Parallelism

Parallelism
- is an evaluation strategy
- deterministic

Concurrency
- is about manging events that happen at the same time
- nondeterministic

Parallel PCF (PPCF) Binary fork-join

Exp $e ::= \ldots$
x
$s(e)$
z
$fix\{\tau\}(x;e)$
$par(e_1;e_2;x_1.x_2.e)$
$par\ x_1 = e_1\ and\ x_2 = e_2\ in\ e$

Statics

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2 \quad \Gamma, x_1 : \tau_1, x_2 : \tau_2 \vdash e : \tau}{\Gamma \vdash par(e_1, e_2; x_1.x_2.e) : \tau}$$

Sequential Dynamics

$$\frac{e_1 \longmapsto e_1'}{par(e_1, e_2; x_1.x_2.e) \longmapsto_{seq} par(e_1', e_2, x_1 x_2 e)}$$

$$\frac{e_1\ val \quad e_2\ val}{par(e_1; e_2; x_1.x_2.e) \longmapsto [e_1, e_2 / x_1, x_2] e}$$

Sequential Dynamics

$$\frac{e_1 \underset{seq}{\longmapsto} e_1'}{par(e_1, e_2; x_1 x_2 e) \underset{seq}{\longmapsto} par(e_1' e_2; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \qquad e_2 \underset{seq}{\longmapsto} e_2'}{par(e_1, e_2; x_1.x_2.e) \underset{seq}{\longmapsto} par(e_1; e_2'; x_1 x_2 e)}$$

$$\frac{e_2 \text{ val} \qquad e_1 \underset{seq}{\longmapsto} e_1'}{par(e_1; e_2; x_1.x_2.e) \underset{seq}{\longmapsto} par(e_1'; e_2; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \qquad e_2 \text{ val}}{par(e_1; e_2; x_1.x_2.e) \underset{seq}{\longmapsto} [e_1, e_2 / x_1, x_2] e}$$

Parallel Dynamics

$$\frac{e_1 \underset{par}{\longmapsto} e_1' \qquad e_2 \underset{par}{\longmapsto} e_2'}{par(e_1, e_2; x_1.x_2.e) \underset{par}{\longmapsto} par(e_1'; e_2'; x_1 x_2 e)}$$

$$\frac{e_1 \text{ val} \qquad e_2 \underset{par}{\longmapsto} e_2'}{par(e_1, e_2\ x_1.x_2.e) \underset{par}{\longmapsto} par(e_1; e_2'; x_1.x_2.e)}$$

$$\frac{e_2 \text{ val} \qquad e_1 \longmapsto e_1'}{par(e_1; e_2; x_1.x_2.e) \longmapsto par(e_1'; e_2; x_1.x_2.e)}$$

$$\frac{e_1 \text{ val} \qquad e_2 \text{ val}}{par(e_1; e_2; x_1.x_2.e) \longmapsto [e_1, e_2 / x_1, x_2] e}$$

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Theorem Assume e is well-typed (so terminates). Then

$$e \mapsto^*_{par} v \quad \text{iff} \quad e \mapsto^*_{seq} v$$

Proof: show 1) $e \mapsto^*_{seq} v$ iff $e \Downarrow v$

2) $e \mapsto^*_{par} v$ iff $e \Downarrow v$

Eval Dynamics

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad [v_1 v_2 / x_1 x_2] e \Downarrow v}{par(e_1; e_2; x_1 x_2 e) \Downarrow v}$$

Lemma 1 $e \Downarrow v$ implies $e \mapsto^*_{seq} v$

Proof: induction on $e \Downarrow v$

Case: Rule for par:

Then $e = par(e_1; e_2; x_1 x_2 e')$

and $e_1 \Downarrow v_1$, $e_2 \Downarrow v_2$, $[v_1 v_2 / x_1 x_2] e' \Downarrow v$

IH: $e_1 \mapsto^*_{seq} v_1$, $e_2 \mapsto_{seq} v_2$ $[v_1 v_2 / x_1 x_2] e' \mapsto_{seq} v$

(Then use the rules above for Sequential Dynamics.)

Show $par(e_1 e_2 x_1 x_2 e) \mapsto^*_{seq} v$

1) By ind. on n_1 $par(e_1 e_2 x_1 x_2 e') \mapsto^{n_1}_{seq} par(v_1 e_2 x_1 x_2 e)$

2) By ind on n_2 $par(v_1 e_2 x_1 x_2 e') \mapsto^{n_2}_{seq} par(v_1 v_2 x_1 x_2 e)$

3) use rule

$par(v_1 v_2 x_1 x_2 e') \mapsto [v_1 v_2 / x_1 x_2] e'$ Then,

$[v_1 v_2 / x_1 x_2] e' \mapsto^*_{seq} v$ by IH

Lemma 2

$e \xrightarrow[\text{seq}]{*} v$ and v val implies $e \Downarrow v$

Proof: show $e \xrightarrow[\text{seq}]{} e'$ and $e' \Downarrow v$ implies $e \Downarrow v$.

