Hart as the dissertation advisor, was obtained in 1951. The 'ten-year

gap' between the two degrees can be explained by war service, research and consultancy activities, which also supplemented his and his wife's meagre, family-based, income. The thesis that underpinned his doctoral degree (2) eventually founded a whole new field of research that became social choice theory; a part of mathematical politics, in Paul Samuelson's felicitous description (Samuelson 1967). Kenneth was quite explicit about why he chose Columbia University for his graduate studies (Feiwel 1987a): 'I went to Columbia also partly because it was difficult for me to go anywhere else; because of financial considerations, so that I could live at home.'

He was still beset with uncertainties about his future career path, partly as a result of the family misfortunes during the depression era. He considered a high-school teaching career in mathematics, actuarial work and so on, but fortunately none of these ever materialized.

Between 1942 and 1946, Arrow did his military service as a weather officer assigned to the Weather Division, Headquarters United States Army Air Forces (20). The first paper he published, 'On the Use of Winds in Flight Planning', is the first chapter of volume five of his collected papers (20). The work that underpinned it led to his scepticism about the superiority of the physical sciences with regard to predictions, as compared to similar activity in economics (see also Heller *et al.* 1986; *The Region* 1995, 15). The article is written in a language that sits comfortably with 'old students of calculus' (Solow 1997). Kenneth deals with the optimality of flight paths, in the same sense as minimal optical paths (24).

Kenneth returned from war service to Columbia University to pursue his doctoral studies, but also as a research associate at the Cowles Commission for Research in Economics, then located at the University of Chicago, where he simultaneously held an assistant professorship in economics. In addition, he was a consultant at the RAND Corporation in Santa Monica, California. During these years of 'interregnum' between the 'east coast' and his eventual move to the 'west coast' (Stanford University), he formed a lasting friendship with Harold Hotelling, to whom he also professed personal and professional admiration and loyalty in numerous writings (e.g. (9)). In particular, he stated (Feiwel 1987b):

> Hotelling was a very kind man. Socially, he was a sort of shy, not very adroit person. It was not that he was lacking in self-confidence, or that he did not think of himself as a person of worth. He was a really sweet man.
>
> In many ways he was quite isolated; he had no one to talk to. He was the one who brought Wald in. . . . Hotelling was very interested in individuals. He was famous for generously giving away ideas. He was always having very good ideas that he was trying to implant in students to work on. . . . It was common for him to suggest that a student work up something. Two weeks later when the student came back, Hotelling would say, 'oh, it was a wonderful idea, it worked out nicely, you developed that on your own'. He would sort of deny the fact that he had anything to do with it.
> . . .
> Originally I had enrolled in mathematics, since I was interested in statistics. I went to him, explaining that I would need a fellowship in order to stay at Colombia after the first year (I had borrowed the money for tuition for the first year), and asking him to write a letter of recommendation. In very careful language he said that he would be glad to write a letter, but that he thought the mathematics department would pay no attention. However, he said, 'if you should think of enrolling in economics, the department has generally allowed me to pick one fellow'. So I was essentially 'bought' into economics! He produced a fellowship, and was my major backer.

With Jacob Marschak and Tjalling Koopmans, Kenneth formed a famous trio of kindred souls who worked with each other, for each other and built their own fundamental theories and methods on those of the other two. His fundamental contributions in the 'astonishing' decade-and-a-half from 1950 to 1965, especially in social choice theory, inventory theory and information economics, on the one hand, and the foundations of (production) exchange theory that was the Arrow–Debreu mathematical theory of general economic equilibrium, on the other, were based on his own mastery of the logic of relations and the axiomatic method, Marschak's many faceted work on the two issues of inventory theory and information economics and on Koopmans' ideas on activity analysis of production (4, 5, 17). Later, he referred copiously to Koopmans (1957) when discussing the existence of general equilibrium, particularly in the presence of non-increasing returns to scale in production (16). Their friendships, begun at Cowles in its Chicago 'days', and at RAND in their respective consultancy times, lasted all their respective lives. Marschak was the only one of the 'trio' who did not win the Nobel award in economics, but he was, surely, on the verge of receiving the prize before death intervened. What Kenneth owed, intellectually, to Abraham Wald, John Hicks and Paul Samuelson was what we all owe to them, but manifested itself in his extraordinary extensions and generalizations of value theory, static and dynamic equilibria, optimal allocation of scarce resources, welfare economics, general equilibrium theory, linear, nonlinear and dynamic programming and optimal control theory as a generalization of the classical calculus of variations, stability and dynamics.

