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Topic:	A Constrained CYK Instances Generator: Implementation and Evaluation
Author:	Andreas Braun <www.github.com/AndreasBraun5> Matrikel-Nr. 1200197
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1. Supervisor:	Prof. Dr. Wim Martens
2. Supervisor:	M.Sc. Tina Trautner

ABC

Abstract

The abstract of this thesis will be found here.

Zusammenfassung

Hier steht die Zusammenfassung dieser Bachelorarbeit.

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1 Introduction

1.1 Motivation

The starting point of this thesis is to get a tool to automatically generate a suitable 4-tuple $exercise = (grammar,\ word,\ parse\ table,\ derivation\ tree)$, that is used to test if the students have understood the way of working of the CYK algorithm.
Various implementations and small online tools of the Cocke-Younger-Kasami (CYK) algorithm can be found [XXX]. Nevertheless it is required to automatically generate suitable *exercise*s, that afterwards can be modified as wanted. This is the reason an own implementation has been made. It is also a task to find a more clever algorithm to automatically generate *exercise*s with a high chance of being suitable as an exam exercise.

1.2 Context Free Grammar

Definition 1. Context Free Grammar (CFG)
We define a CFG as the 4-tuple $G = (V,\ \Sigma,\ S,\ P)$:

- V is a finite set of variables.
- Σ is an alphabet
- S is the start symbol and $S \in V$.
- P is a finite set of rules: $P \subseteq V \times (V \cup \Sigma)^*$.

It is valid that $\Sigma \cap V = \emptyset$.

Definition 2. CFG with restrictions
A CFG $G = (V,\ \Sigma,\ S,\ P)$ is in CNF iff.:

- $P \subseteq V \times (V^2 \cup \Sigma)^*$.

Throughout this thesis a grammar is always synonymous with Definition 2. For further convenience the following default values are always true:

- $V = \{A, B, ...\}$
- $(V^2 \cup\ \Sigma)^* = \{a, b, ...\} \cup \{AA, AB, BB, BA, BS, AC, ...\}$

A rule consists out of a left hand side element (lhse) and a right hand side element (rhse). Example: $lhse \longrightarrow rhse$ applied to $A \longrightarrow c$ and $B \longrightarrow AC$ means that A and B are a $lhse$ and c and AC are a $rhse$.

Definition 3. Word w and language $L(G)$
Word w and language $L(G)$:

- $w \in \Sigma^* = \{w_0,\ w_1,\ ...,\ w_j\}$.
- A language $L(G)$ over an alphabet Σ is a set of words over Σ .

Moreover in the context of talking about sets, a set is always described beginning with an upper case letter, while one specific element of a set is described beginning with a lower case letter. Example: A "*Pyramid*" is a set consisting of multiple "*Cell*"s, whereas a *Cell* is again a subset of the set of variables "V". A "*cellElement*" is one specific element of a "*Cell*". (For further reasoning behind this example see chapter XXX "help data structure")

1.3 General approaches

Two basic approaches, that may help finding a good algorithm are explained informally.

1.3.1 Forward Problem & Backward Problem

The Forward Problem and the Backward Problem are two ways as how to determine if $w \in L(G)$.

Definition 4. Forward Problem ($G \xrightarrow{derivation} w$)
Input: Grammar G in CNF.
Output: Derivation d that shows implicitly $w \subseteq L$.

It is called Forward Problem, if you are given a grammar G and form a derivation from its root node to a final word w. The final word w is always element of $L(G)$.

Definition 5. Backward Problem = Parsing ($w \overset{?}{\subseteq} L(G)$)
Input: w and a grammar G in CNF.
Output: $w \subseteq L(G) \Longrightarrow$ derivation d.

If you are given a word w and want to determine if it is element of $L(G)$, it is called Backward Problem or parsing.

1.3.2 Parsing Bottom-Up & Top-Down

There are again two ways to classify the approach of parsing.

Definition 6. Bottom-Up parsing
Bottom-Up parsing means to start parsing from the leaves up to the root node.