EXERCISE!

Cost Semantics

GOAL: cost semantics $e \Downarrow^{k} v$
where k describes both
seq and parallel costs.

Cost Graph

$c ::= 1$ unit cost
$\quad\quad 0$ zero cost

$c_1 \otimes c_2$ parallel combination

$c_1 \oplus c_2$ sequential combination

Work (seq cost)

$wk(1) = 1$

$wk(0) = 0$

$wk(c_1 \otimes c_2) = wk(c_1) + wk(c_2)$

$wk(c_1 \oplus c_2) = wk(c_1) + wk(c_2)$

Depth (parallel cost)

$dp(1) = 1$

$dp(0) = 0$

$dp(c_1 \otimes c_2) = \max\{dp(c_1), dp(c_2)\}$

$dp(c_1 \oplus c_2) = dp(c_1) + dp(c_2)$

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Eval Rules

$$\frac{}{z \Downarrow^{0} z} \qquad \frac{e \Downarrow^{c} v}{S(e) \Downarrow^{c} S(v)} \qquad \frac{}{\lambda(x:\tau)e \Downarrow^{0} \lambda(x:\tau)e} \qquad \frac{[\mathrm{fix}\{\tau\}(x.e)/x]e \Downarrow^{c} v}{\mathrm{fix}\{\tau\}(x.e) \Downarrow^{c \oplus 1} v}$$

$$\frac{e_1 \Downarrow^{c_1} \lambda(x:\tau)e \quad e_2 \Downarrow^{c_2} v_2 \quad [v_2/x]e \Downarrow^{c_3} v}{e_1(e_2) \Downarrow^{c_1 \oplus c_2 \oplus c_3 \oplus 1} v} \qquad \frac{e_1 \Downarrow^{c_1} v_1 \quad e_2 \Downarrow^{c_2} v_2 \quad [v_1 v_2/x_1 x_2]e \Downarrow^{c_3} v}{\mathrm{par}(e_1\, e_2\; x_1 x_2 e) \Downarrow^{(c_1 \otimes c_2) \oplus c_3 \oplus 1} v}$$

Theorem

a) If $e \Downarrow^{c} v$ then $e \xmapsto[\mathrm{seq}]{wk(c)} v$ and $e \xmapsto[\mathrm{par}]{dp(c)} v$.

b) If $e \xmapsto[\mathrm{seq}]{w} v$ then $e \Downarrow^{c} v$ for some c and $wk(c) = w$.

c) If $e \xmapsto[\mathrm{par}]{d} v$ then $e \Downarrow^{c} v$ for some c and $dp(c) = d$.

Jan Hoffman (Lect 5 conclusion)

Bounded Implementations

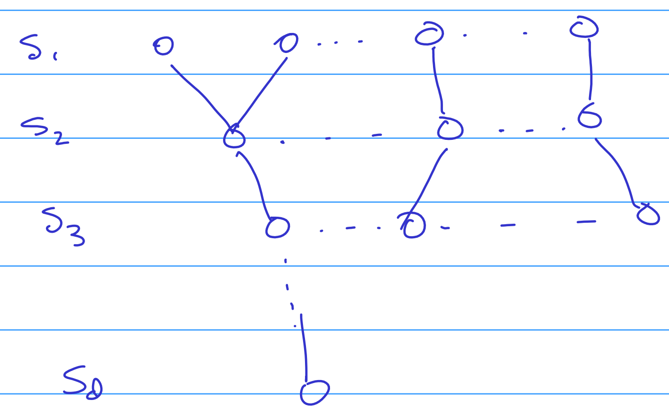
Prototypical Result: Brent's Theorem

Machine model: shared-memory multiprocessor (SMP)

- some fixed $p > 0$ processors
- shared memory w/ constant time access
- constant time synchronization mechanism

Theorem If $e \Downarrow^{c} v$ with $wk(c) = w$ and $dp(c) = d$, then e can be evaluated on an SMP in time $O(\frac{w}{p} + d)$.

"Proof"



$$\sum_{i=1}^{d} \left\lceil \frac{s_i}{p} \right\rceil \leq \sum_{i=1}^{d} \frac{s_i}{p} + 1 = \frac{\sum s_i}{p} + d = \frac{w}{p} + d$$

But what does big O mean here?

what is n??