Kenneth was appointed as (acting) assistant professor of economics and statistics at Stanford University in 1949 and was to remain there, becoming a full professor in these subjects, until 1968. In that year he moved to Harvard University's Department of Economics as a professor of economics and then back to Stanford University 11 years later, as the Joan Kenney Professor of Economics and Operations Research, until he retired in 1991 (figures 3 and 4). He remained an active emeritus professor until his dying day.

The astonishing years

I have called the years from 1950 to 1972 'the astonishing years' partly because during these years, particularly in the first five years of this period, Kenneth made path-breaking, fundamental, theoretical contributions to the broaching of a whole new field, social choice theory (1, 3), and to what became the mathematical standard and methodological norm of standard economic theory, general equilibrium theory (GET) (5). In addition, he made his ground-breaking analysis of the welfare economic underpinnings of GET in the form of the two fundamental theorems of welfare economics (2). For these four jewels of theory he shared the 1972 Sveriges Riksbankens Prize in Economic Sciences in Memory of Alfred Nobel with John R. Hicks (figure 5). The citation stated: ' . . . for their pioneering contributions to general economic equilibrium theory and welfare theory.'

But there was much more: more pioneering theoretical contributions to the economics of information and risk, which, in homage to Keynes and Knight, I shall distinguish from uncertainty, although Kenneth did not seem to; applied work, e.g. (21); edited volumes contributing fundamental essays to novel mathematical methods of optimization, stability, resource allocation and risk-bearing processes; and computation. As a result, this section can

Figure 3. Kenneth in his office, 1984. Photograph by Chuck Painter/Stanford News Service.

only give an impression of a select few of Kenneth's numerous fundamental contributions to theory, applications, extensions and generalizations.

In addition to the above list of achievements, Kenneth mastered—from about the end of the 1930s, through the early 1950s and later as well—mathematical methods that were new even to mathematically-minded economists and that were fundamental and innovative: the axiomatic method; standard mathematical logic for deductive purposes; topological methods and convex analysis, that is, separation theorems, essentially the Hahn–Banach theorem; and compactness, meaning the Brouwer and Kakutani fixed point theorems. This did not mean that he neglected mastery of classical results in dynamical systems, optimization theory and linear and nonlinear programming.

First of all, the sheer quantity of path-breaking (by any standard) contributions to various forms of economic analysis in the form of papers, essays in edited volumes and singly and jointly authored and edited books is quite mind-boggling. Only some of them are listed in the bibliography. Even a cursory glance indicates that Kenneth's productive period, as crudely measured by his written works, including 'oral' histories and interviews, extended from 1949 to 2017, the year of his death, an astonishing period of almost seven decades!

Second, I treat Kenneth's welfare theorems (2) as one whole, with the exception of GET of Arrow and Debreu (5). Thus, Kenneth's impossibility theorem (3) is treated separately because I partly agree with Paul Samuelson (1967) and his constructive critique of the way Kenneth defined a social welfare function in the Bergson–Samuelson sense, when he meant something else.

