# MT Robot Reading Group

## Smooth and Collision-Free Navigation for Multiple Robots Under Differential-Drive Constraints

by Jamie Snape, Jur van den Berg, Stephen J. Guy, and Dinesh Manocha

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# Agenda

1. Introduction
2. Related Work
3. Approach
    - Kinematics of a Differential-Drive Robot
        - Kinematic Model
        - Effective Center and Effective Radius
    - Multi-Robot Navigation
        - Optimal Reciprocal Collision Avoidance
        - Navigation Algorithm
4. Experiment
5. Discussion

# Introduction

- Most mobile robots in practical service have differential-drives
- Robots as distributed system are used in many areas
- Most prior work in smooth and collision-free navigation for single robot

## Related Work

- Early work with kinematics constraints progressed:
  1. With classical Dubins car
  2. Reeds-Shepp car
  3. Simple car

# Related Work

- Single robot navigation through cluttered environment
  - Some based on velocity obstacles and its extensions

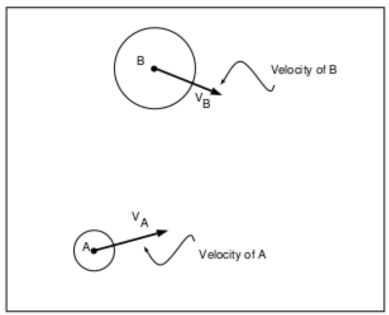



Figure 1: The robot and a moving obstacle.

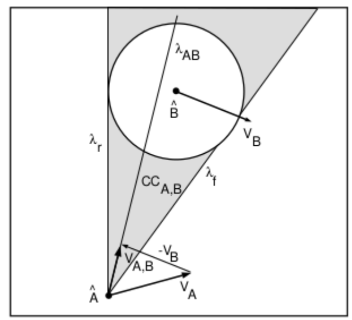



Figure 2: The Relative Velocity $\mathbf{v}_{A,B}$ and the Collision Cone $CC_{A,B}$.

## Related Work

- Decoupled or centralized planners
  - Decoupled: Consider every robot separately
  - Centralized: Combining the degrees of freedom of each robot

# Approach

## Kinematic Model

Differential-drive robot:

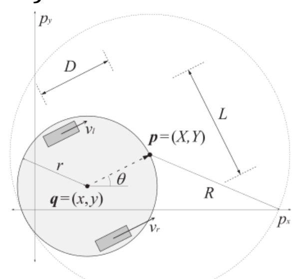
- Position of its center: $\mathbf{q} = (x, y)$
- Orientation: $\theta$
- Transition equations:

$$\dot{x} = \frac{v_l + v_r}{2} cos(\theta),\ \dot{y} = \frac{v_l + v_r}{2} sin(\theta),\ \dot{\theta} = \frac{v_r - v_l}{L}$$

# Approach

## Effective Center and Effective Radius

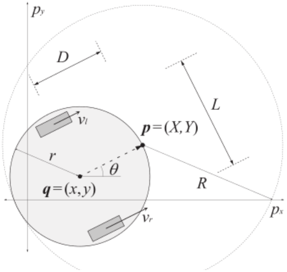
- To enlarge the radius for navigation
  - Increased maneuverability
  - Smooth handling of the kinematic constraints
  - The center $\mathbf{q}$ is not fully controllable



# Approach

## Effective Center and Effective Radius

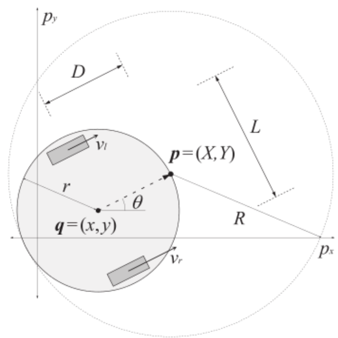
- Effective center: $\mathbf{p} = (X, Y)$
  - May be translated in a direction orthogonal to the wheels
  - Fully controllable
- Effective radius: $R = r + D,\ R > 0$



# Approach

## Effective Center and Effective Radius

$$X = x + Dcos(\theta), \qquad Y = y + Dsin(\theta)$$



# Approach

## Effective Center and Effective Radius

$$X = x + Dcos(\theta), \qquad Y = y + Dsin(\theta)$$

$$\dot{X} = (\frac{cos(\theta)}{2} + \frac{Dsin(\theta)}{L})v_l + (\frac{cos(\theta)}{2} - \frac{Dsin(\theta)}{L})v_r,$$

$$\dot{Y} = (\frac{sin(\theta)}{2} + \frac{Dcos(\theta)}{L})v_l + (\frac{sin(\theta)}{2} - \frac{Dcos(\theta)}{L})v_r,$$

