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Generation Capacity Expansion in a Risky Environment: A Stochastic Equilibrium Analysis

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We cast models of the generation capacity expansion type formally developed for the monopoly regime into equilibrium models better adapted for a competitive environment. We focus on some of the risks faced today by investors in generation capacity and thus pose the problem as a stochastic equilibrium model. We illustrate the approach on the problem of the incentive to invest. Agents can be risk neutral or risk averse. We model risk aversion through the CVaR of plants' profit. The CVaR induces risk-adjusted probabilities according to which investors value their plants. The model is formulated as a complementarity problem (including the CVaR valuation of investments). An illustration is provided on a small problem that captures several features of today's electricity world: a choice often restricted to coal and gas units, a peaky load curve because of wind penetration, uncertain fuel prices, and an evolving carbon market. We assess the potential of the approach by comparing energy-only and capacity market organizations in this risky environment. Our results can be summarized as follows: a deterministic analysis overlooks some changes of capacity structure induced by risk, whether in the capacity market or energy-only organizations. The risk-neutral analysis also misses a shift towards less capital-intensive technologies that may result from risk aversion. Last, risk aversion also increases the shortage of capacity compared to the risk-neutral view in the energy-only market when the price cap is low. This may have a dramatic impact on the bill to the final consumer. The approach relies on mathematical programming techniques and can be extended to full-size problems. The results are illustrative and may deserve more investigation.

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1. Introduction

Generation capacity expansion problems were cast in optimization form more than 50 years ago. Optimization fitted well with the regulated monopoly regime of the time. Companies operated under an obligation to serve, and in compensation were guaranteed a certain rate of return on investment. Optimization ensured demand satisfaction at minimal investment and operation cost computed over a certain horizon. Mathematical programming techniques of all sorts allowed one to improve the representation of the system. This led to a better description of plant characteristics, the inclusion of system criteria such as reliability or the insertion of global economic features such as larger geographic areas and the emergence of new technologies. Fuel price and demand risks existed, but their costs were largely passed to the consumer.

Capacity expansion is quite different today, at least in restructured systems. The industry operates in a competitive environment, even though it remains subject to various regulatory interventions, particularly on the environmental side. Risk is overwhelming, but the customer no longer serves as a cushion against it. This paper examines the question of investment in generation capacity in a competitive system operating in a risky environment and subject to regulatory interventions. It presents a computational approach that remains close to the mathematical programming framework used in former capacity expansion models while accommodating some of the features of today's restructured industry. It therefore departs from alternative approaches that view power plants as financial assets and value them as such. It also departs from industrial economics models that concentrate on particular phenomena of market power and arrive at results that are dense and rich, but difficult to extend. For the sake of clarity and in order to provide a benchmark for our developments and numerical experiments, we base the discussion on an example that is an elaboration of Joskow (2007). It should be clear, however, that the combination of our economic assumptions and mathematical programming approach is not limited to this example, but allows for full-size implementations.

The paper is organized as follows. We account for competition in generation by moving from cost minimization to profit maximization and consider endogenous electricity prices that depend on endogenous investments. This is done through an equilibrium model described in §2. The almost guaranteed generation capacity of the regulatory period has given place today to a general concern of resource adequacy. We take up that question through simple versions of forward capacity and energy-only markets. These are inspired by Joskow (2007) and introduced in §3. We explain in §4 that we disregard questions of market power and concentrate on an economy composed of price-taking agents but subject to imperfections due to regulatory action. We illustrate the latter by a question taken from carbon trading in the EU (the EU-ETS), namely, the allocation of free allowances to entrants in the power system. This mix of investment incentive and climate change issues respectively, presented in §§3 and 4, constitutes the sample of new questions faced by capacity expansion today that we cast in a risk context in the rest of the paper.

Uncertainties now accumulate at an unprecedented pace in the power sector. We consider both standard and new risk factors, namely, fuel prices and climate change policies. Section 5 introduces these risk factors. Mathematical programming traditionally deals with uncertainty through expectations of costs or profits, that is, with a zero price of risk. Current events and new methodological developments justify moving away from this practice. We resort to the relatively recent notion of coherent risk function and its implementation in mathematical programming to model investor's behavior towards risk. We do this in a computable equilibrium model, which we believe is novel in the literature. This is presented in §6. We illustrate the results of our modeling approach in §7 in terms of investments under different assumptions and compare them to those obtained by a more classical deterministic approach. These computations are conducted on the example used throughout the paper. We offer an alternative economic and financial discussion of these results in §8. Examples help intuition but hide generality. We recall that our approach is based on mathematical programming techniques, and hence is amenable to full-scale models, at least within our economic assumption of price-taking agents. The conclusion briefly discusses further points elaborated in forthcoming papers. Two appendixes complete the paper. One is a brief reminder of the CVaR with an illustration of its computation. The second appendix is an assessment of the importance of the different risks appearing in the model. An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/. We explain that fuel risk is less important than the total reduction of emission imposed on the system. This risk is itself less important than the regulatory uncertainty here represented by the future regime of the EU-ETS.

This work is related to different aspects of the literature that we briefly review as we progress through the paper. As mentioned, generation capacity expansion models were among the first applications of mathematical programming. The model of §2 is probably the simplest one of this type. It is constructed to streamline the presentation but the reader interested in generation capacity expansion can find other references in §2. The question of the incentive to invest is rooted in the literature of peak load pricing. As mentioned, we rely on Joskow's (2007) insightful analysis of investments, but mention other work in §3. Market power pervades most of the literature of restructured electricity systems, but is much less present in capacity expansion. We refer to the existing literature to explain in §4 why we assume price-taking behaviour. Free allowances have drawn considerable attention in emission trading; they also raise interesting computational issues. We consider allocation of allowances in §4 to illustrate regulatory interventions in a competitive market. Lastly, the introduction of risk functions in the equilibrium model is the main contribution of the paper. Whereas utility functions are the traditional instruments of economists to model risk aversion, risk functions emerged from the finance literature much later. Their appearance in optimization goes back to 2002, with the full development of the field taking place a few years later. To our knowledge it is the first time that risk functions are used in a computable equilibrium context. We briefly review these questions in §6. We are not aware of analysis relating equilibrium models to the notions of market incompleteness mentioned in §8.

2. The Reference Capacity Expansion Model

Capacity expansion models were introduced by Electricité de france (EdF) in the early days of linear programming (see the collection of early papers in Morlat and Bessiere 1971). Anderson (1972) probably remains the reference survey of these first developments. Extensions of these models to nonlinear, integer, and stochastic programming techniques can be found in many papers (e.g., Sherali et al. 1987, Murphy et al. 1987, Sirikum et al. 2007). This paper reasons on a skeleton of these early tools: we first present a deterministic model in this section and later extend it to a stochastic version when discussing risk.

Consider a two-stage problem: one invests in different types of capacity in stage 0 and operates them in stage 1. Following Joskow (2007), our benchmark example is a three-technology problem involving coal, combined cycle gas turbine (CCGT), and open cycle gas turbine (OCGT). Note that the choice of one agent investing in one technology corresponds to a project finance approach. Each equipment type is characterized by its annual investment and fixed operating costs and a CO_2 emission factor. Because risk premia are endogenous in the stochastic equilibrium model, investment costs are meant to be annualized from overnight construction costs at the risk-free rate (see Table 1). We consider one entity per technology for which

Table 1. Fixed annual cost and emission in a three-technology world.

	Coal	CCGT	OGGT
I: annual capacity and fixed operating cost (euro/KW)	160	80	60
E: emission t/MWh	1	0.35	0.6

costs are assumed to be linear in capacity or operations levels, implying that we neglect economies of scale in generation. This has been the usual assumption in models of restructured electricity systems since Anderson (1972). It could, however, be questioned in view of the nuclear renaissance; past experience with the French nuclear program has indeed shown that important economies of scale exist with this technology. The same will hold for Carbon Capture and Sequestration (CCS). Embedding economies of scale may thus be important for future developments. Emission factors for each plant are in tons of CO_2 per MWh. These figures are stylized views on costs and emission factors found in the European industry at the time of this writing. They do not correspond to particular projects, but are realistic. The operating costs will be derived from fuel prices in §5. The example is small, but the model is general.

Former capacity expansion models as well as several analyses of resource adequacy suppose a price-insensitive demand represented by a load duration curve. We follow suit, more to simplify the presentation than because of technical necessity. Introducing price-sensitive demand is technically straightforward provided a demand function of electricity is available, which is unfortunately often not the case. The load duration curve is segmented in six demand blocks in order to keep the model simple, while still guaranteeing sufficient detail for arriving at meaningful results. The three highest demand segments and the lowest segment exhibit a rather peaky pattern that accounts for the penetration of wind power requested by EU policy. Demand variability is different from long-term structural changes in that it is repeated and cancels out over a longer period rather than branching into different evolutions of state of nature. On the other hand, policies on renewable support or the introduction of smart grids scenarios might change the shape of the load duration curve and hence add an additional risk factor. Table 2 gives the relevant figures and units.