Third, although Kenneth was well acquainted with a variety of theories of probability (e.g. see chapter 1 of (16), which appeared first in 1951) and incomparability as a case of partial
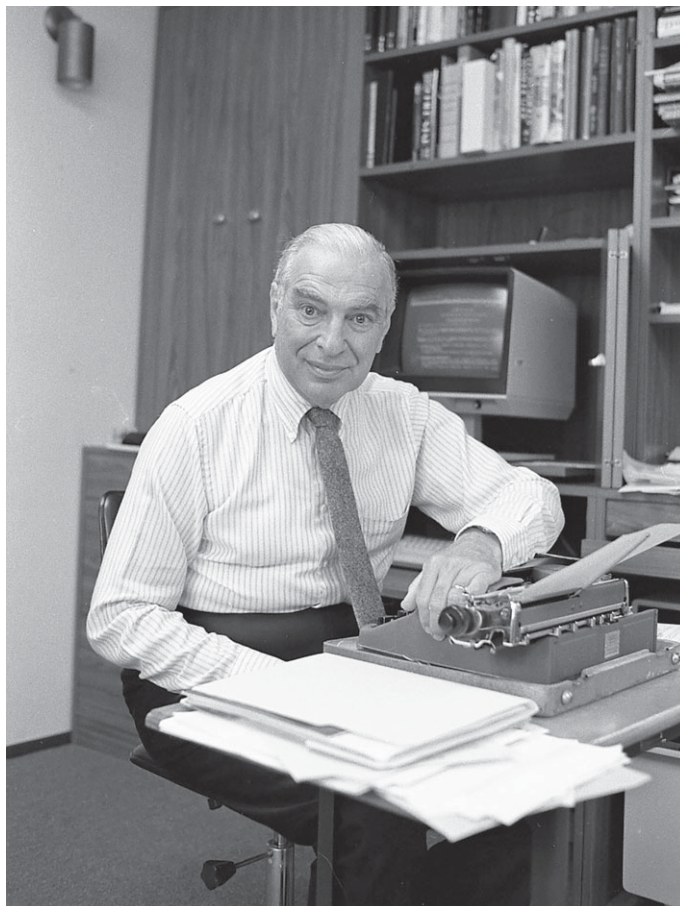
Figure 4. Kenneth during his time at Stanford. Photograph by Chuck Painter/Stanford News Service.

ordering, I shall emphasize the notion of Knightian uncertainty as utilized by Aaronson (2016). I do this only because Aaronson makes a good case for the assumption of convexity in such cases, which was important for Kenneth's mathematization of the social sciences. All kinds of radical or fundamental uncertainty, the generation of 'black swans' and so on, are for me—and, I dare say, also for Kenneth—due to partial ordering and, therefore, partial recursive functions, and they result in formulating algorithms in the sense of computability theory, subject to the halting problem for Turing machines.

Finally, I state Kenneth's results (1, 3) in terms of concepts discussed by Abramsky (2015), again partly because it facilitates implicitly what I would like to say on computability. The arguments thus avoid reliance on Tarski and orderings and set theory. I also use this approach partly because of what Abramsky has to say about proofs of the Arrow impossibility theorem.

Of Kenneth's four pioneering contributions, only one was a joint work, the paper with Debrau (5) (see Feiwel 1987a for the genesis of this joint work). The four papers appeared in
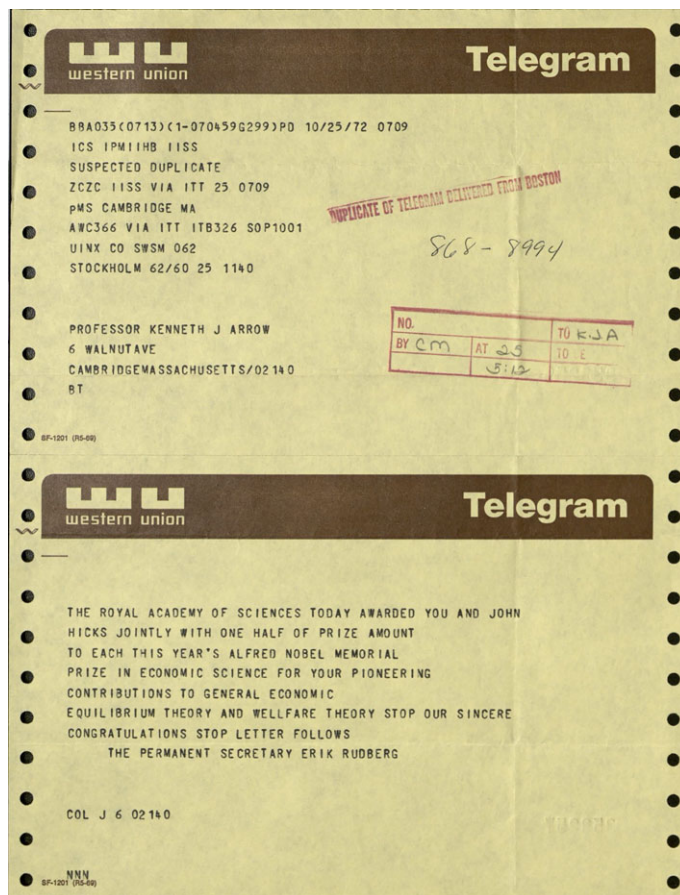
Figure 5. A copy of the telegram informing Kenneth that he was the recipient of the Riksbankens Prize in the Economic Sciences, in Memory of Alfred Nobel, for 1972, jointly with Sir John Hicks. Source unknown. (Online version in colour.)

