OPLSS-2018-Foundations-day3

Friday, July 6, 2018 9:02 AM

Lecture 9: Parallelism
Parallelism

- Evaluation strategy
- Deterministic

Concurrency

- Managing events that happen at the same time
- None-deterministic

PPCF: Binary Fork-Join
Cost semantics

- Goal, cost semantics

7/6/2018 10:52 AM
Bounded Implementations
Prototypical result, Brent's Theorem
Machine model: shared-memory, multiprocessor (SMP)

- Some fixed P>0 processors
- Shared memory with constant time access
- Constant-time synchronization mechanism

Paul
7/6/2018 11:13 AM
CBN
7/6/2018 3:15 PM
Jan
Encoding of types in System F

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Saturday, July 7, 2018 3:30 AM

Lecture 9.
7/6/2018
#1

~~Cost Dynamics~~

~~$e \Downarrow^n_v$ expr. e evaluates to value v with cost n~~

PPCF

Exp e ::=

x

$s(e)$

$fix\{\tau\}(x.e)$

$par(e_1; e_2; x_1.x_2.e)$ in e   par $x_1 = e_1$ and $x_2 = e_2$

Statics

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2 \quad \Gamma, x_1 : \tau_1, x_2 : \tau_2 \vdash e : \tau}{\Gamma \vdash par(e_1; e_2; x_1.x_2.e) : \tau}$$

Sequential dynamics

$$\frac{e_1 \mapsto_{seq} e_1'}{par(e_1; e_2; x_1.x_2.e) \mapsto_{seq} par(e_1'; e_2; x_1.x_2.e)} \qquad \frac{e_1 \text{ val} \quad e_2 \mapsto_{seq} e_2'}{par(e_1; e_2; x_1.x_2.e) \mapsto_{seq} par(e_1; e_2'; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \quad e_2 \text{ val}}{par(e_1; e_2; x_1.x_2.e) \mapsto_{seq} [e_1, e_2 / x_1, x_2]e}$$

Parallel dynamics

$$\frac{e_1 \mapsto_{par} e_1' \quad e_2 \mapsto_{par} e_2'}{par(e_1; e_2; x_1.x_2.e) \mapsto_{par} par(e_1'; e_2'; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \quad e_2 \mapsto_{par} e_2'}{par(e_1; e_2; x_1.x_2.e) \mapsto_{par} par(e_1; e_2'; x_1.x_2.e)}$$

$$\frac{e_2 \text{ val} \quad e_1 \mapsto_{par} e_1'}{par(e_1; e_2; x_1.x_2.e) \mapsto_{par} par(e_1'; e_2; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \quad e_2 \text{ val}}{par(e_1; e_2; x_1.x_2.e) \mapsto_{par} [e_1, e_2 / x_1, x_2]e}$$

Theorem $e \mapsto^*_{par} v$ iff $e \mapsto^*_{seq} v$

Proof: "$\Longleftarrow$"

show: $e \mapsto^n_{seq} e'$ implies $e \mapsto^*_{par} e'$ (intermediate steps are different) show: 1) $e \xrightarrow[seq]{*} v$ iff $e \Downarrow v$

2) $e \xrightarrow[par]{*} v$ iff $e \Downarrow v$

eval dyn

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad [v_1, v_2/x_1, x_2]e \Downarrow v}{par(e_1, e_2, x_1 x_2 e) \Downarrow v}$$

Lemma 1: $e \Downarrow v$ implies $e \xrightarrow[seq]{*} v$ proof: induction on $e \Downarrow v$

case Rule for par:

Then $e = par(e_1; e_2; x_1.x_2.e)$ and $e_1 \Downarrow v_1$ and $e_2 \Downarrow v_2$ and $[v_1, v_2/x_1, x_2] e \Downarrow v$

IH: $e_1 \xrightarrow[seq]{n_1} v_1$, $e_2 \xrightarrow[seq]{n_2} v_2$, $[v_1, v_2/x_1, x_2]e' \xrightarrow[seq]{*} v$

show: $par(e_1; e_2; x_1.x_2.e') \xrightarrow[seq]{*} v$

1) by ind on n_1 (using IH)

