Naive Method, cf. Letourneau & Stentoft (2022)

1. Create N dispersed stock prices X:

$$X_n = x_0 + \alpha \mathcal{K}_{ISD}(U_n), \tag{21}$$

   in practice we use the Epanechnikov distrubution in our Kernel specification

$$\mathcal{K}_{ISD}(U) = 2 \times \sin\left(\arcsin(2 \times U - 1)/3\right), \tag{23}$$

   where U is a vector of size N uniformly distributed random variables on the unit interval.

2. Simulate GBM stock price paths using above ISDs as initial values.

3. Determine optimal stopping times pathwise using LSMC algorithm.

4. Determine pathwise payoffs using above stopping times and

$$Z_n = Z_n(\tau(1, n)) \tag{22}$$

5. Run OLS regression on pathwise payoff Z and paths' X distance to x_0 to approximate price function

$$\min_{b_i} \sum_{n=1}^{N} \left\{ Z_n - \sum_{i=0}^{M_0} b_i (X_n - x_0)^i \right\}^2 \tag{20}$$

6. Compute Price and Greeks using $M_0 + 1$ basis functions $\{\rho_m(\cdot)\}_{m=0}^{\infty}$

$$\hat{P}^N_{M,M_0}(S(0) = s) = \sum_{m=0}^{M} \rho_m(S(0) = s)\hat{b}_m(0) \tag{12}$$

$$\hat{\Delta}^N_{M,M_0}(S(0) = s) = \sum_{m=0}^{M} \rho'_m(S(0) = s)\hat{b}_m(0) \tag{13}$$

$$\hat{\Gamma}^N_{M,M_0}(S(0) = s) = \sum_{m=0}^{M} \rho''_m(S(0) = s)\hat{b}_m(0) \tag{14}$$