Game Semantics

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1 Game Semantics to Prove Equivalence

```
f: com -> nat -> com ⊢ new s
                       new x
                       f(grab s; x++; rel s)
                       (new y;
                       grab s;
                       y := !x;
                       rel s;
                       !y)
                     ≡
                       new s
                       new x
                       f(grab s; x--; rel s)
                       (new y;
                       grab s;
                       y := -!x;
                       rel s;
                       !y)
```

```
f: com -> nat -> com ⊢ ... : com
```

Two paths to follow:

q'	q
grab s	grab s
okay s	okay s
read x	read x
n x	m x
w(n-1)	rel s
okay x	okay s
rel s	-m
okay s	
0 x	

Which is $(Q \cdot 0)^*(Q' \cdot OK')(Q \cdot 1)^* \cdots$
$\texttt{play } p \to \exists t \texttt{ s.t. } [\![t]\!] = \texttt{strat}(p)$
$P_{\texttt{Nat}_1 \to \texttt{Nat}_2 \to \texttt{Nat}} \ni qq_1 3_1 q_2 4_2 7 \in [\![\lambda x.\lambda y.x + y]\!]$ when composed with 3 4.
In parallel: $\in \lambda x_1 \lambda x_2$ new $\bar{y}$ in $(y_1 := x_1; \| \; y_2 := x_2;); 7$

```
test(x) = new sem s; if x then skip else g(s); g(s)
```

$\in \lambda x_1 \lambda x_2$ new $\bar{y}$ in $(\texttt{test}(y_1 := 3); \| \; \texttt{test}(y_2 := 4);); 7$
All together:

$$\in \lambda x_1 \lambda x_2 \quad \begin{array}{c} \text{sem } \bar{s} \text{ in} \\ \text{new } \bar{y} \text{ in} \end{array} \quad \begin{array}{c} (y_1 := x_1; \\ (\texttt{test}(y_1 := 3); \\ \text{grab(s)} \end{array} \left\| \begin{array}{c} \text{release(s)} \\ y_2 := x_2;) \\ \texttt{test}(y_2 := 4);) \end{array} \right. ; 7$$

2 Applications of GS

Sem Model (GS):

- precise
- elementary
- implemented
- generalized
- compilation
  - Automata
  - Circuits
  - Distributions
  - Heterogenous architecture
  - FFI
- unification
  - Obs = not decidable
  - Deciable equivalence
  - Approximation: CEGAR