OLAC
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Label Buying Decision Engine

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Problem statement

Compute the optimal allocation of labels acquisitions that maximize, under resource constraints, long run utility.

Throughout this document the following notation will be used unless convention or availability dictate otherwise:

Let a indicate a scaler.
Let α indicate a vector.
Let $\mathbf{a}$ indicate a vector.
Let $\mathbf{A}$ indicate a matrix.
Let $\mathcal{A}$ indicate a cluster.

Utility

The total utility is defined as the utility gained from each label bought at time t. Utility consists of the cost of buying a label, $c \in \mathbb{R}_+$, the revenue gained by detecting fraud, $u \in \mathbb{R}_+$ with the assumption that the utility gained from determining fraud is greater than the cost, $c < u$. The utility function is defined as:

$$U(\mathbf{u}, \mathbf{y}, \theta, c) = \mathbf{u} \times (\mathbf{y} \times \theta) - c\theta = \sum_{i=0}^{n} u_i y_i \theta_i - c\theta_i \tag{1}$$

Additional elements can be added such as cost of true negatives, missing fraud, or additional costs for investigating false positives.

Clustering

Information K-means

For each block of time t, $t \in \mathbb{Z}$, the newly observed data is compared to the historical data and evaluate whether the number of clusters needs to be adjusted.
Let $k_t = f(\mathbf{f})$; where g is a function that approximates the optimal number of clusters. Let $\mathcal{K}_t = f(k_t, \mathbf{X})$ where f is the K-means algorithm that approximates the optimal centroid location and assignment of data points.

cluster assignment:

$$K_\eta^{(q)} = \{x_i \ : \ \|x_i - m_\eta\|^2 \leq \|x_i - m_j\|^2 \ \forall \ j, 1 \leq j \leq k\} \tag{2}$$

centroid update:

$$m_\eta^{(q+1)} = \frac{1}{|K_\eta^{(q)}|} \sum_{x_i \in K_\eta^{(q)}} x_i \tag{3}$$

Multi Armed Bandit

In the context of this problem a cluster will be seen as the arm where we know the number of points contained in the cluster but do not know their yield.

Greedy agent

A naive implementation would be to maximise utility by allocating all labels in the cluster(s) that have the highest expected utility. The expected utility of allocating all labels to a particular cluster η is defined as:

$$\phi_\eta = \mathbf{h}_\eta \cdot \mathbb{E}[U(\mathbf{u}, \mathbf{y}, \theta, c)] = \mathbf{h}_\eta \cdot (\mathbb{E}[\mathbf{u}] \times (\mathbf{y} \times \theta) - c\theta) \tag{4}$$

Where the matrix $\mathbf{H} \in \{0,1\}^{nk}$ contains the masks for each cluster, $K_\eta \subset \mathcal{K}$, i.e.:

$$h_{i,\eta} = \begin{cases} 1, & \text{if } \mathbf{x}_i \in K_\eta \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

with $i \in \{0, \ldots, n-1\}$, $\eta \in \{1, \ldots, k\}$.

ADW-MAB

Rough idea:

1. Determine the number of clusters (Information K-means, ...)
2. Determine the windows size dependent of measure of change (Adaptive windowing — ADWIN, ...)
3. Determine optimal allocation label (Multi-armed Bandit, ...)

Problem 1
How do we handle observations that that fall out the current clusters, iėȧnomalies at time t.?

Problem 2
When exploring how can we improve upon random selection of labels to aquire?

- Furthest from K (FFK)
  Select the point that is furthest from any known centroid?

- Mixed entropy and distance — MEAD

  Let tl denote the target label, the label that will be acquired.
  Let Δ denote the distance of the potential label from (a/any centroid).
  Let S denote the entropy gained from the label of the potential point.
  Let $u_i \sim U(0,1)$ be a draw from a uniform distribution.

$$P(\theta_i = 1|\Delta, S) = \text{softmax}((c_0\Delta \cdot c_1 S) \cdot \mathbf{u})_i \; \forall i \in \{0, n-1\} \tag{6}$$

  Where $\Delta \in \mathbb{R}^{nk}$, $S \in \mathbb{R}^k$, $0 \leq i < n$ and $1 \leq \eta \leq k$.

Definitions:

- Let $\mathbf{X} \in \mathbb{R}^{nm}$ be the observed data.
- Let $\mathbf{f} \in \mathbb{R}^{m}$ be the vector of features.
- Let $\mathbf{H} \in \{0,1\}^{nk_t}$ be the cluster mask.
- Let $\mathbf{y} \in \{0,1\}^{n}$ be the vector of true labels.
- Let $\theta \in \{0,1\}^{n}$ be the vector of acquired labels.
- Let $\mathcal{K}_t$ be the set of clusters of size k_t at time t.
- Let $\mathbf{k} \in \{1,2,\ldots,n\}^{t}$ be the vector of optimal number of clusters.
- Let $\mathbf{u} \in \mathbb{R}^{n} \sim LN(\mu,\sigma)$ the vector of utility earned by detecting fraud drawn from a log-normal distribution.