$par(e_1; e_2; x_1 x_2. e') \xrightarrow[seq]{n_1} par(v_1; e_2; x_1 x_2 e')$

2) by ind on n_2 (using IH)

$par(v_1; e_2; x_1.x_2.e') \xrightarrow{n_2} par(v_1; v_2; x_1.x_2.e')$

3) $par(v_1; v_2; x_1.x_2 e') \xrightarrow[seq]{\text{use rule}} [v_1, v_2/x_1 x_2]e'$

4) $[v_1, v_2/x_1, x_2]e' \xrightarrow[seq]{*} v$ by IH ∎

Lemma 2: $e \xrightarrow[seq]{*} v$ and v val implies $e \Downarrow v$

Proof: show $e \xrightarrow[seq]{} e'$ and $e' \Downarrow v$ implies $e \Downarrow v$ → homework exercise

Cost semantics: Goal, cost semantics $e \Downarrow^{k} v$ where k describes both seq and parallel cost

Cost graph:

		work (seq. cost)	Depth (par. cost)
$c ::= 1$	unit cost	$wk(1) = 1$	$dp(1) = 1$
0	zero cost	$wk(0) = 0$	$dp(0) = 0$
$c_1 \otimes c_2$	parallel condition	$wk(c_1 \otimes c_2) = wk(c_1) + wk(c_2)$	$dp(c_1 \otimes c_2) = max(dp(c_1), dp(c_2))$
$c_1 \oplus c_2$	seq combination	$wk(c_1 \oplus c_2) = wk(c_1) + wk(c_2)$	$dp(c_1 \oplus c_2) = dp(c_1) + dp(c_2)$

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Eval Rules

$$\frac{}{z \Downarrow^{0} z} \qquad \frac{e \Downarrow^{c} v}{s(e) \Downarrow^{c} s(v)} \qquad \frac{}{\lambda(x:\tau)e \Downarrow^{0} \lambda(x:\tau)e} \qquad \frac{[\mathrm{fix}\{\tau\}(x.e)/x]e \Downarrow^{c} v}{\mathrm{fix}\{\tau\}(x.e) \Downarrow^{c \oplus 1} v}$$

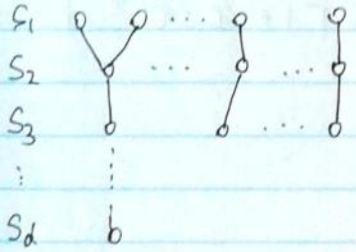
$$\frac{e_1 \Downarrow^{c_1} \lambda(x:\tau)e \quad e_2 \Downarrow^{c_2} v_1 \quad [v_2/x]e \Downarrow^{c_3} v}{e_1(e_2) \Downarrow^{c_1 \oplus c_2 \oplus c_3 \oplus 1} v} \qquad \frac{e_1 \Downarrow^{c_1} v_1 \quad e_2 \Downarrow^{c_2} v_2 \quad [v_1, v_2/x_1, x_2]e \Downarrow^{c_3} v}{\mathrm{par}(e_1; e_2; x_1.x_2.e) \Downarrow^{(c_1 \otimes c_2) \oplus c_3 \oplus 1} v}$$

Theorem
a) If $e \Downarrow^{c} v$ then $e \xrightarrow[seq]{wk(c)} v$ and $e \xrightarrow[par]{dp(c)} v$
b) If $e \xrightarrow[seq]{w} v$ then $e \Downarrow^{c} v$ for some c and $work(c) = w$
c) If $e \xrightarrow[par]{d} v$ then $e \Downarrow^{c} v$ for some c and $dp(c) = d$

Bounded Implementations

Theorem: If $e \Downarrow^{c} v$ with $wk(c) = w$ and $dp(c) = d$ then e can be evaluated on SMP in time $O(\frac{w}{p} + d)$

"proof"



$$\sum_{i=1}^{d} \left\lceil \frac{s_i}{p} \right\rceil \le \sum_{i=1}^{d} \frac{s_i}{p} + 1 = \frac{\sum_{i=1}^{d} s_i}{p} + d = \frac{w}{p} + d$$

$\mathrm{par}(e_1; e_2; x_1.x_2.e)$
$\oplus \dots \oplus id \to e \oplus \dots \oplus \ \mapsto_{p} \oplus \dots \oplus id_1 \hookrightarrow e_1 \oplus id_2$
$\hookrightarrow e_2 \oplus \dots$

