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FULL LENGTH PAPER

On the multiplicity of solutions in generation capacity investment models with incomplete markets: a risk-averse stochastic equilibrium approach

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Abstract Investment in generation capacity has traditionally been evaluated by computing the present value of cashflows accruing from new equipment in a market with globally optimized capacity mix. The competition and risk that now prevail in the sector may require a more refined analysis. We consider a competitive market with agents investing in some mix of capacities: the risk exposure of a plant and the attitude towards risk of the owner depend on the plant and the portfolio of its capacities. They may also depend on hedging contracts acquired by the investor on the market if such contracts exist. We represent these effects through equilibrium models of generation capacity in incomplete markets. The models come in different versions depending on the portfolio of physical plants and hedging contracts. These modify the long-term risk of the plants, the attitude of the owners towards risk, and hence the incentive to invest. The models involve risk-averse producers and consumers, and their behavior is represented by convex risk measures. We use degree theory to prove existence and explore multiplicity of equilibrium solutions.

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1 Introduction

1.1 Problem statement

Generation capacity expansion models were among the first applications of linear programming both in the literature and the industry [53]. The early models involved two types of linked decisions: capacities are decided with respect to a future reference year characterized by fuel cost and demand data; the operation of these capacities in that year is optimized. The model relies on simplifying but standard assumptions: fuel costs and demand in the reference year are known; capacities are selected to minimize the sum of the equivalent annual cost of investment and operating cost.[1] This simple model is still used today under the name of Screening Curve Model (SCM) to provide a rough analysis of the economics of investment in the power sector [66].

This problem was designed as an optimization model for a power utility or a social planner in the days of the regulated industry. Its linear programming formulation quickly led to various developments with the view of increasing the scope and realism of the model. We concentrate on another aspect of this basic formulation namely its interpretation in terms of competitive markets. The economic literature indeed quickly recognized that the primal-dual optimality conditions of linear programs in activity analysis models could be interpreted in terms of perfectly competitive economies [25]. The objective of this paper is to transform the linear programming formulation of capacity expansion into a stochastic partial equilibrium model where different agents invest and operate portfolios of physical assets, possibly completed with financial contracts. Asset pricing in finance is commonly discussed using the same two-stage framework as the early capacity expansion models. We thus keep this simple two-stage framework for dealing with both physical and financial assets throughout the paper.

The interpretation of the optimality conditions of the capacity expansion problem in terms of agents operating in a competitive economy constitutes the starting point of the paper. We extend the initial SCM by introducing random demand and fuel costs in the reference year and assuming risk-averse agents described by risk measures. This model has been introduced in [28] but the treatment presented here is more formalized. Risk aversion in a market economy requires to be specific about asset ownership and owners' objectives. [28] assumes one plant per owner and minimization of the risk measure of the random profit accruing from that plant. This limited generalization from the risk-neutral capacity expansion model already goes beyond what can be represented through a simple optimization problem. This paper considers more general ownership structures including the situation where a producer owns a portfolio of physical assets that it can partially protect with financial contracts. The minimization of the risk measures of the hedged profits of the different producers is then the criterion.

[1] Because the economic life of generation plants is much longer than one year, overnight investment costs are converted into equivalent annual costs (annuities) that are added to annual operating costs to give the total annual costs. This calculation is well known: the discounted sum of annual payments equal to the annuity over the life of the plant must be equal to the overnight investment cost. The calculation requires a discount rate, which for reasons explained in the text is equal to the so-called risk-free rate throughout the paper.

Equilibrium models with both physical and financial assets are uncommon in energy policy but they have been extensively studied in general equilibrium. As briefly discussed in the literature survey, those models treat much more complex issues than the partial equilibrium models presented here; they can thus provide ideas for expanding our simpler models. Jofré et al. [41,42] note that general equilibrium models of incomplete markets are commonly developed using differential topology, which prevents the use of inequalities that are necessary in practical applications. Those authors endeavor to develop general equilibrium models using computable variational methods. We follow suit but with a more modest objective: we work with partial equilibrium models with the view of providing computable capacity expansion tools for testing the impact of risk and hedging instruments on investment in the power economy. We accordingly conduct our developments in the complementarity form inherited from the KKT conditions of the basic SCM model. [24] illustrates the type of insight that can be obtained with that methodology. [38] reports positive prospects for scaling the model using ADMM.

The work arose from discussions on investments in the current European power market. An initial incentive came from a technical analysis of investment criteria in the company that employs two of the authors: a summary of that analysis is presented in Appendix A. A broader motivation arose from the need to understand at technical level the forces at work in the market. The fact that the European power market is in chaos is now recognized by various observers, consultants, academic and public officials [19,46,55]). More than anything else, the downward spiral affecting the sector is testified by the massive impairment of assets by utilities these last years.[2] It is in particular noted that the historically low cost of capital has not been able to restore investment besides those that are explicitly subsidized. The high risk prevailing in the market and insufficient hedging possibilities such as the absence of long-term contracts for supporting investment[3] are mentioned as one of the possible causes of the lack of investment that affects non-subsidized technologies. The role of risk motivates an analysis in terms of stochastic equilibrium. But market incompleteness, as it appears from the lack of long-term contracts, also justifies going beyond a standard analysis of existence of solution to turn the attention to the possible non-uniqueness of equilibrium, which might become a pervasive phenomenon in a chaotic market. Market incompleteness in hedging is indeed just one example of the various micro-economic imperfections that are now uncovered in the market (see the above mentioned reports). These usually take the form of inadequate or missing prices; they may also lead to a multiplicity of equilibrium some of them possibly disastrous (such as when a critical resource is badly priced because of regulation). This would be a much more dramatic risk for investors than the one arising from uncertain costs or demand data in a standard

[2] 102 bn euros between 2010 and 2015 for the 12 biggest European utilities (FT May 22, 2016)

[3] Financial contracts of the necessary long maturity are rare or non-existent even though they have been proposed as an incentive to investment and more specifically in the academic literature under the form of reliability options. To our knowledge the only existing examples to date are the UK contract for difference for the 3200 MWe EDF nuclear plants in Hinkley Point and the French Exeltium contract with large industrial consumers. Those contracts are quite different from a financial contract as they result from negotiations [65]. Both required considerable legal scrutiny before being declared compatible with European competition law.

optimization model representing a well functioning economy. Degree theory and the analysis of the multiply of solutions that it allows is a useful tool for looking at that situation.

1.2 Contribution

The paper elaborates on Ehrenmann and Smeers [28,29]. It extends their model and offers results on existence and multiplicity of solutions.

The attitude of investors in a risky market depends on their portfolios of physical assets and hedging contracts. The models presented in this paper consider a power economy composed of one representative consumer (to simplify the presentation) and several producers. We successively analyze the case where each generator owns a single plant and where it owns a portfolio of plants possibly completed by a portfolio of contracts. The case of a single plant per owner is interpreted as a project finance view where a plant is valued on the basis of its sole merits irrespective of its inclusion in a portfolio. We formulate risk aversion in these different models through convex risk measures, that we sometimes particularize to coherent risk measures.

We treat these models through a common methodology. We adapt an approach introduced by Arrow and Debreu [6] in their proof of existence of general equilibrium and formulate our models as Nash Equilibrium.[4] Those formulations lead to existence proofs through fixed-point arguments provided one can embed the actions of the players in compact sets. Even though Nash Equilibrium is quite common in energy market modeling, it is mainly used in studies of market power [32,60] but is less frequent to address price-taking agents ([63] is an exception). We believe that these formulations are original (even if the project finance model is already presented in [28,29]).

The objective function of each agent is convex in its arguments for given actions of other players, which suggest a convex (monotone) problem. But the combination of the mappings derived from these objectives is not monotone in the Cartesian product of agent's actions because of the incompleteness of risk trading. This lack of monotonicity is common in markets with poorly coordinated policy interventions (and hence departure from the perfect competition paradigm). Existence and uniqueness then require a fresh analysis. It has been mentioned that existence proof can be obtained by fixed-point arguments provided the actions of the agents could remain in a compact set. Because we also want to analyze uniqueness, we follow an alternative approach and resort to degree theory, using essentially the same assumptions as in fixed-point theorems. This approach as well as our emphasis on the number of solutions is motivated by an attempt to construct a methodology for understanding the occurrence of multiple solutions in restructured electricity systems that depart too much from the complete and undistorted markets of the theory. We rely on the concise presentation of Facchinei and Pang [30]. The approach implies constructing an homotopy to move

[4] Arrow and Debreu's proof relies on a reformulation of the general equilibrium problem as a Generalized Nash Equilibrium: this becomes a simple Nash Equilibrium in our partial equilibrium model because budget constraints disappear.

from a known and solvable problem (typically representing the undistorted economy) with a unique solution to the target model. We implement this approach in the project finance model by initiating the homotopy with a standard stochastic capacity expansion problem. We use this same standard stochastic program completed by a simple Variational Inequality problem in the portfolio model. We progressively transform these problems to represent situations where risk-averse agents take positions in an incomplete market. Models with financial contracts present their own difficulties: in contrast with physical positions (capacities) that, by nature are bounded in capacity expansion problems (demand for power is always bounded in given economic situations under reasonable policies), methodological reasons (guaranteeing the absence of arbitrage) prevent imposing ex ante bounds on financial positions. We extend a proof of [23] to show that these positions remain bounded and use the same machinery of degree theory as in the case without contracts. Existence is then proved by arguments from [30].

We explore multiplicity results for both models. We show that equilibrium solutions are isolated, except for possibly degenerate problems, a notion that we formalize by proving that non-isolated equilibria only occur in a set of measure zero of the parameter data. The use of degree theory then implies that the number of solution is finite[5] and odd. The treatment is technically complex and most of it is reported in Appendix B.1. We also give a uniqueness result for the particular case of coherent risk measures. We find that the given (constant) merit order that underpins the SCM guarantees uniqueness in the project finance model with coherent risk measures. This condition is interesting: the constant merit order is an important property of the old power system but it is likely to disappear in the transition to renewable because of short-term gradient constraints. From a longer term perspective, it has already disappeared today in Europe in the competition between gas and coal. Existence of equilibrium with financial contracts adds to technical complexities even though the reasoning follows the same pattern as in project finance. The analysis is thus presented in summary form. Isolated equilibrium lead to a situation similar to the one of general equilibrium where a finite number of isolated equilibrium is the normal situation [33]. Non-monotonicity is the mathematical cause and intervention of public authorities to prevent the economy to move towards a bad equilibrium is the only proposed remedy [18].

1.3 Paper structure

Section 2 coming after this introduction briefly situates our work in the existing literature: we present our models as extensions of former capacity expansion problems that we adapt to the environment of competition and risk prevailing in the industry. Alternatively, they can also be seen as restrictions of general equilibrium models in incomplete markets to a single sector with the consequence that budget constraints disappear. Section 3 describes the generation system and recalls the notion of risk measure as well as the entropic risk measure used for illustrative purpose in the paper. Section 4

[5] This contrasts with continuous set of equilibrium found in problems of missing markets formalized as a Generalized Nash models when a common constraint is un-priced.

states the sub-models of the consumers and producers as well as of the "Walrasian auctioneers" setting the prices in the economy. We discuss models where each plant is owned by a single generator (project finance) and one with portfolios of physical and financial assets. Section 5 combines these sub-models into market (equilibrium) models. Section 6 lists assumptions used for analysing existence and multiplicity of solutions. Degree theory is presented in Sect. 7 together with two auxiliary problems with a single solution (and hence degree one) used to initiate the homotopy leading to our models. Section 8 proves the existence of at least one equilibrium for the project finance model; it presents the proof of isolation of equilibria and mentions one case where uniqueness of solution can be proved. Section 9 proceeds the same way, but in summary form for the problem with a portfolio of physical and financial assets; the section is limited to the proof of a finite set of isolated equilibrium. Section 10 presents a numerical example of a multiplicity of equilibrium, showing that, as in general equilibrium, multiplicity of isolated equilibrium is the general result. Section 11 summarizes and concludes, recalling numerical implementation and experiment with the models in [24,38] that both give promising results. The more technical discussions are provided in appendices.

2 Literature survey

2.1 Cost optimization models

Generation capacity expansion models were initially formulated as the minimization of the total annual cost (the sum of the annual equivalent cost of investment and annual operating cost) of satisfying a given demand. The minimization of the operating cost embedded in that model is a simple short-term optimization problem often referred to as the merit order rule: plants are called upon in order of increasing fuel and variable operating cost until demand is satisfied or insufficient capacity is observed. In this latter case a curtailment cost (VOLL) is registered as operating cost: curtailment is thus equivalent to a fictitious plant of zero investment cost and high variable cost operating after all real capacities are exploited. The investment problem then consists in finding the mix of plants and curtailments that minimizes the sum of capacity and operating costs, including the social cost of curtailments valued at VOLL.

This first model was extensively developed in different directions to expand its scope and realism. The underpinning economic paradigm (present value calculation and merit order) remained essentially unchanged. These extended models are not directly relevant to our subject and hence are left out of the discussion. Surveys of this work can be found in the milestone book of Turvey and Anderson [67] with a broader presentation of capacity expansion problems in [50]. Some of the problems raised by the transition from the regulated monopoly to competition are discussed in [44]. The TIMES model, which is used on a world wide basis (see iea-etsap web site), is a state of the art example of these tools. Conejo et al. [20] discuss a whole range of investment models in generation and transmission.

2.2 Market equilibrium models

The introduction of a non-integrable demand system (that cannot be represented as the gradient of a utility function) in the PIES model (which for the rest was formulated as a linear program) was a milestone step towards computable equilibrium models in the energy sector. PIES was solved by a sequence of optimization problems [4]. [35] replaced the marginal cost pricing of perfect competition by the average cost pricing paradigm of cost plus regulation in a capacity expansion model; this was also solved by a sequence of linear programs. The equivalence between the primal-dual optimality conditions of a linear program and the relations describing a perfectly competitive economy (where agents are price-takers) elaborated in [25] helps understand these developments and the subsequent movement towards complementarity formulations. Because primal-dual conditions of an LP (such as SCM) describe a perfect competition model of the industry, appropriate modifications of these primal-dual conditions offer a method to represent policies that affect perfect competition. The modified model departs from the primal-dual optimality conditions of an LP but retains its so-called complementarity form and hence can benefit from the theory initially developed for extending LP duality theory [21]. Harker [36] constructed various spatial competition models using complementarity problems or the related variational inequalities paradigm. These techniques became standard in energy modeling as can be seen from the book of Gabriel et al. [32] and the cited literature. This paper is conducted using complementarity formulations of risk-averse agents.

2.3 Stochastic cost optimization models

A traditional stochastic programing version of SCM takes the form of a two-stage cost minimization model operated by a social planner: one invests in the first stage before precisely knowing the demand, fuel costs and technological characteristics of the target year; one then operates the system for each target year scenario of the second stage. Investments are based on the expectation of the results of the target year scenarios. These models are risk-neutral in the sense that the social planner minimizes an expectation of the cost of the system (see [12] for an advanced multistage version of these models). Their primal-dual conditions can be interpreted as an equilibrium in a perfectly competitive market under a deep assumption that is almost never mentioned.[6] The only difference of interpretation with the deterministic model for investment purposes is that agents in stage zero now consider the expectation of revenues and costs accruing in stage 1. Specifically, the investment criterion derived from the KKT conditions of the model takes the usual formulation of investment theory except for the replacement of revenue and cost by expected revenue and cost. The stochastic version of the SCM is thus a two-stage stochastic equilibrium model where each generator invests in the first stage to maximize its expected profit computed over the

[6] The assumption is that the probabilities are risk-neutral in a complete market, not otherwise specified (see next section).

two-stages. Conejo et al. [20] gives a comprehensive discussion of these models as well as extensions to transmission and market models.

2.4 Complete markets, optimization and equilibrium

Standard stochastic programs are risk-neutral. Robust programs represent an extreme version of risk aversion. Risk optimization, which covers the range between these two extreme attitudes, is a relatively recent, but already extensively developed, extension of conventional stochastic programming [64]. The two-stage risk optimization model replaces the expectation of the optimal value of the second-stage optimization problem by the risk measure of this optimal value. This is done by embedding the second-stage optimization programs (objective function and constraints) of the different states of the world in a risk function. Risk measures, which began with Artzner et al. [7] are now extensively developed in the mathematical finance literature; they have also received considerable attention in the optimization world [64]. Multistage risk-averse optimization problems are obtained by using muti-period "time consistent" risk measures.

The transition from a risk-neutral equilibrium version of SCM to a risk-averse world is formally easy but raises issues of asset ownership (see [51], Chapter 6). In contrast with deterministic or risk-neutral formulations, these equilibrium models cannot be derived from one risk-averse optimization problem through KKT conditions, except in very special circumstances, that is when the market is "complete" as discussed next. The relation between the stochastic risk-averse equilibrium and a risk-averse optimization problem was first discussed in [28] and extensively explored in [58] where the authors analyze the conditions for which the KKT conditions of a risk-averse version of SCM can effectively be interpreted as representing a risk-averse equilibrium model (see [56] for a similar analysis on hydro management). These conditions are quite demanding; they require the existence of a complete market of risk trading instruments (financial or insurance contracts). Completeness is here taken in the sense of financial economics: intuitively speaking, it requires a sufficiently large number of instruments (contingent claims) so that all risks can be traded among the agents of the economy. In mathematical terms the space of risk exposures generated by the net revenues of the second stage should be covered by the payoffs spanned by the risk trading instruments.

Risk-averse stochastic equilibrium models can also be constructed by resorting to utility functions taken from economics such as in [47], which state a problem through the KKT conditions of the agents maximization problems (this is akin to our project finance model). Schiro et al. [63] states the problem in Nash Equilibrium terms using an exponential utility function. Risk measures are particular cases of utility functions (and the exponential utility function is quite close to the entropic risk function used here): they are grounded in mathematical finance and benefit from a well developed duality theory.

Our models are stochastic Nash games. A whole literature recently developed on monotone stochastic Nash games (e.g. [45,59]). These consider different agents that optimize an objective function, which is convex in the decision variables of the agents when parameterized by the actions of the other agents. This literature adds the assumption that the combination of the mapping derived from individual agents' objectives is monotone in the Cartesian product of agents' actions. The incompleteness of risk trading treated in our paper violates this latter assumption.

2.5 Risk-averse equilibrium in incomplete markets

Ehrenmann and Smeers [29] introduces a risk equilibrium model based on coherent risk measures, which does not rely on the property of completeness. The model is a version of SCM where the expectation of the net revenue of each agent is replaced by its risk measure. It does not contain any risk trading instrument and hence describes a market that is fully incomplete. The problem is not derivable from the KKT conditions of an optimization problem and no monotonicity property can be invoked that would help prove existence and uniqueness of solution. Most energy market models formulated as risk optimization problems have so far implicitly assumed fictitious complete financial markets. In contrast, market incompleteness was extensively studied in the general equilibrium literature (see [52] for comprehensive collections of papers on the subject). These models treat broader problems than ours: consumers allocate their budgets among the different goods of the economy. Asset ownership can be much more complex than what is assumed here (see [51] Chapter 6 for a discussion of different types of ownership in production).

Partial equilibrium models neglect all these aspects and represent consumers by simple expenditure functions and producers by profit functions. Technologies are assigned to producers, which simply maximize the same profit functions as in SCM (but value them through risk measures) without any involvement of stockholders other than direct owners of the plants. The absence of budget constraints also simplifies technical aspects. Some difficulties extensively discussed in the general equilibrium literature remain (and must be taken care of) such as when the dimension of the space (or manifold) spanned by the financial contracts changes as a function of the risk exposure. Partial equilibrium problems are thus easier to state and analyse. Most of the treatment of incomplete markets in general equilibrium is conducted with instruments of differential topology. Jofré et al. [41,42] developed incomplete general equilibrium models treated using variational inequality techniques. We follow their objective in terms of tools while concentrating on simpler partial equilibrium problems.

2.6 Methodological issues

Complementarity models derived from convex optimization problems are by construction monotone and the several risk-neutral stochastic extensions of these models to a risky world are also monotone. The same is true for models where there is a single social planner, modeled by a utility or risk function. Monotonicity is lost when one resorts to a multi-agent model in a risky world. While the behaviour of each agent is still represented by a convex function, the appearance in that function of a nonlinear expressions (a revenue is the product of an endogenous price and an endogenous quantity) and the fact that some of these variables (prices) are common to all agents destroy monotonicity properties when the whole equilibrium problem is considered. This loss of global convexity is not unusual in equilibrium models and can be found both in general equilibrium and Nash Equilibrium problems where one resorts to fixed-point theorems, whether Brower's or Kakutani's, to prove existence. Nash equilibrium is thus the relevant context for this paper: agents' problems are individually monotone but globally non-monotone, which raises questions of existence and uniqueness of equilibrium. While fixed-point arguments can commonly be used to prove existence, provided the problem can be cast in a compact set, the question of uniqueness remains.

The interest in the multiplicity of Nash equilibrium goes back to the early days of that concept. Lemke and Howson [48] proposed an algorithm for finding a Nash equilibrium in bimatrix games and proved that the number of isolated solutions is odd. [68] extended the algorithm to an N-person game and confirmed the oddness of the number of solutions in that more general problem. [62] used the underpinning reasoning in his algorithm for finding the equilibrium prices in Walrasian economies. The approach was later expanded by several authors and led to the so-called homotopy or path method for solving general equilibrium models [27] or industrial economics models with strategic interactions [13]. In both cases, the capability to address the multiplicity of equilibrium was one of the objectives. Path homotopy algorithms solve a target problem by starting with an auxiliary problem that is both easy to solve and has a single solution. The method then progressively modifies this auxiliary problem into the target model. Multiple equilibrium shows up when the path retrogresses (or backtracks) without returning to the initial auxiliary problem. The approach presents several interesting methodological aspects. [15] showed that the path followed in the homotopy could be interpreted (in a limit sense) in terms similar to those of Lemke's algorithm, which the author developed from his early work on bimatrix games. Both the homotopy and mixed complementarity problem approaches [61] use degree theory to obtain similar existence and oddness results on isolated solutions.

The combinatorial character of the original Lemke and Howson's algorithm persisted in the subsequent literature; it offers a treatment of isolated equilibrium. This extends the scope of situations that can be analysed compared to models based on convexity type assumptions. This is of the essence for modeling systems characterized by idiosyncratic regulations and (as in this paper) incomplete financial markets, as these cannot usually be treated by sequences of optimization problems. Needless to say, this more general setup has a cost in complexity. This is investigated through general theories [34], analysis of particular classes of LCP (linear complementarity problems) [2,3] or special classes of LCP.

3 Generalities on the modeling of the power economy

The analysis is conducted in a two-stage power economy adapted from [28]. Producers invest in generation capacities and take financial positions in a first stage $t = 0$. They then collect the revenues accruing from physical and financial assets in the different scenarios of the second stage $t = 1$. Extension to a multistage context is left for future work. Our generation system is standard: plants are represented by their capacity with fixed investment and operating costs on the one hand and variable operating costs on the other. The fixed investment cost is the equivalent annual cost of investment computed at the risk-free rate over the lifetime of the plant. The notion of equivalent annual cost (or annuity) is common in investment theory. We note in the following section that it applies as such in our formalism based on risk measures. To simplify the presentation, plant availabilities are all taken equal to 1.

The common way of modeling yearly electricity demand is to work with a load duration curve decomposed in several load segments (each one characterized by a demand level and duration). Discarding the impact of storage (which so far remains limited in most conventional electricity systems), the load duration curve represents the capacity utilization in the different load segments during a year (or a shorter period depending on the problem). To ease notation, we oversimplify this representation by taking a single load segment per year. This dispenses with introducing the duration of the load segments and permits working with annual operating costs. It should be clear that the models and findings remain valid with a finer description of the load duration curve. We also assume a fixed demand (price insensitive up to a certain cap) that is positive in all states of the world. Except for the one segment decomposition of the load duration curve, these standard assumptions were part of the SCM discussed in the introduction.

