Main Course

$$\frac{\text{beef, bun} \vdash \text{hamburger} \qquad \text{potato, oil} \vdash \text{fries}}{\text{beef, bun, potato, oil} \vdash \text{burger} \otimes \text{fries}} \otimes R$$

[why not?]

$$\frac{\text{beef, oil} \vdash \text{cfs} \qquad \text{potato, bun} \vdash \text{potato sand.}}{\text{beef, bun, oil, potato} \vdash \text{cfs} \otimes \text{potato sand.}}$$

Soup of the Day (Chef's choice)

$$\frac{\text{tomato, cream} \vdash \text{tomato soup}}{\text{tomato, cream} \vdash \text{tomato soup} \oplus \text{chicken noodle soup}} \oplus R_1$$

Desert

$$\frac{\text{strawberries, bananas} \vdash \text{fruit salad} \qquad \text{strawberries, bananas} \vdash \text{smoothie}}{\text{strawberries, bananas} \vdash \text{fruit salad} \,\&\, \text{smoothie}} \,\&R$$

Main = burger $\otimes$ fries.

Soup = tomato $\oplus$ chicken

Desert = fruit salad & smoothie (customer's choice)

Lunch = main $\otimes$ soup $\otimes$ desert

Ingredients = beef, bun, potato, oil, tomato, cream, strawberries, bananas

$$\frac{\text{Ingredients} \vdash \text{Lunch} \qquad \text{Ingredients} \vdash \text{Lunch}}{\text{Ingredients, Ingredients} \vdash \text{Lunch, Lunch}} (\text{Mix})$$

$$\frac{\Gamma \vdash \Delta \qquad \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} (\text{Mix})$$

$$\dfrac{\dfrac{\text{ingred.} \vdash \text{Lunch} \quad \text{ingred.} \vdash \text{Lunch}}{\text{ingred}, \text{ingred} \vdash \text{Lunch}, \text{Lunch}}\,(\text{Mix})}{\text{ingred.ingred} \vdash \text{Lunch} \parr \text{Lunch}}\,(\parr R)$$

$$(\tau_1 \parr \tau_2)^\perp = \tau_1^\perp \oplus \tau_2^\perp, \qquad (\tau_1 \oplus \tau_2)^\perp = \tau_1^\perp \parr \tau_2^\perp$$

$$(\tau_1 \otimes \tau_2)^\perp = \tau_1^\perp \parr \tau_2^\perp, \qquad (\tau_1 \parr \tau_2)^\perp = \tau_1^\perp \otimes \tau_2^\perp$$

$$\top^\perp = 0 \qquad 0^\perp = \top$$

$$\underline{1}^\perp = \underline{\perp} \qquad \underline{\perp}^\perp = \underline{1}$$

Theorem $\tau = \tau^{\perp\perp}$ ($\perp$ is an involution)

Theorem (Duality) If $\Gamma \vdash \Delta$ is derivable, then so is $\Delta^\perp \vdash \Gamma^\perp$.

($\Delta^\perp$ means apply $\perp$ to each thing in Δ)

$$\dfrac{\Gamma, \tau \vdash \Delta}{\Gamma \vdash \tau^\perp, \Delta}\,\text{invL} \qquad \dfrac{\Gamma \vdash \tau, \Delta}{\Gamma\, \tau^\perp \vdash \Delta}\,\text{invR}$$

$$\dfrac{}{\vdash \tau, \tau^\perp}\,\text{Id} \qquad \dfrac{\vdash \tau, \Delta \quad \vdash \tau^\perp, \Delta'}{\vdash \Delta, \Delta'}\,\text{cut}$$

(we could add these rules & omit all the left rules as is sometimes done in the lit.)

Paul Downen (Lecture 7) [2018/07/07]

Some Derivations in Linear Logic

$$\dfrac{\dfrac{}{\tau \vdash \tau}\,(Id)}{\tau, \tau^{\perp} \vdash \cdot}\,(Inv\,L) \qquad \dfrac{\dfrac{}{\tau \vdash \tau}\,(Id)}{\cdot \vdash \tau \multimap \tau}\,(\multimap R)$$

$$\dfrac{\dfrac{\dfrac{}{\tau^{\perp\perp} \vdash \tau}\,(Id)}{\cdot \vdash \tau^{\perp}, \tau}\,(Inv\,R)}{\cdot \vdash \tau^{\perp} \parr \tau}\,(\parr R)$$

"lolipop"

$$\tau_1 \multimap \tau_2 = \tau_1^{\perp} \parr \tau_2$$

$$\tau_1 \parr \tau_2 = \tau_1^{\perp} \multimap \tau_2$$