Bauer 3

Signature $\Sigma = \{ op_i \; P_i \rightsquigarrow A_i \}_{i \in I}$

Trees/terms $\quad Tree_\Sigma(V)$

- return $v \qquad v \in V \quad$ pure
- $op_i(p, k) \qquad p \in P_i \quad k : A_i \to Tree_\Sigma(x)$

effect

Interpretation/model M

carrier $|M|$

$[\![op_i]\!]_M : P_i \times |M|^{A_i} \to |M|$

$Free_T(V) = Tree_{\Sigma_T}(V) / \approx_T \qquad$ computations

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Transformations of computations

$|Free_T(V)| \to |Free_{T'}(V')| \qquad ?$

(maybe can simulate left w/ right)

lots of available structure

not functions;

should be homomorphism for some Theory

T or T' ?

make more sense to respect domain

T homomorphism

$$\mathrm{Free}_T(V) \longrightarrow \underset{T\text{-model}}{M}$$

need extra information on codomain (need to turn it into T model)

where $|M| = |\mathrm{Free}_{T'}(V')|$

and for $op_i : P_i \to A_i$ we need

$$[\![op_i]\!]_M : P_i \times |\mathrm{Free}_{T'}(V')|^{A_i} \to |\mathrm{Free}_{T'}(V')|$$

(such that $\underset{\text{equations}}{\mathcal{E}_T}$ are satisfied by M)

"Handler" need to know how to map generators of V's in $\mathrm{Free}_T(V)$

a <u>Handler</u> H given by:

- the maps $[\![op_i]\!]_M$ as above
- a map $r : V \to |\mathrm{Free}_{T'}(V')|$

e.g. $H([\text{return } v]) = r(v)$

$$H([op_i(p, K)]) = \underset{c_i}{\underset{\parallel}{[\![op_i]\!]_M}}(p, H \circ K)$$

<u>Notation</u>:

$$\text{handler}\, \{ \text{return } x \mapsto r(x), \; \ldots\; op_i(x, K) \mapsto c_i(x, K) \ldots \}$$

$$H\left([\![op_i(p,\kappa)]\!]\right) = [\![op_i]\!]_M\left(p, \underbrace{H \circ \kappa}\right)$$

$\kappa : A_i \to |Free_T(V)|$

$A_i \xrightarrow{\kappa} |Free_T(V)| \xrightarrow{H} |Free_{T'}(V')|$

Notation for $H(c)$ where $c \in |Free_T(V)|$:

with H handle C

with H handle return v $= r(v)$

with H handle $op(p,\kappa)$ $= c_i(p, \lambda x.$

with H handle $\kappa\, x)$

always be suspicious of simple solutions that solve all problems

idea Plotkin, Power

comodels - world

model - computation

not "top-level" handler

but : comodel

Scanned with CamScanner

Comodels

A **T-comodel** in a category $\mathbb{C}$ is a

T-model in $\mathbb{C}^{op}$

In $\mathbb{C} = \underline{Set}$ we get

a T-cointerpretation W

[M for model
W for comodels
(world)]

is given by:

- carrier set $|W|$
- for each $op_i : P_i \to A_i$ a co-operation

$$[\![op_i]\!]^W : P_i \times |W| \to A_i \times |W|$$

Why?

$$[\![op_i]\!]_M : P_i \times |M|^{A_i} \to |M|$$

curry $\quad |M|^{A_i} \to |M|^{P_i}$

dualize $\quad A_i \times |M| \leftarrow P_i \times |M|$ turn arrow around

(exponentials are right adjoint to products)

Extend to interpret trees
know what satisfy eq means
$\Rightarrow$ <u>T - comodel W</u>
as a cointerpretation that
validates the equations

<u>Examples</u>

$\text{print} : \text{String} \rightsquigarrow 1$

$[\![\text{print}]\!]^W : \text{String} \times |W| \to 1 \times |W|$ ← new world (changed) by print

$\text{read} : 1 \rightsquigarrow \text{String}$

$[\![\text{read}]\!]^W : 1 \times |W| \to \text{String} \times |W|$

$\text{rand} : 1 \rightsquigarrow \text{bool}$

Model M and comodel W

Tensor $\underbrace{M}_{\text{software}} \otimes \underbrace{W}_{\text{hardware}} = M \times W /_{\sim_T}$ (least equiv reln such that)

where

$$(\underbrace{[\![op_i]\!]_M (p, \kappa)}_{\text{run this comp}}, \underbrace{w}_{\text{in this world}}) \sim_T (\underbrace{\kappa}_{\text{continuation}}(\underbrace{a}_{\text{return value}}), \underbrace{w'}_{\text{new world}})$$

$$[\![op_i]\!]^W (p, w) = (a, w')$$

Combining theories

T and T'

2 ways to combine

1) Coproduct $T \oplus T'$ (disjoint union)

$$\Sigma_{T \oplus T'} = \Sigma_T + \Sigma_{T'}$$

$$\mathcal{E}_{T \oplus T'} = \mathcal{E}_T + \mathcal{E}_{T'}$$

2) Tensor $T \otimes T'$ (not same tensor as above)

$$\Sigma_{T \otimes T'} = \Sigma_T + \Sigma_{T'}$$

$$\mathcal{E}_{T \otimes T'} = \mathcal{E}_T + \mathcal{E}_{T'} + \text{distributivity}$$

distribute $(x+y)\cdot z = x\cdot z + y\cdot z$

exchange operations ($\cdot$ for $+$)

op_1

combine many states

want distribute