Moving to a market interpretation, we assume price-taking agents (and hence no market power). Because we are in an incomplete market this is not equivalent to a perfectly competitive market where risk is fully traded. The implication of the price-taking assumption is that agents do not see the impact of their actions on prices. This means that investors in physical assets do not account for the impact of their investment on the spot price in the electricity market. In the same way, investors in financial assets do not see the impact of their actions on the prices of the financial assets.

Agents are risk-averse and their behavior is described by risk measures in the sense of Artzner et al. [7] and successive authors. We follow the main literature (e.g. [7,31,64]) by interpreting risk measures as the minimum amount that has to be added at initial date to make a risky project acceptable (a capital requirement). The goal of a risk-averse investor is to minimize this amount.

3.1 A reminder of risk measure

We assume a finite probability space (Ω, P) and denote a scenario and its (natural) probability by ω and $P(\omega)$. A random variable is described by a function $Z : \Omega \to \mathbb{R}$. $\mathcal{Z}$ denotes the space of all bounded real-valued measurable functions (containing the constants) on the space Ω and $\mathcal{P}$ the set of all probability measures on Ω. The risk measure of the random variable Z of an asset is a number $\rho(Z)$ that quantifies the risk and is defined as the minimal amount of capital that needs to be added at initial date to the value of the asset in order to make it acceptable for the investor. We follow here the presentation of [7,31] where the random variable Z represents a payoff (also referred to as net revenue in the rest of the paper).

An agent optimizes its portfolio of assets (financial and/or physical) by choosing a strategy that minimizes the risk measure ρ of the sum of its deterministic and random payoffs Z. This risk measure is assumed to be convex in the sense of Föllmer and Schied [31].

Definition 3.1 A convex (monetary) risk measure is a function $\rho : \mathcal{Z} \to \mathbb{R}$ satisfying the following axioms.

- *Monotonicity:* $\forall Z_1, Z_2 \in \mathcal{Z}$: if $Z_1 \preceq Z_2$, then $\rho(Z_1) \geq \rho(Z_2)$.
- *Cash invariance:* $\forall Z \in \mathcal{Z}$: if $a \in \mathbb{R}$ then $\rho(Z + a) = \rho(Z) - a$.
- *Convexity:* $\forall Z_1, Z_2 \in \mathcal{Z}, \forall t \in [0, 1]$:
  $\rho(tZ_1 + (1 - t)Z_2) \leq t\rho(Z_1) + (1 - t)\rho(Z_2)$.

A consequence of the cash invariance property is that risk aversion is entirely taken care of inside the risk measure and hence that discounting from the second stage to the first stage is done at the risk-free rate (the rate of the risk-free asset). This justifies computing the equivalent annual cost by the standard annuity formula at the risk-free rate.

We extensively use the dual formulation of convex risk measures, coming from the well-known representation theorem [31].

Theorem 3.2 (Föllmer and Schied [31]) *Any convex risk measure ρ has a dual representation $(\mathcal{M}, \alpha)$*

$$\rho(Z) = \max_{Q \in \mathcal{M}} \left\{ \mathbb{E}_Q[-Z] - \alpha(Q) \right\} ,$$

where $\mathcal{M} = \{Q \in \mathcal{P} : \alpha(Q) < +\infty\}$ is a closed and convex set of probability measures. The functional $\alpha(\cdot) : \mathcal{P} \to \mathbb{R} \cup \{+\infty\}$ is a penalty function; it is convex and continuous on $\mathcal{P}$.

This representation theorem states that every convex risk measure ρ can be represented as an expectation computed with a probability measure Q selected from the feasible set of subjective risk probabilities $\mathcal{M}$ and penalized by the function α.

We sometimes further specialize our results by taking a coherent risk measure.

Definition 3.3 A coherent risk measure is a risk function $\rho : \mathcal{Z} \to \mathbb{R}$ that is convex and satisfies the positive homogeneity axiom

- *Positive Homogeneity:* $\forall Z \in \mathcal{Z}, \forall \lambda \in \mathbb{R}^+ : \rho(\lambda Z) = \lambda \rho(Z)$.

In the coherent case, the penalty function $\alpha(\cdot)$ becomes a characteristic function of the closed convex set $\mathcal{M}$ and the dual representation simplifies to $\rho(Z) = \max_{Q \in \mathcal{M}} \mathbb{E}_Q[-Z]$.

The sub-differential of a convex risk measure is given by

$$\partial\rho(Z) = \arg \max_{Q \in \mathcal{M}} \left\{ \mathbb{E}_Q[-Z] - \alpha(Q) \right\} . \tag{3.1}$$

The risk measure ρ is differentiable when the solution to the above problem reduces to a singleton (i.e. the subjective risk probabilities are unique). This would be the case when the function α is continuous and strictly convex. We call its solution the subjective probability measure at which the payoff is evaluated.

Remark 3.4 (Absence of arbitrage) The fundamental theorem on absence of arbitrage requires the existence of a risk-neutral measure, i.e. a probability measure that is equivalent to the natural probability measure P and such that asset prices are equal to the discounted expected value of the future payoff under this measure. In a market model without friction, the subjective probability measure $\bar{Q}_i$ of an agent usually constitutes such a measure under the condition that all its probabilities are positive (equivalent to P). For example, this is the case with the entropic risk measure (given in definition 3.5) or, more generally, with any risk measure constructed as the convex combination (with positive coefficient) of the expectation under P and another convex risk measure $\tilde{\rho}(Z)$, i.e. $\rho(Z) = \lambda \mathbb{E}_P[Z] + (1-\lambda)\tilde{\rho}(Z)$.

3.2 Risk exposure and notation

The random payoff of an agent i is noted $Z_i(\omega)$. This cashflow results from a portfolio of assets, where the composition of the portfolio is the vector of decision variables $v_i \in \mathbb{R}^n$. Let $\rho_i(\cdot)$ be the risk measure of agent i and $Z_i = F_i(v_i) : \mathbb{R}^n \to \mathcal{Z}$ be the mapping giving the payoff resulting from the decision variables v_i. The risk measure of the random cash-flow is given by the composite function $\phi_i(v_i) = \rho_i \circ F_i(v_i) : \mathbb{R}^n \to \mathbb{R}$. We write $f_i(v_i, \omega)$ for $[F_i(v_i)](\omega)$, and view $f_i(v_i, \omega)$ as a random function defined on the measurable space $(\mathbb{R}^n, \Omega)$. We say that the mapping F_i is concave and differentiable if the function $f_i(\cdot, \omega)$ is concave and differentiable for every $\omega \in \Omega$.

Let ρ_i be a convex risk measure, F_i be a concave (profit functions being typically concave with capacities) and differentiable mapping at v_i and $Z_i := F(v_i)$, then the sub-differential of the composite function $\phi = \rho \circ F$ is given by:

$$\partial\phi(v_i) = \cup_{\bar{Q}_i \in \partial\rho_i(Z_i)} \mathbb{E}_{\bar{Q}_i}\left[-\nabla f_i(v_i, \omega)\right] . \tag{3.2}$$

It is differentiable when the composite function is differentiable and $\{\bar{Q}_i\} = \{\nabla\rho_i(Z_i)\}$ reduces to a singleton.

We assume in this paper that agents' risk measures are continuously differentiable. We write KKT condition $0 = \nabla\phi_i(v_i)$ as:

$$0 = \mathbb{E}_{\bar{Q}_i(Z_i)}\left[-\nabla_{v_i} f(v_i, \omega)\right] , \tag{3.3}$$

where $\bar{Q}_i(Z_i) \in \mathcal{Z}$ is equal to the singleton $\nabla\rho_i(Z_i)$.

We elaborate in Sect. 6 on the suitability of this differentiability assumption. We here give the example of the entropic risk measure that is widely used in economics and financial mathematics. It is based on the exponential utility function and exhibits constant absolute risk aversion (CARA). We resort to this particular risk measure in Sect. 10 to provide a numerical example on multiplicity of solutions.

Definition 3.5 *(Entropic risk measure)* The entropic risk measure $e_\theta(Z)$ is given by

$$e_\theta(Z) = \frac{1}{\theta} \ln\left(\mathbb{E}_P\left[\exp(-\theta \cdot Z(\omega))\right]\right) . \tag{3.4}$$

The associated feasible set of subjective risk probabilities is $\mathcal{M} = \{Q \in \mathcal{P}$ such that $\forall \omega \in \Omega,\ Q(\omega) > 0\}$ and its penalty function $\alpha(Q)$ is given by:

$$\alpha(Q) = \sum_{\omega \in \Omega} \frac{1}{\theta} \frac{Q(\omega)}{P(\omega)} \ln \frac{Q(\omega)}{P(\omega)} . \tag{3.5}$$

The entropic risk measure is continuously differentiable and its gradient is given by

$$\bar{Q}_{e_\theta}(Z; \omega) = \frac{P(\omega) \cdot \exp(-\theta Z(\omega))}{\mathbb{E}_P\left[\exp(-\theta Z(\omega))\right]} . \tag{3.6}$$

4 The agents in the power economy

We introduce the economic equilibrium in its equivalent Nash game formulation, adding a market agent (also known as a Walrasian auctioneer) whose objective is to minimize the value of excess supply. The economy comprises a single good, electricity, that is traded on a spot market in the second stage. We denote by $\mathbf{p}_{el}$ the vector of the electricity prices in the different scenarios $\mathbf{p_{el}} := (p_{el}(\omega))_{\omega \in \Omega} \in \mathbb{R}^{|\Omega|}$.

When considered, the financial market comprises C different contracts. Each financial contract $c = 1, \ldots, C$ has price p_c^f in the first stage and gives a (finite-valued) stochastic payoff $p_c^s(\omega)$ in the second stage. The vector $\mathbf{p^f}$ is the vector of all contract prices $\mathbf{p^f} := (p_c^f)_{c=1}^C \in \mathbb{R}^C$ and $\mathbf{p_c^s}$ is the matrix of all the random payoffs $\mathbf{p_c^s} := (p_c^s)_{c=1;\omega\in\Omega}^C \in \mathbb{R}^{C\times|\Omega|}$. Economic theory (see the introductory Chapter of Magill and Quinzii [52]) distinguishes two basic types of securities: *nominal* assets (such as Arrow-Debreu securities, e.g. weather securities in our case) that do not have intrinsic value and whose payoff is entirely determined by the occurrence of a certain scenario and *real* assets (as futures contracts) whose payoff depends on the outcome of a market. To model real assets in our simplified power economy, the following equation defines the payoff structure of each contract $c = 1, \ldots C$ for each scenario $\omega \in \Omega$

$$p_c^s(\omega) = h_{c,\omega}(p_{el}(\omega)) , \tag{4.1}$$

where $h_{c,\omega}$ is a mapping $\mathbb{R} \to \mathbb{R}$. Nominal assets of the Arrow-Debreu type such as weather or fuel derivatives (whose payoffs are exogenous in this partial equilibrium setting) could also be treated.

Given the hypothesis of price-taking agents, this payoff structure is known (taken as parameters) to all trading agents.

4.1 The consumer

4.1.1 Consumer model without financial contract

Electricity consumers are modeled by a representative agent d facing in $t = 1$ an exogenous positive demand of energy denoted by LOAD(ω) for each scenario ω. This agent values the electricity consumed at PC (ideally equal to the value of lost load [43]) and can curtail the consumption by $s(\omega)$ when the electricity price $p_{el}(\omega)$ is too high. Let $\mathbf{s}$ be the vector of the demand curtailments $\mathbf{s} := (s(\omega))_{\omega\in\Omega} \in \mathbb{R}^{|\Omega|}$. The net profit of agent d in scenario ω is given by

$$Z_d(\omega) = \big(PC - p_{\mathrm{el}}(\omega)\big) \cdot \big(\mathrm{LOAD}(\omega) - s(\omega)\big) \,, \tag{4.2}$$

where $(PC - p_{el}(\omega))$ represents the surplus accruing from a unit of consumption. The following risk optimization problem gives the optimal risk measure of the consumer's use of electricity in a world without financial contracts. Given the electricity prices $\mathbf{p}_{el}$, the consumer chooses the level of load curtailment that solves:

$$\mathcal{C}_d(\mathbf{p}_{el}) \equiv \arg \min_{\mathbf{s}:s(\omega)\geq 0} \;\rho_d\left(\big(PC - p_{\mathrm{el}}(\omega)\big) \cdot \big(\mathrm{LOAD}(\omega) - s(\omega)\big)\right). \tag{4.3}$$

The model states that the consumer minimizes the risk measure of the surplus accruing from consumption. An interruption of supply $s(\omega) > 0$ sets the price at PC, which leaves a zero surplus to the consumer. The model contains no first-stage decision. The corresponding KKT conditions are

$$0 \leq PC - p_{el}(\omega) \;\perp\; s(\omega) \geq 0 \quad \forall \omega \in \Omega \,. \tag{4.4}$$

4.1.2 Consumer model with financial contracts

Financial contracts require the consumer to take first-stage positions $x_{d,c}$ in the contracts $c = 1, \ldots, C$. A positive $x_{d,c}$ corresponds to a purchase and *vice versa*. The consumer pays p_c^f for contract c in the first stage and receives $p_c^s(\omega)$ in the second stage in scenario ω. The vector of the consumer's financial positions is denoted $\mathbf{x_d} = (x_{d,c})_{c=1}^C \in \mathbb{R}^C$. Given the electricity prices $\mathbf{p}_{el}$ and the financial contract prices $\mathbf{p^f}$ (recall that $p_c^s(\omega) = h_{c,\omega}(p_{el}(\omega))$, the consumer solves:

$$\begin{aligned}\mathcal{C}_d^f(\mathbf{p}_{el}, \mathbf{p^f}) \equiv \arg \min_{\substack{\mathbf{s}:s(\omega)\geq 0\\ \mathbf{x_d}}} \;\rho_d\bigg(&(PC - p_{\mathrm{el}}(\omega)) \cdot (\mathrm{LOAD}(\omega) - s(\omega)) \\ &- \sum_{c=1}^{C} x_{d,c} \cdot (p_c^f - p_c^s(\omega))\bigg) \,.\end{aligned} \tag{4.5}$$

Using the cash invariance property of convex risk measures, the problem can be rewritten as:

$$\begin{aligned}\mathcal{C}_d^f(\mathbf{p}_{el}, \mathbf{p^f}) \equiv \mathrm{Min}_{\substack{\mathbf{s}:s(\omega)\geq 0\\ x_{d,c}\in\mathbb{R}^C}} \bigg\{ &\max_{Q_d\in\mathcal{M}_d} \mathbb{E}_{Q_d}\Big[- (PC - p_{\mathrm{el}}(\omega)) \cdot (\mathrm{LOAD}(\omega) - s(\omega)) \\ &- \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega)\Big] - \alpha_d(Q_d) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^f \bigg\}.\end{aligned} \tag{4.6}$$

The KKT conditions of the above optimization problem are:

$$0 \leq PC - p_{el}(\omega) \perp s(\omega) \geq 0 \qquad \forall \omega \in \Omega \tag{4.7}$$

$$\mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f = 0 \qquad \forall c = 1, \dots, C\,, \tag{4.8}$$

where the term

$$Z_d^f(\omega) = (PC - p_{\mathrm{el}}(\omega)) \cdot (\mathrm{LOAD}(\omega) - s(\omega)) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega) \tag{4.9}$$

is the second stage net profit of the consumer, comprising payoffs coming from the electricity market as well as from the financial market. Condition (4.8) states that the contract prices at stage 0 are equal to the expectation of their payoffs in stage 1 under the subjective probability measure. This property combined with positive subjective probabilities (cf. remark 3.4) implies that a market equilibrium, if it exists, is free of arbitrage.

4.2 The producers

There are $\nu = 1, \dots, N$ producers that invest in $k = 1, \dots, K$ physical assets (differing by technologies, e.g. nuclear, coal, gas,...) at $t = 0$ and optimally operate them in $t = 1$. They evaluate those investment options by the risk measure of the cash-flow accruing from these assets in $t = 1$. Let $u_{\nu,k}$ be the capacity invested in each technology $k = 1, \dots, K$, $\nu = 1, \dots, N$ and $y_{\nu,k}(\omega)$ be the operating level of this technology for each scenario $\omega \in \Omega$. Those operational decisions are constrained in $t = 1$ by the capacity invested in $t = 0$: $0 \leq y_{\nu,k}(\omega) \leq u_{\nu,k}$. For each producer $\nu = 1, \dots, N$, we denote $\mathbf{u}_\nu := (u_{\nu,k})_{k=1}^K \in \mathbb{R}^K$ the vector of invested capacities and $\mathbf{y}_\nu := (y_{\nu,k}(\omega))_{k=1,\omega\in\Omega}^K \in \mathbb{R}^{K|\Omega|}$ the vector of their production levels. The vector of all these physical investments is denoted $\mathbf{u} := (\mathbf{u}_\nu)_{\nu=1}^N \in \mathbb{R}^{NK}$ and the vector of all physical operations is $\mathbf{y} := (\mathbf{y}_\nu)_{\nu=1}^N \in \mathbb{R}^{NK|\Omega|}$.

The equivalent annual costs (computed at the risk-free rate) of investment for the producer ν are denoted by $I_\nu = (I_{\nu,1}, \dots, I_{\nu,K}) \in \mathbb{R}^{+,K}$ and $C_\nu(\omega) = (C_{\nu,1}(\omega), \dots, C_{\nu,K}(\omega)) \in \mathbb{R}^{+,K}$ are the operating costs of the technologies at scenario $\omega \in \Omega$. As we consider a single demand level in the year, those operating costs are expressed as yearly operating costs. We assume that parameters $I_{\nu,k}$ and $C_{\nu,k}(\omega)$ are all positive.

In this section we present two different investment models. The first one describes a world without financial instruments and where each plant is valued on the basis of its sole merits. It can also be interpreted as a market where each producer invests in a single type of plant. The second model allows for companies investing in a portfolio of plants and hedging their payoffs by some financial contracts. This accounts for the portfolio effect of owning a diversified generating system as well as a set of financial contracts. Note that all models refer to incomplete markets in the sense that risk cannot be fully hedged.

4.2.1 Model without financial contracts: project finance valuation

This first model describes a world without financial instruments where each plant is evaluated on the basis of its own merits (project finance). Given the electricity prices $\mathbf{p}_{el}$, each investment is valued separately, leading to the following risk-averse stochastic program:

$$\mathcal{G}_\nu(\mathbf{p}_{el}) \equiv \arg \min_{\mathbf{u}_\nu : u_{\nu,k} \geq 0} \sum_{k=1}^{K} \rho_\nu \left(\max_{\substack{y_{\nu,k} \in \mathbb{R}^{|\Omega|} : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}} \left\{ \left(p_{el}(\omega) - C_{\nu,k}(\omega) \right) \cdot y_{\nu,k}(\omega) \right\} - I_{\nu,k} u_{\nu,k} \right) , \tag{4.10}$$

where the constraints of the second-stage problem appear under the risk measure for each asset $k = 1, .., K$. We denote by $\mu_{\nu,k}(\omega)$ the dual variables associated to the constraints $y_{\nu,k}(\omega) \leq u_{\nu,k}$. The following primal-dual relation holds at the optimum of the second-stage sub-problem defined inside each ρ_ν function:

$$\left(p_{el}(\omega) - C_{\nu,k}(\omega) \right) \cdot y_{\nu,k}(\omega) = u_{\nu,k} \cdot \mu_{\nu,k}(\omega) , \tag{4.11}$$

where all variables are taken at the value of the optimal solution to the pair of primal and dual problems. The variable $\mu_{\nu,k}(\omega)$ can be interpreted as the payment by the operator of the plant to the investor (tolling agreement) and the equality states that the revenue accruing from operating a specific plant is entirely redistributed to the owner of this plant. This is the standard condition of perfect competition (price-taking in a deterministic spot market in each state of the world ω implies perfect competition in that state of the world). For each physical asset $k = 1, \ldots, K$, the risk measure is valued at the payoff in the second-stage

$$Z_{\nu,k}(\omega) = u_{\nu,k} \cdot \mu_{\nu,k}(\omega). \tag{4.12}$$

Using the dual representation of convex risk measures, the investor's problem can be reformulated as:

$$\mathrm{Min}_{\mathbf{u}_\nu : u_{\nu,k} \geq 0} \sum_{k=1}^{K} \left\{ \max_{Q_\nu \in \mathcal{M}_\nu} \mathbb{E}_{Q_\nu} \Big[- u_{\nu,k} \cdot \mu_{\nu,k}(\omega) \Big] - \alpha_\nu(Q_\nu) \right\} + \sum_k I_{\nu,k} \cdot u_{\nu,k} . \tag{4.13}$$

The investment behavior is then expressed in the following KKT conditions:

$$0 \leq I_{\nu,k} - \mathbb{E}_{\bar{Q}_\nu(Z_{\nu,k})} \left[\mu_{\nu,k}(\omega) \right] \perp u_{\nu,k} \geq 0 \qquad \forall k = 1, \ldots, K \tag{4.14}$$

where the subgradients of the risk measure $\bar{Q}_\nu(Z_{\nu,k}) \in \partial \rho_\nu(Z_{\nu,k})$ are evaluated at the second-stage stochastic payoff $Z_{\nu,k}(\omega)$ of the producer in $t = 1$.

Condition (4.14) only differs from a standard investment criterion by the fact that net revenues appear in a risk measure. An usual stochastic programming expression would state that the investment cost of a profitable plant is recovered by the expectation of the net revenues (the $u.\mu$) accruing from the plant [37]. Except for that adjustment, the condition states that the risk measure of the cashflow from the investment covers the capital cost of that investment.

The full complementarity formulation of the investor problem then consists of the concatenation of these investment conditions with the KKT conditions of the operational decisions,

$$\begin{array}{lll} \text{Max}_{\mathbf{y}_\nu} & \sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega)\Big) \cdot y_{\nu,k}(\omega) & \\ \text{s.t.} & y_{\nu,k}(\omega) \leq u_{\nu,k} & (\mu_{\nu,k}(\omega)) \\ & y_{\nu,k} \geq 0 \ , & \end{array} \tag{4.15}$$

that are stated as

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \quad \perp \ \mu_{\nu,k}(\omega) \geq 0 \quad \forall k = 1, \ldots, K, \omega \in \Omega \tag{4.16}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \quad \perp \ y_{\nu,k}(\omega) \geq 0 \quad \forall k = 1, \ldots, K, \omega \in \Omega \ . \tag{4.17}$$

4.2.2 *Model with financial contracts: diversified portfolio of physical and financial contracts*

The investor in generation capacity simultaneously takes positions in financial assets. For each producer $\nu = 1, \ldots, N$, we let $x_{\nu,c}$ be its positions in the different contracts $c = 1, \ldots, C$ and $\mathbf{x}_\nu$ be the vector of those financial positions $\mathbf{x}_\nu := (x_{\nu,c})_{c=1}^{C} \in \mathbb{R}^C$. Given the electricity prices $\mathbf{p}_{el}$ and the financial contract prices $\mathbf{p^f}$ (recall that $p_c^s(\omega) = h_{c,\omega}(p_{el}(\omega))$, the investor solves:

$$\begin{aligned} \mathcal{G}_\nu^f(\mathbf{p}_{el}, \mathbf{p^f}) = \arg \min_{\substack{\mathbf{u}_\nu : u_{\nu,k} \geq 0 \\ \mathbf{x}_\nu}} \ & \sum_{k=1}^{K} I_{\nu,k} \cdot u_{\nu,k} + \sum_{c=1}^{C} p_c^f \cdot x_{\nu,c} \\ & + \rho_\nu \Bigg(\max_{\substack{\mathbf{y}_\nu : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}} \left\{ \sum_{k=1}^{K} \Big(p_{el}(\omega) - C_{\nu,k}(\omega)\Big) \cdot y_{\nu,k} \right\} \\ & + \sum_{c=1}^{C} x_{\nu,c} \cdot p_c^s(\omega) \Bigg). \end{aligned} \tag{4.18}$$

As before, we have at the optimum of the second-stage problem (optimal operation of the plants)

$$\sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega)\Big) \cdot y_{\nu,k} = \sum_k u_{\nu,k} \cdot \mu_{\nu,k}(\omega) \ , \tag{4.19}$$

which gives the following KKT conditions,

$$0 \leq I_{\nu,k} - \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[\mu_{\nu,k}(\omega)\right] \perp \quad u_{\nu,k} \geq 0 \quad \forall k = 1, \dots, K \tag{4.20}$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \perp \quad \mu_{\nu,k}(\omega) \geq 0 \quad \forall k = 1, \dots, K, \omega \in \Omega \tag{4.21}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \perp \quad y_{\nu,k}(\omega) \geq 0 \quad \forall k = 1, \dots, K, \omega \in \Omega\,. \tag{4.22}$$

These conditions are formally identical to those of the model without contracts except that the subgradient of the risk measure $\bar{Q}_\nu(Z_\nu^f) \in \partial\rho_\nu(Z_\nu^f)$ is now evaluated at the (second-stage) payoff Z_ν^f that takes into account the cash-flows accruing from the physical power market and the financial contracts of the producer:

$$Z_\nu^f(\omega) = \sum_{k=1}^{K} u_{\nu,k} \cdot \mu_{\nu,k}(\omega) + \sum_{c=1}^{C} x_{\nu,c} \cdot p_c^s(\omega)\,. \tag{4.23}$$

These expressions are complemented by the conditions on the prices of the financial products:

$$0 = -\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[p_c^s(\omega)\right] + p_c^f \qquad \forall c = 1, \dots, C\,. \tag{4.24}$$

As for the consumer, positive subjective probabilities for producer ν ensure the absence of arbitrage opportunity in the financial market (cf. remark 3.4).

