Game Semantics

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1 Recap via Example

T_* : $\texttt{Nat} \to \texttt{Nat} \to \texttt{Nat}$
For the purposes of this lecture will denote what is happening in which column by the use of 'primes' (each represents a different arena):
T_* : $\texttt{Nat}' \to \texttt{Nat}'' \to \texttt{Nat}$
then the order of moves is: $q; q'; n'; q''; m''; p = m * n$
Where $p \to q; m'' \to q''; q'' \to q; n' \to q'; q' \to q$
Notation: $\langle x \rangle$ denotes new variable being introduced
$q\langle b \rangle \cdot g' b \langle c \rangle \cdot n' c \langle d \rangle \cdots$
$\hat{T}(P^O) = m^P$ - "automata-like"

2 Semantic View of State

2.1 Local State

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Int x in M ≝ new(λx.M)
new : (var -> command) -> command
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$(q \vdash a) \vdash (q' \vdash a') \vdash (\text{write(n)}^{Q_0} \vdash \text{OK}^{PA}, \text{read}^{Q_0} \vdash n^{PA})$
Steps:

1. 0
2. $p \cdot wr(n) \mapsto \text{OK}$
3. $p \cdot rd() \cdot n \cdot rd() \mapsto n$
4. $p \cdot wr(n) \cdot \text{OK} \cdot rd \mapsto n$
5. $q \mapsto q'$
6. $p \cdot a' \mapsto a$
7. $q \cdot q' \cdot rd \mapsto 0$

2.2 Control

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label x in M : (1 + Nat) 'or' Option Nat
≝ catch(λx.M) : (command -> Nat) -> (1+Nat)
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3 Composing Strategies

$A \xrightarrow{T} B \xrightarrow{\tau} C$ turns into $A \xrightarrow{T;\tau}$

1. Hiding
   $P \downarrow X = (P', \Pi)$ with P' new seq. and $\Pi : \mathbb{A} \to \mathbb{A}$
   $\mathcal{E} \downarrow X = \mathcal{E}$
   $P \cdot m(a\langle b\rangle) \downarrow X = (P' \cdot m(\Pi(a)) \cdot \langle b\rangle, \Pi)\ m \notin X$
   $P \cdot m(a\langle b\rangle) \downarrow X = (P' \cdot (\Pi|b \mapsto \Pi(a))\ m \in X$

2. Selecting/Threading
   $P \uparrow X = (P', X')$
   $\mathcal{E} \uparrow X = \mathcal{E}$
   $P \cdot m(a\langle b\rangle) \uparrow X = (P' \cdot m(a\langle b\rangle), X \cup \{b\}\ a \in X$
   $P \cdot m(a\langle b\rangle) \uparrow X = (P', X)\ a \notin X$

3.1 Definition: Synchronizing

$T \subseteq J_M, \tau \subseteq J_N$
$T \underset{M,N}{\|} \tau = \{P \in J_{M \cup N} | P \downarrow (M \backslash N) \in \tau \land P \downarrow (N \backslash M) \in T\}$

3.2 Definition: Iteration

$T \subseteq J_M, N \subseteq M$
$!_N T = \{P \in J_M | \forall m(a\langle b\rangle) \in P, m \in N \implies P \uparrow \{b\} \in T\}$

3.3 Definition: Composition

$T : A \to B,\ \tau : B \to C$
$\tau \circ T = T, \tau = (!_{I_B} T \underset{\substack{M_A \to B \\ M_B \to C}}{\|} \tau) \downarrow M_B$

3.4 Is composition sensible?

Is it well formed? $T, \tau : A \to C$ well justified? YES
$(T;\tau);\mathcal{V} = T;(\tau;\mathcal{V})$
$T \subseteq T' \implies \mathcal{V};T \subseteq \mathcal{V};T'$ and $T;\tau \subseteq T', \tau$

4 Exercises/Lemmas

- $P \in P_{A \times B \to C}$ then $P \downarrow M_A \in P_{B \to C}$
- $\forall m(a\langle b \rangle) \in P, m \in I_A$ then $P \uparrow \{b\} \in P_A$
- $P \downarrow M_A \downarrow M_B = P \downarrow M_B \downarrow M_A$
- $P \downarrow M_A \downarrow \{b\} = P \downarrow \{b\} \downarrow M_A$