We now formally generalize this simple example to accommodate an arbitrary number of plant types and time

Table 2. Reference load duration curve and its decomposition in time segments.

	Power level and utilisation					
D: MW	86,000	83,000	80,000	60,000	40,000	20,000
T: duration (1,000 hours)	0.01	0.04	0.31	4.4	3	1

Table 3. Sets.

$\ell \in L$	Demand segments
$f \in F$	Fuel scenarios
$n \in N$	Nap scenarios
$b \in B$	Allocation scenarios
$k \in K$	Plant type

segments. Because most of the literature refers to deterministic capacity expansion models, a deterministic approach constitutes the natural benchmark for assessing the possible additional insight brought by stochastic equilibrium models. Notation is given in Tables 3, 4, and 5 for the stochastic model elaborated on later and particularized in this section to the deterministic version.

The deterministic model supposes one fuel price scenario (set F) and one scenario of total allowances available to the power sector (set N for National Allocation Plan or NAP). It also supposes full auctioning as the single allowance allocation method (set B). The sets F, N, and B of Table 3 therefore each only contain a single element in the deterministic model. Table 4 specializes as follows. I, D, T, and E are defined in Tables 1 and 2 and require no further explanation. The amount of free allowances (in tons of CO_2/year MW) received in allocation scenario b by an investor building a plant k is noted $A(k, b)$; it is set to zero in this section, where we assume full auctioning of allowances. PC can be interpreted as a Price Cap or a Value of Lost Load (VOLL) set by the regulator. The stochastic model involves three probability spaces (F, N, and B), and hence three probability measures that we, respectively, note P_f, P_n, and P_b. The probabilities $P_f(F)$, $P_n(N)$, and $P_b(B)$ are all one in the deterministic model. Some coefficients and variables are only relevant in the risk model presented from §5 on and will be explained there. This is the case for the coefficient of risk aversion α, the "risk-adjusted" probabilities $q(K; F, N, B)$ and the Value at Risk (VaR(K)).

Given these particularities, we introduce the reference capacity expansion model by first stating the profit-maximization problems of the individual generator and then deriving the KKT conditions and concatenating them into

Table 4. Parameters.

$I(K)$	Annual investment and maintenance cost in euro/KW
$D(L)$	Demand in MWh
$T(L)$	Duration in 1,000 h
$E(K)$	Emission level in t/MWh
$A(K, B)$	Allocation by technology and scenario in 1,000 allowance/MW per year
$P_f(F)$	Probabilities
$P_n(N)$	Probabilities
$P_b(B)$	Probabilities
NAP(n)	CO_2 system cap in million tons
$C(K, F)$	Fuel costs euro/MWh
PC	Price cap in euro/MWh
$\alpha(k)$	Risk aversion

Table 5. Full set of variables.

$x(K)$	Capacity of technology K in MW
$y(K, L, F, N, B)$	Production in MWh
$z(L, F, N, B)$	Shortage in MWh
$\mu(K, L, F, N, B)$	Marginal value of capacity in euro per KW per h
$\pi(L, F, N, B)$	Price in euro per MWh
$\lambda(F, N, B)$	Allowance price in euro per ton
ν	Marginal value of capacity in the forward capacity market in euro per KW
$u(K; F, N, B)$	Value of the loss with respect to VaR in euro per KW
$q(K; F, N, B)$	Risk-adjusted probability
$VaR(K)$	VaR in euro per KW

Note. Equilibrium variables are included.

a mixed complementarity problem (see Gürkan et al. 2009 for a more extensive discussion of this approach). Market clearing conditions for the electricity and the CO2 price are added to arrive at a square system. The individual generator maximizes her profit by choosing the optimal capacity $x(k)$ and operation $y(k, l)$ for given electricity, fuel, and CO2 prices, respectively, $C(k)$, $\pi(\ell)$, and λ.

Dual variables appear at the right of the constraints. Recall that indices f, n, and b are dropped in this deterministic (single-scenario) model.

$$\max_{w(k),\, y(k)} \sum_{\ell \in L} T(\ell)\left[\sum_{k\in K} \pi(\ell) - (E(k) + C(k))y(k, \ell)\right] - I(k)x(k) \qquad (1)$$

$$\text{s.t.} \quad 0 \leqslant x(k) - y(k, \ell) \quad \mu(k, \ell) \qquad (2)$$

$$0 \leqslant y(k, \ell). \qquad (3)$$

The interpretation of the agents' profit-maximization model is straightforward. The objective function (1) is the time-weighted sum over the different time segments ℓ of the market price minus the plant-operating costs times the duration of operation of the plant. Operating costs include the fuel cost $C(k)$ as well as the emission cost of the plant $E(k)\lambda$. The latter refers to the EU Emission Trading Scheme (EU-ETS) operating in the EU since January 2005. The EU-ETS is a cap-and-trade system that requires firms from different economic sectors to surrender emission allowances for every emitted ton of CO_2.

Applying standard duality theory, we convert the optimization problem into complementarity form. It is this form, generalized to the case of several scenarios, that we shall refer to in the following section when discussing the equilibrium model. Relations (4), (5), and (6) are the complementary slackness conditions of the primal problem (1) to (3).

$$0 \leqslant I(k) - \sum_{\ell \in L} T(\ell)\mu(k, \ell) \perp x(k) \geqslant 0. \qquad (4)$$

The economic interpretation of (4) is straightforward: one invests in plant k when its investment cost is equal to the marginal value of the capacity. One does not invest when it is larger.

$$0 \leqslant C(k) + \mu(k, \ell) + E(k)\lambda - \pi(\ell) \perp y(k, \ell) \geqslant 0 \qquad (5)$$

$$0 \leqslant x(k) - y(k, \ell) \perp \mu(k, \ell) \geqslant 0. \qquad (6)$$

Relation (5) states that one operates a plant whenever the market price is higher than its operating costs and (6) sets $\mu(k, \ell)$ equal to the marginal value of capacity in time segment ℓ.

Conditions (7) set the price of electricity to the sum of the fuel, emission, and capacity costs of the operating plants or to the price cap if there is insufficient capacity. Specifically, the price of electricity is equal to the sum of the fuel and emission allowance costs of the most expensive running plant if this latter is not at capacity. It is useful to point out that ramping rates can invalidate that principle and require a more complex model. This is beyond the scope of this paper, where we retain this standard simplification of calling upon the most expensive running plant (merit order rule). The complementarity condition (8) caps the electricity price by PC:

$$0 \leqslant \sum_{k \in K} y(k, \ell) + z(\ell) - D(\ell) \perp \pi(\ell) \geqslant 0 \qquad (7)$$

$$0 \leqslant \mathrm{PC} - \pi(\ell) \perp z(\ell) \geqslant 0 \qquad (8)$$

$$0 \leqslant \mathrm{NAP} - \sum_{\ell \in L} T(\ell) \sum_{k \in K} E(k)y(k, \ell) \perp \lambda \geqslant 0. \qquad (9)$$

(9) states that the total emission over the year does not exceed the total amount of allowances NAP determined by the National Allocation Plan. The amount is exogenous and only applies here to the power sector. The total emission is computed by summing the production of CO_2 over all time segments. The dual variable λ of constraint (9) is interpreted as the allowance price. The system of Equations (4) to (9) constitutes our reference model.

It is straightforward to modify this model into one with demand response. This requires a set of demand functions, one for each time segment, and some slight adaptations of the equations to replace the fixed demand by the price dependent one. We avoid that discussion in order not to overload the presentation.

3. Resource Adequacy: Capacity Markets and Energy-Only Markets

The theory of peak load pricing is instrumental for understanding investments in generation capacity. The first developments took place in Electricité de france (see the collection of early papers in Morlat and Bessiere 1971), together with the construction of capacity expansion models. The two are closely related as capacity expansion models can be seen as the computational version of peak load pricing. Although the theory was initially constructed for monopolies, it quickly became clear that it also applied to competitive markets (Crew et al. 1995). The theory further developed to account for uncertainty in demand (Carlton 1977) or uncertainty in demand and supply (Chao 1983). Panzar (1976) shows that the particular payment of capacity of the theory of peak load pricing derives from a property of electricity and would take a more usual form with standard neoclassical technologies. Stoft (2002) was probably the first one to adapt peak load pricing to capacity expansion in restructured systems. The subject has since seen an explosion of interest in both the literature and in practice (see in particular Hogan 2005, Oren 2005, Cramton and Stoft 2006). We here follow Joskow's (2007) analysis of the subject.

