Tutorial Questions Week 5

4.1 Answer the following questions:
a. Can rotations and translations alter the shape of an object?

a. No combination of rotations and translations can alter the shape of an object; they can alter only the object's location and orientation. Rotations and translations alone cannot lead to an affine transformation. If we take a homogeneous coordinate matrix for rotation and translation and multiply it with the object matrix in any combination, it will generate no change in the shape of the object.

b. Can scaling alter the shape of an object?

b. For a given fixed point, there are three degrees of freedom—the two angles necessary to specify the orientation of the vector and the angle that specifies the amount of rotation about the vector.

c. Is there any point that remains unchanged by a rotation?

c. There is one point called the origin that remains unchanged by a rotation. This point is known as the fixed point of the transformation. If we take a homogeneous coordinate matrix for rotation and multiply it with a fixed point, such as the origin (0,0), it will generate the same point.

4.23 Given two nonparallel, three-dimensional vectors u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors?

The vector $a = u \times v$ is orthogonal to u and v. The vector $b = u \times a$ is orthogonal to u and a. Hence, u, a and b form an orthogonal coordinate system.

4.21 We defined an instance transformation as the product of a translation, a rotation, and a scaling. Can we accomplish the same effect by applying these three types of transformations in a different order? Check whether the following sequences commute:
a. A rotation and a uniform scaling

b. Two rotations about the same axis
c. Two translations
d. A rotation and a translation

4.4 If we are interested in only two-dimensional graphics, we can use three dimensional homogeneous coordinates by representing a point as p = [xy1] T and a vector as v = [ab0] T . Find the 3 X 3 rotation, translation, scaling, and shear matrices. How many degrees of freedom are there in an affine transformation for transforming two dimensional points?

Translation:

$$\begin{vmatrix} 1 & 0 & x2 \\ 0 & 1 & y2 \\ 0 & 0 & 1 \end{vmatrix}$$

Rotation

$$\begin{vmatrix} \cos & \sin & 0 \\ -\sin & \cos & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Scale

$$\begin{vmatrix} W & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Shear

$$\begin{vmatrix} 1 & \tan(x) & 0 \\ \tan(y) & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Degrees of freedom in 2D: 6
Degrees of freedom in 3D: 12