OLAC
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Label Buying Decision Engine

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Problem statement

Compute the optimal allocation of labels acquisitions that maximize, under resource constraints, long run utility.

Throughout this document the following notation will be used:

Let a indicate a scaler.
Let α indicate a vector.
Let $\mathbf{a}$ indicate a vector.
Let A indicate a function.
Let $\mathbf{A}$ indicate a matrix.
Let $\mathcal{A}$ indicate a cluster.

Utility

The total utility is defined as the utility gained from each label bought at time t. Utility consists of the cost of buying a label, $c \in \mathbb{R}_+$, the revenue gained by detecting fraud, $u \in \mathbb{R}_+$ with the assumption that the utility gained from determining fraud is greater than the cost, $c < u$. The utility function is defined as:

$$U(\mathbf{u}, \mathbf{y}, \theta, c) = \mathbf{u} \times (\mathbf{y} \times \theta) - c\theta = \sum_{i=0}^{n} u_i y_i \theta_i - c\theta_i \tag{1}$$

Additional elements can be added such as cost of true negatives, missing fraud, or additional costs for investigating false positives.

Clustering

Information K-means

For each block of time t, $t \in \mathbb{Z}$, the newly observed data is compared to the historical data and evaluate whether the number of clusters needs to be adjusted.
Let $k_t = G(\mathbf{f})$; where g is a function that approximates the optimal number of clusters.
Let $\mathcal{K}_t = F(k_t, \mathbf{X})$ where F is the K-means algorithm that approximates the optimal centroid location and assignment of data points.

cluster assignment:

$$K_\eta^{(q)} = \{x_i \ : \ \|x_i - m_\eta\|^2 \leq \|x_i - m_j\|^2 \ \forall \ j, 1 \leq j \leq k\} \tag{2}$$

centroid update:

$$m_\eta^{(q+1)} = \frac{1}{|K_\eta^{(q)}|} \sum_{x_i \in K_\eta^{(q)}} x_i \tag{3}$$

Multi Armed Bandit

Greedy agent

A naive implementation would be to maximise utility by allocating all labels in the cluster(s) that have the highest expected utility. The expected utility of allocating all labels to a particular cluster η is defined as:

$$\phi_\eta = \mathbf{h}_\eta \cdot \mathbb{E}[U(\mathbf{u}, \mathbf{y}, \theta, c)] = \mathbf{h}_\eta \cdot (\mathbb{E}[\mathbf{u}] \times (\mathbf{y} \times \theta) - c\theta) \tag{4}$$

Where the matrix $\mathbf{H} \in \{0,1\}^{nk}$ contains the masks for each cluster, $K_\eta \subset \mathcal{K}$, i.e.:

$$h_{i,\eta} = \begin{cases} 1, & \text{if } \mathbf{x}_i \in K_\eta \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

with $i \in \{0, \ldots, n-1\},\ \eta \in \{1, \ldots, k\}$.

ADW-MAB

Rough idea:

1. Determine the number of clusters (Information K-means, ...)
2. Determine the windows size dependent of measure of change (Adaptive windowing — ADWIN, ...)
3. Determine optimal allocation label (Multi-armed Bandit, ...)

Definitions:

- Let $\mathbf{X} \in \mathbb{R}^{nm}$ be the observed data.
- Let $\mathbf{f} \in \mathbb{R}^{m}$ be the vector of features.
- Let $\mathbf{H} \in \{0,1\}^{nk_t}$ be the cluster mask.
- Let $\mathbf{y} \in \{0,1\}^{n}$ be the vector of true labels.
- Let $\theta \in \{0,1\}^{n}$ be the vector of acquired labels.
- Let $\mathcal{K}_t$ be the set of clusters of size k_t at time t.
- Let $\mathbf{k} \in \{1,2,\ldots,n\}^{t}$ be the vector of optimal number of clusters.
- Let $\mathbf{u} \in \mathbb{R}^{n} \sim LN(\mu,\sigma)$ the vector of utility earned by detecting fraud drawn from a log-normal distribution.