Consider a perfectly competitive power exchange that remunerates each operating plant at the short-run marginal cost, taken as the fuel cost, of the most expensive operating unit. Joskow's presentation needs to be slightly adapted here to account for allowance trading: plants are still operated in merit order, but the short-run marginal cost now contains both the fuel cost and the value of the marginal allowance $E(k)\lambda$ surrendered because of emissions. It is easily verified that complementarity conditions (5) and (6) reflect this mechanism. Relation (6) introduces the capacity rent $\mu(k, \ell)$ of plant k in time segment ℓ. Relation (7) states that the electricity price $\pi(\ell)$ is no greater than the sum of the fuel cost $C(k)$, the emission cost $E(k)\lambda$, and the scarcity rent $\mu(k, \ell)$ when plant k does not operate in time segment ℓ. It is equal to that sum when plant k operates in time segment ℓ. This relation takes a particular interpretation when there is no plant left to set the price $\pi(\ell)$ (when all capacity is used up). The standard reasoning in perfectly competitive markets is to assume a downward-sloping curve that intersects the capacity limit and sets the price at this intersection level: the scarcity rent rations the demand so as to make it compatible with the existing capacity. The reality is that, absent demand-side bidding, the short-term demand function of electricity is considered inelastic, implying that both demand and the vertical line may not intersect. The market then determines neither the price of electricity nor the scarcity rent. This is the origin of the market failure initially described in Stoft (2002). This also justifies a regulatory intervention. We consider two solutions, an energy-only market and a capacity market.

The energy-only organization prices electricity at the (high) value of lost load (VOLL) when demand is curtailed. This principle is applied in the Australian and Texan markets: it is obtained in the model by assuming that PC is set at VOLL. A more sophisticated method is to increase the price of electricity when the operating reserve margin decreases (Hogan 2005). An alternative approach is to implement a capacity (forward) market whereby the TSO or the regulator imposes some capacity target in line with the expected demand and conducts an auction to procure the needed capacity. In New York, capacity is auctioned in the year it is delivered, whereas New England auctions capacity forward three years in advance, and new plant can select a commitment period of 1–5 years to bid for.

None of these market organizations has been formally implemented in Europe, with the result that EU electricity systems implicitly operate energy-only markets, without having explicitly decided on a price cap PC. In contrast European competition and regulatory authorities pay considerable attention to possible monopoly pricing practices and tend to interpret markups with respect to fuel cost more as exercises of market power than as scarcity rents. Prices anywhere close to the 10,000 Dollar/MWh as in Australia during curtailment are thus unlikely in Europe. Taking stock of these different elements, we consider three cases of energy-only markets where PC is, respectively, set to 10,000, 1,000, and 250 euros/MWh. We justify these choices as follows (considering the currencies as equal): 10,000 is a reference figure in Australia; 1,000 appears in pools of the East Coast of the United States, and was a technical limit in European exchanges for many years; lastly, 250 is lower than the 300 of California and higher than the 180 of Spain.

A capacity market requires a modification of the investment condition (4), which is now replaced by two complementarity constraints. Condition

$$0 \leqslant \sum_{k \in K} x(k) - \max_{\ell \in L} D(\ell) \perp \nu \geqslant 0 \tag{10}$$

relates the capacity target and the capacity value ν. Capacity is auctioned before the investment decision is made, with the result that the price of the capacity is known at the time of investment. This differs from real implementations, where because of market design the price of the capacity is only known after long lead-time plants have been committed. This is a limitation of the model, and additional work is required to remove it. We also neglect random outages and demand uncertainty and hence set the capacity requirement to the maximal demand. The reality is also different. Existing capacity markets in the United States require that installed capacity exceeds peak demand by a certain reserve margin to cover outages and demand uncertainty. Also, the regulator is unlikely to set the capacity target choosing the highest demand among all scenarios. These two assumptions simplify the presentation and can easily be replaced by a slightly more technical modeling.

The second condition

$$0 \leqslant I(k) - \nu - \sum_{\ell \in L} T(\ell)\mu(k) \perp x(k) \geqslant 0 \tag{11}$$

specifies the new investment criterion. It is identical to relation (4) except for the addition of the capacity remuneration ν appearing in (10). Note that we only consider a two-stage model, and hence we assume that the capacity payment is certain for the lifetime of the plant (whereas in the real world there remains uncertainty about the amount of revenue accruing from the subsequent auctions).

4. Market Imperfection and Regulation

The above capacity market and energy-only models assume price-taking firms and consumers. This is the usual perfect competition counterfactual. Real markets depart from perfect competition. The literature on electricity restructuring commonly supposes that generators exercise market power. We first justify why we do not follow that path. Real markets can be affected by other imperfections such as average cost pricing (e.g., Zhao et al. 2010, Oggioni and Smeers 2009), free allocation of allowances, or price caps. These do not detract from the assumption of price-taking firms and consumers. We explain in the second part of this section that we concentrate on free allocation of allowances as an example of these latter market imperfections and treat price caps in sensitivities.

4.1. On Market Power

The analysis of monopoly or oligopoly power in short-run electricity markets is abundant in the economic and computational literature dealing with the sector; its treatment in investment problems is more limited. We restrict ourselves to a few elements that we believe are relevant for computational models. We discard at the outset strategic models of the real option type; their embedded dimension of investment timing is a subject in itself, and their assumption that new investments are fully utilized is incompatible with the capacity margin formation that underlies capacity market and energy-only models. In more classical approaches, von der Fehr and Harbord (1997) were probably the first to offer a treatment of investments in generation capacity under imperfect competition. They consider players investing in a mix of technologies, and find a distortion of investments towards peak units. Arellano and Serra (2007) find similar results. More recently, Zöttl (2008) presented an application of techniques derived from supply function equilibrium to this investment problem and finds a bias towards base load equipment. These models assume symmetric players and concentrate on symmetric equilibrium. They differ by their assumptions of competition on the spot and investment markets, and accordingly arrive at different, sometimes contradictory, results. Other models restrict their investigation to symmetric equilibria with a single technology and a single time segment (e.g., Gabszewicz and Poddar 1997, Reynolds and Wilson 2000, Boom and Buehler 2007, Grimm and Zoettl 2008, de Frutos and Fabra 2007, and de Frutos et al. 2008). The results again depend on the underlying assumptions of competition. Joskow and Tirole (2007) consider different states of the world but assume strategic behaviour in a single time segment (the peak); their treatment of price-sensitive and price-insensitive consumers places their paper somewhat apart in the literature. Murphy and Smeers (2005) probably offer the only available treatment of asymmetric equilibria for asymmetric players confronted with a full load duration curve. They consider a very stylized model and assume Cournot competition at both the investment and operations stages. Their results are limited, but again at variance with some of the above literature: they find an increase of based load capacity with respect to a classic open-loop Cournot model. Their paper also shows the considerable technicalities that one needs to get into for dealing with the inherent nonconvexities arising in asymmetric equilibria of games with asymmetric players. We believe that these economic theory papers allow one to delineate the possibilities of computational models for analyzing investments under market power. The price-taking counterfactual invoked in this paper serves its standard benchmarking role and allows for the introduction of new ideas without venturing into market power. There is hope to extend these ideas to market power with symmetric players, even though the treatment of their nonconvexities is already a challenge and the results are likely to be unstable with respect to the underlying assumptions of competition. Realistic asymmetric games remain hopeless because of the nonconvexities that they imply. We here concentrate on the counterfactual perfectly competitive market.

4.2. On Regulation

Although the insertion of market power in investment models will probably remain a challenge for some time, we are today capable of casting important market imperfections observed in the real world in computational form. We illustrate this point on a question taken from the EU Emission Trading Scheme. The allocation of free allowances in cap-and-trade systems such as the EU-ETS is a controversial subject. This is particularly true for those granted to entrants or new investments. Free allowances are true subsidies that depend on the prevailing carbon price. It is generally accepted that subsidies distort investments, and an extensive literature elaborates on this subject for the case of emission allowances (e.g., Burtraw and Palmer 2008, Cramton and Kerr 2002, Neuhoff and Matthes 2008). The alternative is to auction allowances. The first version of the EU-ETS provided for the allocation of free allowances to entrants according to certain rules. Hobbs et al. (2006) analyze the impact of these rules on investment in a deterministic computational model. We here introduce a simple deterministic version of the problem that we expand in a stochastic model in §5. We therefore consider price-taking agents operating in imperfectly competitive markets (Oggioni and Smeers 2009 give another illustration of market imperfections in investments).

We therefore retain the cases of both auctioned and free allowances. In this latter case we suppose that their number can be proportional to the installed capacity irrespective of the technology, or benchmarked with respect to the best available technology for each type of plant (computed on the basis of a certain number of hours of operations predetermined by the regulator). The number of free allowances is thus adapted to the investment, but not to the operation of the equipment. Allowances that are not allocated for free are auctioned. Each policy is summarized by the coefficient a (see Table 4) specifying the amount of free allowances per unit of installed capacity. Full auctioning implies setting a to zero. Proposals of these types have all been part of the discussion.

The introduction of free allowances in an energy-only market requires a slight modification of (4) into:

$$0 \leqslant I(k) - A(k)\lambda - \sum_{\ell \in L} T(\ell)\mu(k,\ell) \perp x(k) \geqslant 0 \qquad (12)$$

where $A(k)\lambda$ is the value of free allowances received by plant k. Note that (4) and (12) are equivalent when $A(k)$ are zero. Relation (12) can be interpreted as follows: one invests when the investment cost of the plant is equal to the sum of the value of allowances received free and the marginal value of the capacity.