4.3 The market agent

We complete the description of the power economy by introducing market agents that choose market prices so as to minimize excess supplies.

4.3.1 The spot market agent

Considering only the electricity market and given decisions $y_{\nu,k}(\omega)$ and $s(\omega)$ of the producers and consumer, the market agent chooses the electricity price $\mathbf{p_{el}}$ for the scenarios $\omega \in \Omega$ by solving the following optimization problem:

$$\mathcal{A}(\mathbf{y}, \mathbf{s}, \omega) \equiv \arg \min_{\mathbf{p_{el}}: p_{el}(\omega) \geq 0} p_{el}(\omega) \cdot \left(\sum_{\nu=1}^{N} \sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) - \mathrm{LOAD}(\omega) \right). \tag{4.25}$$

In a Nash equilibrium solution to the game, if it exists, the electricity prices cannot be unbounded as the consumer curtails its demand for prices higher than PC. The KKT condition of the market agent at each scenario ω is hence given by:

$$0 \leq -\mathrm{LOAD}(\omega) + \sum_{\nu=1}^{N} \sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) \perp p_{el}(\omega) \geq 0 \;\; \forall \omega \in \Omega, \tag{4.26}$$

which is the standard market clearing condition of the spot electricity market with price-taking agents.

4.3.2 The financial market agent

We now consider both the electricity and the financial markets. The vector of all financial positions is denoted by $\mathbf{x} := \left(\mathbf{x}_d, (\mathbf{x}_\nu)_{\nu=1}^N\right) \in \mathbb{R}^{(N+1)C}$. We introduce two market agents: the spot market agent chooses electricity prices $p_{el}(\omega)$ for each scenario $\omega \in \Omega$, given variables $y_{\nu,k}(\omega)$ and $s(\omega)$ by solving the optimization programs (4.25) for each scenario ω. Given variables $\mathbf{x}$, the financial market agent chooses contract prices $\mathbf{p}^f$ in stage 0 so as to minimize the value of excess supply of financial positions:

$$\mathcal{A}^f(\mathbf{x}) \equiv \arg\min_{\mathbf{p^f}} \sum_{c=1}^{C} p_c^f \cdot \left(\sum_{\nu=1}^{N} x_{\nu,c} + x_{d,c}\right). \tag{4.27}$$

In a Nash equilibrium of the game, if it exists, the contract prices $\mathbf{p^f}$ cannot be unbounded (see [23] for a proof). The KKT conditions of the spot and financial market agents are given by:

$$\begin{aligned} &0 \leq -\mathrm{LOAD}(\omega) + \sum_{\nu=1}^{N}\sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) \perp p_{el}(\omega) \geq 0 && \forall \omega \in \Omega \\ &0 = \sum_{\nu} x_{\nu,c} + x_{d,c} && \forall c = 1, \ldots, C. \end{aligned} \tag{4.28}$$

The second condition expresses the clearing of the financial market, i.e. that the sum of financial positions of producers and the consumer balances to zero.

5 Two equilibrium problems

5.1 Problem P: the project finance equilibrium

This model describes a world without financial instruments where each plant is valued separately with no concern for the portfolio diversification effect of owning a mix of generation assets. A version of this model was first introduced in [29].

Definition 5.1 (*Problem P as a Nash game*) The vector $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ of capacities, operational decisions, load curtailments and electricity prices constitutes a Nash equilibrium of the project finance game P if:

(i) For the consumer, the load curtailments minimize the risk measure of its surplus: $\mathbf{s} \in \mathcal{C}_d(\mathbf{p_{el}})$, presented in Sect. 4.1.1.
(ii) For every producer $\nu = 1, \ldots, N$, the investment and operational decisions minimize the risk measure of the producer's net profit: $(\mathbf{u}_\nu, \mathbf{y}_\nu) \in \mathcal{G}_\nu(\mathbf{p_{el}})$, presented in Sect. 4.2.1.

(iii) For the market agent, the electricity price minimizes the value of excess supply: $p_{el}(\omega) \in \mathcal{A}(\mathbf{y}, \mathbf{s}, \omega)$, for every $\omega \in \Omega$, presented in Sect. 4.3.1.

Proposition 5.2 (Problem P as a complementarity problem)

A Nash equilibrium of the project finance game P consists of a tuple $(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}) \in \mathbb{R}^{M_P}$, *where* $M_P = (NK) + 2|\Omega| + 2(NK|\Omega|)$, *satisfying the following complementarity conditions:*

$$0 \leq PC - p_{el}(\omega) \quad \perp \ s(\omega) \geq 0 \ \ \forall \omega \in \Omega \tag{5.1}$$

$$0 \leq I_{\nu,k} - \mathbb{E}_{\bar{Q}_\nu(Z_{\nu,k})}\Big[\mu_{\nu,k}(\omega)\Big] \quad \perp \ u_{\nu,k} \geq 0 \ \ \forall \nu = 1, \ldots, N, k = 1, \ldots, K \tag{5.2}$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \quad \perp \ \mu_{\nu,k}(\omega) \geq 0 \ \ \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{5.3}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \quad \perp \ y_{\nu,k}(\omega) \geq 0 \ \ \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{5.4}$$

$$0 \leq -LOAD(\omega) + \sum_{\nu=1}^{N}\sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) \quad \perp \ p_{el}(\omega) \geq 0 \ \ \forall \omega \in \Omega. \tag{5.5}$$

The complementarity problem presented in proposition 5.2 is obtained by concatenating the KKT conditions of the different optimization programs of the agents presented in Sects. 4.1.1, 4.2.1 and 4.3.1.

We already argued before that the investment criterion obtained in this formulation is identical to the standard investment rule except for taking the risk measure of uncertain cashflows instead of their mathematical expectation.

Remark 5.3 (The clearing of the spot electricity market) The model reproduces the standard outcome of short-term electricity markets: at an equilibrium of problem P, the generation capacities are dispatched in the second-stage according to their merit order, i.e. a generation unit can produce only if the spot electricity price is higher than or equal to its variable operating cost; it is producing at full capacity if this price is strictly higher than this cost. The complementarity conditions (5.1), (5.3)–(5.5) also reflect the clearing of a spot electricity market organized by an ISO. In such an organization, the ISO collects the technical characteristics of the plants (assuming truthful revelation from the producers), i.e. the capacities $u_{\nu,k}$ and the variable operating costs $C_{\nu,k}(\omega)$ in our simple setting. It then clears the market in every scenario $\omega \in \Omega$ by solving the following mathematical program:

$$\begin{aligned} ISO \equiv \min_{\mathbf{y}(\omega), s(\omega)} & \sum_{\nu=1}^{N}\sum_{k=1}^{K} C_{\nu,k}(\omega) y_{\nu,k}(\omega) + PCs(\omega) \\ \text{s.t. } & 0 \leq y_{\nu,k} \leq u_{\nu,k} \\ & s(\omega) \geq 0 \\ & \sum_{\nu=1}^{N}\sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) - \text{LOAD}(\omega) \geq 0. \end{aligned} \tag{5.6}$$

The market price $p_{el}(\omega)$ is then the dual variable associated with the market clearing constraint:

$$\sum_{\nu=1}^{N}\sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) - \mathrm{LOAD}(\omega) \geq 0. \tag{5.7}$$

5.2 Problem P^f: the spot-financial equilibrium

The spot-financial equilibrium model comprises a financial market where the consumer and the producers can purchase financial contracts to redistribute revenues across stages.

Definition 5.4 (*Problem P^f as a Nash game*) The vector $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{x}, \mathbf{p_{el}}, \mathbf{p^f})$ of capacities, operational decisions, load curtailments, financial positions, electricity prices and contract prices constitutes a Nash equilibrium of the spot-financial game P^f if:

(i) For the consumer, the load curtailments and the financial positions minimize the risk measure of its surplus: $(\mathbf{s}, \mathbf{x_d}) \in \mathcal{C}_d^f(\mathbf{p_{el}}, \mathbf{p^f})$, presented in Sect. 4.1.2.
(ii) For every producer $\nu = 1, \ldots, N$, the investment, the operating decisions and the financial positions minimize the risk measure of the producer's payoff: $(\mathbf{u}_\nu, \mathbf{y}_\nu, \mathbf{x}_\nu) \in \mathcal{G}_\nu^f(\mathbf{p_{el}}, \mathbf{p^f})$, presented in Sect. 4.2.2.
(iii) For the spot market agent, the electricity price minimizes the excess supply: $p_{el}(\omega) \in \mathcal{A}(\mathbf{y}, \mathbf{s}, \omega)$, for every $\omega \in \Omega$, presented in Sect. 4.3.1.
(iv) For the financial market agent, the contract prices minimize the excess supplies of financial positions: $\mathbf{p^f} \in \mathcal{A}^f(\mathbf{x})$, presented in Sect. 4.3.2.

Proposition 5.5 (Problem P^f as a mixed complementarity problem)
A solution to the spot-financial equilibrium problem P^f consists of a tuple $\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}}$, *where* $M_{Pf} = NK + 2|\Omega| + 2NK|\Omega| + (N+1)C + C(1+|\Omega|)$, *satisfying the following complementarity conditions:*

$$0 = \mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f \quad \forall c = 1, \ldots, C \tag{5.8}$$

$$0 \leq PC - p_{el}(\omega) \quad \perp \; s(\omega) \geq 0 \quad \forall \omega \in \Omega \tag{5.9}$$

$$0 = \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[p_c^s(\omega)\right] - p_c^f \quad \forall \nu = 1, \ldots, N, c = 1, \ldots, C \tag{5.10}$$

$$0 \leq I_{\nu,k} - \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[\mu_{\nu,k}(\omega)\right] \quad \perp \; u_{\nu,k} \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K \tag{5.11}$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \quad \perp \; \mu_{\nu,k}(\omega) \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{5.12}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \quad \perp \; y_{\nu,k}(\omega) \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{5.13}$$

$$0 \leq -LOAD(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega) \quad \perp \; p_{el}(\omega) \geq 0 \quad \forall \omega \in \Omega \tag{5.14}$$

$$0 = \sum_{\nu} x_{\nu,c} + x_{d,c} \quad \forall c = 1, \ldots, C \tag{5.15}$$

$$0 = p_c^s(\omega) - h_{c,\omega}(p_{el}(\omega)) \quad \forall c = 1, \ldots, C, \omega \in \Omega. \tag{5.16}$$

In that equilibrium the risk measure is evaluated at the second-stage payoffs, i.e. $Z_\nu^f(\omega)$ defined in (4.23) for the producers and $Z_d^f(\omega)$ defined in (4.9) for the consumer.

The mixed complementarity problem presented in Proposition 5.5 is obtained by concatenating the KKT conditions of the different optimization programs of the agents presented in 4.1.2, 4.2.2, 4.3.1 and 4.3.2. We also note that the functioning of the spot electricity market in the second stage (described by complementarity conditions (5.9), (5.12)–(5.14)) is identical for problems P and P^f and hence the same interpretation given in remark 5.3 prevails. Equations (5.8) and (5.10) impose for each market participant that the price of each contract at stage 0 should be equal to the risk measure of its random payoff of stage 1. Also, if at an equilibrium solution to problem P^f, there exists an agent with positive subjective probabilities (cf. remark 3.4), then there should be no arbitrage opportunity in the financial market.

6 Assumptions

This section assembles important assumptions that intervene in the rest of the developments.

(H1) We simplify the presentation by making the blanket assumption that the convex risk measures ρ_i used in this paper are continuously differentiable.

Some well-known risk measures satisfy this property such as the entropic risk measure, the Epstein-Zin utility or the good-deal risk measure (when taking a sufficiently small Sharpe ratio [1]). On the contrary, polyhedral risk measures (and in particular the commonly used CVaR) fail to satisfy this property .

Assumption H1 *Risk measures are convex and (once) continuously differentiable.*

The rationale of this assumption is that we must guarantee continuity of the functions involved in the statements of the equilibrium problems in order to use degree theory. [49] shows how to overcome non-differentiability by defining a sequence of approximating differentiable variational inequalities based on smoothing the non-differentiable risk measures in the agents' problems. They notably prove that this approximation converges, which gives some hope to get rid of the continuous differentiability assumption in future work.

(H2) Let $\overline{\mathcal{M}} = \left(\cap_{\nu=1}^N \mathcal{M}_\nu\right) \cap \mathcal{M}_d$ be the intersection of the feasible sets of subjective risk probabilities of the different agents. Different papers [23,57] have drawn the attention to the role of the interior of this set for proving the existence of equilibrium with financial contracts. We rely on the same considerations here and impose that "the risk measures of the players are sufficiently similar".

Assumption H2 *The agents' risk measures are sufficiently similar, i.e. the intersection of the agents' feasible set of subjective risk probabilities has a non-empty interior:*

$$\textbf{int}\ \overline{\mathcal{M}} \neq \emptyset. \tag{6.1}$$

Assumption H2 is critical for the portfolio model to show that, in an equilibrium solution, financial positions remain bounded.

(**H3**) The following property is crucial in discussions of incomplete markets [52]. It states that, functions $h_{c,\omega}$ ensure that the subspace spanned by the existing financial products always has a fixed dimension equal to the number of contracts C.

Assumption H3 *Functions $h_{c,\omega}$ are such that, for all electricity prices $p_{el}(\omega)$, financial contracts are never redundant, also with the risk-free asset:*

$$\forall \omega \in \Omega : \quad \sum_{c} x_c \cdot p_c^s(\omega) + y = 0 \Rightarrow \forall c = 1, \ldots, C : \; x_c = 0 \textit{ and } y = 0. \tag{6.2}$$

Duffie and Schaffer [26] also use this assumption in their analysis of general equilibrium in incomplete markets. In our case, this assumption is necessary to avoid degeneracy of the contract positions.

7 Degree theory and the auxiliary problems

It is well known that the complementarity problem $0 \leq x \perp y \geq 0$ can be rewritten as an element-wise minimization $\min(x, y) = 0$. It is also clear that our assumption of continuously differentiable risk measures implies that all the expressions appearing in the complementarity conditions are continuous. The complementarity conditions associated to problems P and P^f can thus be seen as finding a zero of continuous, non-differentiable functions. We construct the functions Φ^P and Φ^{P^f}, associated to the equilibrium problems P, P^f respectively, as the element-wise minimization $\min(\cdot, \cdot)$ for the complementarity conditions (with the equations of the mixed complementarity problem P^f inserted as such in Φ^{P^f}). In other words, solving P (P^f) is equivalent to finding the zeros of function Φ^P (Φ^{P^f}). We will further elaborate on this in Sects. 8 and 9.

We now resort to a standard application of degree theory (more specifically Sect. 2.1 of Facchinei and Pang [30]), i.e. the study of the existence of a solution to the equation $\Phi(x) = p$, where Φ is a continuous function defined on the closure of a bounded open set Ξ of $\mathbb{R}^n$ and taking values in $\mathbb{R}^n$, and p is a vector not in $\Phi(\mathbf{bd}\,\Xi)$. Let Γ be the collection of such triples (Φ, Ξ, p). The following definition is taken from [30].

Definition 7.1 Let an integer $\deg(\Phi, \Xi, p)$ be associated with each triple (Φ, Ξ, p) in the collection Γ. The function deg is called a *topological degree* if the following three axioms are satisfied (I_n is the identity function of $\mathbb{R}^n$).

(A1) $\deg(I_n, \Xi, p) = 1$ if $p \in \Xi$;

(A2) $\deg(\Phi, \Xi, p)=\deg(\Phi, \Xi_1, p)+\deg(\Phi, \Xi_2, p)$ if Ξ_1 and Ξ_2 are two disjoint open subsets of Ξ and $p \notin \Phi((\mathrm{cl}\Xi)\backslash(\Xi_1 \cup \Xi_2))$;

(A3) $\deg(H(\cdot, t), \Xi, p(t))$ is independent of $t \in [0, 1]$ for any two continuous functions $H : cl\Xi \times [0, 1] \to \mathbb{R}^n$ and $p : [0, 1] \to \mathbb{R}^n$ such that

$$p(t) \notin H(\mathbf{bd}\,\Xi, t) \quad \forall t \in [0, 1].$$

One can show that such a function exists. The following theorem from [30] makes the link between the topological degree and the existence of a solution to $\Phi(x) = p$.

Theorem 7.2 *Let Ξ be a nonempty, bounded open subset of $\mathbb{R}^n$ and let $\Phi = cl\,\Xi \to \mathbb{R}^n$ be a continuous function. Assume that $p \notin \Phi(bd\,\Xi)$. If $deg(\Phi, \Xi, p) \neq 0$, then there exists an $\bar{x} \in \Xi$ such that $\Phi(\bar{x}) = p$. Conversely if $p \notin \Phi(cl\Xi)$, then $deg(\Phi, \Xi, p) = 0$.*

We use the following idea presented in [30] to prove existence of a solution to $\Phi(x) = 0$. We construct a homotopy function: $H : cl\,\Xi \times [0, 1] \to \mathbb{R}^n$ such that $H(x, 0) = \Phi(x)$ for all $x \in cl\,\Xi$ and the degree of $H(x, 1)$ is known and nonzero. It follows that if 0 does not belong to $H(bd\,\Xi, t)$ for all $t \in [0, 1]$, then by the homotopy invariance property, the degree of the original triple $(\Phi, \Xi, 0)$ is equal to the degree of the auxiliary triple $(H(\cdot, 1), \Xi, 0)$; hence there exists a solution to $\Phi(x) = 0$. The construction of the homotopy can be transformed into a true algorithm as discussed in [13,27] or [34]. We limit ourselves in this paper to using the theory to explore solution existence, uniqueness or multiplicity.

The following proposition tells something about the oddness of the number of solutions.

Proposition 7.3 *Let Ξ be a non-empty, bounded open subset of $\mathbb{R}^n$ and let $\Phi : cl\,\Xi \to \mathbb{R}^n$ be a continuous mapping. If $p \notin \Phi(\mathbf{bd}\ \Xi)$, if Φ is continuously differentiable at every solution $\bar{x} \in \Phi^{-1}(p)$, if the Jacobian matrix $J\Phi(\bar{x})$ is nonsingular at every solution $\bar{x} \in \Phi^{-1}(p)$, the following properties hold:*

- *The set of solutions $\Phi^{-1}(p)$ is a finite set;*
- *The degree $deg(\Phi, \Xi, p)$ can be computed as:*

$$deg(\Phi, \Xi, p) = \sum_{\bar{x} \in \Phi^{-1}(p)} sgn\ det\ J\Phi(\bar{x}). \tag{7.1}$$

Proof First, let us assume by contradiction that $\Phi^{-1}(p)$ is not finite. We can then construct a sequence of solutions $\bar{x}_n \in \Phi^{-1}(p)$ satisfying:

$$\forall n,\ n' \in \mathbb{N},\ n \neq n' \Longrightarrow \bar{x}_n \neq \bar{x}_{n'}.$$

We already know that $(\bar{x})_n$ is a sequence of clΞ, which is compact. Therefore, one can extract a sub-sequence $(\bar{x})_{\psi(n)}$ of $(\bar{x})_n$, that converges toward $\bar{x} \in$ clΞ. Without any loss of generality, we can assume that $(\bar{x})_n$ converges toward $\bar{x}$. Since Φ is continuous, one can deduce that $\Phi(\bar{x}) = p$ or that $\bar{x} \in \Phi^{-1}(p)$. Therefore, we have:

$$\forall \epsilon > 0,\ \exists n \in \mathbb{N},\ \text{such that } \bar{x}_n \in \Phi^{-1}(p),\ \ \|\bar{x}_n - \bar{x}\| < \epsilon \text{ and } \bar{x}_n \neq \bar{x},$$

which means that the solution $\bar{x}$ is not isolated, which contradicts the fact that the Jacobian $J\phi(\bar{x})$ is nonsingular. Therefore, one can conclude that $\Phi^{-1}(p)$ is a finite set.

Using Proposition 2.1.6 of [30], one can deduce immediately that:

$$\deg(\Phi, \Xi, p) = \sum_{\bar{x}\in\Phi^{-1}(p)} \text{sgn det } J\Phi(\bar{x}). \tag{7.2}$$

□

In order to apply these ideas, this section introduces two equilibrium problems that have a single solution under a standard assumption of LP non-degeneracy and strict convexity. The first problem AP consists of the KKT conditions of a simple stochastic linear program where all agents are risk-neutral.[7] We then introduce a second auxiliary problem with financial contracts AP^f, which is obtained by concatenating the KKT conditions of problem AP with those of a hedging problem.

7.1 AP, the auxiliary equilibrium problem of P

We construct the auxiliary problem AP that has the same structure (in terms of variables and constraints) as the project finance model P but where all agents are risk-neutral (i.e. evaluate their payoffs by taking their expectation under the natural probability measure $P(\omega)$). The solution to such an equilibrium model can be computed by solving the following standard stochastic capacity expansion problem.

$$\begin{aligned} &\underset{\mathbf{u}:u_{\nu,k}\geq 0}{\text{Min}} \sum_{\nu=1}^{N}\sum_{k=1}^{K} I_{\nu,k}\cdot u_{\nu,k} \\ &+\mathbb{E}_P\left[\underset{\substack{\mathbf{y},\mathbf{s} \\ \text{s.t. } 0\leq y_{\nu,k}(\omega)\leq u_{\nu,k} \\ \sum_{\nu=1}^{N}\sum_{k=1}^{K} y_{\nu,k}+s(\omega)\geq \text{LOAD}(\omega)}}{\min} PC\cdot s(\omega)+\sum_{k=1}^{K} C_{\nu,k}(\omega)\cdot y_{\nu,k}(\omega)\right]. \end{aligned} \tag{7.3}$$

The auxiliary problem AP is then given by the KKT conditions of the stochastic program.

[7] One could have considered an alternative auxiliary problem with risk-averse agents, which all have the same risk measure ($\overline{\rho}$) that they value at the same position. This type of equivalence between complementarity and optimization was first noted in [28] and has now been elaborated in different papers [23,56,57] This problem is closer to our model P, which could be useful if the homotopy was used as an algorithm. This pure LP problem simplifies the presentation.