print in the first four years of the 1950s: 1950, 1951 and 1954 (1, 2, 3, 5). In what follows, I give the mathematical presentation of the fundamental results of these four works:

I. *The Arrow impossibility theorem* (1, 2, 3)
   a. A: (finite) set of alternatives
   b. ϑ: (finite) set of individuals
   c. P (A): (finite) set of preference relations on A
   d. P (A)ϑ: set of profiles
   e. social welfare function (SWF) is a MAP, σ: P (A)$\vartheta \to$ P (A)

Given the two conditions:

   i. independence of irrelevant alternatives (IIA)
   ii. the Pareto principle (PP)

Then, Arrow's impossibility theorem is:

Theorem 1:

If $|A| > 2$ and given ϑ is finite, then any SWF, satisfying IIA and PP is a DICTATORSHIP; i.e. for some $\iota \in \vartheta$, $\forall \rho \in P(A)\vartheta$ and a, b $\in$ A:

$$a\,\sigma(\rho)\,b \Leftrightarrow a\,\rho i\,b$$

Thus, the SWF, under IIA and PP, is that of one fixed individual—called the dictator.

II. *The two fundamental theorems of welfare economics* (2)

 i. If a competitive equilibrium exists, and if all commodities are priced in the market, then the equilibrium is optimal in the Pareto sense—i.e. $\exists$ no other allocation of resources (or services), which will make all participants in market activities better off (see (16), p. 178).

 ii. If $\nexists$ increasing returns to scale (in production), and if certain other minor conditions are satisfied (exactly worked in (16)), then every optimal state is a competitive equilibrium corresponding to some initial distribution.

For a general economist, the most famous result in Kenneth's oeuvre is the Arrow–Debreu notion of the existence, but not uniqueness, of a GET. It is an imaginative generalization of Nash's concept of an equilibrium for a game (Nash 1950) to what Arrow and Debreu called an abstract economy (5). Nash was awarded the Nobel Prize in economics in 1994 (see also Nasar 1998).

III. *Existence of a competitive equilibrium* (Theorem 1, (5)).

Given mathematical assumptions on production (Koopmans 1951), consumption, utility functions, initial holdings (determining the budget constraint) and non-satiation (formalizing the notion of scarcity), then:

$\exists$ a competitive equilibrium for any economic system satisfying the assumptions.

Arrow and Debreu offer many generalizations of the assumptions, and thereby the theorem too; in subsequent writings they also suggested alternative proofs of the generalized theorems (15):

Also, in most cases, we have offered new proofs of the known results in competitive equilibrium theory.

This is exactly a mirror image of Abramsky (2015: 16), in the context of the impossibility theorem:

An extraordinary number of different proofs, as well as innumerable variations, have appeared in the (huge) literature.

In addition to these pioneering results, Arrow contributed to a variety of 'middlebrow theories' (*pace* Solow 1997):

• growth, innovations, technological progress and capital theory (11, 13) (Feiwel 1989);
• inventory theory (4, 8);

- dynamic programming and optimal control theory in public investment and optimal fiscal policy (14);
- optimization of the mathematical programming variety—i.e. linear and nonlinear programming (7);
- crystallization of selected aspects of mathematical methods in economics (10);
- the dynamics and stability of resource allocation processes under many assumptions about the criterion function (18).