"Bottom-Up parsing is the general method used in the Cocke-Younger-Kasami(CYK) algorithm, which fills a parse table from the "bottom up""[Duda 8.6.3 page 426].

Definition 7. Top-Down parsing
Top-Down parsing means to start parsing from the node down to the leaves.

"Top-Down parsing starts with the root node and successively applies productions from P, with the goal of finding a derivation of the test sentence w." [XXX] (The so called test sentence is synonymous to an word w.) Reasonably criteria to guide the choice of which rewrite rule to apply could include to begin the parsing at the first (left) or last (right) character of the word w [XXX][Duda 8.6.3 page 428]

1.4 Data Structure Pyramid

To be able to describe the way of working of the different algorithms easier the help data structure *Pyramid* will be defined – note that *Pyramid* starts with upper case and therefore is a set). But before that:

Definition 8. $[i, j]$
$[i,\ j] := \{i,\ i+1, ..., j-1,\ j\} \subseteq \mathbb{N}_{\geq 0}$.

Definition 9. $Cell_{i,j}$
$Cell_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\}$

Now *Pyramid* can be defined as following:

Definition 10. *Pyramid*
$Pyramid := \{Cell_{i,j} \mid i \in [0,\ i_{max}],\ j \in [0,\ j_{max,i}],\ i_{max} = |w|-1,\ j_{max,i} = i_{max}-i\}$.

The following is the visual representation of a *Pyramid* that additionally has written the word w above it:

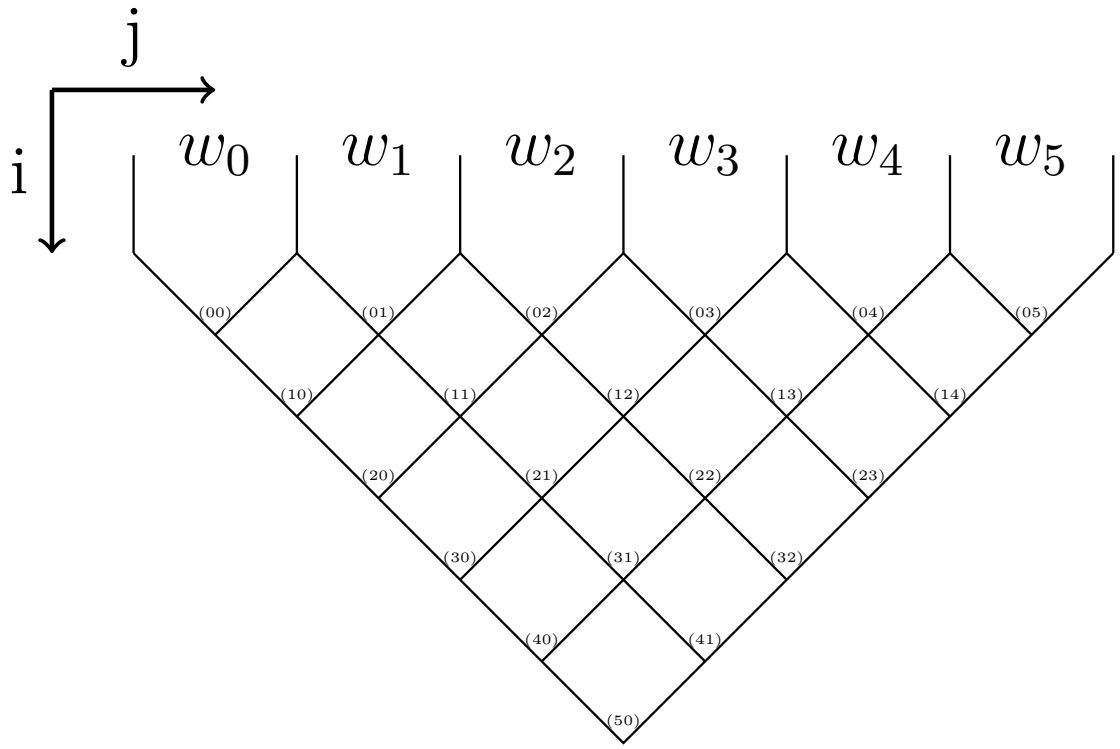


Figure 1: Visual representation of a *Pyramid* with the word w above it.