Definition 7.4 (*Problem AP*) A solution to the equilibrium problem AP consists of a tuple $\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\right) \in \mathbb{R}^{M_P}$ satisfying the following complementarity conditions

$$0 \leq I_{\nu,k} - \mathbb{E}_P[\mu_{\nu,k}(\omega)] \quad \perp \ u_{\nu,k} \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K \tag{7.4}$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \quad \perp \ \mu_{\nu,k}(\omega) \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{7.5}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \quad \perp \ y_{\nu,k}(\omega) \geq 0 \quad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{7.6}$$

$$0 \leq PC - p_{el}(\omega) \quad \perp \ s(\omega) \geq 0 \quad \forall \omega \in \Omega \tag{7.7}$$

$$0 \leq -\mathrm{LOAD}(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega) \quad \perp \ p_{el}(\omega) \geq 0 \quad \forall \omega \in \Omega. \tag{7.8}$$

The problem AP has the same structure as problem P except for one major point: all agents are risk-neutral.

Proposition 7.5 *For almost all parameter data, the problem AP has a unique solution.*

Proof The linear program is obviously feasible as it suffices to invest in any plant up to the maximum of the LOAD to be feasible. It is also bounded since all costs are positive. It is thus primal-dual feasible. For almost all parameter data, it is also non-degenerate as a perturbation can avoid degeneracy. This argument is standard in linear programming but will need to be refined later for dealing with nonlinear relations in proofs of isolated solutions. ◻

7.2 The problem AP^f, the auxiliary equilibrium problem of P^f

We construct the auxiliary equilibrium problem with financial contracts AP^f by completing the problem AP with a financial equilibrium model. This model is one where agents optimally hedge an exogenous payoff (identical for all agents), noted $Z_{\text{exo}}(\omega)$. In this case we cannot simply assume risk-neutral agents as this may introduce unbounded financial positions. We rather construct this auxiliary equilibrium problem by taking the same entropic risk measure (given in definition 3.5) $e_\theta(Z)$ for all economic agents. The hedging optimization problem for the producer $\nu = 1, \ldots, N$ is given by:

$$\mathcal{H} - \mathcal{AP}_\nu \equiv \mathrm{Min}_{x_\nu} \ e_\theta\left(Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{\nu,c} \cdot p_c^s(\omega) - p_c^f \cdot x_{\nu,c}\right), \tag{7.9}$$

where the $p_c^s(\omega)$ are computed from the solution to AP and the p_c^f are endogenous. Using the cash-invariance property of the risk measure, the KKT conditions of this problem are:

$$\mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}} + \sum_{c=1}^{C} x_{\nu,c} p_c^s)}\left[p_c^s(\omega)\right] - p_c^f = 0 \quad \forall c = 1, \ldots, C. \tag{7.10}$$

The consumer solves a similar hedging problem:

$$\mathcal{H}-\mathcal{AP}_d \equiv \text{Min}_{x_d}\ e_\theta \left(Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega) - p_c^f \cdot x_{d,c} \right). \tag{7.11}$$

This problem has KKT conditions similar to (7.10):

$$\mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}}+\sum_{c=1}^{C} x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] - p_c^f = 0 \quad \forall c = 1, \ldots, C. \tag{7.12}$$

Note that the expression $\bar{Q}_{e_\theta}(.)$ of the subjective probabilities of all agents is the same, because we have used the same convex risk measure e_θ for all agents.

Definition 7.6 presents the complementarity problem obtained by concatenating the KKT conditions of the producers (7.10) and the consumer (7.12), and completing them with the market clearing conditions of the financial contracts market (5.15).

Definition 7.6 (*Problem AP^f*) A solution to the equilibrium problem AP^f consists of a tuple $\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}}$ satisfying the following complementarity conditions

$$0 = \mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}}+\sum_{c=1}^{C} x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] - p_c^f \qquad \forall c = 1, \ldots, C \tag{7.13}$$

$$0 = \mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}}+\sum_{c=1}^{C} x_{\nu,c} p_c^s)}\left[p_c^s(\omega)\right] - p_c^f \qquad \forall \nu = 1, \ldots, N, c = 1, \ldots, C \tag{7.14}$$

$$0 = \sum_{\nu} x_{\nu,c} + x_{d,c} \qquad \forall c = 1, \ldots, C \tag{7.15}$$

$$0 \leq I_{\nu,k} - \mathbb{E}_P\left[\mu_{\nu,k}(\omega)\right] \ \perp\ u_{\nu,k} \geq 0 \qquad \forall \nu = 1, \ldots, N, k = 1, \ldots, K \tag{7.16}$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \ \perp\ \mu_{\nu,k}(\omega) \geq 0 \qquad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{7.17}$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \ \perp\ y_{\nu,k}(\omega) \geq 0 \qquad \forall \nu = 1, \ldots, N, k = 1, \ldots, K, \omega \in \Omega \tag{7.18}$$

$$0 \leq PC - p_{el}(\omega) \ \perp\ s(\omega) \geq 0 \qquad \forall \omega \in \Omega \tag{7.19}$$

$$0 \leq -\text{LOAD}(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega) \ \perp\ p_{el}(\omega) \geq 0 \qquad \forall \omega \in \Omega \tag{7.20}$$

$$0 = p_c^s(\omega) - h_{c,\omega}(p_{el}(\omega)) \qquad \forall c = 1, \ldots, C,\ \omega \in \Omega. \tag{7.21}$$

The obtained equilibrium model has the same structure as problem P^f except that, as in the comparison between P and AP, all agents have the same risk measure e_θ that they all assess at the same initial position. This position is equal to the variable cost of the system in the different scenarios, which is the global risk of the system.

The following proposition states that the solution to AP^f is the same as the solution to AP. The intuition is rather straightforward: because AP^f describes an economy where all agents are exposed to the same risk with the same risk aversion, they have no interest in financial trading and zero trading is thus an equilibrium; it remains to prove that it is the unique equilibrium, something that results from the strict convexity of the risk function e_θ.

Proposition 7.7 *For almost all parameter data, problem AP^f has then a unique solution where $x_{\nu,c} = x_{d,c} = 0,\ \forall c = 1, \ldots, C, \nu = 1, \ldots, N$. For the variables of the problem related to the electricity sector, i.e.* $(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu})$*, this unique solution is the one of problem AP.*

Proof Equations (7.16)–(7.20) are exactly the KKT conditions of problem AP given in (7.4)–(7.8). We then already know that they have a unique solution.

We now fix the contract prices $\mathbf{p_c^s}$ and demonstrate that the problem constituted by Eqs. (7.13)–(7.15) is integrable. The reason behind this assertion is that all players use the same risk measure e_θ to hedge their stochastic payoffs and therefore, their behavior can be modeled by a unique risk-averse agent optimizing the system and having the same risk aversion as we show.

Consider the following optimization program (recall that $\mathbf{p_c^s}$ are given). Dual variables are written between parenthesis next to their corresponding constraints:

$$\begin{aligned}
\mathrm{Min}_{\mathbf{x}\in\mathbb{R}^{(N+1)C}} \quad & R\left(x_{\nu,c}, x_{d,c}\right) = \sum_{\nu=1}^{N} e_\theta\left(Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{\nu,c} \cdot p_c^s(\omega)\right) \\
& + e_\theta\left(Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega)\right) \\
\text{s.t.} \quad & \sum_{\nu=1}^{N} x_{\nu,c} + x_{d,c} = 0 \qquad \left(p_c^f\right).
\end{aligned} \tag{7.22}$$

The optimization program (7.22) is convex because $e_\theta(.)$ is strictly convex. Its KKT conditons are hence necessary and sufficient to ensure optimality. These KKT conditions are exactly (7.13)–(7.15). In other words, for a given matrix of contract prices $\mathbf{p_c^s}$, the problem constituted by (7.13)–(7.15) is integrable and is equivalent to optimization program (7.22).

Optimization program (7.22) is strictly convex because $e_\theta(.)$ is strictly convex and the contracts are never redundant (using assumption H3, we know that the contract prices matrix (p_c^s) is of full rank). Therefore it has at most one solution. Besides, we know that:

$$\forall N \in \mathbb{N}^*,\ \forall x_1, x_2, \ldots, x_n, x_{N+1} \in \mathbb{R}^\Omega,\ e_\theta\left(\frac{\sum_{i=1}^{N+1} x_i}{N+1}\right) \leq \frac{1}{N+1}\sum_{i=1}^{N+1} e_\theta(x_i). \tag{7.23}$$

Hence, by replacing $x_i(\omega)$ by $Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{i,c} \cdot p_c^s(\omega),\ i = 1, \ldots, N$ and $x_{N+1}(\omega)$ by $Z_{\text{exo}}(\omega) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega)$ in Eq. (7.23), one obtains:

$$
\begin{aligned}
&\forall (x_{\nu,c}, x_{d,c}) \in \mathbb{R}^{NC} \times \mathbb{R}^{C} \text{ such that } \sum_{\nu=1}^{N} x_{\nu,c} + x_{d,c} = 0, \\
&R\left(x_{\nu,c}, x_{d,c}\right) \geq (N+1)e_\theta \left(Z_{\text{exo}}(\omega) + \frac{1}{N+1} \sum_{c=1}^{C} \left(\sum_{\nu=1}^{N} x_{\nu,c} + x_{d,c} \right) \cdot p_c^s(\omega) \right) \\
&\qquad\qquad = R(0,0).
\end{aligned}
\tag{7.24}
$$

Therefore, $x_{\nu,c} = x_{d,c} = 0$ is the minimum of function R. Program (7.22) has a unique solution and so does the financial equilibrium (7.13)–(7.15). To summarize, for given contract prices $\mathbf{p}_\mathbf{c}^\mathbf{s}$, the equilibrium problem (7.13)–(7.20) has a unique solution: its physical part is the solution to problem AP and its financial one corresponds to zero contractual positions. Finally, it suffices to select the prices from the solution to AP, $\mathbf{p}_{\mathbf{el}}$, and impose that contract prices $\mathbf{p}_\mathbf{c}^\mathbf{s}$ satisfy equation (7.21) to complete the proof. □

Remark 7.8 (Interpretation of parameter $Z_{\text{exo}}(\omega)$) The exogenous stochastic payoff $Z_{\text{exo}}(\omega)$ introduced in the auxiliary problem AP^f does not have particular economic interpretation in terms of P^f. The choice of $Z_{\text{exo}}(\omega)$ is arbitrary. This payoff only serves to define the auxiliary problem AP^f intervening in the definition of the homotopy function leading to the calculation of the degree of problem P^f. One could also have considered a zero exogenous payoff $Z_{\text{exo}}(\omega) = 0,\ \omega \in \Omega$, without altering the proof and conclusions of proposition 7.7. One can indeed easily see that the choice of $Z_{\text{exo}}(\omega)$ does not modify the continuity and compactness arguments used in the homotopy presented in Sect. 9. One can also show that in that case, at the equilibrium solution of problem AP^f, the contract positions are all zero and the subjective probabilities of the agents are equal to the natural probability $P(\omega)$:

$$
\bar{Q}_{e_\theta}\left(Z_{\text{exo}} + \sum_{c=1}^{C} x_{d,c} p_c^s \right) = P(\omega),\ \forall \omega \in \Omega.
$$

There is thus a single solution to problem AP^f whatever the choice of $Z_{\text{exo}}(\omega)$.

8 Solution existence and multiplicity for the project finance problem *P*

8.1 The existence of a solution to *P*

As explained in Sect. 7, solving the project finance problem P is equivalent to finding the zeros of a function Φ^P, defined on the set $\mathbb{R}^{M_P}$. Function Φ^P is then defined by:

$$
\left(\mathbf{u}, \mathbf{p}_{\mathbf{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\right) \in \mathbb{R}^{M_P} \text{ is a solution to problem } P \iff \Phi^P\left(\mathbf{u}, \mathbf{p}_{\mathbf{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\right) = 0.
\tag{8.1}
$$

To simplify notation, we denote by X the vector $\left(\mathbf{u}, \mathbf{p}_{\mathbf{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\right) \in \mathbb{R}^{M_P}$.

We now show how function Φ^P is constructed from the complementarity problem P given in proposition 5.2. First, we define a new mapping $F^P : \mathbb{R}^{M_P} \longrightarrow \mathbb{R}^{M_P}$ as follows:

$\forall \big(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\big) \in \mathbb{R}^{M_P}$,

$$F^P\big(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\big) = \begin{pmatrix} I_{\nu,k} - \mathbb{E}_{\bar{Q}_\nu(Z_{\nu,k})}\Big[\mu_{\nu,k}(\omega)\Big] & \nu = 1, \dots, N,\ k = 1, \dots, K \\ u_{\nu,k} - y_{\nu,k}(\omega) & \nu = 1, \dots, N,\ k = 1, \dots, K,\ \omega \in \Omega \\ C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) & \nu = 1, \dots, N,\ k = 1, \dots, K,\ \omega \in \Omega \\ PC - p_{el}(\omega) & \omega \in \Omega \\ -\text{LOAD}(\omega) + \sum_{\nu=1}^{N} \sum_{k=1}^{K} y_{\nu,k}(\omega) + s(\omega) & \omega \in \Omega \end{pmatrix}. \tag{8.2}$$

Then, for a fixed integer n, we generalize the definition of the minimum function defined on $\mathbb{R}$ to $\mathbb{R}^n$ as follows: given two vectors ζ and $\xi \in \mathbb{R}^n$, we denote by $\min(\zeta, \xi)$ the vector of $\mathbb{R}^n$ defined by:

$$\forall i = 1, \dots, n,\ [\min(\zeta, \xi)]_i = \min(\zeta_i, \xi_i).$$

Finally, recalling that the complementarity form $0 \le x \perp y \ge 0$ can be rewritten as an element-wise minimization $\min(x, y) = 0$, one can define the function Φ^P as follows:

$$\forall X \in \mathbb{R}^{M_P},\ \Phi^P(X) = \min(X, F^P(X)). \tag{8.3}$$

Similarly, functions F^{AP} and Φ^{AP} defined on $\mathbb{R}^{M_P}$ are constructed from the complementarity problem AP defined in 7.4 such that:

$$\forall X \in \mathbb{R}^{M_P},\ \Phi^{AP}(X) = \min(X, F^{AP}(X)) \tag{8.4}$$

and

$$\big(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\big) \in \mathbb{R}^{M_P} \text{ is a solution to problem } AP \iff \Phi^{AP}\big(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\big) = 0. \tag{8.5}$$

The functional form of the complementarity problems P and AP makes the functions F^P, Φ^P, F^{AP} and Φ^{AP} continuous.

We construct the homotopy function $H^P(X, \lambda)$ obtained from problems P and AP by modifying the element-wise minimization as follows:

$$\forall X \in \mathbb{R}^{M_P},\ \forall \lambda \in [0, 1],\ H^P(X, \lambda) = \min\big(X, \lambda F_{AP}(X) + (1 - \lambda) F_P(X)\big). \tag{8.6}$$

For a fixed $\lambda \in [0, 1]$, solving $H^p(., \lambda) = 0$ can also be viewed as solving a complementarity problem obtained by:

– replacing (5.2) in problem P by:

$$0 \leq I_{\nu,k} - \lambda \mathbb{E}_P\big[\mu_{\nu,k}(\omega)\big] \\ - (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_{\nu,k})}\big[\mu_{\nu,k}(\omega)\big] \quad \perp u_{\nu,k} \geq 0 \quad \forall \nu = 1,\ldots,N, \forall k = 1,\ldots,K \tag{8.7}$$

– and keeping the other complementarity conditions of problem P unchanged.

It should be clear that $H^P(\cdot, 1) = 0$ is identical to problem AP and $H^P(\cdot, 0) = 0$ to problem P. To apply the degree theory to our case, we first need to check that H^P is continuous.

Lemma 8.1 *Function H^P viewed as function of variables $(X, \lambda) \in \mathbb{R}^{M_P} \times [0, 1]$ is continuous on $\mathbb{R}^{M_P} \times [0, 1]$.*

Proof The use of continuously differentiable convex risk measures (assumption H1) implies that all the expressions appearing in the complementarity conditions are continuous, which makes functions F^P and F^{AP} continuous. Therefore, function $(X, \lambda) \longrightarrow (\lambda F_{AP}(X) + (1-\lambda)F_P(X))$ is also continuous. As a consequence, function $(X, \lambda) \longrightarrow \min\big(X, \lambda F_{AP}(X) + (1-\lambda)F_P(X)\big)$ is continuous.

To be able to use Theorem 7.2, one needs to identify a bounded open set Ξ^P such that there is no solution to $H^P(\cdot, \lambda) = 0$ on $\mathrm{bd}\Xi^P$. This is done in the following lemma.

Lemma 8.2 *Consider the solutions to $H^P(\cdot, \lambda) = 0, \quad \forall \lambda \in [0, 1]$. There exists a bounded open set Ξ^P independent of λ such that any solution to $H^P(\cdot, \lambda) = 0$ for some λ is in the interior of the closure of that set.*

$$0 \notin H^P(\mathbf{bd}\, \Xi^P, \lambda), \quad \forall \lambda \in [0, 1].$$

Proof It should be obvious that

– Relations (5.1) and (5.5) imply $p_{el}(\omega) \in [0, PC]$.
– Relation (5.4) implies $\mu_{\nu,k}(\omega) \in [0, PC]$ if $y_{\nu,k}(\omega)$ is positive.
– We now want to prove that

$$y_{\nu,k}(\omega) \in [0, \max_{\omega' \in \Omega} \mathrm{LOAD}(\omega')], \quad \text{for all } \omega \in \Omega, \ \nu = 1,\ldots,N, k = 1,\ldots,K.$$

Suppose that $y_{\nu',k'}(\omega') > \max_{\omega \in \Omega} \mathrm{LOAD}(\omega)$ for a producer ν', a technology k' in scenario ω'. Then, by (5.5), $p_{el}(\omega') = 0$ which implies by (5.4) that $y_{\nu,k}(\omega') = 0$ for all ν, k, which is a contradiction.

– We also need

$$s(\omega) \in [0, \max_{\omega' \in \Omega} \mathrm{LOAD}(\omega')], \quad \text{for all } \omega \in \Omega .$$

Suppose it is not true and $s(\omega') > \max_{\omega \in \Omega} \mathrm{LOAD}(\omega)$. Then by (5.5), $p_{el}(\omega') = 0$ and by (5.1) $s(\omega') = 0$ which is a contradiction.

– It remains to show that

$$u_{\nu,k} \in [0, \max_{\omega\in\Omega} \text{LOAD}(\omega)] \qquad \text{for all } \nu = 1, \dots, K \ , k = 1, \dots, K \ .$$

Suppose $u_{\nu',k'} > \max_{\omega\in\Omega} \text{LOAD}(\omega)$ for some ν', k'. One also has $u_{\nu',k'} > y_{\nu',k'}(\omega)$ for all $\omega \in \Omega$ and hence $\mu_{\nu',k'}(\omega) = 0$ for all $\omega \in \Omega$. This implies that the left part of (8.7) holds as a strict inequality and hence $u_{\nu',k'} = 0$, which is again a contradiction.

We can now define the bounded open set Ξ^P that does not contain, for every $\lambda \in [0, 1]$, a solution to $H^P(\cdot, \lambda) = 0$ on its closure. The scalar Δ is a positive number.

$$\Xi^P := \left\{ (\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}) \in \mathbb{R}^{M_P} \,\middle|\, \begin{array}{l} u_{\nu,k} \in \ ]-\Delta, \max_{\omega'\in\Omega} \text{LOAD}(\omega') + \Delta[\\ p_{el}(\omega) \in \ ]-\Delta, PC + \Delta[\\ s(\omega) \in \ ]-\Delta, \max_{\omega'\in\Omega} \text{LOAD}(\omega') + \Delta[\\ y_{\nu,k}(\omega) \in \ ]-\Delta, \max_{\omega'\in\Omega} \text{LOAD}(\omega') + \Delta[\\ \mu_{\nu,k}(\omega) \in \ ]-\Delta, PC + \Delta[\end{array} \right\} . \tag{8.8}$$

□

We can then state the following theorem.

Theorem 8.3 *The degree of* $(H^P(\cdot, 0), \Xi^p, 0)$ *is one. Therefore, the project finance problem* P *has at least one solution.*

Proof H^P is continuous in $\Xi^P \times [0, 1]$ and we just showed that $H^P(\cdot, \lambda) = 0$ has no solution on the boundary of Ξ^P. Using the definition of the degree (the homotopy invariance axiom A3) for H^P, we have that $\deg(H^P(\cdot, 0), \Xi^p, 0)$ is equal to $\deg(H^P(\cdot, 1), \Xi^p, 0)$, which is 1 because problem $H^p(\cdot, 1) = 0$ corresponds to problem AP that has a unique solution by Proposition 7.5. By Theorem 7.2, there exists a solution to the project finance equilibrium problem P. □

This shows that the problem P has at least one solution and that its degree is equal to one.

Remark 8.4 (Existence using fixed-point theorem) This existence result can be obtained by standard fixed-point arguments, given our analysis of the boundedness of Ξ^P and our assumptions on continuity. In fact our assumptions are identical to those of the fixed-point approach. As discussed in the introduction, our choice of the degree theory stems from the need to analyze the possibility of multiple equilibria: the current environment and the market distortions implied by various policies make the market different from the perfect competition paradigm (that can guarantee existence and uniqueness of equilibrium). Being able to go beyond pure existence appears as a requirement in the current context and degree theory offers an instrument to do so.

Remark 8.5 (Extension to multistage problems) The use of time-consistent risk measures makes it possible to extend the existence result to multistage incomplete markets. We base our claim on early considerations of time-consistent measures made by

Artzner et al. (2002) [8] where it is explained in Sects. 4 and 5 that a multistage risk function constructed with time-consistent risk measures has equivalent backward and forward formulations. The forward formulation implies that the problem can be stated as a single-stage model (very much like a standard multistage stochastic problem can be stated as a single-stage optimization problem) to which the two-stage developments presented in this paper apply. In other words, the time-consistent risk function makes it possible to bypass the backward recursion and to treat the problem as two-stage. In that case anything that relies on properties of continuity would remain true, and hence the proof of existence.

8.2 The multiplicity of solutions to *P*

8.2.1 The general case: isolated solutions and the possibility of multiple solutions (in odd numbers)

We first analyze the structure of a given equilibrium solution. We call $\overline{KN}$ the set of plants with positive capacity[8] in this solution and we partition the set of scenarios in two subsets. The marginal[9] unit operates at partial capacity or there is demand curtailment in the scenarios of the first subset: these are called *non-characteristic scenarios*. All active units are effectively generating at full capacity in a scenario of the second subset and the sum of their production is exactly equal to the load; they are defined as *characteristic scenarios*. This is restated below.

Definition 8.6 In an equilibrium solution to problem P, a characteristic scenario $\omega' \in \Omega$ satisfies the property that all producing (active) plants are effectively generating at full capacity and that the sum of their production is exactly equal to LOAD(ω'). The set of characteristic scenarios is denoted by $\Omega_c \subset \Omega$ and the set of active plants in a characteristic scenario ω_c by $\overline{KN}_c(\omega_c)$.

The rationale behind this partitioning is twofold: the electricity price in a characteristic scenario can vary in an interval (the vertical segment between two operating costs in Figure 1) and this degree of freedom is central to satisfying the investment constraint (5.2) with equality. On the contrary, given the equilibrium capacities $\mathbf{u}$, the prices in the non-characteristic scenarios are entirely fixed by the merit order.[10] The following lemma relates the number of characteristic scenarios to the number of built plants.