The investment criterion for the capacity market is similarly written:

$$0 \leqslant I(k) - A(k)\lambda - \nu - \sum_{\ell \in L} T(\ell)\mu(k,\ell) \perp x(k) \geqslant 0 \qquad (13)$$

One should note that the complementarity models with free allowances are no longer equivalent to optimization problems. From a mathematical programming point of view, they involve nonmonotone mappings, and hence are nonconvex problems.

5. Overview of Power Sector Risks

The power sector evolved from an almost risk-free environment in the days of regulation (except for the prudence reviews in the United States) to one of extreme uncertainty today. The following sketches the situation faced by European investors in generation capacity. We identify two types of risk that we characterize as exogenous and regulatory. Fuel price risk comes from worldwide movements and can be seen as the paradigm of exogenous risks. Uncertainties due to the implementation of the EU-ETS illustrate regulatory risks. They are generated by the institutional process and are thus endogenous. We emphasize again the illustrative character of the discussion and our focus on methodology: a company will consider more scenarios (e.g., on penetration of renewables) than what is discussed here. Our mathematical programming approach would not be immediately affected by more scenarios, technologies, and time segments.

5.1. Exogenous Risk

Fuel price and demand risks have accompanied the electricity sector since the first oil crisis in the early seventies. We here adopt a very simplified view and only consider two scenarios of gas prices, which we assume occur with equal probability. We convert them into fuel cost scenarios through plant efficiencies. They are depicted in Table 6. Production from coal power stations is assumed to cost 30 euro/MWh.

Table 6. Plant-operating cost scenarios (in euro/MWh).

	Scenario f1	Scenario f2
CCGT	45	68
OCGT	80	120
Prob.	0.5	0.5

In order to simplify the discussion, we do not include demand uncertainty in this paper. The introduction of this risk factor is not technically complicated, but it would require a significant expansion of the discussion of the results. We leave this extension to another paper.

5.2. Regulatory Risk

Policy implementation often proceeds by trial and error, and hence creates risks. We here refer to the state of the EU-ETS at the beginning of 2008 and single out two major risk factors faced by investors at that time.

The forthcoming required reduction of emissions was (and still is at the time of this writing) one of these risks. We stylize the uncertainty on that total amount in the scenarios depicted in Table 7. These figures only cover the power sector and have been computed by applying reductions of 20% and 30% to emissions of the sector simulated without ETS.

Allowances can be purchased or received at no cost. The possibility of still receiving free allowances was the second major uncertainty in early 2008. The proposals for the revision of the law strongly suggested a movement towards full auctioning, but this was not certain. Extensive discussions were still taking place as to the best mechanism for allocating allowances that would be granted at no cost. We summarize these possibilities in the coefficient A. The scenarios corresponding to these policies are represented in Table 8, together with their assumed probabilities. Recall that figures are in 1,000 allowance per MW of installed capacity. Full auctioning simply sets a to zero. Its higher probability reflects the proposals of the European Commission at the time. (Full auctioning for the power sector was finally passed into law, but with significant exceptions).

The first allocation method might lead to allocating more allowances than the cap (in this case the regulator would buy back allowances from the market to allocate them for free). Recall that allowances that are not allocated for free are auctioned.

Table 7. NAP scenarios.

	Scenario n1	Scenario n2
In million ton	200	240
Prob.	0.5	0.5

Table 8. Scenarios of free allocation by unit of capacity (1,000-allowance/MW).

	Scenario b1	Scenario b2	Scenario b3
Coal	6	2.1	0
CCGT	2.1	2.1	0
OCGT	1.2	2.1	0
Prob.	0.1	0.3	0.6

5.3. Summing Up and Modeling Implications

The above discussion introduces two price scenarios (set F), three allowance allocation modes (set B), and two NAP scenarios (set N). This amounts to a total of 12 scenarios identified by (f, n, b). The probabilities of these scenarios are obtained by straight multiplication of $P_f(f)$, $P_n(n)$, and $P_b(b)$, because we assume that events on fuel costs, NAPs, and allocation modes are independent. The following complementarity conditions extend the deterministic relations (6) to (11) to the uncertain world by indexing variables and constraints on scenarios. For technical reasons, we also make demand $D(\ell)$ dependent on the scenarios and write $D(\ell, f, n, b)$: using the same peak demand in all scenarios introduces primal degeneracy in the model, and hence a multiplicity of dual variables. One could (and should if data were available) also see this dependence from an economic point of view: different fuel price and EU-ETS policies will effectively result in different levels of power demand. Whatever the argument, the $D(\ell, f, n, b)$ are constructed as small perturbations of the base demand $D(\ell)$ with the scenarios, but should not be seen as additional demand risk factors. The equilibrium of the short-run market is then represented as follows:

for all (f, n, b)

$$0 \leqslant x(k) - y(k, \ell, f, n, b) \perp \mu(k, \ell, f, n, b) \geqslant 0 \tag{14}$$

for all (f, n, b)

$$0 \leqslant \sum_{k\in K} y(k, l, f, n, b) + z(\ell, f, n, b) - D(\ell, f, n, b) \perp \pi(\ell, f, n, b) \geqslant 0 \tag{15}$$

for all n

$$0 \leqslant \mathrm{NAP}(n) - \sum_{\ell\in L} T(\ell) \sum_{k\in K} E(k) y(k, \ell, f, n, b) \perp \lambda(\ell, f, n, b) \geqslant 0 \tag{16}$$

for all (f, n, b)

$$0 \leqslant C(k, f) + \mu(k, \ell, f, n, b) + e(k)\lambda(f, n, b) - \pi(\ell, f, n, b) \perp y(k, \ell, f, n, b) \geqslant 0 \tag{17}$$

for all (f, n, b)

$$0 \leqslant \mathrm{PC} - \pi(\ell, f, n, b) \perp z(\ell, f, n, b) \geqslant 0. \tag{18}$$

The investment criterion in the energy-only market is given as follows:

for all k

$$0 \leqslant I(k) - \sum_{f\in F,\, n\in N,\, b\in B} P_b(b) A(k, b) P_f(f) P_n(n) \lambda(f, n, b)) - \sum_{\ell\in L,\, f\in F,\, n\in N} T(\ell) P_b(b) P_f(f) P_n(n) \mu(k, \ell, f, n, b) \perp x(k) \geqslant 0. \tag{19}$$

The investment criterion in the capacity market is stated as follows:

$$0 \leqslant \sum_{k\in K} x(k) - \max_{\ell\in L,\, f\in F,\, n\in N, b\in B} D(\ell, f, n, b) \perp \nu \geqslant 0 \tag{20}$$

for all k

$$0 \leqslant I(k) - \sum_{f\in F,\, n\in N,\, b\in B} P_f(f) P_n(n) P_b(b) A(k, b) \lambda(f, n, b)) - \nu - \sum_{\ell\in L,\, f\in F,\, n\in N,\, b\in B} T(\ell) P_f(f) P_n(n) P_b(b) \mu(k, \ell, f, n, b) \perp x(k) \geqslant 0. \tag{21}$$

6. Attitude Towards Risk

6.1. Risk-Neutral and Risk-Adjusted Versions of the Investment Model

The capacity market and energy-only models discussed so far assume risk-neutral investors; they describe a market where the risk premium is zero. Relations (14) to (19) refer to the energy-only market. The combination of relations (14) to (18) with (20) and (21) gives the capacity market model. In both organizations, investors base their decisions solely on their expected profit and disregard the distribution of these profits. We now extend the economic interpretation of the complementarity conditions to alternative behaviors towards risk and begin by revisiting the investment criteria (19) (energy-only model) and (23) and (21) (capacity market). First consider relation (19). It states that one invests in plant k if the expected gross margin accruing to that technology is equal to its investment cost. Relation (21) has a similar interpretation after adapting the gross margin to account for the capacity price ν. Gross margins are computed for each scenario (f, n, b) and equal the revenue from sales minus fuel and emission costs. Adapting Joskow (2007) to the case with an allowance market, the electricity price is equal to the sum of the fuel and allowance opportunity costs of the most expensive plant in operation when there remains some spare capacity. The electricity price is equal to the price cap when all capacity is used up.

Abusing the language of mathematical finance, we refer to the probability $P_b(b)P_f(f)P_n(n)$ of scenario (b, f, n) introduced in §5 as the "statistical probability" and note it P. Risk-neutral investors decide by taking expectations on the basis of this statistical probability in their investment criteria (19) or (21). We model risk-averse investors by assuming that they behave according to a modified probability that we refer to as the "risk-adjusted probability" and denote it by Q (in the model we denote the risk-adjusted probability by q to highlight that, unlike P, it is endogenous). Before explaining the construction of this risk-adjusted probability, we observe that it allows one to formally replace the statistical probability P by any risk-adjusted probability Q in relations (18) or (21) without changing the rest of the model. We thus refer to the risk-neutral (RN, using the probability P) and the risk-averse (RA, using the probability Q) versions of the equilibrium model. We complete the risk-averse model by specifying how the Q probability is obtained. We do this by resorting to the theory of coherent risk functions.