Definition 11. *CellDown, CellUpperLeft and CellUpperRight*
Let there be a $Cell_{i,j}$ then the following is true:

- $CellDown = Cell_{i,j}$.
- $CellUpperLeft = Cell_{i-1,j}$.
- $CellUpperRight = Cell_{i-1,j+1}$.

1.5 Cocke-Younger-Kasami Algorithm

The Cocke-Younger-Kasami Algorithm (CYK) has been developed independently in the 1960s by Itiroo Sakai, John Cocke, Tadao Kasami, Jacob Schwartz and Daniel Younger that uses the principle of dynamic programming. [wiki and the four sources] The description of the algorithm follows [TI Hofmann] adjusted to the help data structure *Pyramid*.

Algorithm 1: CYK

Input: Grammar $G = (V,\ \Sigma,\ S,\ P)$ and word $w \in \Sigma^* = \{w_0,\ w_1,\ ...,\ w_j\}$

Output: true $\Leftrightarrow w \in L(G)$

```
1  Pyramid = ∅;
2  for j := 0 → i_max do
3  |  Pyramid ∪ Cell_{0,j} = {(X, j) | X ⟶ w_j}
4  end
5  for i := 1 → i_max do
6  |  for j := 0 → j_{max,i} do
7  |  |  for k := i − 1 → 0 do
8  |  |  |  Pyramid ∪ Cell_{i,j} = {X | X ⟶ YZ, Y ∈ Cell_{k,j}, Z ∈
   |  |  |   Cell_{i−k−1,k+j+1}};
9  |  |  end
10 |  end
11 end
12 wInL = false;
13 if (S, i) ∈ Cell_{i_max,0} then
14 |  wInL = true;
15 end
16 return wInL;
```

Line 2: First row.
Line 5: All rows except the first.
Line 6: All cells in each row.
Line 2: All possible cell combinations for each cell.
Line 14: True iff $Cell_{i_{max},0}$ contains the start variable.

1.6 Success Rates

Success Rates (SR) are used to compare the algorithms accounting to their performance of the different requirements. $N \in \mathbb{N}$ is the count of all generated grammars of the examined algorithm.

Success Rate: An generated *exercise* contributes to the Success Rate (SR) iff it contributes to the SR-Producibility, to the SR-Cardinality-Rules and to the SR-Pyramid at the same time.
It holds: $SR = n/N$, whereas n is the count of *exercises* that fulfil the requirements in this case.

Success Rate Producibility: An generated *exercise* contributes to the SR-Producibility iff the CYK algorithm's output (Algorithm 1) is true.

It holds: SR-Producibility $= p/N$, whereas p is the count of *exercises* that fulfil the requirement.

Success Rate Cardinality-Rules An generated *exercise* contributes to the SR-Cardinality-Rules iff the grammar has got less than a certain amount of productions.
It is true: SR-Cardinality-Rules $= cr/N$, whereas cr is the count of *exercises* that fulfil this requirement.

Success Rate Pyramid An generated *exercise* contributes to the SR-Pyramid iff the following conditions are met:

1. At least one cell forces a right cell combination.
2. There are less than a certain amount of variables in the entire pyramid, per default 100.
3. There are less than a certain amount of variables in each cell of the pyramid, per default 3.