Lemma 8.7 *In an equilibrium solution to problem P, the number of characteristic scenarios is lower than or equal to the number of built plants:* $|\Omega_c| \leq |\overline{KN}|$, *for almost all data of the problem.*

[8] i.e. at the equilibrium solution $\overline{KN} := \{(\nu, k) | \, u_{\nu,k} > 0, \nu = 1, 2, \ldots, N, \; k = 1, 2, \ldots, K\}$

[9] The marginal unit is the operating plant with the highest variable cost.

[10] They can be obtained by solving the optimal dispatch problem presented in (5.6) for the non-characteristic scenario ω, which is a linear program. By definition of a non-characteristic scenario, one has one of the two situations: there exists a plant (i.e. the marginal unit) not producing at full capacity or load curtailment is activated. This implies that the optimization problem (5.6) is non-degenerate (unique primal-dual solutions). The electricity price is then equal to the marginal cost of a plant or to the value of load curtailment PC.

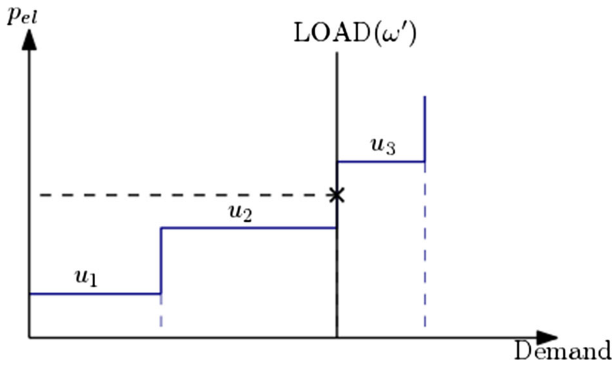

Fig. 1 Illustration of a characteristic scenario ω'

Proof By definition, for every characteristic scenario $\omega_c \in \Omega_c$, all active plants are effectively producing at capacity. This implies that the market clearing condition in this characteristic scenario ω_c becomes

$$\sum_{(\nu,k)\in\overline{KN}_c(\omega_c)} u_{\nu,k} = \text{LOAD}(\omega_c)\,. \tag{8.9}$$

A solution with more characteristic scenarios than the number of built plants $|\overline{KN}|$, implies a solution to an over-determined linear system that cannot hold for almost all LOAD parameter data. □

We now demonstrate that any solution to problem P is isolated, possibly by perturbing the problem in an infinitesimal way. More precisely, we show that the subset of the problem parameter values where the model would have non-isolated equilibrium is of measure zero. This implies that an infinitesimal perturbation of the data will always make the equilibrium solutions isolated.

Proposition 8.8 *For almost all parameter data, any solution to problem P is isolated*

Proof Consider a solution $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ to problem P, with a set Ω_c of characteristic scenarios. Given invested capacities $\mathbf{u}$, the operation levels $\mathbf{y}$ and curtailments $\mathbf{s}$ are uniquely determined for each scenario $\omega \in \Omega$ by solving the optimization problem (5.6) (the merit-order is uniquely determined by variable operating costs in each scenario, assuming that plant costs $C_{\nu,k}(\omega)$ are different, possibly after an infinitesimal perturbation). Prices in scenarios $\omega \notin \Omega_c$ that are not characteristic are also uniquely determined. It is then sufficient to prove that investments $\mathbf{u}$ and prices in the characteristic scenarios $p_{el}(\omega_c)$, $c \in \Omega_c$ are isolated. We achieve this result by explicitly analyzing the equations defining variables $\mathbf{u}$ and $p_{el}(\omega_c)$ and proving that it is non-singular (for almost all parameter data). The formal proof is given in Sect. B.1 of the appendix. □

The following theorem summarizes all previous results regarding problem P:

Theorem 8.9 *For almost all parameter data, problem P has a finite and odd number of solutions.*

Proof We have seen that for almost all parameter data of problem P, the Jacobian of Φ^p is nonsingular at all solutions of problem P (see the proof of proposition 8.8). Besides, we know that the degree of problem P is 1 by theorem 8.3. Proposition 7.3 can then be invoked to conclude. □

8.2.2 A uniqueness result

The following propositions specialize the results to the case of coherent risk measures, when the merit order (the ordering of technologies with respect to their operating cost) remains unchanged across scenarios. This characterization is instrumental in proving our only uniqueness result (and there is indeed no monotonicity argument that would suggest a general uniqueness result). It is also noteworthy that the constant merit order has largely been the rule in the history of the sector but is seriously questioned these days for various fundamental reasons.

Lemma 8.10 *Suppose that producers value their investment by a coherent risk measure (ensuring positive subjective probabilities[11]) and that the merit order is unchanged in all scenarios, then all solutions are isolated and, for every solution to the problem $\Phi^P(.) = 0$, the determinant of the Jacobian is positive:*

$$\forall X \in \left(\Phi^P\right)^{-1}(0),\ det(J\Phi^p(X)) > 0. \tag{8.10}$$

Proof The proof can be found in Sect. B.2 of the appendix. □

Theorem 8.11 *Suppose that producers value their investment by a coherent risk measure (ensuring positive subjective probabilities) and that the merit order is unchanged in all scenarios, then the project finance equilibrium P has only one solution.*

Proof The degree of P is equal to 1 which is also the sum of the signs of the determinants of the Jacobian matrix calculated at all its solutions (proposition 7.3) that are isolated. Because these signs are the same for all solutions, there can only be a single solution. □

9 Existence and multiplicity of solutions for problem P^f

We now turn to the spot-financial equilibrium problem where generators invest in portfolios of plants and can hedge them by taking positions in financial contracts. The discussion is parallel to the one of the project finance case, but with more technicalities. It is presented here in summary version.

[11] As already mentioned in remark 3.4, any coherent risk measure constructed as a convex combination (with positive coefficient) of the expectation under the natural probability measure P and another coherent risk measure would satisfy this requirement.

9.1 The existence of a solution to P^f

We first recall that solving problem P^f (see Sect. 7) is equivalent to finding the zeros of a function Φ^{Pf}, defined (*a priori*) on the set $\mathbb{R}^{M_{Pf}}$. This function Φ^{Pf} is defined as follows:

$$\begin{aligned}\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}} &\text{ is a solution to problem } P^f \\ &\Longleftrightarrow \\ \Phi^{Pf}(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s) &= 0.\end{aligned} \tag{9.1}$$

Function Φ^{Pf} is constructed from the mixed complementarity problem P^f given in proposition 5.5 in a way similar to what has been done for problem P in Sect. 8. The detail of this derivation is given in Sect. B.3.1 of the appendix.

Similarly, we construct the function Φ^{APf} on $\mathbb{R}^{M_{Pf}}$ from the KKT conditions of problem AP^f given in definition 7.6 such that:

$$\begin{aligned}\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}} &\text{ is a solution to problem } AP^f \\ &\Longleftrightarrow \\ \Phi^{APf}(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s) &= 0.\end{aligned} \tag{9.2}$$

To simplify notation, we denote the vector $\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right)$ as Y.

As in Sect. 8, one constructs the homotopy function $H^{Pf}(Y, \lambda)$ that continuously links problems P^f and AP^f. Its expression is given in Sect. B.3.2 of the appendix. The homotopy H^{Pf} satisfies the property that for a fixed $\lambda \in [0, 1]$, solving $H^{Pf}(., \lambda) = 0$ can also be viewed as solving an equilibrium problem obtained by three replacements in problem P^f:

- One first modifies the producers' investment rules by replacing (5.11) by

$$\begin{aligned}0 \le I_{\nu,k} &- \lambda \cdot \mathbb{E}_P\left[\mu_{\nu,k}(\omega)\right] \\ &- (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[\mu_{\nu,k}(\omega)\right] \quad \perp \quad u_{\nu,k} \ge 0 \quad \forall \nu = 1, \dots, N, \forall k = 1, \dots, K.\end{aligned} \tag{9.3}$$

- One similarly modifies the financial decisions of the consumer by replacing (5.8) by

$$\begin{aligned}0 = \lambda \cdot &\mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}} + \sum_{c=1}^C x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] \\ &+ (1-\lambda)\mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f \qquad \forall\, c = 1, \dots, C.\end{aligned} \tag{9.4}$$

- Last, one applies the same transformation to the financial decisions of the producers by replacing (5.10) by

$$\begin{aligned}0 = \lambda \cdot &\mathbb{E}_{\bar{Q}_{e_\theta}(Z_{\text{exo}} + \sum_{c=1}^C x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] \\ &+ (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[p_c^s(\omega)\right] - p_c^f \qquad \forall\, \nu = 1, \dots, N,\; c = 1, \dots, C.\end{aligned} \tag{9.5}$$

– The other equations of problem P^f are unchanged.

Obviously, solving problem AP^f is equivalent to solving problem $H^{p^f}(., 1) = 0$ and solving problem P^f is equivalent to solving problem $H^{p^f}(., 0) = 0$.

Lemma 9.1 *The function H^{P^f} viewed as function of variables (Y, λ) belonging to the set $\mathbb{R}^{M_{P^f}} \times [0, 1]$ is continuous.*

Proof The use of continuously differentiable risk measures (assumption H1) implies that all the expressions in the mixed complementarity problems P^f and AP^f are continuous, which makes homotopy function H^{P^f} also continuous. □

Lemma 9.2 *Suppose a solution to $H^{P^f}(\cdot, \lambda) = 0$ exists, then this solution is in the interior of the closure of a bounded open set Ξ^{P^f} that does not depend on λ.*

$$0 \notin H^{P^f}(\mathbf{bd}\, \Xi^{P^f}, \lambda), \quad \forall \lambda \in [0, 1].$$

Proof The reasoning used in lemma 8.2 for variables $\mathbf{u}$, $\mathbf{y}$, $\mathbf{p_{el}}$, $\boldsymbol{\mu}$ and $\mathbf{s}$ applies unchanged here. It remains to show that the financial positions $\mathbf{x}$ and prices $\mathbf{p^f}$ also belong to the closure of a bounded open set that is independent of λ. This proof is similar to the one in de Maere d'Aertrycke and Smeers [23] and can be found in Sect. B.3.3 of the appendix. □

Theorem 9.3 *The degree of $(H^{P^f}(., 0), \Xi^{P^f}, 0)$ is equal to one, and there exists at least a solution to the spot-financial equilibrium problem P^f.*

Proof This is a direct application of degree theory presented in 7. H^{P^f} is continuous in $\Xi^{P^f} \times [0, 1]$ and $H^{P^f}(., \lambda) = 0$ has no solution on the boundary of Ξ^{P^f}. It is also obvious that deg$(H^{P^f}(., 1), \Xi^{P^f}, 0)$=1, because the associated problem AP^f has a unique solution (cf. proposition 7.7). By theorem 7.2, there exists a solution to the spot-financial equilibrium problem P^f. □

As argued for problem P, it is also possible to extend the homotopy argument to spot-financial multistage incomplete markets to prove existence.

9.2 Equilibrium solutions to P^f are isolated and come in odd number

Because problem P^f is analytically more complicated and exhibits the same lack of monotonicity as model P, it seems reasonable to give up any attempt to provide uniqueness results for this problem. But it is still a valid objective to prove that its equilibrium solutions are isolated (for almost all parameter data) as this would conclude with a finite and odd number of solutions. It is relatively easy to extend the analysis presented in Sect. 8.2 for problem P to problem P^f. We briefly sketch the main steps of the proof here.

First of all, the notion of characteristic scenarios extends as such together with lemma 8.7 as moving to P^f does not change the clearing of the spot electricity market. Similarly, Sect. B.1.2 in the appendix applies as such. The rest consists in adapting the proof after the following remark.

Remark 9.4 (Financial positions as implicit functions of physical decisions) In order to make the analysis of problem P^f as close as possible to the one of problem P, we first note that agents take financial positions $\mathbf{x}$ to hedge physical decisions (investments, operational decisions and curtailments). Financial positions are thus individually optimized for each agent. The first order conditions determining those optimal contract positions are given in equations (5.8) and (5.10), that we recall here:

$$\begin{aligned} p_c^f &= \mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] \qquad \forall c = 1, \ldots, C \\ p_c^f &= \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[p_c^s(\omega)\right] \qquad \forall \nu = 1, \ldots, N, c = 1, \ldots, C\ , \end{aligned} \tag{9.6}$$

where the contracts payoffs depend on the electricity prices, i.e. $p_c^s(\omega) = h_{c,\omega}(p_{el}(\omega))$, and the second stage net profits are given by:

$$\begin{aligned} Z_d^f(\omega) =& (PC - p_{el}(\omega)) \cdot (\text{LOAD}(\omega) - s(\omega) + \sum_{c=1}^{C} x_{d,c} \cdot p_c^s(\omega) \\ Z_\nu^f(\omega) =& \sum_{k=1}^{K} u_{\nu,k} \cdot \mu_{\nu,k}(\omega) + \sum_{c=1}^{C} x_{\nu,c} \cdot p_c^s(\omega) \qquad \forall \nu = 1, \ldots, N. \end{aligned} \tag{9.7}$$

For the consumer, this implies that the vector of financial positions $\mathbf{x}_d$ varies as a function of the contract prices $\mathbf{p}^f$ and the electricity prices $\mathbf{p_{el}}$. Similarly, for a producer ν, $\mathbf{x}_\nu$ depends also on the contract prices $\mathbf{p}^f$ and the electricity prices $\mathbf{p_{el}}$, but also on the vector of capacities $\mathbf{u}_\nu$.

We assume here that risk measures are twice continuously differentiable and strictly convex. We know that for almost all operating costs $C_{\nu,k}(\omega)$, the financial payoffs $(p_s^c(\omega))_{\omega\in\Omega}$ and the capacity margins $(\mu_{\nu,k}(\omega))_{\omega\in\Omega}$ are linearly independent. Using assumption H3, one can invoke the implicit function theorem. Indeed, it can be shown that the functions intervening in relations (9.6) always have nonsingular Jacobians with respect to variables $\mathbf{x}_d$ and $\mathbf{x}_\nu$. Therefore, $\mathbf{x}_d$ and $\mathbf{x}_\nu$ can locally be written as continuously differentiable functions of $\mathbf{p}^f$ and $\mathbf{p_{el}}$ (for the consumer), and $\mathbf{p}^f$, $\mathbf{p_{el}}$ and $\mathbf{u}_\nu$ (for producer ν): $\mathbf{x}_d := x_d(\mathbf{p_c^f}, \mathbf{p_{el}})$ and $\mathbf{x}_\nu := x_\nu(\mathbf{p_c^f}, \mathbf{p_{el}}, \mathbf{u}_\nu)$.

One can then formally extend the analysis done for problem P by also analyzing, in a neighborhood of an equilibrium solution, the equations that define the equilibrium. We showed for problem P in Sect. B.1.3 that for almost all investment costs and load parameters, the Jacobian of these constitutive equations is nonsingular. For problem P^f, given the preliminary discussion above, we have to add to these constitutive equations, a new one: $f_{\texttt{contract}}(\mathbf{p_{el}^c}, \mathbf{p_c^f}, \mathbf{u}) : \mathbb{R}^{C|KN|\cdot|\Omega_c|} \to \mathbb{R}^C$,

$$f_{\texttt{contract}}(\mathbf{p_{el}^c}, \mathbf{p_c^f}, \mathbf{u}) := \left(\sum_{\nu=1}^{N} x_{\nu,c}(\mathbf{p_c^f}, \mathbf{p_{el}}, \mathbf{u}_\nu) + x_{d,c}(\mathbf{p_c^f}, \mathbf{p_{el}}, \mathbf{u}_\nu) - B_c \right)_{c=1,\ldots,C} \tag{9.8}$$

that gives, for each contract c, the sum of the contract positions minus an exogenous term B_c, that we add to our model. This parameter $B \in \mathbb{R}^C : (B_c)_{c=1}^C$ should in principle be set to zero in order to reflect that all contract positions balance to zero. However, it may as well be taken equal to some net position of speculators (either exogenous or function of the contracts or spot prices) without invalidating neither the realism nor the framework of partial equilibrium models. The proof can then proceed as for problem P by showing that the Jacobian of those new constitutive equations (relative to problem P^f) with respect to $LOAD$, I and B parameters has full rank. One can hence invoke the transversality theorem 2 (see Appendix B.1) to prove that all equilibria are isolated, for almost all parameter data.

The following theorem summarizes all previous results regarding problem P^f:

Theorem 9.5 *Assuming twice continuously differentiable and strictly convex risk measures, Problem P^f has a finite and odd number of solutions, for almost all parameter data.*

Proof We have just shown that for almost all parameter data, the solutions of problem P^f are isolated. As the degree of problem P^f is 1 by theorem 9.3, we can directly use proposition 7.3 to conclude. □

10 A numerical example

This section illustrates that one cannot expect a general result on uniqueness and that a finite and odd number of isolated equilibria is the best general result one can achieve. We develop a numerical illustration on the project finance problem P and highlight that the market incompleteness may lead to a multiplicity of equilibrium when the merit order of technologies varies across scenarios (a now very well admitted possibility in the current context of uncertainty on the energy transition). Note that neither a capacity optimization nor a partial equilibrium model in a complete market can uncover that possibility.

We construct an example with two producers, each having the possibility to invest in one technology. For simplicity, we aggregate indexes ν and k into one index $\nu \in \{\nu_1, \nu_2\}$. We take $\Omega := \{\omega_1, \omega_2, \omega_3\}$ and make agents value the risk of their payoff by the entropic risk measure.[12] given in definition 3.5 Table 1 reports the data used for the example. Figures are normalized and presented in no particular units.

The merit order of plants changes across scenarios: in scenario ω_2, producer ν_2 has the cheapest plant whereas in ω_1 and ω_3, producer ν_1 has the least expensive technology. Because $\theta_1 > \theta_2$, producer ν_1 is more risk-averse than producer ν_2.

To find the solutions to problem P, we enumerate all possible combinations of characteristic scenarios and positive investments. More details about this procedure are provided in Appendix C. We find three isolated solutions to problem P. We report the equilibrium solutions in Table 2, as well as the corresponding sign of the determinant of the Jacobian matrix of Φ^P and the list of characteristic scenarios of each solution.

[12] This choice of the entropic measure is motivated by the simplicity of its numerical implementation (it has a simple closed form) and the fact that it is commonly used in finance.

Table 1 Parameter data used for the example

$PC = 40$

	ω_1	ω_2	ω_3		ν_1	ν_2
Load	10	20	50	I	0.8	3.2
C_{ν_1}	0.01	8.0	0.50	θ	0.005	0.0015
C_{ν_2}	7.0	0.04	9.0			
P	0.3	0.3	0.4			

Table 2 Solutions to problem P

	u_{ν_1}	u_{ν_2}	$p_{el}(\omega_1)$	$p_{el}(\omega_2)$	$p_{el}(\omega_3)$	Sign$(det J\Phi^P)$	Characteristic scenarios
Solution 1	50.00	0	0.01	8	5.66	1	ω_3
Solution 2	29.42	20.58	0.01	0.04	18.67	1	ω_3
Solution 3	30.00	20.00	0.01	0.42	18.08	-1	$\{\omega_2, \omega_3\}$

Table 3 The subjective probabilities of producer ν_1 and ν_2

$\bar{Q}_{\nu_1}$	ω_1	ω_2	ω_3	$\bar{Q}_{\nu_2}$	ω_1	ω_2	ω_3
Solution 1	0.42	0.42	0.16	Solution 1	0.30	0.30	0.40
Solution 2	0.45	0.45	0.05	Solution 2	0.34	0.34	0.32
Solution 3	0.48	0.48	0.04	Solution 3	0.33	0.32	0.35

These results illustrate the theoretical development on the degree of problem P, i.e. the number of solutions is odd and the equilibrium solutions are isolated. Also one can check that the sum of the signs of the determinants of the Jacobian of Φ^p in all solutions is equal to 1, i.e. the degree of problem P (cf. theorem 8.3).

Solutions 2 and 3 are quite similar in terms of investment and prices. However, they fundamentally differ by the fact that solution 3 has one more characteristic scenario ω_2. We report in Table 3 the subjective probabilities of the producers in each solution:

Finally, we analyze which of the three solutions can be characterized as best. To do so, we focus on the risk measure of the consumer's surplus (recall that the risk measure of a producer's profit is always zero at an equilibrium) and compute it for each solution (the consumer's surplus at scenario ω is given in equation (4.2)). The consumer is also risk-averse and also values its risk using the entropic risk measure with parameter θ_d. However, since the consumer does not take any first-stage decision, its risk aversion does not change the formulation of problem P. In other words, the level of the consumer's risk aversion does not influence the investment decisions of problem P. Therefore, it is worth analyzing which solution is best for the consumer depending on its risk-aversion. Figure 2 gives the evolution of the risk measure of the consumer's surplus with respect to θ_d for the three solutions.

Up to a certain limit of risk aversion $\theta_d^0 = 0.0026$, solution 1 is the best one. Between θ_d^0 and $\theta_d^1 = 0.0056$, solution 3 becomes the best for the consumer, even though the difference with solution 2 in terms of risk for the consumer is tiny. This is

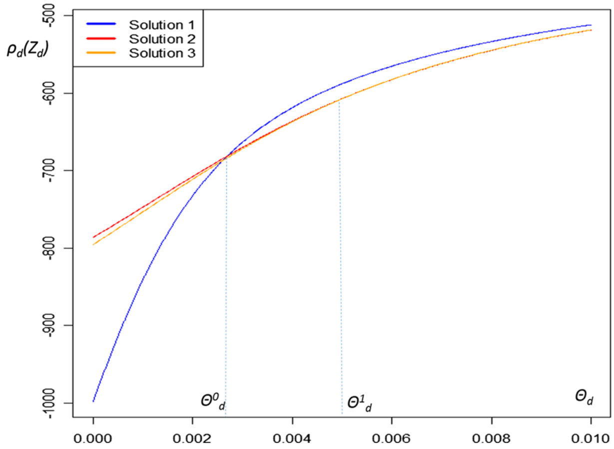

Fig. 2 Evolution of the risk of the consumer with respect to θ_d

explained by the fact that solutions 2 and 3 are very close to each other. Finally, if the consumer is very risk-averse, $\theta_d > \theta_d^1$, solution 2 is the best one.

11 Conclusion

Optimization has ruled the electricity industry for more than 50 years both for short-term (unit commitment and optimal economic dispatch) and long-term (investment) issues. Market designs and the restructuring process confirmed the role of optimization models for short-term operations organized around an ISO or a Power Exchange. The case of long-term capacity optimization models (simulating a perfect competition paradigm) is less evident, at least in the European context that is recognized to be in chaos. It thus makes sense to investigate models that adapt the early tools to the current realities of competition in the restructured environment.

Market imperfections can explain inefficiencies in the investment market. They can roughly be classified in three categories: agents may exercise market power; particular policy objectives may lead to prices that significantly depart from marginal cost (this includes missing markets where some services are simply not priced); last the market has become very risky and did not create the tools to adequately manage that risk. The paper only deals with the latter problem but we believe that it could be extended to distorted price signals.[13] These problems are methodologically interesting: they involve many long-term uncertainties that are today almost impossible to hedge and are further complicated by distorted price signals. They are also relevant: the "energy transition" is demanding in terms of investment and we know very little on the impact of the combination of different signals sent by different policies.

We propose two models that both represent a competitive market where agents are price-takers but where risk is important and imperfectly traded. The models are based

[13] Our analysis is indeed based on an homotopy that progressively transforms a perfectly competitive market into a situation where risk is imperfectly traded. Similarly one could also construct an homotopy that progressively transforms marginal cost pricing to a distorted price signals.

on standard capacity expansion tools but the investment process is adapted to reflect a situation where individual agents invest in a risky environment. Our first model, referred to as project finance, probably best fits the current market situation where long-term hedging possibilities essentially do not exist. The second model implements the emerging idea that long-term contracts are necessary to remedy[14] the current lack of spontaneous investment.

