Tutorial Questions Week 6

4.9 In two dimensions, we can specify a line by the equation y = mx + h. Find an affine transformation to reflect two-dimensional points about this line.

If we do a translation by -h we convert the problem to reflection about a line passing through the origin. From m we can find an angle by which we can rotate so the line is aligned with either the x or y axis. Now reflect about the x or y axis. Finally we undo the rotation and translation so the sequence is of the form $T^{-1}R^{-1}SRT$.

4.12 Find a homogeneous-coordinate representation of a plane, which passes through a point P0 and perpendicular to the vector n.

Point (P0) is an object of zero dimensions, vector (n) is an object of one dimension and a Plane (P) is an object of two dimensions

$$n(p-P0) = 0 \Leftrightarrow n\,p = nP0 \Leftrightarrow np/n = nP0/n \Leftrightarrow p = P0$$

5.0 Not all projections are planar geometric projections. Give an example of a projection in which the projection surface is not a plane.

Answer :

Projection is mapping a set on a subset

Cross product is a projection from a one dimension to a one dimension

Dot product is a projection from a one dimension to a zero dimension

5.17 Find the projection of a point onto the plane ax + by + cz + d = 0 from a light source located at infinity in the direction (dx, dy, dz).

All the points on the projection of the point (x.y, z) in the direction
dx, dy, dz) are of the form (x + dx, y + dy, z + dz). Thus the shadow of
the point (x, y, z) is found by determining the for which the line intersects the plane, that is
$ax_s + by_s + cz_s = d$

Substituting and solving, we find $\alpha = d - ax - by - cz / adx + bdy + cdz$

However, what we want is a projection matrix, Using this value of we find
$xs = z + \alpha dx = x(bdy + cdx) - dx(d - by - cz) / adx + bdy + cdz$

with similar equations for ys and zs. These results can be computed by multiplying the homogeneous coordinate point (x, y, z, 1) by the projection matrix

$$\mathrm{M} = \begin{bmatrix} bd_y + cd_z & -bd_x & -cd_x & -dd_x \\ -ad_y & ad_x + cd_z & -cd_y & -dd_y \\ -ad_z & -bd_z & ad_x + bd_y & -dd_z \\ 0 & 0 & 0 & ad_x + bd_y + cd_z \end{bmatrix}$$