My own favourite is what Solow called a 'middlebrow theory', 'The Economic Implications of Learning by Doing' (11), and Solow's own analysis of it, particularly as a 'forerunner' of at least one kind of endogenous growth theory. But the other 'middlebrow theory', the book by Arrow and Kurz (14) on dynamic programming and optimal control theory applied to public policy, comes a very close 'second best' in the true welfare economics tradition.

Then there are the important—and interesting—applied economic papers (16, 21). The 'norm' remained the three fundamental theories outlined above, which were Kenneth's pioneering 'highbrow' contributions. On the basis of these, he constructed applicable, applied, policy-oriented theories on the medical care industry, insurance, risk-aversion, moral hazard, discrimination, urban economic development, environment sustainability, economic development, natural and extractive resource economics and much else.

Kenneth was certainly fully aware of both Keynes' and Knight's work on the incomparability of probabilistic judgements; he was also well aware of Keynes' emphasis on the logical basis of probability and Knight's vagueness on the varieties of probabilistic theories that needed to be considered to make formal sense of the distinction between uncertainty and risk, as pointed out above. For reasons of tractability, I believe he confined his analysis of the unknowability of future events, particularly when it came to applied and applicable topics, to risk-taking situations.

The device of an 'external observer', similar to oracles as defined by Turing (1938 [2012]) was utilized imaginatively by Aaronson (2016). In the context of his interpretation of Knightian uncertainty, convexity was resurrected and would enable Kenneth to use as 'norms' the three pioneering contributions discussed above.

My interpretation of Kenneth's work on *The limits of organization* (17) is that these 'limits' are to be found in the first of his three fundamental results described above, which leads to the Arrow impossibility theorem. However, he does not deal explicitly with the considerations of March and Simon (1958).

Finally, to give some substance to the earlier remarks concerning Emil Post, some aspects of Kenneth's thoughts on computation and computability should be mentioned. It is necessary, however, to take into account Royden's work (1985) on constructive versus classical mathematics and Mines' work (1988) on polynomial time algorithms, lest one appears to be too harsh in interpreting Kenneth's approach to constructivity, computability and computational mathematics.

The earliest statement by Kenneth on computational methods was made in 1957 (6); this was consistent with his interests in programming problems in the 1950s and later. The next is squarely in the Emil Post sense of computability (8) (see also Feiwel 1989), which impinges on Kenneth's later interest in computational complexity theory and his association with the work at the Santa Fe Institute (23); but it was with Karlin and Scarf that he pointed out (8):

> We want to stress that solutions which are not effectively computable are not properly solutions at all. Existence theorems and equations which must be satisfied by optimal solutions are useful tools toward arriving at effective solutions, but the two must not be confused.

By 1986, his 'vision' of computability had crystallized. First, he was explicit about the role of computability in economic analysis (22):

> The next step in economic analysis, I would conjecture, is a more consistent assumption of computability in the formulation of economic hypotheses. This is likely to have its own difficulties because, of course, not everything is computable, and there will be in this sense an inherently unpredictable element in rational behavior.

Related to this is the letter Kenneth wrote to Alain Lewis, on 21 July 1986 (Arrow Papers, Box 23, Perkins Library, Duke University):

> The claim that excess demands are not computable which is 'easy' to prove using the theorem of the Halting Problem for Turing Machines, but not constructively, is a much profounder question for economics than the claim that equilibria are not computable. The former challenges economic theory itself; if we assume that human beings have calculating capacities not exceeding those of Turing machines, then the non-computability of optimal demands is a serious challenge to the theory that individuals choose demands optimally.

Finally, in Feiwel (1987a: 198), with reference to GET Kenneth points out that:

> There is another benefit of the existence theorem that I should mention. It turns out that the existence theorem implies an algorithm for solving general equilibrium. This is the line that Herb Scarf first took up. So that, in fact, we have now actually people who are solving general equilibrium systems and not merely postulating them. So as it turns out the existence theorem has a second implication of a practical nature.