6.2. Coherent Risk Functions and Equilibrium with Risk-Averse Investors

Coherent risk functions were introduced in the seminal paper of Artzner et al. (1989) and extensively developed later by several authors. Artzner et al. (1989) define "coherence" by four axioms and presents examples of risk measures that satisfy them. The authors explain that the widely used "value at risk" (or VaR) is not coherent. They also implicitly introduce a duality theory of coherent risk measures. Rockafellar and Uryasev (2002) were the first to relate coherent risk measures and mathematical programming: they showed that one of these risk measures, namely the conditional value at risk or CVaR, can be computed by solving a linear program. Eichhorn and Römisch (2005) expanded this finding in their theory of polyhedral risk functions. More recently, Ruszczynski and Shapiro (2006) extended Artzner et al.'s (1989) duality of risk functions and used it to develop a new theory of optimization under risk functions that generalizes stochastic programming to risk-averse behaviour. Stochastic equilibria with risk aversion in power markets are studied by Kannan et al. (2009) using a CVAR approach, whereas Fan et al. (2010) use utility functions. Cabero et al. (2010) apply a CVaR approach to the risk management of a hydro producer.

We call upon these different elements as follows. We first describe risk-averse investors as valuing the profit (the gross margin minus the investment cost) accruing from an investment according to a CVaR that can be computed by solving a linear program. We then call upon the duality of coherent risk functions to formulate this valuation in complementary form and note that this step introduces the measure Q as a set of dual variables of the CVaR linear program. Finally, we insert these CVaR-based complementary relations in the stochastic equilibrium capacity market and energy-only models and obtain the corresponding models of risk aversion. In so doing, we implicitly expand the optimization with risk function to the equilibrium paradigm. The CVaR is only one of several coherent risk measures that can be used in this fashion. Any other coherent risk measure in the sense of Artzner et al. (1989) (that satisfies their four axioms) would lead to a similar development. We now elaborate on these steps in more details.

Consider a risk-averse investor who wants to value a new plant according to the CVaR of its profit. Like their risk-neutral counterparts, risk-averse investors are price takers, and hence reason in terms of marginal costs and profits. This implies that they decide on the basis of the prices π, μ, and λ, but see these prices as independent of their investment decisions. Assume now that an investor values new capacity as the expectation of the profits conditional on being lower or equal to their VaR_α. The VaR_α is the smallest margin that is strictly exceeded with a probability of at most $1-\alpha$: all retained scenarios have a margin smaller than or equal to VaR_α. The investor therefore disregards the set of higher margins that would occur with a probability smaller than $1-\alpha$, which expresses her risk aversion or prudence. Note, that for an α close to 1 we are not concentrating on the lower tail, but we exclude the beneficial scenarios.

Future research should address the more common formulation of investors maximizing a convex combination of expected margins and a CVaR of the lower tail of the margins. Such an approach has the additional benefit that the resulting risk-adjusted probabilities would be an equivalent risk measure, i.e., all possible outcomes that have a positive probability in P would have a positive probability in Q.

6.3. Duality and Complementarity Representation of Risk Measures

We now apply this reasoning to the two market designs. The treatments of the capacity and the energy-only models are identical, and we simplify the notation by introducing the profit$(k; f, n, b)$ (subtracting investment costs from the gross margin) of an investment in plant k in scenario (f, n, b) defined as

$$\mathrm{profit}(k; f, n, b) \equiv \sum_{\ell\in L} T(\ell)\mu(k, \ell, f, n, b) + \nu + A(k, b)\lambda(f, n, b) - I(k) \qquad (22)$$

for the capacity market and

$$\mathrm{profit}(k; f, n, b) \equiv \sum_{\ell\in L} T(\ell)\mu(k, \ell, f, n, b) + A(k, b)\lambda(f, n, b)x(k) - I(k) \qquad (23)$$

for the energy-only market.

Following Rockafellar and Uryasev (2002), we write the CVaR(k) of the profits generated by plant k as a linear programming problem where one maximizes with respect to VaR(k) (see Appendix 1 for a numerical illustration of this expression).

$$\max_{\mathrm{VaR}(k)} +\mathrm{VaR}(k) - \frac{1}{\alpha(k)} \sum_{f\in F,\, b\in B,\, n\in N} P_f(f)P_b(b)P_n(n),$$
$$\max(\mathrm{VaR}(k) - \mathrm{profit}(k; f, n, b), 0). \qquad (24)$$

Note that because different technologies are subject to different fuel and CO2 risks, one may want to assume that the risk attitude of the investor depends on the technology. Hence, we introduce a technology-specific $\alpha(k)$.

Rockafellar and Uryasev also showed that the optimal solution $\mathrm{VaR}(k)$ of this problem is the value at risk ($\mathrm{VaR}(k)$) of the distribution of profits of equipment k for the technology-specific risk aversion $\alpha(k)$. Next, we reformulate problem (24) by introducing nonnegative variables $u(k; f, n, b)$ that we restrict to be no smaller than ($\mathrm{VaR}(k)$-profit $(k; f, n, b)$). We also introduce dual variables $q(k; f, n, b)$ and rewrite (24) together with the dual variables of these constraints as

$$\max_{\mathrm{VaR(k)}} \ \mathrm{VaR}(k) - \frac{1}{\alpha(k)} \cdot \sum_{f\in F,\, b\in B,\, n\in N} P_f(f)P_b(b)P_n(n)u(k; f, n, b), \quad (25)$$

$$\text{s.t.} \quad u(k; f, n, b) \geqslant \mathrm{VaR(k)} - \mathrm{profit}(k; f, n, b) \quad (q(k; f, n, b)), \quad (26)$$

$$u(k; f, n, b) \geqslant 0. \quad (27)$$

The duality conditions of this problem at optimum can be written in complementarity form as

$$0 \leqslant u(k; f, n, b) - \mathrm{VaR}(k) + \mathrm{profit}(k; f, n, b) \perp q(k; f, n, b) \geqslant 0, \quad (28)$$

$$0 = -\sum_{f\in F,\, b\in B,\, n\in N} q(k; f, n, b) + 1, \quad (29)$$

$$0 \leqslant -q(k; f, n, b) + \frac{1}{\alpha(k)} P_f(f)P_n(n)P_b(b) \perp u(k; f, n, b) \geqslant 0. \quad (30)$$

The dual variables q of the LP (25), (26), and (27) appearing in these conditions are probabilities as shown by their nonnegativity and relation (29). We refer to them as the investor's risk-adjusted probabilities. To justify this interpretation, note from the LP that $\mathrm{profit}(k; f, n, b) > \mathrm{VaR}$ implies $q(k; f, n, b) = 0$ and $u(k; f, n, b) = 0$. This amounts to discarding the scenarios where the profit exceeds the VaR threshold level. Conversely, a profit strictly lower than the threshold ($\mathrm{VaR}(k) > \mathrm{profit}(k; f, n, b)$) implies strictly positive $u(k; f, n, b)$, and hence strictly positive $q(k; f, n, b)$ given by relation (30). Also, a profit equal to the threshold ($\mathrm{VaR}(k) = \mathrm{profit}(k; f, n, b)$) allows (but does not imply) a strictly positive $u(k; f, n, b)$, and hence strictly positive $q(k; f, n, b)$ in relation (30). Taking the $q(k; f, n, b)$ weighted sum of the left part of (28) and equating it to (25) to reflect that primal and dual optimal values are equal therefore implies

$$\sum_{f\in F,\, b\in B,\, n\in N} q(f, n, b)\mathrm{profit}(f, n, b) = \frac{1}{\alpha} \sum_{f\in F,\, b\in B,\, n\in N} P_f(f)P_n(n)P_b(b)\mathrm{profit}(f, n, b) \equiv \mathrm{CVaR}_\alpha(\mathrm{profit}(f, n, b)). \quad (31)$$

In other words, the expected profit of plant k computed with the q probabilities associated with that plant is also the CVaR of these profits.

6.4. Reformulation of the Equilibrium Problem for Risk-Averse Investors

We can now convert the P version (based on the statistical probabilities P_n, P_b, and P_f) of the energy-only and capacity market models into corresponding Q versions that represent risk-averse, CVaR-behaving agents. Expressing that these agents maximize their profit by investing in plant k according to the risk-adjusted probabilities $q(k; f, n, b)$) derived from the duality conditions of the $\mathrm{CVaR}(k)$, we first modify the stochastic equilibrium model of risk-neutral investors by adding the following conditions that define the risk-adjusted probabilities $q(k; f, n, b)$: for all $k \in K$ we add (28), (29), and (30) We then modify the investment criteria (19) (energy-only market) and (21) (capacity market) to represent investors behaving according to CVaR-driven risk-adjusted probabilities and write

$$0 \leqslant -\sum_{f\in F,\, b\in B,\, n\in N} q(k; f, n, b)\mathrm{profit}(k; f, n, b) \perp x(k) \geqslant 0. \quad (32)$$

The rest of the models (relations (14) to (18), not (19) for the energy-only model (augmented by (20) in the case of the capacity market model)) remain unchanged.