$f \in O(g)$ iff $\exists n_0$ and c such that for all $n \ge n_0$ $f(n) \le g(n)$

Paul Downen

$id\ Bool = \lambda(x : Bool)\, x : Bool \to Bool$
$id\ Num = \lambda(x : Num).\, x : Num \to Num.$
$id\ Alpha = \lambda(x : a).\, x : a \to a$

System F (Girard), Polymorphic λ-calculus (Reynolds)
$\tau ::= a \mid arr(\tau_1; \tau_2) \mid all(a.\tau)$

$\tau ::= \alpha \mid \tau_1 \to \tau \mid \forall\alpha.\tau$

$e ::= x \mid lam\{\tau\}(x.e) \mid lam\{\tau\}(x_e) \mid app(e_1; e_2)$

$\quad \mid Lam(\alpha.e) \mid App(e; \tau)$

$e ::= x \mid \lambda x.\tau.e \mid e_1\, e_2$

$\quad \mid \Lambda\alpha.e \mid e\tau$

$\Gamma ::= x_1.x_1 \cdots x_n : \tau_n \quad \Theta ::= \alpha_1, \cdots, \alpha_2$

$J ::= \Theta ; \Gamma \vdash e : \tau$

$\quad \mid \Theta \vdash \tau : \star \quad FV(e) \subseteq \Theta$

$$\frac{}{\Theta, \alpha \vdash \alpha : \star} \qquad \frac{}{\Theta : \Gamma, x:\tau \vdash x : \tau}$$

$$\frac{\Theta \vdash \tau_1 : \star \quad \Theta \vdash \tau_2 \star}{\Theta \vdash \tau_1 \to \tau_2 : \star} \qquad \frac{\Theta, \alpha \vdash \tau : \star}{\Theta \vdash \forall\alpha\, \tau : \star} \qquad \frac{\Theta : \Gamma \vdash e : \tau \to \tau' \quad \Theta : \Gamma \vdash e_2 : \tau}{\Theta . \Gamma \vdash e_1\, e_2 : \tau'}$$

$$\frac{\Theta . \Gamma . x:\tau \vdash e : \tau'}{\Theta : \Gamma \vdash \lambda x\tau : e : \tau \to \tau'} \qquad \frac{\Theta ; \Gamma \vdash e : \forall\alpha\, \tau \quad \Theta \vdash \tau' \star}{\Theta\ \Gamma \vdash e\ \tau' : \tau[\tau'/\alpha]}$$

$$\frac{\Theta . \alpha ; \Gamma \vdash e : \tau}{\Theta ; \Gamma \vdash \Lambda\alpha.e : \forall\alpha\tau}$$

Countex

$$\frac{}{\cdot ; x:\alpha \vdash x:\alpha}$$

Theorem

if $\Theta \vdash \tau : \star$ then $FV(\tau) \subseteq \Theta$

if $\Theta ; \Gamma \vdash e : \tau$ then $\Theta \vdash \tau \star$ ~~al~~

$\Theta \vdash T$

2:00pm

CBN

$$\frac{}{\lambda x.\tau.e \text{ val}} \qquad \frac{}{x \text{ val}} \qquad \frac{}{\Lambda\alpha.e \text{ val}} \qquad \frac{}{(\lambda x.\tau e)\, e' \mapsto e[e'/x]}$$

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$(\alpha \to \alpha)[Num/\alpha] = Num \to Num$

$id : \forall\alpha.\ \alpha \to \alpha$

$id : \Lambda\alpha.\ \lambda x\alpha.\ x$

$\underbrace{id:\ Num}_{Num \to Num \text{ (is derivable)}}\ 1 \quad \mapsto^* 1$

$$\frac{}{(\lambda x{:}\tau.e)e' \mapsto e[e'/x]}$$

$(\Lambda\alpha.e)\tau \mapsto e[\tau/\alpha]$

Lemma (Progress): if $\Theta; \cdot \vdash e : \tau$ then either e is a value or $e \mapsto e'$ for sure!