I should like to mention, first, that the algorithms Scarf ingeniously constructed are non-constructive, but not in the sense meant by him. He uses the *tertium non datur* in his proofs, even if he avoids the retraction theorem; the algorithms Scarf uses are not implied by the kind of GET of Arrow and Debreu (5). However, it is possible to show the existence of algorithms that are implied by the GET theories Kenneth has in mind. Whether they can be constructed is not known. The same applies to the Arrow impossibility theorem in its original form, but not in the Abramsky formulation, which is why I prefer to state my version of Arrow's pioneering result in the latter terms.

'Almost' endless years of reflections

In his later years, long after 1972, the year of his Nobel Prize, Kenneth became more reflective and pensive. His thoughtful reflections of his own youthful contributions, particularly those of the 'astonishing years' of the 1950s and early 1960s, are constructively critical. In the white heat of 'doing' and 'succeeding' in these activities and in the mathematical conceptualization of optimal decision making in economics and politics, he sometimes seemed to have 'failed' in the mission he set out to accomplish.

According to my characterization, Kenneth felt that his work 'until 1960 ... had been totally abstract', but he was convinced, as he moved to theoretically underpinned empirical work, that 'theory does not feed on itself' (Feiwel 1987a). I include in this the 'middlebrow

theory', which has almost immediate applicable, empirical content, as in the latter parts of Solow (1997).

On the other hand, pure theoretical work, such as his 'jewels' of the previous section, are several layers removed from empirical applications. As shown in the previous section, they need someone, such as Scarf in the case of GET, or Abramsky in the case of social choice theory, to provide coherent mathematical frameworks to implement them empirically—algorithms to compute GETs and category theory to formalize social choice theory.

His relatively mature thoughts on his early beliefs and his enthusiasm for economic planning and pure empirical work, as described by Feiwel (1987a), put into perspective Kenneth's changing attitudes to being a 'cautious socialist' (19).

In answer to the question 'Would you care to classify by order of importance the most significant contribution to economics in the last 50 years?' (Feiwel 1987a), Kenneth's response was to place 'Keynesian economics at the top of the list', followed by 'the economics of uncertainty and information' and only in third place 'the elaboration of GET'; fourth place was occupied by applied and applicable economics, grounded in solid economic theory, and fifth came pure empirical data gathering exercises, but always, I presume, on the basis of foundational underpinning in theory.

He never thought much of the core beliefs of new classical economics 'that says there is no such thing as unemployment—a point I find completely unacceptable'; and he objected to the vagueness inherent in the concept of the 'neoclassical synthesis' (12). He was also sceptical of the Simon school's bounded rationality-satisficing approach to economics as a decision science; his criticism was based on the information costs of being rational and taking optimal decisions.

He was, even more than Wicksell (Samuelson 1962), a saintly economist and, through his work and influence on social choice theory, a saintly political scientist. It is rare to be sanctified twice.

Achievements, influences and personality

With the benefit of hindsight, one is tempted to say that Kenneth Arrow has always, in his education and research career, been blessed with affiliations and associations with institutions of outstanding quality—beginning with the undergraduate and postgraduate education at the CCNY and Columbia University in the city of New York, through research and consultancies at the Cowles Foundation for Research at Chicago and RAND, all the way through to appointments at various levels at Stanford and Harvard universities.

He was active, even actively vocal, on many economic and social issues, including the Organization of the Petroleum Exporting Countries (OPEC) cartel's oil embargo, Israeli policy promulgated by the likes of Shamir and Netanyahu against the two-nation solution with the Palestinians and the South Africa of the apartheid regime, going to the extent of requesting Stanford University to rethink its ties with the South African companies of the time, even using the dubiousness of a 'unique internal rate of return' in arguing publicly against the global warming deniers, against every kind of nuclear power for atomic stockpiling purposes, and so on.

His influences, theoretical, applied, methodological and the use of mathematics in economics and social choice theory, were fundamental and many. They were usually transmitted through his own written works, although he professed to a dislike of this form of communication, but also through his colleagues in many countries and continents without racial or gender biases of any sort and, above all, by generations of students, many of whom were themselves Nobel Prize winners.