Note that each model generates as many vectors of risk-adjusted probabilities as the number of plant types. This has important economic and financial interpretations that we shall briefly touch upon in §8.

7. Numerical Results and Policy Implications

We illustrate the application of the stochastic equilibrium models and the possible insight that they can bring with a comparison of energy-only and capacity markets. The relative merits of these two market organizations have been discussed for many years, both in the literature and in practice. The debate was intense in the United States, but remains almost nonexistent in Europe. The work is conducted on the test problem progressively elaborated in the paper. As in Joskow (2007), there is no representation of demand randomness or machine failure, and hence no need to introduce reserve capacity in the model. Because demand is a function of the scenario (in order to eliminate degeneracy concerns), we set the capacity target at the maximal demand over all scenarios (f, n, b). We also simplify the energy-only model into one where the regulator allows for a high electricity price in case of curtailment. We consider three caps equal, respectively, to 250, 1,000, and 10,000 euros/MWh. Moving from this simple energy-only organization to an operations reserve pricing

Table 9. Deterministic scenario results: consumer costs include energy and capacity market prices.

	Coal	CCGT	OCGT	Total	Shortfall	Hours	Consumer cost in bn euro
CM/10,000	11,985	68,103	6,007	86,095	0	0	33.831
EO/10,000	11,985	68,103	6,007	86,095	0	0	33.831
CM/1,000	11,985	68,103	6,007	86,095	0	0	33.831
EO/1,000	11,988	68,100	3,003	83,092	3,003	10	34.039
CM/250	11,985	68,103	6,007	86,095	0	0	33.831
EO/250	12,004	68,084	0	80,089	6,007	50	35.420

mechanism (e.g., Hogan 2005) requires a much more complex model (see Gurkan et al. 2009). We compare investments under these different assumptions of market design (capacity or energy-only), regulation (price cap level), and attitude towards risk (P or Q). In order to illustrate the insight brought by the stochastic view, we also compare these results to those of a standard deterministic analysis. When demand has to be curtailed, we value the cost incurred by the consumer because of the unserved electricity at the VOLL taken at 10,000 euros/Mh.

The reference deterministic model involves a single scenario where fuel, NAP, and free allowances are taken as averages over their respective scenarios. Results are given in Table 9 for the two market designs and the three price caps.

CM and EO refer to capacity market and energy-only models, respectively. The first three columns report investments (in MW) in coal, CCGT, and OCGT capacity, respectively. The fourth column, "Total" gives the total investment. The two latter columns describe the curtailment in two different ways: "Shortfall" indicates the maximal shortage of capacity with respect to the peak; "Hours" is the maximal number of hours of curtailment. Finally, the last column reports the total bill paid by the consumer, computed as the sum of three terms: the payment for delivered energy is the product of the market price (possibly equal to PC) by demand; the cost of unserved energy is the product of VOLL and curtailed energy; the capacity price is only paid in the capacity market model. Note that the capacity constraint is not binding for the case of a price cap at 10,000 euro/MWh, and hence both market arrangements lead to the same investment mix.

The results of the stochastic equilibrium models are reported in Table 10. RN and RA, respectively, designate simulations conducted with risk-neutral (zero risk premium or P probability) and risk-averse (CVaR or Q probability) agents. Computations are conducted on the basis of a CVaR with risk aversion $\alpha(k) = 0.9$ for all technologies. Here the total cost for the consumers is calculated as the expectation in the statistical probability.

Although there is only one statistical probability P, each technology has its own risk-adjusted probabilities $q(k)$. For instance, in the case of a price cap of 10,000 euro/MWh and no capacity market technology, CCGT excludes scenarios $(f2, n1, b1)$ and $(f2, n1, b2)$ by setting their respective risk-adjusted probabilities to zero, whereas GT excludes $(f2, n1, b2)$ and reduces the risk-adjusted probability of scenario $(f1, n2, b3)$.

7.1. Capacity Market

The analysis of the capacity market shows that both the deterministic and stochastic equilibrium models entail the

Table 10. Results for price caps of 10,000, 1,000, and 250 euro/MWh.

	Coal	CCGT	OCGT	Total	Shortfall	Hours	Consumer cost in bn euro
Price cap: 10,000 euro/MWh							
CM/RN	15,442	64,655	6,180	86,277	0	0	34.425
CM/RA	15,439	64,650	6,188	86,277	0	0	34.982
EO/RN	15,442	64,655	6,171	86,268	10	10	34.425
EO/RA	15,438	64,650	6,179	86,268	10	10	34.629
Price cap:1,000 euro/MWh							
CM/RN	15,442	64,655	6,180	86,277	0	0	34.425
CM/RA	15,128	45,297	25,852	86,277	0	0	34.943
EO/RN	15,461	64,636	161	80,258	6,019	50	36.080
EO/RA	15,147	45,261	19,849	80,258	6,019	50	36.596
Price cap: 250 euro/MWh							
CM/RN	15,442	64,655	6,180	86,277	0	0	34.425
CM/RA	15,128	45,297	25,852	86,277	0	0	35.107
EO/RN	15,467	64,623	0	80,090	6,187	50	36.387
EO/RA	15,905	44,289	0	60,193	26,084	360	108.309

necessary investments when future demand can be perfectly forecast. This is true whatever the price cap and whether agents are risk neutral or risk averse. The capacity value ν gives the adequate additional incentive to invest. A change of price cap in the capacity market model does not influence the structure of the generation system in the deterministic and risk-neutral stochastic equilibrium model. In contrast, the generation structure changes for risk-averse investors when the price cap is low. One indeed observes that risk-averse investors shift a significant capacity of CCGT into OCGT when the price cap lowers to 1,000 or 250 euro/MWh. Such a move towards less capital-intensive plants is normally considered as increasing the costs of the generation system compared to the one in a less risky situation. Reduction of regulatory risk is an obvious way to limit this loss. It reminds one of the period where U.S. investors subject to an obligation to serve moved investments towards gas turbine in response to the risk created by prudence reviews. Lastly, we also report the expected payment of the consumer in the different situations. Uncertainty increases the expected bill of the consumer with respect to the deterministic model, and risk aversion increases it even further.

7.2. Energy-Only Market

The energy-only market exhibits a more diversified behaviour. The deterministic model shows the expected phenomenon: the energy-only model entails the necessary capacity when the price cap is high, but there is curtailment under a low cap. Capacity shortfalls obtained in the risk-neutral cases are higher or of the same order of magnitude than those computed by the deterministic model. As in the capacity market, generation structures also differ by the amount of coal between the deterministic and stochastic energy-only models when investors are risk neutral.

Comparing now the energy-only and capacity market models in the risk-neutral case, one observes that a high cap leads to almost the same generation structure and curtailment in both (the energy-only market shows a small curtailment of 10 MW). However, total consumer costs are lower in the energy-only market in this case because a substantial capacity price is needed to ensure the adequate investment in the capacity market. Curtailment dramatically increases with risk-averse agents operating under a low cap. The combination of a low price cap and risk aversion therefore seriously deteriorates security of supply. One also observes that risk-averse agents decrease CCGT capacity and eliminate OCGT investments when the price cap is low. Again we measure this loss of efficiency by the expected bill of customers. The high curtailment appearing under low price cap and risk aversion in the energy-only market reveals catastrophic consequences for the customers that are not captured by the deterministic analysis. The catastrophic result is not explained by larger producer profits, but is due to the large amount of unserved energy that is priced at 10,000 euro/MWh and is a real loss of social welfare.

7.3. Summing Up

The overall message is thus threefold. The deterministic analysis overlooks some change of capacity structure that may be induced by risk whether in the capacity market or energy-only organizations. The risk-neutral analysis also misses a shift towards less capital-intensive technologies that may result from risk aversion. Risk aversion also increases the shortage of capacity in the energy-only market when the price cap is low. These may have dramatic impacts on the bill of the final consumer. Needless to say, these results are purely illustrative, but they reveal phenomena that may deserve more investigation.

Appendix 2 reports the impact of replacing each average parameter by its associated event tree. In stochastic programming terms, fuel and NAP uncertainties correspond to standard randomness in the objective function and right hand sides. Uncertainty on free allowances corresponds to the much more unusual randomness in the matrix of a complementarity problem. For our example we find that the uncertainty about the allocation leads to the biggest distortion in terms of technology mix (under a capacity market) and the largest risk of shortfall (energy-only market).

8. Financial Interpretation and Extension

8.1. Risk Premium

Risk-neutral agents base their decisions solely on the expected profit without concern for its distribution. We casually referred to risk premia in the preceding sections. We now explicitly compute the risk premium embedded in the CVaR decision (or in its equivalent risk-adjusted probabilities Q) using standard notions of finance.

