Paul Downen (Lecture 4) [2018/07/06]

Polymorphism

id Bool = $\lambda(x{:}\text{Bool}).x$: Bool $\to$ Bool

id Num = $\lambda(x{:}\text{Num}).x$: Nat $\to$ Nat

id Alpha = $\lambda(x{:}\alpha).x$: $\alpha \to \alpha$

We want to avoid this redundancy.
What is the identity function?

id = ... ? ... : $\forall\alpha.\ \alpha \to \alpha$

How do we add this quantifier to the language?

$\boxed{\text{System F}}$ (The polymorphic λ-calculus) aka
(Girard) logic (Reynolds) PL

(Basically λ-calc plus something for $\forall$)

Syntax for System F

$\tau ::= \alpha \mid \text{app}(\tau_1, \tau_2) \mid \text{all}(\alpha.\tau)$

Alternative Syntax

$\tau ::= \alpha \mid \tau_1 \to \tau_2 \mid \forall\alpha.\tau$

Expressions for System F

Recall, in STLC, we had

$e ::= x \mid \text{lam}\{\tau\}(x.e) \mid \text{app}(e_1; e_2)$

For system F, we add:

$e ::= \ldots \mid \text{Lam}(\alpha.e) \mid \text{App}(e;\tau)$

(polymorphism) (instantiation)

Alternative syntax

$e ::= x \mid \lambda x{:}\tau.e \mid e_1 e_2$

$e ::= \ldots \mid \Lambda\alpha.e \mid e\,\tau$

$\tau ::= \alpha \mid \tau_1 \to \tau_2 \mid \forall \alpha . \tau$

$\Gamma ::= x_1 : \tau_1, \ldots, x_n : \tau_n \qquad \Theta ::= \alpha_1, \ldots, \alpha_m$

$J ::= \Theta ; \Gamma \vdash e : \tau \quad (FV(e) \subseteq \Gamma)$
$\quad \mid \Theta \vdash \tau : \star \quad (FV(\tau) \subseteq \Theta)$ (judgments)

Typing Rules

$$\frac{}{\Theta, \alpha \vdash \alpha : \star} \qquad \frac{\Theta \vdash \tau_1 : \star \quad \Theta \vdash \tau_2 : \star}{\Theta \vdash \tau_1 \to \tau_2 : \star}$$

$$\frac{\Theta, \alpha \vdash \tau : \star}{\Theta \vdash \forall \alpha . \tau : \star}$$

(says → doesn't bind any FV's.)

Now incorporate Θ into rules for STLC.

$$\frac{}{\Theta, \Gamma, x : \tau \vdash x : \tau} \qquad \frac{\Theta, \Gamma \vdash e : \tau_1 \to \tau_2 \quad \Theta, \Gamma \vdash e' : \tau_1}{\Gamma \vdash e\, e' : \tau_2}$$

$$\frac{\Theta, \Gamma \vdash e : \forall \alpha . \tau \quad \Theta \vdash \tau' : \star}{\Theta, \Gamma \vdash e\, \tau' : \tau[\tau'/\alpha]} \qquad \frac{\Theta, \Gamma, x : \tau \vdash e : \tau'}{\Theta, \Gamma \vdash \lambda x : \tau . e : \tau \to \tau'}$$

$$\frac{\Theta, \alpha, \Gamma \vdash e : \tau}{\Theta, \Gamma \vdash \Lambda \alpha . e : \forall \alpha . \tau}$$

$$\frac{}{\circ \,;\, x:\alpha \vdash x:\alpha}$$

$\lambda x. \lambda x. x =_\alpha \lambda x. \lambda y. y$ oops...

we can fix this by eg

$$\frac{\Theta \vdash \Gamma, x:\tau}{\Theta, \Gamma\; x:\tau \vdash x:\tau}\ ,\ \text{etc} \ldots$$

Theorem: If $\Theta \vdash \tau : \star$ then $FV(\tau) \subseteq \Theta$

If $\Theta, \Gamma \vdash e : \tau$ then $\Theta \vdash \tau : \star$ and $\Theta \vdash \Gamma$