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Encoding Types in System F

Unit

$$\text{unit} \triangleq \forall t.\, t \to t$$
$$\langle\rangle \triangleq \Lambda(t).\, \lambda(x:t)\, x$$

Pairs

$$\tau_1 \times \tau_2 \triangleq \forall r\, (\tau_1 \to \tau_2 \to r) \to r$$
$$\langle e_1, e_2 \rangle \triangleq \Lambda(r)\ \lambda(f : \tau_1 \to \tau_2 \to r)\, f(e_1)(e_2)$$
$$e \cdot l \triangleq e(\tau_1)(\lambda x_1{:}\tau_1.\, \lambda x_2{:}\tau_2.\, x_1)$$
$$e \cdot r \triangleq e(\tau_2)(\lambda x_1{:}\tau_1.\, \lambda x_2{:}\tau_2.\, x_2)$$

Binary Sums

$$\tau_1 + \tau_2 \triangleq \forall r\, (\tau_1 \to r) \to (\tau_2 \to r) \to r$$

aka $\text{inl}\, e_1$ — $l \cdot e_1 \triangleq \Lambda(r)\ \lambda(f_1 : \tau_1 \to r).\, \lambda(f_2 : \tau_2 \to r).\, f_1 e_1$

$\text{inr}\, e_2$ — $r \cdot e_2 \triangleq \Lambda(r)\ \lambda(f_1 : \tau_1 \to r).\, \lambda(f_2 : \tau_2 \to r).\, f_2 e_2$

$$\text{case}(e; x_1.e_1, x_2.e_2) \triangleq e(\tau)\,(\lambda(x_1:\tau_1).e_1)(\lambda(x_2:\tau_2).e_2)$$
$$\cdot\ e_1 : \tau \wedge e_2 : \tau$$

Natural Numbers

$$\text{nat} \triangleq \forall r.\, (\text{nat} \to r) \to (r \to r) \to r$$

or just $\forall r.\ r \to (r \to r) \to r$

$$z \triangleq \Lambda r.\, \lambda(z:r)\, \lambda(s : r \to r).\, z.$$
$$S(e) \triangleq \Lambda r.\, \lambda(z:r)\, \lambda(s : r \to r).\, s\,(\underbrace{e[r](z)(s)}_{\text{computes result for predecessor.}})$$

$e : \text{nat}$

$$\text{iter}\{e_1,\, x.e_2\}(e) \triangleq e[\tau](e_1)(\lambda(x:\tau).e_2)$$