Risk premia can be computed in various ways depending on the utility or risk function selected to model risk aversion. We here use two measures commonly referred to in the well-established capital asset pricing model (CAPM; see any textbook of finance), namely, the excess return and the Sharpe ratio. These are defined as follows. Consider a unit investment in year 0 giving a random payoff R in period 1. Let $E(R)$ and $\sigma(R)$ be, respectively, the expectation and standard deviation of this random payoff. Let R_f also be the payoff of a unit investment at the risk-free rate. The excess return is the incremental expected return $E(R)$ with respect to R_f. The Sharpe ratio refers the excess return to the total risk measured by $\sigma(R)$. It is equal to

$$SR \equiv \frac{E(R) - R_f}{\sigma(R)}.$$

The CVaR-driven models presented here are not of the CAPM type. CAPM investors measure risk by the variance of the return, whereas the CVaR supposes that they only pay attention to one tail of the distribution. Notwithstanding these quite different views on risk, a CVaR-driven behaviour implies a mean and a variance of returns, and hence can be gauged on the basis of an excess return and

Table 11. Computation of risk premium of the whole generation system.

	Investment	Expected profits	Standard deviation	Excess return (%)	Sharpe ratio
10,000/CM/RA	8,013,498	1,137,929	4,409,761	14.2	0.26
10,000/EO/RA	8,012,931	785,197	3,655,756	9.8	0.21
1,000/CM/RA	7,595,363	1,193,433	3,671,642	15.7	0.32
1,000/EO/RA	7,235,364	1,030,506	3,564,989	14.2	0.29
250/CM/RA	7,595,363	1,193,473	3,671,791	15.7	0.32
250/EO/RA	6,087,818	840,866	2,697,266	13.8	0.31

Table 13. Computation of risk premium of the CCGT plant.

	Investment	Expected profits	Standard deviation	Excess return (%)	Sharpe ratio
10,000/CM/RA	5,171,998	700,850	3,291,790	13.6	0.21
10,000/EO/RA	5,171,997	436,407	2,963,516	8.44	0.15
1,000/CM/RA	3,623,783	558,362	1,847,590	15.4	0.30
1,000/EO/RA	3,620,918	496,689	1,971,958	13.7	0.25
250/CM/RA	3,623,783	558,383	1,847,659	15.4	0.30
250/EO/RA	3,543,094	449,939	1,633,644	12.7	0.28

a Sharpe ratio. This is of interest because these notions are commonly used by financial analysts. In short, we evaluated CVaR-driven investments in standard physical units (MW) in §7 and reports their value in standard financial terms in this section.

We report two different types of result. Table 11 gives the risk premia of the whole generation system. This corresponds to the common practice of measuring financial results with respect to the "market" or to a particular sector (here the power sector).

Tables 12, 13, and 14 report the results by individual plant types. This is a "project finance" view: We suppose three types of firms or investors that assess projects on their individual merits without considering their impact on the portfolio of plants. In other words, an investor in a coal unit makes her decision solely on the basis of the profit and risk incurred by this unit, without accounting for the fact that it may benefit other plants. Again, taking another view, this amounts to assuming that each firm specializes in a particular type of plant (which companies effectively do when they rely on project finance) without trading the risk implied by these different plants. In financial terms, this is an incomplete market. Tables 12, 13, and 14 therefore report the benefits accruing to each type of plant as viewed by each investor, whereas Table 11 reports them for the whole market. Future research should address the case of a market where agents invest in the whole portfolio of plants.

The overall message of the tables is that the energy-only market is less profitable, whether expressed by the excess return or the Sharpe ratio. Comparing the results of the total generation system with the individual investments, one finds higher Sharpe ratios for the portfolio than for the individual assets. In several cases, excess returns are often lower for the portfolio than for each individual plants, but low standard deviation of the portfolio leads to higher excess returns.

One also observes that the obtained values are in line with orders of magnitudes observed in the market (Standard & Poors reported a three-year Sharpe ratio for the S&P 500 of 0.3293 on the 09/30/2007 (http://www2.standardand-poors.com/spf/pdf/index/500factsheet.pdf). This suggests a way to calibrate the risk function: selects α such that the $CVAR_\alpha$ reproduces an observed risk premium. Our choice of 0.9 here is purely incidental, but it gives quite reasonable results.

8.2. Technology-Specific Risk Aversion

Because different technologies are subject to different fuel and CO_2 risks, they may also be subject to other risks not explicitly represented in the model. Stoft (2002) elaborates on the demand risk that affects open-cycle gas units. Risks of gas disruption are also commonly mentioned in Europe. For these different reasons, one may want to assume that the risk attitude of the investor depends on the technology. An investor may be more risk averse when investing in a gas turbine than in a combined-cycle unit because the demand risk of the former is higher. More generally, all investors need not have the same risk aversion.

The only difference with the preceding numerical results lies in using technology-specific $\alpha(k)$.

Table 15 introduces assumptions where investors are progressively less risk averse as one goes from the open cycle gas turbine to combined-cycle gas turbine and coal.

Table 12. Computation of risk premium of the coal plant.

	Investment	Expected profits	Standard deviation	Excess return (%)	Sharpe ratio
10,000/CM/RA	2,470,213	380,189	1,774,020	15.4	0.21
100,00/EO/RA	2,470,214	317,039	1,509,082	12.9	0.21
1,000/CM/RA	2,420,484	395,015	1,709,952	16.3	0.23
1,000/EO/RA	2,423,486	376,350	1,719,114	15.5	0.22
250/CM/RA	2,420,484	395,022	1,709,965	16.3	0.23
250/EO/RA	2,544,724	390,927	1,781,496	15.4	0.22

Table 14. Computation of risk premium of the OCGT plant.

	Investment	Expected profits	Standard deviation	Excess return (%)	Sharpe ratio
10,000/CM/RA	371,287	56,889	271,085	15.3	0.21
10,000/EO/RA	370,720	31,751	258,025	8.6	0.12
1,000/CM/RA	1,551,097	240,056	832,757	15.5	0.29
1,000/EO/RA	1,190,960	157,466	718,937	13.2	0.22
250/CM/RA	1,551,097	240,068	832,805	15.5	0.29
250/EO/RA	0	0	0	0	0

Table 15. Risk aversion for different technologies.

α Coal	α CCGT	α GT
1	0.8	0.5

Table 16. Technology-specific risk aversion in combination with price caps of 10,000, 1,000, and 250 euro/MWh.

	Coal	CCGT	OCGT	Total	Shortfall	Hours
		Price cap: 10,000 euro/MWh				
CM/RA	15,468	64,789	6,020	86,277	0	0
EO/RA	15,450	67,642	3,090	86,182	94	10
		Price cap: 1,000 euro/MWh				
CM/RA	15,130	45,297	25,849	86,277	0	0
EO/RA	15,513	64,744	0	80,256	6,020	360
		Price cap: 250 euro/MWh				
CM/RA	15,131	45,297	25,849	86,277	0	0
EO/RA	15,929	44,264	0	60,193	26,084	360

Tables 16 reports the results for the three price caps. Curtailments are not significantly affected compared to the results obtained with a common risk aversion $\alpha(k) = 0.9$ reported in Table 10. In contrast the movement from CCGT to OCGT observed in the energy-only model with a price cap of 1,000 disappears when investors are more reluctant to invest in this latter technology.

9. Conclusions

Capacity expansion assessment was technically rather easy in the regulated power industry. The problem could be cast in optimization form, and hence benefit from mathematical programming advances. Uncertainties were moderate, if only because the monopoly regime protected the utilities, limiting risk significantly and facilitating passing some of its costs to consumers. The guaranteed rate of return on equities that prevailed in most cases facilitated the financing of the huge capital required by investments.

Restructuring and environmental concerns have changed all this. The optimization model no longer fits companies in competition. Risks, whether driven by world events or domestically generated by new regulations, are now piling up from everywhere, without the protection of the old regulatory regime. As if this were not enough, we have somehow lost confidence in the market to send the right signals to invest.

We look at this novel situation by trying to move the old techniques of capacity expansion models into this new environment. Equilibrium substitutes optimization to account for competition; we stick to the former fixed demand model but could easily include demand response if adequate information were available. We explicitly introduce a risky environment and model its impact through risk functions that we take here as CVaR. Although the approach does not call ex ante upon the CAPM to compute the cost of capital, it allows one to calculate CAPM-type information. Interestingly, the CVaR is just one risk function opening the way to alternative models of risk.