Conceptually, these models are related to two different areas of knowledge. As already mentioned, they are extensions of the traditional capacity expansion models. They can therefore benefit from future research regarding the representation of the technologies (e.g. variable renewable technologies or storage) or the development of efficient algorithms. Alternatively, as partial equilibrium models, they represent a simplification of general equilibrium models, and are also susceptible of benefiting from ideas developed in that area. This paper relies on both sources: on one hand, degree theory has become standard for exploring existence and multiplicity in general equilibrium (notably in incomplete markets), on the other hand, it implements these ideas through complementarity formulations, which are now the *lingua franca* in energy modeling. We show existence of a solution for both models and that all equilibrium solutions are isolated except for a set of measure zero of the parameters (which formalizes the usual notion of non-degeneracy). We also have that the number of solutions is odd.

A key issue related to this property is whether one can really find a finite set of different equilibria. This question is methodologically interesting and intriguing in practice. The multiplicity of equilibria signals that the economy may end up in bad situations. It also raises some unformalized questions about the validity of the standard approach to corporate investment without the idea of a single equilibrium; last but not least it may explain why several utilities decided to freeze their investment outside of subsidized niches. These questions are left for further analysis. We only provide an illustration of this phenomenon.

While the driving ideas of the paper are taken from basic economics,[15] their implementation in complementarity problems may be unusual. As to the economic aspect of the work, extensive analysis has been conducted with simple versions of the two models with the view of exploring how the incentive to invest changes with risk aversion and hedging instruments [24]. The results are very much in line with economic expectations and they show the relevance of long-term contracts. As to computational aspects, another set of experiments [38] has been performed to examine to which extent techniques such as ADMM (to decentralize the problem by investors) could help scale the problems. These experiments are also promising.

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[14] The principle is debated among those that argue that these contracts are necessary to deal with risk and others arguing that they will foreclose the market and hinder competition.

[15] Most materials can be found in microeconomics graduate textbooks [18,52] for the degree theory and the transversality theorems.

A Discussion on capital budgeting

This work grew out from discussions with business developers of power plants in the European market. Their approach to investment valuation is standard: the cost of capital is computed as the WACC , with the cost of equities based on an unconditional CAPM and cost of debt derived from the rating of the company or the project. An "adder" to the WACC accounts for the "conditions of the market" (see [40] for a survey of this approach in practice and for an in depth analysis of the "adder"). Even though the approach is well established, its different steps have often been questioned in the literature. [5] gives a comprehensive general discussion that we briefly summarize together with a few recent references.

- First *the robustness of the CAPM* (and of so-called β models) which is the corner stone of the evaluation of the cost of capital has been a continual controversy since its inception: there exists an extensive discussion on the need to include other factors than "the market" (see [16] for a general discussion). [17] gives a more recent presentation and concludes with a "zoo" of risk factors that should usefully complete the classical model and an inherent difficulty of identifying the elements of that zoo.
- The need to move *from an unconditional to a conditional CAPM* (making the β conditional on information of the period) has also been extensively discussed; [9] offers a concise survey of the previous literature and an in depth empirical analysis of the conditional CAPM: they conclude that the conditional CAPM is able to reproduce individual stock price evolution but that additional risk factors (very much like Cochrane's zoo) would be needed to reproduce cross section variations of stocks' expected returns (and hence differentiated cost of capital in a transposition to physical assets).
- The CAPM was initially developed for financial assets and *its use to value physical assets* also raised questions. Bogue and Roll [11], Brennan [14] gave the first elements but Myers and Turnbull [54] quickly mentioned difficulties. Jagannathan and Meier [39] gives a global view of the problem and Jagannathan et al. [40] provides an in depth empirical analysis including a discussion of the adder. [22] explains why the very nature of projects (options embedded in projects) makes the CAPM of a project different from the CAPM of a firm.
- Last but not least, the use of the CAPM in the European "energy transition" policy, which requires a complete change of physical structure of the generation mix in the EU, raises *an issue of chicken and eggs*. [5] gives a detailed discussion of that problem: risk underpins the computation of the Cost of Capital; the Cost of Capital influences investments, which because they are meant to be massive in the "energy transition", influences the risk exposure of plants. There is thus a feedback effect that should be tackled by a fixed-point approach and is not reflected in the standard econometric calibration of the CAPM from past data.

Observation of the European power market confirms that the situation is not well understood in practice: while cost of capital and WACC from classical approaches are reported to be low (not least because of quantitative easing adopted by the European Central Bank), investments only take place in subsidized renewable power. This work is an attempt to propose an alternative approach to the question less dependent on the above problems. Its main methodological step is to cast the problem as a stochastic equilibrium problem (Rational Expectation Equilibrium in economic parlance). The model is constructed using the now well-developed theory of risk measures. This bypasses the difficulties of the choice of the discount factor discussed above: it sets the discount rate to the risk-free rate as all risk premium are embedded in the risk measures. It replaces the "zoo" of risk factors by the risk factors that we know effectively affect investment (as determined by the standard and numerous sensitivity analysis going on in the industry). It casts the problem of the required return on investment directly as a fixed point and hence makes risk premium endogenous. These gains are obtained at a cost. The numerical treatment of the stochastic equilibrium is one, but this should be seen in the context of on-going developments in stochastic optimization. The calibration of the risk measures is another difficulty for which we have little to say except a general remark: the gradient of risk measures (which drives our investment criteria) has a natural interpretation of stochastic discount factor that ties in with econometric estimation in classical approaches.

B Elements of proof

B.1 Proof of isolated solutions to equilibrium problem P

Proposition *For almost all parameter data, any solution to problem P is isolated.*

Proof The proof is structured in three parts: one first gives two fundamental theorems that are instrumental in the overall demonstration. The second part shows that all equilibrium in a neighborhood of any non-isolated solution to problem P would share the same structure in terms of built plants and characteristic scenarios. The third part concludes the proof.

B.1.1 Two transversality theorems

Transversality theorems have been extensively used in economics to demonstrate that a relation holds or does not hold almost always, in proofs of local equilibrium ([26], see also Magill and Quinzii [51] Chapter 6 for a textbook treatment). We first present a transversality theorem given in Colell et al. [18] that we state as follows: consider the problem $g(a; b) = 0$ where a is the vector of variables and b the vector of parameters. The theorem gives conditions under which the Jacobian matrix of g with respect to the variable a is of full rank, at almost every data point b.

Theorem B.1 *Transversality theorem 1. Let $A \subset \mathbb{R}^J$ and $B \subset \mathbb{R}^T$ be open sets and let $g : A \times B \longrightarrow \mathbb{R}^V$ be a smooth function. If:*

- *$\forall(a; b) \in A \times B$ satisfying $g(a; b) = 0$, $rank(Jg_{a;b}(a; b)) = V$ (where $Jg_{a;b}(a; b)$ is the Jacobian matrix of g with respect to the vector $(a; b)$, evaluated at $(a; b)$),*

then there exists $B' \subset B$ such as B/B' is a set of measure zero and:

$$\forall b \in B',\ rank(Jg_a(a;b)) = V,\ whenever\ g(a;b) = 0,$$

where $Jg_a(a;b)$ *is the Jacobian matrix of* g *with respect to the variable* a *evaluated at* $(a;b)$.

The interpretation of the Transversality theorem is quite simple: if at each data b and at each equilibrium a, solution to $g(.;b) = 0$, the Jacobian matrix of function g with respect to the whole vector $(a;b)$ (composed of the variable and the data of the problem) is of full rank, then for almost every point data point b, the Jacobian matrix of g with respect to a at any solution to $g(.;b) = 0$ is also of full rank.

The second transversality theorem (Magill and Quinzii [51] Chapter 6) used in our proofs can be stated as follows: suppose one solves problem $g(a;b) = 0$ where a is the vector of unknown variables, b is a vector of parameters and g induces more equations than variables. The theorem provides conditions under which $g(a;b) = 0$ holds almost nowhere in the space of the parameters.

Theorem B.2 *Transversality theorem 2. Let* $A \subset \mathbb{R}^J$ *and* $B \subset \mathbb{R}^T$ *be open sets and let* $g : A \times B \longrightarrow \mathbb{R}^V$ *be a smooth function. If:*

- $J < V$,
- $\forall (a;b) \in A \times B$ *satisfying* $g(a;b) = 0$, $rank(Jg_{a;b}(a;b)) = V$ *(where* $Jg_{a;b}(a;b)$ *is the Jacobian matrix of* g *with respect to* $(a;b)$ *evaluated at* $(a;b)$*),*

then there exists $B' \subset B$ *such as* B/B' *is a set of measure zero, such that:*

$$\forall b \in B',\ g(.;b) = 0 \textit{ does not have a solution.} \tag{B.1}$$

We now consider an equilibrium of problem P and assume that it not isolated. We show that this leads to a contradiction. We first demonstrate that the structure of the equilibrium is the same in an neighborhood of the equilibrium.

B.1.2 A common structure in a neighborhood of any non-isolated solution

Recall that at a solution $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ to problem P, $\overline{KN}$ denotes the set of plants that have positive capacity: $\overline{KN} = \{(\nu, k) | u_{\nu,k} > 0,\ \nu = 1, 2, \ldots, N,\ k = 1, 2, \ldots, K\}$. To ease the notation, we will also write that $\mathbf{u} \in \overline{KN}$ to denote that $\forall (\nu, k) \in \overline{KN},\ u_{\nu,k} > 0$ and $\forall (\nu, k) \notin \overline{KN},\ u_{\nu,k} = 0$ (this holds also for the sets $\overline{KN}_c(\omega_c)$).

Proposition B.3 *For almost all parameter data, if an equilibrium solution to problem* P *is not isolated, then there exists in a neighborhood other equilibrium solutions that share the same structure, i.e. share*

1) *The same set of positive capacity* $\overline{KN}$.
2) *The same electricity price in the non-characteristic scenarios* $\Omega \backslash \Omega_c$.
3) *The same set of characteristic scenarios* Ω_c *and the same set of plants active in the characteristic scenarios* $\overline{KN}_c$.

Those equilibrium solutions mainly differ by the level of capacity $\mathbf{u} \in \overline{KN}$ *and the price in the characteristic scenarios, denoted* $\mathbf{p_{el}^c} := (p_{el}(\omega_c))_{\omega_c \in \Omega_c}$.

Proof Consider an equilibrium solution $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ to problem P, characterized by $\overline{KN}, \Omega_c$ and $\overline{KN_c}$, and suppose that this solution is not isolated, i.e. $\forall\epsilon > 0, \exists(\mathbf{u}^\epsilon, \mathbf{y}^\epsilon, \mathbf{s}^\epsilon, \mathbf{p_{el}}^\epsilon)$ solution to problem P: $(\mathbf{u}^\epsilon, \mathbf{y}^\epsilon, \mathbf{s}^\epsilon, \mathbf{p_{el}}^\epsilon) \neq (\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ and $||(\mathbf{u}^\epsilon, \mathbf{y}^\epsilon, \mathbf{s}^\epsilon, \mathbf{p_{el}}^\epsilon) - (\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})|| < \epsilon$ (we use the Euclidian norm). We denote $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}}) \subset \mathbb{R}^{KN+(KN+2)|\Omega|}$ the open ball of radius ϵ and center $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$.

We first concentrate on claim number 1. We know that $\forall(\nu, k) \in \overline{KN}$, $u_{\nu,k} > 0$. Therefore, we have that $\exists\epsilon_0 > 0$ small enough such that $\forall\epsilon$ satisfying $0 < \epsilon \leq \epsilon_0$, $\forall(\nu, k) \in \overline{KN}$, $u^\epsilon_{\nu,k} > 0$. We want to proof now that $\exists\epsilon_0 > 0$ small enough such that $\forall\ 0 < \epsilon \leq \epsilon_0$ and $\forall(\nu, k) \notin \overline{KN}$, $u^\epsilon_{\nu,k} = 0$. If this was not the case, then we can construct a sequence $(\mathbf{u}^n, \mathbf{y}^n, \mathbf{s}^n, \mathbf{p_{el}}^n)$ of solutions to problem P converging toward $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ such that for one element $(\nu, k) \notin \overline{KN}$ and $\forall n \in \mathbb{N}, u^n_{\nu,k} > 0$. Because $\mathbf{u}^n$ converges toward $\mathbf{u}$, we can write that $u^n_{\nu,k} \longrightarrow u_{\nu,k}$. Using the investment equation for $u^n_{\nu,k} > 0$, one can write that:

$$I_{\nu,k} = \mathbb{E}_{\bar{Q}_\nu(Z^n_{\nu,k})}\Big[\mu^n_{\nu,k}(\omega)\Big], \tag{B.2}$$

and by making n go to $+\infty$, one would have (recall that function $\bar{Q}_\nu(.)$ is assumed to be continuous):

$$I_{\nu,k} = \mathbb{E}_{\bar{Q}_\nu(Z_{\nu,k})}\Big[\mu_{\nu,k}(\omega)\Big], \tag{B.3}$$

which means that the investment condition for solution $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ holds with equality for this $(\nu, k) \notin \overline{KN}$. Therefore, by increasing in an infinitesimal way $I_{\nu,k}$, the vector $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ remains a solution to problem P (but this time, the investment criterion will hold with strict inequality) but equation (B.3) will not be satisfied anymore. This can be formalized using transversality theorem 2 to prove that relation (B.3) occurs only in a set of parameters of measure zero. As a consequence, one can conclude that for almost all parameter data, $\exists\epsilon_0 > 0$ small enough such that $\forall\ 0 < \epsilon \leq \epsilon_0$, $\mathbf{u}^\epsilon \in \overline{KN}$.

We now focus on claim number 2 and show that for some $\epsilon_0 > 0$ small enough, $\forall 0 < \epsilon \leq \epsilon_0$, the prices of any solution belonging to $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ do not change in the non-characteristic scenarios of the initial solution $(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$. In every non-characteristic scenario $\omega \in \Omega\backslash\Omega_c$, the price $p_{el}(\omega)$ is given by the operating cost of the marginal plant or by the value of load curtailment PC. This price can be obtained by solving the optimal dispatch problem presented in (5.6), which is a linear program depending on capacities $\mathbf{u}$, LOAD and operating costs C. For a non-characteristic scenario, this optimization problem (5.6) is non-degenerate (unique primal-dual solution) at this initial solution. In the previous paragraph, we showed that for some ϵ_0 the vector of capacities $\mathbf{u}^\epsilon \in \overline{KN}$, as long as $0 < \epsilon \leq \epsilon_0$. Therefore, for some other $\epsilon_0 > 0$ small enough, the optimal basis of the LP associated with any solution in $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$, with $0 < \epsilon \leq \epsilon_0$, cannot change. This implies that the prices $p^\epsilon_{el}(\omega) = p_{el}(\omega)$ for the non-characteristic scenarios $\omega \notin \Omega_c$. Also for some $\epsilon_0 > 0$ small enough, the scenario ω remains non-characteristic (as defined by relations holding with strict inequality in the initial solution) for any solution belonging to $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ with $0 < \epsilon \leq \epsilon_0$.

We use the Transversality theorem 2 to prove claim 3 as follows: it can easily be shown that in a characteristic scenario $\omega_c \in \Omega_c$, the price cannot be equal to the operating cost of a plant or to the price cap PC, for almost all parameters (LOAD, I, C, PC). Hence for almost all those parameters, the plants producing in ω_c are making a positive profit. This ensures that for some $\epsilon_0 > 0$ small enough, $\forall\, 0 < \epsilon \leq \epsilon_0$, all solutions in $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ satisfy that the producing plants are producing at full capacity in the characteristic scenarios: $\forall \omega_c \in \Omega_c, \forall(\nu, k) \in \overline{KN}_c(\omega_c):\ y^{\epsilon}_{\nu,k}(\omega_c) = u^{\epsilon}_{\nu,k}$.

For the non-active plants, this price is strictly lower than their operating cost. So for some $\epsilon_0 > 0$, the solutions in $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$ (with $0 < \epsilon \leq \epsilon_0$) have the same set of active plants in the characteristic scenarios $\overline{KN}_c$. Combining the two situations, this proves that, for almost all parameter data, for some $\epsilon_0 > 0$ small enough, the set of characteristic scenarios is the same across all solutions belonging to $B_\epsilon(\mathbf{u}, \mathbf{y}, \mathbf{s}, \mathbf{p_{el}})$, when $0 < \epsilon \leq \epsilon_0$ and is equal to Ω_c.

To conclude, the main differences between those solutions are the prices in the characteristic scenarios $\mathbf{p^c_{el}}$ and the vector of capacity $\mathbf{u}$. So far, we just proved that prices in the non-characteristic scenarios do not change in a small enough neighborhood of the initial non-isolated solution. The operation levels $(\mathbf{y}, \mathbf{s})$ in the non-characteristic scenarios can vary across those equilibrium solutions, but they don't change the valuation of the plant (not appearing in the equations (5.2)) and are uniquely determined by the vector of installed capacities $\mathbf{u}$. □

B.1.3 The proof of isolated solutions

We now show that the existence of an non-isolated equilibrium leads to an absurdity, except for a set of zero measure of the parameters of the problem. Consider a non-isolated solution and the corresponding neighborhood of solutions sharing Ω_c, $\overline{KN}$, and $\overline{KN}_c$ (cf. proposition B.3).

The neighborhood equilibrium solutions vary by the level of capacities $\mathbf{u} \in \overline{KN}$, and the prices in the characteristic scenarios $\mathbf{p^c_{el}}$. To avoid confusion in the notation, we denote in this part of the proof $\mathbf{u}_0$ and $\mathbf{p^c_{el,0}}$ our non-isolated solution (we do not consider here operating levels as they can directly be deduced from the capacities $\mathbf{u}^0$).

The equations defining such equilibrium solutions are given by $f_{\texttt{clearing}}(\mathbf{u}) : \mathbb{R}^{|\overline{KN}|} \to \mathbb{R}^{|\Omega_c|}$ and $f_{\texttt{inv}} : \mathbb{R}^{|\overline{KN}|\cdot|\Omega_c|} \to \mathbb{R}^{|\overline{KN}|}$, where

$$f_{\texttt{clearing}}(\mathbf{u}) \equiv \left(\sum_{(\nu,k)\in\overline{KN}_c(\omega_c)} u_{\nu,k} - \text{LOAD}(\omega_c) \right)_{\omega_c \in \Omega_c} \tag{B.4}$$

gives the excess supply in the characteristic scenarios, and (we denote by x^+ the positive part of $x \in \mathbb{R}$: $x^+ = \max(x, 0)$)

$$f_{\texttt{inv}}(\mathbf{u}, \mathbf{p^c_{el}}) \equiv \begin{pmatrix} I_{\nu_k} - \displaystyle\sum_{\substack{\omega_c\in\Omega_c \\ (\nu,k)\in\overline{KN}_c(\omega_c)}} Q_\nu(Z_{\nu,k}; \omega_c)(p_{el}(\omega_c) - C_{\nu,k}(\omega_c)) \\ - \displaystyle\sum_{\omega\in\Omega\setminus\Omega_c} Q_\nu(Z_{\nu,k}, \omega)(\overline{p_{el}}(\omega) - C_{\nu,k}(\omega))^+ \end{pmatrix}_{(\nu,k)\in\overline{KN}} \tag{B.5}$$

measures for each power plants the additional capital asked by the generator company to invest. An equilibrium solution is hence given by the equation $f_{\mathtt{eq}} : \mathbb{R}^{|\overline{KN}|+|\Omega_c|} \to \mathbb{R}^{|\overline{KN}|+|\Omega_c|} : \begin{pmatrix} f_{\mathtt{clearing}} \\ f_{\mathtt{inv}} \end{pmatrix} = \mathbf{0}_{(|\overline{KN}|+|\Omega_c|)\times 1}$. We have seen in proposition B.3 that prices in the non-characteristic scenarios are fixed (they will be considered as constants).

We apply the transversality theorem 1 to function $f_{\mathtt{eq}}$. For that sake, we look at function $f_{\mathtt{eq}}$ as a function of variables $(\mathbf{u}, \mathbf{p_{el}}^c)$ but also of some of the parameters of the problem: the loads in the characteristic scenarios $\mathbf{LOAD}^c \in \mathbb{R}^{|\Omega_c|}$ and investment costs of the built plants $\mathbf{I}^{\overline{KN}} \in \mathbb{R}^{|\overline{KN}|}$. Function $f_{\mathtt{eq}}(\mathbf{u}, \mathbf{p_{el}}^c; \mathbf{LOAD}^c, \mathbf{I}^{\overline{KN}})$ is now a smooth function from $\mathbb{R}^{2(|\Omega_c|+|\overline{KN}|)}$ into $\mathbb{R}^{|\Omega_c|+|\overline{KN}|}$.

The Jacobian of $f_{\mathtt{eq}}$ is the following block matrix (all our expressions are evaluated at point $(\mathbf{u}, \mathbf{p_{el}}^c; \mathbf{LOAD}^c, \mathbf{I}^{\overline{KN}})$. To ease the notation, we do not explicitly write this vector in the following equations):

$$J_{(\mathbf{u},\mathbf{p_{el}}^c;\mathbf{LOAD}^c,\mathbf{I}^{\overline{KN}})} f_{\mathtt{eq}} = \begin{pmatrix} \frac{\partial f_{\mathtt{clearing}}}{\partial \mathbf{u}} & \frac{\partial f_{\mathtt{clearing}}}{\partial \mathbf{p_{el}}^c} & \frac{\partial f_{\mathtt{clearing}}}{\partial \mathbf{LOAD}^c} & \frac{\partial f_{\mathtt{clearing}}}{\mathbf{I}^{\overline{KN}}} \\ \frac{\partial f_{\mathtt{inv}}}{\partial \mathbf{u}} & \frac{\partial f_{\mathtt{inv}}}{\partial \mathbf{p_{el}}^c} & \frac{\partial f_{\mathtt{inv}}}{\partial \mathbf{LOAD}^c} & \frac{\partial f_{\mathtt{inv}}}{\mathbf{I}^{\overline{KN}}} \end{pmatrix}. \tag{B.6}$$

Matrix $J_{(\mathbf{u},\mathbf{p_{el}}^c;\mathbf{LOAD}^c,\mathbf{I}^{\overline{KN}})} f_{\mathtt{eq}}$ is of size $\left(|\Omega_c| + |\overline{KN}|\right) \times 2\left(|\Omega_c| + |\overline{KN}|\right)$. It is straightforward to see that:

$$\frac{\partial f_{\mathtt{clearing}}}{\partial \mathbf{LOAD}^c} = Id_{|\Omega_c|}, \tag{B.7}$$

where $Id_{|\Omega_c|}$ is the identity matrix of size $|\Omega_c|$.

It is also straightforward to see that that:

$$\frac{\partial f_{\mathtt{inv}}}{\partial \mathbf{I}^{\overline{KN}}} = Id_{|\overline{KN}|}, \tag{B.8}$$

where $Id_{|\overline{KN}|}$ is the identity matrix of size $|\overline{KN}|$. Therefore, one can write that:

$$\forall (\mathbf{u}, \mathbf{p_{el}}^c; \mathbf{LOAD}^c, \mathbf{I}^{\overline{KN}}) \in \mathbb{R}^{2\left(|\Omega_c|+|\overline{KN}|\right)}, \text{rank}\left(J_{(\mathbf{u},\mathbf{p_{el}}^c;\mathbf{LOAD}^c,\mathbf{I}^{\overline{KN}})} f_{\mathtt{eq}}\right) = |\overline{KN}| + |\Omega_c|. \tag{B.9}$$

Therefore, using the transversality theorem 1, we know that: for almost any data $(\mathbf{LOAD}^c, \mathbf{I}^{\overline{KN}}) \in \mathbb{R}^{|\Omega_c|+|\overline{KN}|}$, for any equilibrium $(\mathbf{u}, \mathbf{p_{el}}^c) \in \mathbb{R}^{|\Omega_c|+|\overline{KN}|}$ verifying $f_{\mathtt{eq}}(\mathbf{u}, \mathbf{p_{el}}^c; \mathbf{LOAD}^c, \mathbf{I}^{\overline{KN}}) = 0$, we must have

$$\text{rank}(J_{(\mathbf{u},\mathbf{p_{el}}^c)} f_{\mathtt{eq}}) = |\overline{KN}| + |\Omega_c|. \tag{B.10}$$

Since $J_{(\mathbf{u},\mathbf{p_{el}}^c)} f_{\mathtt{eq}}$ is a square matrix of size $|\overline{KN}| + |\Omega_c|$, relation (B.10) ensures that it is nonsingular for almost all data point. This implies that our initial equilibrium solution $(\mathbf{u}_0, \mathbf{p_{el}}_0^c)$ has to be isolated (for almost all data point), which is a contradiction. □

The proof of the following lemma is long and technical: a few preliminary remarks may help clarify its logic. The "project finance model" is a transformation of the standard capacity expansion problem (as the SCM), which is a standard linear programming problem. The complementarity conditions of the project finance model are accordingly also transformations of the primal-dual conditions of the SCM. After some preliminaries, a first part of the proof analyzes the structure of the complementarity conditions of problem P with the view of separating what remains identical to the primal-dual conditions of SCM so as to be able to only concentrate on the changes. The second part of the proof analyzes the part that has really changed compared to the SCM, namely the investment conditions. The third and last part concludes with the proof of the positivity of the determinant.