Lemma (Preservation): if $\Theta; \Gamma \vdash e : \tau$ $e \mapsto e'$ then $\Theta; \Gamma \vdash e' : \tau$.

Theorem (Type Safety) if $\Theta; \Gamma \vdash e : \tau$ is derivable and $e \mapsto^* e'$ then e' is not Stuck (either have result or taking more steps)

Lemma (Type Substitution): if $\Theta\ \alpha; \Gamma \vdash e : \tau$ and $\Theta, \alpha; \Gamma \vdash e : \tau'$ $\because$ ✱ and derivable,

then so is $\Theta; \Gamma[\tau/\alpha] \vdash e[\tau/\alpha] : \tau[\tau/\alpha]$

$E ::= \Box \mid E\ e$

CBV

$$\frac{e' \cancel{val}}{(\lambda x{:}\tau.e)e' \mapsto e[e'/x]} \qquad \frac{e_1 \mapsto e_1'}{e_1 e_2 \mapsto e_1' e_2} \qquad \frac{e \mapsto e'}{e\tau \mapsto e'\tau}$$

$$\frac{}{(\Lambda\alpha.e)\tau \mapsto e[\tau/\alpha]} \qquad \frac{e_1\ val \quad e_2 \mapsto e_2'}{e_1 . e_2 \mapsto e_1 e_2'}$$

$erase(x) = x$

$erase(\lambda x{:}\tau.e) = \lambda x.erase(e)$ $\quad erase(e_1, e_2) = erase(e_1) . erase(e_2)$

$erase(\Lambda a.e) = erase(e)$

$erase(e\ \tau) = erase(e)$

Theorem (Type Erase)

if $e \mapsto^* v$ then $erase(e) \mapsto^* erase(v)$

CBN Fst: if $\Theta; \cdot \vdash e : \tau$ and $\tau \neq \forall a.\tau'$

$erase(\Lambda a.(\lambda\alpha.x)(\lambda x.x)) = (\lambda x.x)(\lambda x x)$

Jan

Unit

$\text{unit} \triangleq \forall t.\ t \to t$

$\langle\rangle \triangleq \Lambda(t)\ \lambda(x{:}t)\,x$

Pairs

$\tau_1 \times \tau_2 \triangleq \forall r\ (\tau_1 \to \tau_2 \to r) \to r$

$\langle e_1; e_2\rangle \triangleq \Lambda(r)\ \lambda(f{:}\tau_1 \to \tau_2 \to r)\ f(e_1)(e_2)$

$e\cdot l \triangleq e[\tau_1]\,(\lambda(x_1{:}\tau_1)\,\lambda(x_2{:}\tau_2)\,x_1)$

$e\cdot r \triangleq e[\tau_2]\,(\lambda(x_1{:}\tau_1)\,\lambda(x_2{:}\tau_2)\,x_2)$

application

Binary Sums

$\tau_1 + \tau_2 \triangleq \forall r\ (\tau_1 \to r) \to (\tau_2 \to r) \to r$

$l\cdot e_1 \triangleq \Lambda r\ \lambda(f_1{:}\tau_1 \to r)\ \lambda(f_2{:}\tau_2 \to r)\ f_1(e_1)$

$r\cdot e_2 \triangleq \Lambda r\ \lambda(f_1{:}\tau_1 \to r)\ \lambda(f_2{:}\tau_2 \to r)\ f_2(e_2)$

$\text{case}\,(e; x_1.e_1; x_2.e_2) \triangleq e[\tau]\,(\lambda(x_1{:}\tau_1)e_1)\,(\lambda(x_2{:}\tau_2)e_2)$

$\Gamma \vdash e_1 : \tau,\quad \Gamma \vdash e_2 : \tau$

Nature numbers N

$\text{nat} \triangleq \forall r.\ \cancel{r} \to (r \to r) \to r$

$z \triangleq \Lambda r.\ \lambda(z{:}r)\,\lambda(s{:}r \to r)\ z$

$s(e) \triangleq \Lambda r\ \lambda(z{:}r)\,\lambda(s{:}r \to r)$

not $\quad s(e[r](z)(s))$

$\text{iter}\,\{e_1; x.e_2\}(e) \triangleq e[\tau]\,(e_1)\,(\lambda(x{:}\tau)e_2)$