Models such as these can be used to quantitatively assess the impact of policies in discussion today. Policies that provide incentives to invest in capacity are crucial, because a lack of incentive to invest could damage security of supply. Even though our analysis is mainly illustrative it points to results that may deserve more investigation. Referring to capacity policy we find that a deterministic analysis overlooks changes of capacity structure induced by risk whether in the capacity market or energy-only organizations. Not surprisingly, we observe that a capacity market is better suited to guarantee security of supply than an energy-only market when the electricity price cap is low. Possibly less obvious, we also find that risk aversion enhances this shortage of capacity in energy-only markets with low price caps. We also observe that risk aversion distorts investments towards less capital-intensive equipment, whether in energy-only or capacity markets. This effect combines with the shortage of capacity induced by a low cap in an energy-only market to result in a dramatic impact on the bill to the final consumer. The simple policy conclusion is that although energy-only and capacity markets can be argued to be similar when looked at in a deterministic framework, they can significantly differ as soon as risk and risk aversion are taken on board.

The paper overlooks many issues that we briefly mention here. The first one is that the example is small. This may be justified for pedagogical reasons, but it also raises questions. We optimistically dismissed all numerical discussion in the course of the paper, but should mention potential difficulties: all equilibrium problems presented here are nonconvex, a subject that we did not discuss, but should at least mention for further reference. This begins with the introduction of free allowances. Less usually, the introduction of differentiated risk aversion for different agents implies nonmonotone mapping in the equilibrium problem. Economies of scale in some technologies (e.g., nuclear or CCS) would introduce other nonconvexities. A second point is that the model is static or, in stochastic programming parlance, of the two-stage type. Extending a two-stage stochastic program to multistage is not conceptually difficult with risk-neutral investors. However, the matter is different with risk-averse agents. Artzner et al. (2002, 2007) discuss the problem of coherent multiperiod risk functions. We should also mention that we assumed that there is no demand risk. Our current research suggests that this assumption should be removed for policy analysis: demand risk is a subject in itself that we shall treat in another paper. Last but not least, the EU-ETS evolves and capacity policies will finally also be taken on board in the EU. The small example that serves a basis of discussion corresponds to the early 2008 context. Some of these uncertainties have now been eliminated, others remain, and new ones are and will be emerging.

10. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

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References

Anderson, D. 1972. Models for determining least-cost investments in electricity supply. *Bell J. Econom. Management Sci.* **3**(1) 267–299.

Arellano, M. S., P. Serra. 2007. A model of market power in electricity industries subject to peak load pricing. *Energy Policy* **35**(10) 5130–5135.

Artzner, P., F. Delbaen, J.-M. Eber, D. Heath. 1989. Coherent measures of risk. *Math. Finance* **9** 203–228.

Artzner, P., F. Delbaen, J.-M. Eber, D. Heath, H. Ku. 2002. Coherent multiperiod risk measurement. Working paper, ETH, Zurich, http://www.math.ethz.ch/~delbaen.

Artzner, P., F. Delbaen, J.-M. Eber, D. Heath, H. Ku. 2007. Coherent multiperiod risk adjusted values and Bellman's principle. *Ann. Oper. Res.* **152**(1) 5–22.

Boom, A., S. Buehler. 2007. Restructuring electricity markets when demand is uncertain: Effects on capacity investments, retail prices and welfare. Discussion paper 2007–09, Center for Industrial Economics, Department of Economics, University of Copenhagen, Copenhagen.

Burtraw, D., K. L. Palmer. 2008. Compensation rules for climate policy in the electricity sector. *J. Policy Anal. Management* **27**(4) 819–847.

Cabero, J., M. Ventosa, S. Cerisola, A. Baillo. 2010. Modeling risk management in oligopolistic electricity markets: A Benders decomposition approach. *IEEE Trans. Power Systems* **25**(1) 263–271.

Carlton, D. 1977. Peak load pricing with stochastic demand. *Amer. Econom. Rev.* **67**(5) 1006–1010.

Chao, H.-P. 1983. Peak load pricing and capacity planning with demand and supply uncertainty. *Bell J. Econom.* **14**(1) 179–190.

Cramton, P., S. Kerr. 2002. Tradable carbon permit auctions: How and why to auction not grandfather. *Energy Policy* **30**(4) 333–345.

Cramton, P., S. Stoft. 2006. The convergence of market designs for adequate generating capacity with special attention to the CAISO's resource adequacy program. White paper for California Electricity Oversight Board, April 2006.

Crew, M., S. F. Chitru, P. R. Kleindorfer. 1995. The theory of peak-load pricing: A survey. *J. Regulatory Econom.* **8**(3) 215–248.

de Frutos, M.-A., N. Fabra. 2007. Endogenous capacities and price competition: The role of demand uncertainty. CEPR Discussion Paper 6096, Centre for Economic Policy Research, London.

de Frutos, M.-A., N. Fabra, N.-E. von der Fehr. 2008. Investment incentives and auction design in electricity markets. CEPR Discussion Paper 6626, Centre for Economic Policy Research, London.

Eichhorn, A., W. Römisch. 2005. Polyhedral risk measures in stochastic programming. *SIAM J. Optim.* **16**(1) 69–95.

Fan, L., B. F. Hobbs, C. S. Norman. 2010. Risk aversion and CO_2 regulatory uncertainty in power generation investment: Policy and modeling implications. *J. Environ. Econom. Management* **60**(3) 193–208.

Gabszewicz, J., S. Poddar. 1997. Demand fluctuation and capacity utilization under duopoly. *Econom. Theory* **10**(1) 131–146.

Grimm, V., G. Zoettl. 2008. Strategic capacity choice under uncertainty: The impact of market structure on investment and welfare. Working paper, University of Cologne, Cologne, Germany.

Gürkan, G., O. Ozdemir, Y. Smeers. 2009. Capacity investment in electricity markets: Perfect competition. Working paper, Department of Econometrics and Operations Research, Tilburg University, Tilburg, The Netherlands.

Hogan, W. W. 2005. On an "energy-only" electricity market design for resource adequacy. HEPG, Harvard University, Cambridge, MA.

Joskow, P. 2007. Competitive electricity markets and investment in new generation capacity. D. Helm, ed. *The New Energy Paradigm*. Oxford University Press, Oxford, UK.

Joskow, P., J. Tirole. 2007. Reliability and competitive electricity markets. *RAND J. Econom.* **38**(1) 60–84.

Kannan, A., U. V. Shanbhag, H. M. Kim. 2009. Risk-based generalized Nash games in power markets: Characterization and computation of equilibria. University of Illinois. Submitted for publication.

Morlat, G., F. Bessière. 1971. Vingt-cinq ans d'économie électrique. *Monographie de Recherche Opérationnelle* 11. Dunod, Paris.

Murphy, F. H., Y. Smeers. 2005. Generation capacity expansion in imperfectly competitive restructured electricity markets. *Oper. Res.* **53**(4) 646–661.

Murphy, F. H., S. Sen, A. L. Soyster. 1987. Electric utility expansion planning in the presence of existing capacity: A nondifferentiable, convex programming approach. *Comput. Oper. Res.* **14**(1) 19–31.

Neuhoff, K., F. C. Matthes. 2008. The role of auctions for emissions trading. Report, Climate Strategies, http://www.climatestrategies.org/research/our-reports/category/1.html.

Oggioni, G., Y. Smeers. 2009. Evaluating the impact of average cost based contracts on the industrial sector in the European emission trading scheme. *Central Eur. J. Oper. Res.* **17**(2) 181–217.

Oren, S. 2005. Generation adequacy via call option obligations: Safe passage to the promised land. *Electricity J.* **18**(9) 28–42.

Panzar, J. C. 1976. A neoclassical approach to peak-load pricing. *Bell J. Econom.* **7**(2) 521–530.

Reynolds, S. S., B. J. Wilson. 2000. Bertrand-Edgeworth competition, demand uncertainty, and asymmetric outcomes. *J. Econom. Theory* **92**(1) 122–141.

Rockafellar, R. T., S. Uryasev. 2002. Conditional value-at-risk for general loss distributions. *J. Banking Finance* **26**(7) 1443–1471.

Ruszyński, A., A. Shapiro. 2006. Optimization of convex risk functions. *Math. Oper. Res.* **31**(3) 433–452.

Sherali, H. D., K. Staschus, J. M. Huacuz. 1987. An integer programming approach and implementation for an electric utility capacity planning problem with renewable energy sources. *Management Sci.* **33**(7) 831–847.

Sirikum, J., A. Techanitisawad, V. Kachitvichyanukul. 2007. A new efficient GA-Benders' decomposition method for power generation expansion planning with emission controls. *IEEE Trans. Power Systems* **22**(3) 1092–1100.

Stoft, S. 2002. *Power System Economics*. IEEE Press, New York.

von der Fehr, N.-H., D. C. Harbord. 1997. Capacity investment and competition in decentralised electricity markets. Memorandum, 27, Department of Economics, University of Oslo, Oslo.

Zhao, J., B. F. Hobbs, J.-S. Pang. 2010. Long-run equilibrium modeling of emissions allowance allocation systems in electric power markets. *Oper. Res.* **58**(3) 529–548.

Zöttl, G. 2008. Investment decisions in liberalized electricity markets: A framework of peak load pricing with strategic firms. CORE Discussion Paper 2008/41, Université Catholique de Louvain, Louvain-la-Neuve, Belgium.