B.2 Proof of Lemma 8.10

Lemma *Suppose that the producers value their investment by a coherent risk measure (ensuring positive subjective probabilities) and that the merit order is unchanged in all scenarios. Then all solutions are isolated and for every solution to the problem* $\Phi^P(.) = 0$*, the determinant of the Jacobian is positive:*

$$\forall X \in \left(\Phi^P\right)^{-1}(0),\ det(J\Phi^p(X)) > 0. \tag{B.11}$$

B.2.1 The structure of problem P for coherent risk measures

Proof Modeling risk-aversion with coherent risk measures in problem P leads to a number of characteristic scenarios that is equal to the number of built plants. The demonstration of lemma 8.7 can be applied unchanged here, so we have less characteristic scenarios than the number of built plants: $|\Omega_c| \leq |\overline{KN}|$.

We then show that there exists at least $|\overline{KN}|$ characteristic scenarios in a solution.
Consider the equation:

$$I_{\nu,k} = \sum_{\omega} \bar{Q}_{\nu,k}(Z_{\nu,k};\omega)\mu_{\nu,k}(\omega)\ ,$$

associated to each plant with positive capacity. By the complementarity condition $0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \perp y_{\nu,k}(\omega) \geq 0$, the vector $\mu_{\nu,k}(\omega)$ consists of:

- $\mu_{\nu,k}(\omega) = 0$, for scenarios ω where the plant does not operate or operates at partial load.
- $\mu_{\nu,k}(\omega) = \left(C_{\nu',k'}(\omega) - C_{\nu,k}(\omega)\right)$, for scenarios ω where the plant ν, k operates at full capacity but ω is not a characteristic scenario. The marginal plant ν', k' sets the price but does not make any margin.
- $\mu_{\nu,k}(\omega) = p_{el}(\omega) - C_{\nu,k}(\omega)$, for all characteristic scenarios ω where the plant operates (at full capacity). The price $p_{el}(\omega)$ is not fixed by the operating cost of a plant.

The margin vector $\boldsymbol{\mu}$ is hence entirely determined by the problem's parameters except for the characteristic scenarios. Recall that agents use coherent risk measures. Because of the homogeneity property, one can write (when $u_{\nu,k} > 0$):

$$\bar{Q}_{\nu,k}(Z_{\nu,k};\omega) = \bar{Q}_{\nu,k}(u_{\nu,k}\mu_{\nu,k}(\omega);\omega) = \bar{Q}_{\nu,k}(\mu_{\nu,k}(\omega);\omega), \tag{B.12}$$

which means that investment equations:

$$I_{\nu,k} = \sum_{\omega} \bar{Q}_{\nu,k}(Z_{\nu,k};\omega)\mu_{\nu,k}(\omega) \ ,$$

are independent on variable $u_{\nu,k}$. To satisfy the $|\overline{KN}|$ investment equations, the only degrees of freedom are the electricity prices at the characteristic scenarios of the solution. So, If the number of characteristic scenarios is lower than $|\overline{KN}|$, this system of equation is over-determined and does not have a solution for almost all investment costs data.

B.2.2 Preliminaries on the Jacobian matrix of P

Let us now reformulate the project finance equilibrium problem P for the case of coherent risk measures. Without loss of generality and to simplify the notation of this technical proof, we concatenate the indices ν and k in a unique index k (the project finance valuation looks at the plants individually) and introduce a new variable $\text{slack}_k(\omega)$ that represents the unused capacity of technology k in scenario ω. Recall that each player/plant k uses a coherent risk measure, with a subjective probability denoted $\bar{Q}_k(\mu_k u_k)$:

The KKT system is modified into:

$$u_k = y_k(\omega) + \text{slack}_k(\omega) \qquad \forall k = 1,\ldots,K, \forall\omega \in \Omega \tag{B.13}$$

$$0 \le -\text{LOAD}(\omega) + \sum_k y_k(\omega) + s(\omega) \quad \perp \quad p_{el}(\omega) \ge 0 \qquad \forall\omega \in \Omega \tag{B.14}$$

$$0 \le \text{slack}_k(\omega) \quad \perp \quad \mu_k(\omega) \ge 0 \qquad \forall k = 1,\ldots,K, \forall\omega \in \Omega \tag{B.15}$$

$$0 \le C_k(\omega) + \mu_k(\omega) - p_{el}(\omega) \quad \perp \quad y_k(\omega) \ge 0 \qquad \forall k = 1,\ldots,K, \forall\omega \in \Omega \tag{B.16}$$

$$0 \le PC - p_{el}(\omega) \quad \perp \quad s(\omega) \ge 0 \qquad \forall\omega \in \Omega \tag{B.17}$$

$$0 \le I_k - \mathbb{E}_{\bar{Q}_k(\mu_k u_k)}\left[\mu_k(\omega)\right] \quad \perp \quad u_k \ge 0 \qquad \forall k = 1,\ldots,K. \tag{B.18}$$

B.2.3 Structuring the complementarity conditions

This part of the proof partitions the above complementarity conditions so as to identify those that are directly inherited from the primal-dual conditions of the SCM, and those that concentrate on the modified investment criterion.

The problem can be separated in three groups of equations: a first one (B.13) - (B.14) involves only "primal variables"[16] ($\mathbf{y}$, $\mathbf{slack}$, $\mathbf{s}$, $\mathbf{u}$). The second one (B.15) - (B.17) involves only "dual variables" ($\boldsymbol{\mu}$, $\mathbf{p_{el}}$) and a last group constituted by equation (B.18) that mixes primal and dual variables because of the *a priori* dependence of the subjective probabilities on investment decisions $\mathbf{u}$. However, as we have seen above, the use of coherent risk measures implies that these subjective probabilities actually do not depend on $\mathbf{u}$ (see equation (B.12)), which will put to zero some terms of the Jacobian matrix as we will see in the remainder of this proof.

Consider now a basis of the problem, i.e. the set of positive variables defining a solution to the reformulated problem P. We define the sets $\Omega_k := \{\omega \in \Omega |\, y_k(\omega) > 0\}$ (a generic element of Ω_k will be denoted ω_k), $\Omega'_k \subset \Omega_k := \{\omega \in \Omega |\, \text{slack}_k(\omega) > 0 \text{ and } y_k(\omega) > 0\}$ (a generic element of Ω'_k will be denoted ω'_k) and $\Omega_s := \{\omega \in \Omega |\, s(\omega) > 0\}$ (a generic element of Ω_s will be denoted ω_s). We denote the primal variables belonging to the basis by $(y'_k(\omega_k), \text{slack}'_k(\omega'_k), s'(\omega_s), u'_k)$. We will assume that the basis variable u'_k ranges from $k = 1$ to $k = K'$ (in other words, to ease the notation, we now denote the number of built plants $|\overline{KN}|$ by K'). This implies that we have K' characteristic scenarios in the solution. The set of equations, taken from (B.13)–(B.14), defining the basis for the primal variables can be written using block matrices:

$$
\begin{pmatrix}
-Id_1 & 0 \cdots & 0 & -A_1 & 0 \cdots & 0 & 0 & e_1 & 0 \cdots & 0 \\
 & \ddots & & & \ddots & & & & \ddots & \\
0 & -Id_k & 0 & 0 & -A_k & 0 & 0 & 0 & e_k & 0 \\
 & \ddots & & & \ddots & & & & \ddots & \\
0 & \cdots 0 & -Id_{K'} & 0 & 0 \cdots & -A_{K'} & 0 & 0 & 0 \cdots & e_{K'} \\
B_1 & \cdots B_k & \cdots B_{K'} & 0 & 0 & 0 & D_s & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
y'_1(\omega_1) \\ \vdots \\ y'_{K'}(\omega_{K'}) \\ \text{slack}'_1(\omega'_1) \\ \vdots \\ \text{slack}'_{K'}(\omega'_{K'}) \\ s'(\omega_s) \\ u'_1 \\ \vdots \\ u'_{K'}
\end{pmatrix}
=
\begin{pmatrix}
0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \text{LOAD}
\end{pmatrix},
\tag{B.19}
$$

where, for every plant k in the basis, $Id_k := I^{|\Omega_k| \times |\Omega_k|}$ is an identity matrix of size $|\Omega_k|$ and e_k is a column vector defined by $e_k \in \mathbb{R}^{|\Omega_k|} := (1, \cdots, 1)^T$. Furthermore, for every plant k in the basis, the matrix A_k defines scenarios where the plant k does not operate at full capacity, and the matrix B_k defines scenarios where the plant operates, i.e.

$$
A_k \in \mathbb{R}^{|\Omega_k| \times |\Omega'_k|} := \begin{cases} a_{\omega_k, \omega_k} = 1 & \text{if slack}_k(\omega) > 0 \\ a_{\omega_k, \omega'_k} = 0 & \text{otherwise} \end{cases},
$$

$$
B_k \in \mathbb{R}^{|\Omega| \times |\Omega_k|} := \begin{cases} b_{\omega, \omega} = 1 & \text{if } y_k(\omega) > 0 \\ b_{\omega, \omega_k} = 0 & \text{otherwise} \end{cases}. \tag{B.20}
$$

[16] This abuse of terminology is justified by the optimization problem (7.3).

The matrix D_s defines scenarios where the load is curtailed,

$$D_s \in \mathbb{R}^{|\Omega|\times|\Omega_s|} := \begin{cases} d_{\omega,\omega} = 1 & \text{if } s(\omega) > 0 \\ d_{\omega,\omega_s} = 0 & \text{otherwise} \end{cases}. \tag{B.21}$$

B.2.4 Analysis of the Jacobian matrix regarding the investment conditions

The Jacobian matrix of the primal part of the basis (i.e. the matrix in (B.19)) is denoted $J_{\mathcal{P}}$. Let us now calculate its size. Given the definition of matrices Id_k, A_k, B_k and D_s, one can calculate the number of rows and columns of $J_{\mathcal{P}}$:

$$\#rows = |\Omega_1| + \cdots + |\Omega_{K'}| + |\Omega|, \tag{B.22}$$

$$\#columns = |\Omega_1| + \cdots + |\Omega_{K'}| + |\Omega'_1| + \cdots + |\Omega'_{K'}| + |\Omega_s| + K'. \tag{B.23}$$

Considering a scenario $\omega \in \Omega$ and a plant k in the basis, we know that exactly one of the following is true:

- ω is a characteristic scenario for k.
- ω is not a characteristic scenario for k and the slack variable is positive. In other words, $\omega \in \Omega'_k$. This means that k is the marginal plant and any non-degeneracy assumption implies that it is the only plant that is marginal (which is the case for almost all parameter data).
- ω is a scenario where curtailment happened: $\omega \in \Omega_s$.

Therefore, one will conclude that:

$$|\Omega| = K' + |\Omega'_1| + \cdots + |\Omega'_{K'}| + \Omega_s \tag{B.24}$$

and that $\#rows = \#columns$. Matrix $J_{\mathcal{P}}$ is square and is of dimension $|\Omega_1| + \cdots + |\Omega_{K'}| + |\Omega|$.

We denote the dual variables belonging to the basis by $(\mu'_k(\omega), p_{el}(\omega))$. For the gross margin μ'_k, we include all scenarios for which the plant is producing (not only those where the μ'_k are positive) i.e. $\mu_k \in \mathbb{R}^{|\Omega_k|}$. By equation (B.15), one has that $\mu'_k(\omega) = 0$ if $\omega \in \Omega'_k$ (these are the marginal plants that do not operate at capacity and hence set the price in the scenarios ω). The set of equations defining the dual variables in the basis can be decomposed in two parts: the first part deals with equations (B.15)–(B.17) and the second with equation (B.18).

$$\begin{pmatrix} -Id_1 & 0 & \cdots & & 0 & B_1^T \\ & \ddots & & & & \vdots \\ 0 & & -Id_k & & 0 & B_k^T \\ & & & \ddots & & \vdots \\ 0 & & \cdots & & -Id_{K'} & B_{K'}^T \\ -A_1^T & 0 & \cdots & & 0 & 0 \\ & \ddots & & & & \vdots \\ 0 & & -A_k^T & & 0 & \vdots \\ & & & \ddots & & \vdots \\ 0 & & \cdots & & -A_{K'}^T & 0 \\ 0 & & \cdots & & 0 & D_s^T \end{pmatrix} \begin{pmatrix} \mu_1'(\omega_1) \\ \vdots \\ \mu_k'(\omega_k) \\ \vdots \\ \mu_{K'}'(\omega_{K'}) \\ p_{el}(\omega) \end{pmatrix} = \begin{pmatrix} C_1(\omega_1) \\ \vdots \\ C_k(\omega_k) \\ \vdots \\ C_{K'}(\omega_{K'}) \\ 0 \\ \vdots \\ PCC \end{pmatrix} \tag{B.25}$$

$$\begin{pmatrix} \mathbb{E}_{\bar{Q}_1(\mu_1 u_1)}\left[\mu_1(\omega)\right] \\ \vdots \\ \mathbb{E}_{\bar{Q}_k(\mu_k u_k)}\left[\mu_k(\omega)\right] \\ \vdots \\ \mathbb{E}_{\bar{Q}_{K'}(\mu_{K'} u_{K'})}\left[\mu_{K'}(\omega)\right] \end{pmatrix} = \begin{pmatrix} I_1 \\ \vdots \\ I_k \\ \vdots \\ I_{K'} \end{pmatrix}, \tag{B.26}$$

where PCC is the column vector defined by $PCC \in \mathbb{R}^{|\Omega_s|} := (PC, \ldots, PC)^T$. The Jacobian of the risk measure $\left(\mathbb{E}_{\bar{Q}_1(\mu_1 u_1)}\left[\mu_1(\omega)\right], \cdots, \mathbb{E}_{\bar{Q}_{K'}(\mu_{K'} u_{K'})}\left[\mu_{K'}(\omega)\right]\right)^T$ is composed of two parts. The first is obtained by differentiating $\left(\mathbb{E}_{\bar{Q}_1(\mu_1 u_1)}\left[\mu_1(\omega)\right], \cdots, \mathbb{E}_{\bar{Q}_{K'}(\mu_{K'} u_{K'})}\left[\mu_{K'}(\omega)\right]\right)^T$ with respect to the primal variables ($\mathbf{y}$, $\mathbf{slack}$, $\mathbf{s}$, $\mathbf{u}$), which always gives zero because the subjective probabilities $\bar{Q}_i(\mu_i u_i)$ do not depend on investment decisions in the case of coherent risk measures (see relation (B.12)). The matrix of the non-diagonal part of the whole Jacobian matrix of problem (B.13)–(B.18) is then zero. The second is obtained by differentiating $\left(\mathbb{E}_{\bar{Q}_1(\mu_1 u_1)}\left[\mu_1(\omega)\right], \cdots, \mathbb{E}_{\bar{Q}_{K'}(\mu_{K'} u_{K'})}\left[\mu_{K'}(\omega)\right]\right)^T$ with respect to the dual variables ($\boldsymbol{\mu}$, $\mathbf{p_{el}}$). We now do this calculation:

For $k = 1, 2, \ldots K'$, let us denote function $f_k(\mu_k) = \mathbb{E}_{\bar{Q}_k(\mu_k u_k)}\left[\mu_k(\omega)\right]$:

$$f_k(\mu_k) = \sum_{\omega \in \Omega} \bar{Q}_k(\mu_k u_k; \omega)\mu_k(\omega) \tag{B.27}$$

and using the fact that risk measures are coherent, on gets:

$$\forall \omega_k \in \Omega_k, \frac{\partial f_k}{\partial \mu_k(\omega)} = \bar{Q}_k(\mu_k u_k; \omega). \tag{B.28}$$

The Jacobian of $\left(\mathbb{E}_{\bar{Q}_1(\mu_1 u_1)}\left[\mu_1(\omega)\right], \cdots, \mathbb{E}_{\bar{Q}_{K'}(\mu_{K'} u_{K'})}\left[\mu_{K'}(\omega)\right]\right)^T$ with respect to the dual variables $(\boldsymbol{\mu}, \mathbf{p_{el}})$ is then the following matrix of $\mathbb{R}^{K' \times (|\Omega_1|+\cdots+|\Omega_{K'}|+|\Omega|)}$:

$$\begin{pmatrix} \bar{Q}_1 & 0 & \cdots & & 0 & 0 \\ & \ddots & & & & \vdots \\ 0 & & \bar{Q}_k & & & \vdots \\ & & & \ddots & & \vdots \\ 0 & & \cdots & & \bar{Q}_{K'} & 0 \end{pmatrix}, \tag{B.29}$$

where $\bar{Q}_k = \bar{Q}_k(\mu_k u_k, \omega_k), \omega_k \in \Omega_k$ is written as a line vector. Recall that it is calculated only for scenarios $\omega_k \in \Omega_k$, since we consider only the basis variables $(\mu'_k(\omega_k), \omega_k \in \Omega_k)$.

We then define the Jacobian matrix of the dual part $J_{\mathcal{D}}$ by concatenating the matrices in (B.25) and in (B.29) as follows:

$$J_{\mathcal{D}} = \begin{pmatrix} -Id_1 & 0 & \cdots & & 0 & B_1^T \\ & \ddots & & & & \vdots \\ 0 & & -Id_k & & 0 & B_k^T \\ & & & \ddots & & \vdots \\ 0 & & \cdots & & -Id_{K'} & B_{K'}^T \\ -A_1^T & 0 & \cdots & & 0 & 0 \\ & \ddots & & & & \vdots \\ 0 & & -A_k^T & & 0 & \vdots \\ & & & \ddots & & \vdots \\ 0 & & \cdots & & -A_{K'}^T & 0 \\ 0 & & \cdots & & 0 & D_s^T \\ \bar{Q}_1 & 0 & \cdots & & 0 & 0 \\ & \ddots & & & & \vdots \\ 0 & & \cdots & & \bar{Q}_{K'} & 0 \end{pmatrix}. \tag{B.30}$$

It can be shown that like for matrix $J_{\mathcal{P}}$, the Jacobian matrix of the dual part of the problem $J_{\mathcal{D}}$ is square and of dimension $|\Omega_1|+\cdots+|\Omega_{K'}|+|\Omega|$. One sees that, except for the subjective "risk" part of the matrix (linked to the $\bar{Q}_k$ vriables), the Jacobian matrix of the dual is the transpose of the primal one.

B.2.5 Positivity of the determinant

The Jacobian matrix of the whole problem (B.13)–(B.18) is then the square matrix J of size $2\left(|\Omega_1| + \cdots + |\Omega_{K'}| + |\Omega|\right)$ defined by blocks as follows:

$$J = \begin{pmatrix} J_{\mathcal{P}} & 0 \\ 0 & J_{\mathcal{D}} \end{pmatrix}. \tag{B.31}$$

Therefore, one can deduce that:

$$det(J) = det(J_{\mathcal{P}}).det(J_{\mathcal{D}}). \tag{B.32}$$

The determinant of the full Jacobian J will be positive if $det(J_{\mathcal{P}})$ and $det(J_{\mathcal{D}})$ have the same sign. This condition was already noted for linear complementarity problems [10].

We first focus on the primal Jacobian matrix that we factorize in two triangular matrices for which we will be able to evaluate the determinant. We do so by pre-multiplying the primal basis by the following lower triangular square matrix of size $|\Omega_1| + \cdots + |\Omega_{K'}| + |\Omega|$:

$$LT := \begin{pmatrix} Id_1 & 0 & \cdots & & 0 & 0 \\ & \ddots & & & & 0 \\ 0 & & Id_k & & 0 & 0 \\ & & & \ddots & & 0 \\ 0 & & \cdots & & Id_{K'} & 0 \\ -B_1 & \cdots & -B_k & \cdots & B_{K'} & -I_{|\Omega|} \end{pmatrix}. \tag{B.33}$$

The product of $(LT \;\; J_{\mathcal{P}})$ gives the following upper triangular matrix.

$$LT\,J_{\mathcal{P}} = \begin{pmatrix} -Id_1 & 0 \cdots & 0 & -A_1 & 0 \cdots & 0 & 0 & e_1 & 0 \cdots & 0 \\ & \ddots & & & \ddots & & & & \ddots & \\ 0 & -Id_k & 0 & 0 & -A_k & 0 & 0 & 0 & e_k & 0 \\ & \ddots & & & \ddots & & & & \ddots & \\ 0 & \cdots 0 & -Id_{K'} & 0 & 0 \cdots & -A_{K'} & 0 & 0 & 0 \cdots & e_{K'} \\ 0 & \cdots & 0 & B_1 A_1 & \cdots B_k A_k & \cdots B_{K'} A_{K'} & -D_s & -B_1 e_1 & \cdots - B_k e_k & \cdots - B_{K'} e_{K'} \end{pmatrix}. \tag{B.34}$$

Similarly the multiplication of the LT matrix with the transpose of the dual matrix $(LT\,J_{\mathcal{D}}^T)$ gives

$$LT\,J_{\mathcal{D}}^T = \begin{pmatrix} -Id_1 & 0\cdots & 0 & -A_1 & 0\cdots & 0 & 0 & \bar{Q}_1^T & 0\cdots & 0 \\ & \ddots & & & \ddots & & & & \ddots & \\ 0 & -Id_k & 0 & 0 & -A_k & 0 & 0 & 0 & \bar{Q}_k^T & 0 \\ & \ddots & & & \ddots & & & & \ddots & \\ 0 & \cdots 0 & -Id_{K'} & 0 & 0\cdots & -A_{K'} & 0 & 0 & 0\cdots & \bar{Q}_{K'}^T \\ 0 & \cdots & 0 & B_1A_1 & \cdots B_kA_k & \cdots B_{K'}A_{K'} & -D_s & -B_1\bar{Q}_1^T & \cdots - B_k\bar{Q}_k^T & \cdots - B_{K'}\bar{Q}_{K'}^T \end{pmatrix}. \tag{B.35}$$

The auxiliary matrix LT is lower triangular and its determinant can be calculated by blocks:

$$det(LT) = (-1)^{|\Omega|}.$$

Thus, one can easily deduce that:

$$det\left((LT\,J_{\mathcal{P}})\right).det\left(LT\,J_{\mathcal{D}}^T\right) = (-1)^{|\Omega|}.det(J_{\mathcal{P}}).(-1)^{|\Omega|}.det(J_{\mathcal{D}}) = det(J). \tag{B.36}$$

Let us then first focus on the calculation of the determinant of $LT\,J_{\mathcal{P}}$. The determinants of all diagonal blocks except the last one are easy to compute:

$$\forall k = 1\ldots K',\; det(-Id_k) = (-1)^{|\Omega_k|}. \tag{B.37}$$

It remains to analyze the following block of size $|\Omega|$ whose determinant needs to be calculated:

$$\left(B_1A_1 \cdots B_kA_k \cdots B_{K'}A_{K'} \; -D_s \; -B_1e_1 \cdots - B_ke_k \cdots - B_{K'}e_{K'}\right). \tag{B.38}$$

We first note that the matrix $(B_1A_1 \;\; \cdots \;\; B_kA_k \;\; \cdots \;\; B_{K'}A_{K'} \;\; - D_s)$ (of size $\Omega \times (\Omega - K')$) is equivalent [17] (with an appropriate re-ordering of the matrix) to the identity matrix of dimension $|\Omega| - K'$ and some zeros, i.e.

$$\begin{pmatrix} I_{|\Omega|-K'} \\ 0 \end{pmatrix}.$$

With this reordering, we are left with the scenarios where no plant is at partial capacity (these are the characteristic scenarios defined before) and there is no load curtailment (or the scenario where the price is not fully defined by the marginal cost of a technology). This number of scenarios is equal to K'.