Interestingly, Kenneth did not espouse the insights one can get from a variety of approaches to quantum physics: thought experiments, such as Schrödinger's cat; Dirac's delta function; Feynman diagrams and the difficulty of encapsulating the theory within the framework of some kind of axiomatization; or formalizing Brouwer and Bishop on constructive mathematics. Many followers of Brouwer have done so—the prime example being Heyting (1966)—starting from Whitehead and Russell (1927), with which Kenneth was very familiar (see Feiwell 1989).

Kenneth's son, David, has characterized him by quoting Shakespeare: Hamlet's tribute to his father before seeing his ghost on the battlements (Act I, scene ii, lines 187–188):

> He was a man. Take him for all in all.
> I shall not look upon his like again.

My own characterization of Kenneth Arrow would be in terms of Mark Antony on Brutus, in Shakespeare's *Julius Caesar*:

> This was the noblest Roman of them all, ...

Kenneth Arrow died on 21 February 2017 in Palo Alto, California. His two sons, David and Andrew, grandson, Charles Benjamin, and his sister, Anita, survive him.

Appointments (academic and research)

1946–1949	(part-time) research associate, Cowles Commission for Research in Economics; assistant professor of economics, University of Chicago
1946–1949	consultant at the RAND Corporation, Santa Monica, California
1950–1968	acting assistant professor to professor of economics and statistics, Stanford University
1969–1978	professor of economics, Harvard University
1979–1991	the Joan Kenney professor of economics and professor of operations research (to retirement)

Honours and awards (selection)

1940	awarded Gold Pell Medal for graduating with the highest grade as an undergraduate at CCNY
1951	elected fellow, Econometric Society
1956	elected president, Econometric Society
1956–57	elected fellow, the Center for Advanced Study in the Behavioral Sciences, California, USA

1957	awarded John Bates Clark Medal of the American Economic Association
1952–55	elected honoured fellow, treasurer and donor (jointly with Selma), Blackwell Lecture Fund, Institute of Mathematical Statistics
1959	elected fellow, American Academy of Arts & Sciences
1962	appointed economist on the staff of the United States Council of Economic Advisors
1963	elected president, Institute of Management Sciences
1967	awarded honorary doctoral degree, University of Chicago, USA
1968	member, National Academy of Sciences
1968	member, American Philosophical Society
1971	awarded honorary doctoral degree, University of Vienna, Austria
1972	awarded honorary doctoral degree, City University of New York, USA
1972	The Riksbankens Award in Memory of Alfred Nobel ('Nobel Prize in Economics', shared with Sir John Hicks)
1972	elected president, American Economic Association
1974	elected member, Institute of Medicine, USA
1983–86	elected president, International Economic Association
1986	awarded von Neumann Prize of The Institute for Management Sciences (TIMS) and Operations Research Society of America (ORSA)
1963, 1970, 1973, 1986	overseas fellow, Churchill College, Cambridge, UK
1988	appointed external faculty and member of the Science Board, Santa Fe Institute, New Mexico, USA
1994	founding member, the Pontifical Academy of Social Sciences
1995	awarded honorary doctoral degree, Uppsala University, Sweden
1995	appointed Fulbright professor, University of Siena, Italy
1996	visiting fellow, All Souls College, Oxford, UK
2002	elected fellow, Institute of Operations Research and the Management Sciences
2004	awarded National Medal of Science, USA
2005	inducted into Hall of Fame, International Federation of Operational Research Societies

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Of course, none of the above are even remotely responsible for the remaining infelicities, which are my sole responsibility. The portrait was kindly provided by Stanford University and is © Linda A. Cicero/Stanford News Service.

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Kumaraswamy (Vela) Velupillai is an emeritus professor of economics. He was formerly the John E. Cairnes professor of economics at the National University of Ireland, Galway; professore di Chiara Fama at the University of Trento, in Trento; and a distinguished—non-tenured—professor of economics at The New School for Social Research, in New York. He has had a series of permanent and visiting appointments at various Cambridge colleges: Peterhouse, Clare Hall and Girton, in particular. He came to know Kenneth Arrow personally in the early years of this century, but also as a result of many years of correspondence.

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(3) 1951 *Social choice and individual values*. New York: John Wiley & Sons (there were three editions of this ground-breaking monograph in Kenneth's own lifetime: 1951, 1963, 2012).

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