$$\mathbf{v} = M(\theta) \cdot \mathbf{u}$$

# Approach

## Effective Center and Effective Radius

$$\mathbf{v} = M(\theta) \cdot \mathbf{u}$$

Obtain wheel speeds $v_l,\ v_r$ from velocity $\mathbf{v}$ by solving

$$\mathbf{u} = M^{-1}(\theta) \cdot \mathbf{v}$$

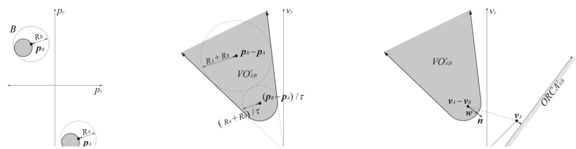
# Approach

## Multi-Robot Navigation

### Optimal Reciprocal Collision Avoidance

$$VO^{\tau}_{A|B} = \{\mathbf{v} \mid \exists t \in [0, \tau] \ :: \ t(\mathbf{v} - \mathbf{v}_B) \\ \in D(\mathbf{p}_B - \mathbf{p}_A, R_A + R_B\}$$

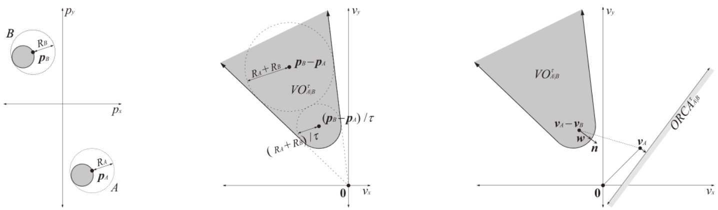
where $D(\mathbf{p}, R)$ is an open disc of radius $R$ centered at $\mathbf{p}$



# Approach

## Multi-Robot Navigation

### Optimal Reciprocal Collision Avoidance



$$CA^{\tau}_{A|B} = \{\mathbf{v} \mid \mathbf{v} \notin VO^{\tau}_{A|B}\}$$

# Approach

## Multi-Robot Navigation

### Optimal Reciprocal Collision Avoidance

$$CA^{\tau}_{A|B} = \{\mathbf{v} \mid \mathbf{v} \notin VO^{\tau}_{A|B}\}$$

- A and B will not collide but their trajectories may not be smooth
- Optimal reciprocal collision avoidance resolves this situation

$$ORCA^{\tau}_{A|B} \subset CA^{\tau}_{A|B}$$

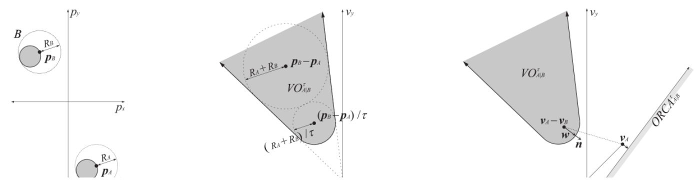
# Approach

## Multi-Robot Navigation

### Optimal Reciprocal Collision Avoidance

$$\mathbf{w} = (\operatorname{argmin}_{v \in \partial VO^{\tau}_{A|B|}} ||\mathbf{v} - (\mathbf{v}_A - \mathbf{v}_B)||_2) - (\mathbf{v}_A - \mathbf{v}_B)$$

$$ORCA^{\tau}_{A|B} = \{\mathbf{v} \mid (\mathbf{v} - (\mathbf{v}_A + \frac{1}{2}\mathbf{w})) \cdot \mathbf{n} \geq 0\}.$$



# Approach

## Multi-Robot Navigation

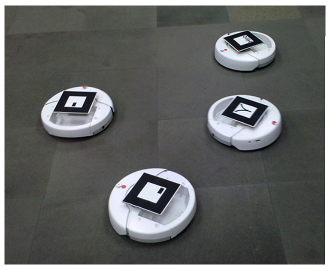
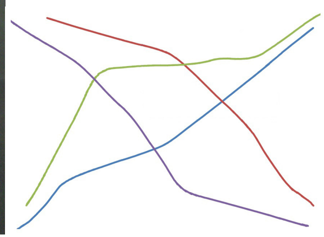
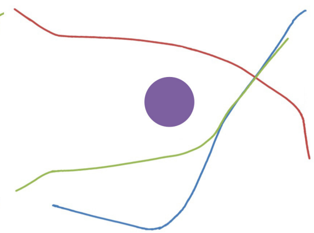
### Navigation Algorithm

- Incorporating each of the previous components together

$$ORCA^{\tau}_{A_i} = \bigcap_{A_j \in A, i \neq j} ORCA^{\tau}_{A_i | A_j}$$

- Preferred velocity: $\mathbf{v}^{\text{pref}}_{A_i}$ velocity if no other robots in its way
- New velocity: $\mathbf{v}^{\text{new}}_{A_i} = \text{argmin}_{\mathbf{v} \in ORCA^{\tau}_{A_i}} ||\mathbf{v} - \mathbf{v}^{\text{pref}}_{A_i}||_2$
    - Calculated efficiently using linear programming

# Experiments

# Discussion

## Questions

- Strong assumption:

  Each robot is aware of the exact position and velocity of all the other robots at all times.

- ORCA works without communication with each other, requiring knowledge of only the radius, position, and velocity of each other.