Suppose now again that the merit order does not change with the scenarios. We can reorder the plants such that $C_{k1}(\omega) \geq C_{k2}(\omega)$ for all $k2 > k1$, $\omega \in \Omega$, and the scenarios such that the matrix $(-B_1e_1 \cdots - B_ke_k \cdots - B_{K'}e_{K'})$ becomes an upper triangular matrix with 1 on the diagonal. The determinant of the matrix $LT\;\; J_{\mathcal{P}}$ is then

[17] Indeed, $(B_kA_k)_{\omega,\omega'} = 1$ only when $\omega \in \Omega'_k$. Also, for every $\omega \in \Omega$, one can have only one technology (plants or curtailment) that can operate at partial capacity.

(+/-) 1 depending on the number of operations carried out to reorder the plants and scenarios. This sign will be denoted σ and we obtain:

$$det((LT J_{\mathcal{P}})) = \sigma(-1)^{|\Omega_1|+\cdots+|\Omega_{K'}|}. \tag{B.39}$$

We now analyze the sign of the determinant of the matrix $LT J_{\mathcal{D}}^T$ that has a similar structure as matrix $LT J_{\mathcal{P}}$. The interesting block to study is the following:

$$\left(B_1 A_1 \cdots B_k A_k \cdots B_{K'} A_{K'} \; -D_s \; -B_1 \bar{Q}_1^T \cdots - B_k \bar{Q}_k^T \cdots - B_{K'} \bar{Q}_{K'}^T \right). \tag{B.40}$$

We will now apply exactly the same calculation that we did for the block matrix defined in (B.38) to affirm that (we denote by $\overline{\omega_k}$ the scenario that is characteristic for plant k):

$$\begin{aligned} &det\left(B_1 A_1 \cdots B_k A_k \cdots B_{K'} A_{K'} \; -D_s \; -B_1 \bar{Q}_1^T \cdots - B_k \bar{Q}_k^T \cdots - B_{K'} \bar{Q}_{K'}^T \right) \\ &\quad = \sigma \Pi_{k=1}^{K'} \bar{Q}_k(\overline{\omega_k}) \end{aligned} \tag{B.41}$$

and that

$$det\left(LT J_{\mathcal{D}}^T\right) = \sigma(-1)^{|\Omega_1|+\cdots+|\Omega_{K'}|} \Pi_{k=1}^{K'} \bar{Q}_k(\overline{\omega_k}). \tag{B.42}$$

Using relations (B.36), (B.39) and (B.42), we can write that

$$det(J) = det\left((LT J_{\mathcal{P}})\right).det\left(LT J_{\mathcal{D}}^T\right) = \left(\sigma(-1)^{|\Omega_1|+\cdots+|\Omega_{K'}|}\right)^2 \Pi_{k=1}^{K'} \bar{Q}_k(\overline{\omega_k}) > 0. \tag{B.43}$$

Therefore, the sign of the determinant of the full Jacobian matrix is positive. □

B.3 Existence of solution to problem P^f

B.3.1 Expression of the mapping $\Phi^{\mathbf{P^f}}$

We here give the expression of function Φ^{Pf} introduced in Sect. 9 appearing in the definition of problem P^f. Function Φ^{Pf} is defined on the set $\mathbb{R}^{M_{Pf}}$ and takes its values also in $\mathbb{R}^{M_{Pf}}$. Its expression is given below:

$$\forall\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}}, \;\; \Phi^{Pf}\left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) =$$

$$\left(\begin{array}{cc} \mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f & c = 1, \ldots, C \\ \mathbb{E}_{\bar{Q}_v(Z_v^f)}\left[p_c^s(\omega)\right] - p_c^f & v = 1, \ldots, N, \; c = 1, \ldots, C \\ \sum_v x_{v,c} + x_{d,c} & c = 1, \ldots, C \\ \min\left(\left(I_{v,k} - \mathbb{E}_{\bar{Q}_v(Z_v^f)}\left[\mu_{v,k}(\omega)\right]\right) \;\; , \;\; u_{v,k}\right) & v = 1, \ldots, N, \; k = 1, \ldots, K \\ \min\left(\left(u_{v,k} - y_{v,k}(\omega)\right) \;\; , \;\; \mu_{v,k}(\omega)\right) & v = 1, \ldots, N, \; k = 1, \ldots, K, \omega \in \Omega \\ \min\left(\left(C_{v,k}(\omega) + \mu_{v,k}(\omega) - p_{el}(\omega)\right) \;\; , \;\; y_{v,k}(\omega)\right) & v = 1, \ldots, N, \; k = 1, \ldots, K, \; \omega \in \Omega \\ \min\left(\left(PC - p_{el}(\omega)\right) \;\; , \;\; s(\omega)\right) & \omega \in \Omega \\ \min\left(\left(-\text{LOAD}(\omega) + \sum_{v,k} y_{v,k}(\omega) + s(\omega)\right) \;\; , \;\; p_{el}(\omega)\right) & \omega \in \Omega \\ p_c^s(\omega) - h_{c,\omega}(p_{el}(\omega)) & c = 1, \ldots, C, \; \omega \in \Omega \end{array} \right). \tag{B.44}$$

B.3.2 Expression of the homotopy function H^{Pf}

$$\forall Y = \left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \in \mathbb{R}^{M_{Pf}},\ \forall \lambda \in [0,1],\ H^{Pf}\left(Y, \lambda\right) =$$

$$\left(\begin{array}{cc}
\lambda \cdot \mathbb{E}_{\bar{Q}_{e_\theta}(\mathrm{Zexo}+\sum_{c=1}^{C} x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] + (1-\lambda)\mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f & c = 1,\ldots,C \\
\lambda \cdot \mathbb{E}_{\bar{Q}_{e_\theta}(\mathrm{Zexo}+\sum_{c=1}^{C} x_{d,c} p_c^s)}\left[p_c^s(\omega)\right] + (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[p_c^s(\omega)\right] - p_c^f & \nu = 1,\ldots,N,\ c = 1,\ldots,C \\
\sum_\nu x_{\nu,c} + x_{d,c} & c = 1,\ldots,C \\
\min\left(\left(I_{\nu,k} - \lambda \cdot \mathbb{E}_P\left[\mu_{\nu,k}(\omega)\right] - (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[\mu_{\nu,k}(\omega)\right]\right) \quad , \quad u_{\nu,k}\right) & \nu = 1,\ldots,N,\ k = 1,\ldots,K \\
\min\left(\left(u_{\nu,k} - y_{\nu,k}(\omega)\right) \quad , \quad \mu_{\nu,k}(\omega)\right) & \nu = 1,\ldots,N,\ k = 1,\ldots,K, \omega \in \Omega \\
\min\left(\left(C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega)\right) \quad , \quad y_{\nu,k}(\omega)\right) & \nu = 1,\ldots,N,\ k = 1,\ldots,K,\ \omega \in \Omega \\
\min\left(\left(PC - p_{el}(\omega)\right) \quad , \quad s(\omega)\right) & \omega \in \Omega \\
\min\left(\left(-\mathrm{LOAD}(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega)\right) \quad , \quad p_{el}(\omega)\right) & \omega \in \Omega \\
p_c^s(\omega) - h_{c,\omega}(p_{el}(\omega)) & c = 1,\ldots,C,\ \omega \in \Omega
\end{array}\right). \tag{B.45}$$

B.3.3 Proof of Lemma 9.2 and expression of Ξ^{Pf}

Lemma *Suppose a solution to $H^{Pf}(\cdot, \lambda) = 0$ exists, then this solution is in the interior of the closure of a bounded open set Ξ^{Pf} that does not depend on λ.*

$$0 \notin H^{Pf}(\mathbf{bd}\ \Xi^{Pf}, \lambda), \quad \forall \lambda \in [0,1].$$

Proof The reasoning used in lemma 8.2 to show that the variables $\mathbf{u}$, $\mathbf{y}$, $\mathbf{p_{el}}$, $\boldsymbol{\mu}$ and $\mathbf{s}$ belong to the closure of a bounded open set defined independently of λ applies unchanged here.

It remains to show that the financial positions $\mathbf{x}$ and prices $\mathbf{p^f}$ also belong to the closure of a bounded open set that is independent of λ. Because there is no natural way to bound the $\mathbf{x}$ variable, we adapt an argument of de Maere d'Aertrycke and Smeers [23] where a similar problem was posed. As $\mathbf{p_{el}}$ belongs to a closed and bounded set, the payoffs $\mathbf{p^s}$ of the financial contracts also belong to a closed convex set

$$\left\{ p_c^s(\omega) \in [\min_{p_{el} \in [0,PC]} h_{c,\omega}(p_{el}) \ , \ \max_{p_{el} \in [0,PC]} h_{c,\omega}(p_{el})], \forall c \in 1,\ldots,C,\ \omega \in \Omega \right\} .$$

The price of financial contracts at $t = 0$, $\mathbf{p^f}$, also belongs to the following compact set

$$\left\{ p^f \in \mathbb{R}^C \middle| p_c^f \in [\min_{\omega \in \Omega, p_{el} \in [0,PC]} h_{c,\omega}(p_{el}) \ , \ \max_{\omega \in \Omega, p_{el} \in [0,PC]} h_{c,\omega}(p_{el})], \forall c \in 1,\ldots,C \right\} , \tag{B.46}$$

otherwise all market agents would exploit the obvious arbitrage opportunity (i.e. a contract that always gives a higher (lower) payoff than its price at $t = 0$) created, making it impossible to clear the financial positions (all agents have a convex risk measure that satisfies monotonicity).

It remains to prove that the financial positions $\mathbf{x}$ also belong to the closure of a bounded open set that is independent of λ. This proof will extensively use the sets of the subjective probabilities of the producers $\mathcal{M}_\nu$ and the consumer $\mathcal{M}_d$, that were defined in the representation theorem 3.2. We recall that this representation theorem holds for the general case of convex risk measures, which is what we consider in problem P^f.

We first define "not too attractive" prices of financial products for producers ν in the homotopy problem $H^{P^f}(., \lambda) = 0$. A similar definition applies to the consumer.

Definition B.4 Prices of financial products in problem $H^{P^f}(\cdot, \lambda) = 0$ are "not too attractive" for the producer ν if they belong to the set $\mathtt{P}_\nu(\lambda) \subset \overline{\mathtt{P}}$ defined as:

$$\mathtt{P}_\nu(\lambda) = \left\{ p^f \in \mathbb{R}^C \middle| \exists Q \in \mathcal{M}_\nu(\lambda), \forall c = 1, \ldots, C : p_c^f = \mathbb{E}_Q[p_c^s] \right\}, \tag{B.47}$$

where $\mathcal{M}_\nu(\lambda) = \lambda \mathcal{P} + (1 - \lambda)\mathcal{M}_\nu$.

The rationale for this definition is that prices that are too attractive immediately create a "subjective" arbitrage opportunity for the producer and hence lead to unbounded transactions. Assumption H2 states that $\overline{\mathcal{M}}$ has a non-empty interior. Let us focus on the set

$$\overline{\mathcal{M}}(\lambda) = \cap_{\nu=1,\ldots,N} \mathcal{M}_\nu(\lambda) \cap \mathcal{M}_d(\lambda). \tag{B.48}$$

As $\mathcal{M}_\nu \subset \mathcal{P}$, one has that its interior is non-empty as $\textbf{int}\, \overline{\mathcal{M}} \subset \textbf{int}\, \overline{\mathcal{M}}(\lambda)$. Accordingly the set $\overline{\mathtt{P}}(\lambda) = \cap_{\nu=1,\ldots,N} \mathtt{P}_\nu(\lambda) \cap \mathtt{P}_d(\lambda)$ has also non-empty interior, i.e.

$$\forall \lambda \in [0, 1] : \quad \textbf{int}\, \overline{\mathtt{P}}(\lambda) \neq \emptyset. \tag{B.49}$$

We are now ready to prove that the financial position in problem $H^{p^f}(., \lambda) = 0$, denoted by $\mathbf{x}(\lambda)$ belongs to a compact set that is independent of λ. This is done by applying the reasoning of proposition 3.12 of [23] with some adaptation.

Suppose the set of positions $x_\nu(\lambda)$ of the producer[18] ν of problem $H^{P^f}(\cdot, \lambda) = 0$ is unbounded. We can take an unbounded sequence of the optimal solutions to $H^{P^f}(\cdot, \lambda) = 0$:

$$\left(\mathbf{u}^l(\lambda^l), \mathbf{p_{el}}^l(\lambda^l), \mathbf{s}^l(\lambda^l), \mathbf{y}^l(\lambda^l), \boldsymbol{\mu}^l(\lambda^l), \mathbf{x}^l(\lambda^l), \mathbf{p^f}^l(\lambda^l), \mathbf{p^s}^l(\lambda^l), \lambda^l \right), \tag{B.50}$$

that we order such that $||x_\nu^l(\lambda^l)|| \leq ||x_\nu^{l+1}(\lambda^{l+1})||$. We take a subsequence of that sequence such that λ^l and all other variables than $x^l(\lambda^l)$ converge (we have already showed that these belong to a compact set):

$$\begin{aligned} &\left(\mathbf{u}^l(\lambda^l), \mathbf{p_{el}}^l(\lambda^l), \mathbf{s}^l(\lambda^l), \mathbf{y}^l(\lambda^l), \boldsymbol{\mu}^l(\lambda^l), \mathbf{p^f}^l(\lambda^l), \mathbf{p^s}^l(\lambda^l), \lambda^l \right) \\ &\quad \to \left(\mathbf{u}^*(\lambda^*), \mathbf{p_{el}}^*(\lambda^*), \mathbf{s}^*(\lambda^*), \mathbf{y}^*(\lambda^*), \boldsymbol{\mu}^*(\lambda^*), \mathbf{p}^{\mathbf{f},*}(\lambda^*), \mathbf{p}^{\mathbf{s},*}(\lambda^*), \lambda^* \right). \end{aligned} \tag{B.51}$$

[18] The reasoning would not be changed by introducing the consumer d.

The subsequence of optimal financial positions $x^l_\nu(\lambda^*)$ for the agent ν is still unbounded. By the dual representation of the risk measure, the valuation of the payoffs in problem $H^{Pf}(., \lambda^*) = 0$ is the lowest value compatible with the set of subjective probabilities. This implies, if x^l_ν is unbounded, that:

$$\forall Q \in \mathcal{M}_\nu(\lambda^*), \forall l \in \mathbb{N} : \qquad \sum_c x^l_{\nu,c}\Big(\mathbb{E}_Q[p^{s,*}_c] - p^{f,*}_c\Big) \geq 0, \tag{B.52}$$

or, equivalently, dividing through by $||x^l_\nu||$ to remain in a compact set,

$$\forall Q \in \mathcal{M}_\nu(\lambda^*), \forall l \in \mathbb{N} : \qquad \sum_c \frac{x^l_{\nu,c}}{||x^l_\nu||}\Big(\mathbb{E}_Q[p^{s,*}_c] - p^{f,*}_c\Big) \geq 0. \tag{B.53}$$

This proves that there exists a limit point $\tilde{x_\nu}$ of a subsequence of $\dfrac{x^l_\nu}{||x^l_\nu||}$ such that

$$\forall Q \in \mathcal{M}_\nu(\lambda^*) : \quad \sum_c \tilde{x}_{\nu,c}\Big(\mathbb{E}_Q[p^{s,*}_c] - p^{f,*}_c\Big) \geq 0. \tag{B.54}$$

Because the financial market clears in problem $H^{Pf}(., \lambda^*) = 0$, there exists a set of market agents $\mathcal{I} \subset \{1, \dots, N, d\} \backslash \{\nu\}$ also taking unbounded positions, and hence for some subsequence

$$\sum_{i\in\mathcal{I}} x^l_{i,c} = -x^l_{\nu,c} \;\text{ or }\; \sum_{i\in\mathcal{I}} \frac{x^l_{i,c}}{||x^l_\nu||} = -\tilde{x_{\nu,c}}. \tag{B.55}$$

Thus, there exists a (non-empty) subset of players $\mathcal{I}' \subset \mathcal{I}$ such that for all $i \in \mathcal{I}'$,

$$x^l_i \to \tilde{x}_i \neq 0, \tag{B.56}$$

$$\sum_{i\in\mathcal{I}'} \tilde{x}_i = -\tilde{x}_\nu, \tag{B.57}$$

$$\forall Q_i \in \mathcal{M}_i(\lambda^*) : \; \sum_c \tilde{x}_{i,c}\Big(\mathbb{E}_{Q_i}[p^{s,*}_c] - p^{f,*}_c\Big) \geq 0, \tag{B.58}$$

where the last expressions are required by the optimality of the unbounded positions x^l_i. These hold for simultaneously $\forall \overline{Q} \in \mathcal{M}_{\mathcal{I}'}(\lambda^*) = \cap_{i\in I'}\mathcal{M}_i(\lambda^*)$. Summing up these conditions on $i \in \mathcal{I}'$, one gets:

$$\begin{aligned} \forall \overline{Q} \in \mathcal{M}_{\mathcal{I}'}(\lambda^*) \quad &: \sum_c \left(\sum_{i\in\mathcal{I}'} \tilde{x}_{i,c}\right)\Big(\mathbb{E}_{\overline{Q}}[p^{s,*}_c] - p^{f,*}_c\Big) \\ &= \sum_c -\tilde{x}_{\nu,c}\Big(\mathbb{E}_{\overline{Q}}[p^{s,*}_c] - p^{f,*}_c\Big) \geq 0. \end{aligned} \tag{B.59}$$

This latter condition (B.59) combined with (B.52) means that $\tilde{x}_\nu$ is a separating hyperplane between $\mathtt{P}_\nu(\lambda^*)$ and $\mathtt{P}_{\mathcal{I}'}$ (where $\mathtt{P}_{\mathcal{I}'} = \cap_{i\in\mathcal{I}'}\mathtt{P}_i(\lambda^*)$). Hence, the intersection **int** $\mathtt{P}_\nu(\lambda^*) \cap$ **int** $\mathtt{P}_{\mathcal{I}'} = \emptyset$, which contradicts condition (B.49).

As the financial position of each agent is bounded, there exists finite $r \in \mathbb{R}$ defining open bounded sets $\widehat{X}_\nu := \{x_\nu : \forall c = 1\dots C,\ |x_{\nu,c}| < r\}$ for $\nu = 1,\dots,N$ and $\widehat{X}_d := \{x_d : \forall c = 1\dots C,\ |x_{d,c}| < r\}$ such that there exists no solution on their boundary.

We can finally define the bounded open set Ξ^{Pf} that does not contain, for every $\lambda \in [0,1]$, a solution to $H^{Pf}(.,\lambda) = 0$ on its closure. The scalar Δ is a positive number.

$$\Xi^{Pf} := \left\{ \left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s\right) \left| \begin{array}{l} \left(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}\right) \in \Xi^P \\ p_c^s(\omega) \in\ ]-\Delta + \min\limits_{p_{el}\in[0,PC]} h_{c,\omega}(p_{el})\ ,\ \Delta + \max\limits_{p_{el}\in[0,PC]} h_{c,\omega}(p_{el})[\\ p_c^f \in]-\Delta + \min\limits_{\omega\in\Omega,\, p_{el}\in[0,PC]} h_{c,\omega}(p_{el})\ ,\ \Delta + \max\limits_{\omega\in\Omega,\, p_{el}\in[0,PC]} h_{c,\omega}(p_{el})[\\ x_\nu \in \widehat{X}_\nu \\ x_d \in \widehat{X}_d \end{array} \right. \right\}. \tag{B.60}$$

□

C Enumeration of the solutions of the numerical example

To find all possible solutions to the numerical example of problem P presented in Sect. 10, we explore all possible combinations of characteristic scenarios and positive investments of both plants. Given the merit order of plants in each scenario, we search for the equilibria that satisfy (recall that one should have more built plants than characteristic scenarios):

- ω_1 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} = 0$. We then solve problem P imposing that $u_{\nu_1} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_2} = 0$.
- ω_1 is a characteristic scenario and $u_{\nu_2} > 0,\ u_{\nu_1} = 0$. We then solve problem P imposing that $u_{\nu_2} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_1} = 0$.
- ω_1 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} > 0$. We then solve problem P imposing that $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_1)$.
- ω_2 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} = 0$. We then solve problem P imposing that $u_{\nu_1} = \mathrm{LOAD}(\omega_2)$ and $u_{\nu_2} = 0$.
- ω_2 is a characteristic scenario and $u_{\nu_2} > 0,\ u_{\nu_1} = 0$. We then solve problem P imposing that $u_{\nu_2} = \mathrm{LOAD}(\omega_2)$ and $u_{\nu_1} = 0$.

- ω_2 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} > 0$. We then solve problem P imposing that $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_2)$.
- ω_3 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} = 0$. We then solve problem P imposing that $u_{\nu_1} = \mathrm{LOAD}(\omega_3)$ and $u_{\nu_2} = 0$.
- ω_3 is a characteristic scenario and $u_{\nu_2} > 0,\ u_{\nu_1} = 0$. We then solve problem P imposing that $u_{\nu_2} = \mathrm{LOAD}(\omega_3)$ and $u_{\nu_1} = 0$.
- ω_3 is a characteristic scenario and $u_{\nu_1} > 0,\ u_{\nu_2} > 0$. We then solve problem P imposing that $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_3)$.
- ω_1 and ω_2 are characteristic scenarios and $u_{\nu_2} > 0,\ u_{\nu_1} > 0$. In this case, given the merit order in both scenarios, two possibilities arise: $u_{\nu_1} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_2)$, or $u_{\nu_1} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_2} = \mathrm{LOAD}(\omega_2)$. We then solve two instances of problem P imposing the previous conditions.
- ω_1 and ω_3 are characteristic scenarios and $u_{\nu_2} > 0,\ u_{\nu_1} > 0$. The only case where this is possible is when $u_{\nu_1} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_3)$ (given the merit order in both scenarios, the other possibility would be $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_3)$, which is not possible given that $\mathrm{LOAD}(\omega_1) \neq \mathrm{LOAD}(\omega_3)$). We then solve problem P imposing that $u_{\nu_1} = \mathrm{LOAD}(\omega_1)$ and $u_{\nu_1} + u_{\nu_1} = \mathrm{LOAD}(\omega_3)$.
- ω_2 and ω_3 are characteristic scenarios and $u_{\nu_2} > 0,\ u_{\nu_1} > 0$. In this case, given the merit order in both scenarios, two possibilities arise: $u_{\nu_2} = \mathrm{LOAD}(\omega_2)$ and $u_{\nu_1} + u_{\nu_2} = \mathrm{LOAD}(\omega_3)$, or $u_{\nu_1} = \mathrm{LOAD}(\omega_3)$ and $u_{\nu_2} = \mathrm{LOAD}(\omega_2)$. We then solve two instances of problem P imposing the previous conditions.
- Note that all combinations do not necessarily lead to a solution.

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