It holds: SR-Pyramid $= p/N$, whereas p is the count of *exercises* that fulfil the requirements above.
While checking 1., 2. and 3. a simplification of $Cell_{i,j}$ is done:
$Cell_{i,j} \subseteq \{(V,k) \mid k \in \mathbb{N}\} \longrightarrow Cell_{i,j} \subseteq V$ See the following example:

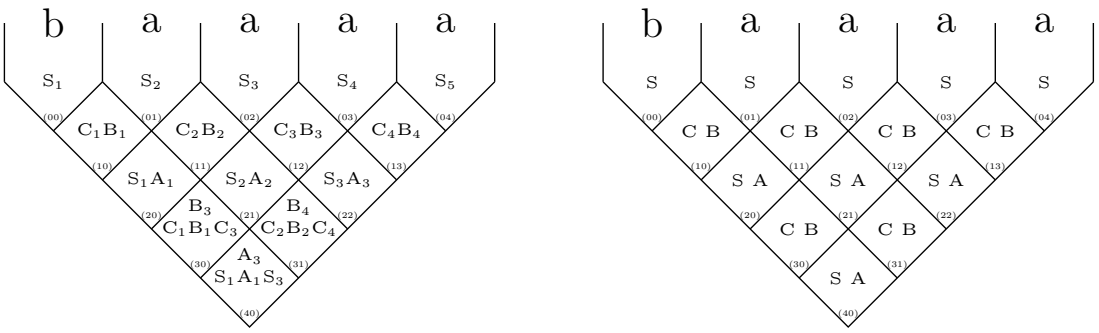


Figure 2: The simplification of cells in a pyramid.

For more detail to how a cell forces a right cell combination (1.) see the following algorithm. But note that a right cell combination can only be forced of cells with index $i > 1$. This restriction while calling Algorithm 2 is only thoughtful in the context of a exercise. Students easily find a pattern of how to fill the first two rows of the *pyramid* but do more mistakes beginning at row $i \geq 2$. The approach of finding only patterns and not thoroughly understanding the algorithm is countered that way.

Algorithm 2: checkForceCombinationPerCell

Input: $CellDown,\ CellUpperLeft,\ CellUpperRight \subseteq V,\ P \subseteq V \times (V^2 \cup \Sigma)$

Output: $true \iff$ *at least one variable* $\in CellDown$ *forces*

```
1  VarsForcing = ∅;  // VarsForcing ⊆ V
2  VarComp = {xy | x ∈ CellUpperLeft ∧ y ∈ CellUpperRight};
3  foreach v ∈ CellDown do
4  |   Prods = {p | p ∈ P ∧ p = (v_1, rhse_1) ∧ v_1 = v};
5  |   Rhses = {rhse | p ∈ Prods ∧ p = (v_1, rhse_1) ∧ rhse_1 = rhse};
6  |   if Rhses ⊈ VarComp then
7  |   |   VarsForcing = VarsForcing ∪ v;
8  |   end
9  end
10 return |VarsForcing| > 0;
```

Line 4: Get all rules of P that have v on their left side.
Line 5: Get the rhse of each element of $Prods$.
Line 6: If no $rhse \in Rhse$ can be found in VarComp, then this variables forces, concluding that this cell as a hole forces.

As seen in Figure 3 the variables in $Cell_{2,0}$ and in $Cell_{2,1}$ force each a right cell combination and in both cases $VarComp = \{SS\}$. The variable $v = C$ doesn't have SS as one of its rhses. Therefore the variable C forces. $Cell_{3,0}$ doesn't force because $VarComp = \{CC\}$ and the variable $v = S$ has CC as its rhse. Remember that cells with index $i \leq 1$ can't force at all.

Grammar :
$C \to CS \mid a \mid b$
$S \to CC$

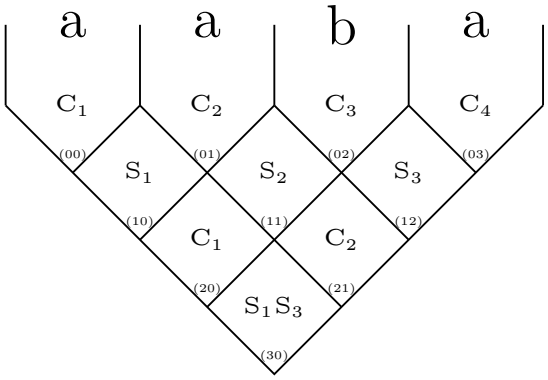


Figure 3: Application of Algorithm 2 onto a entire pyramid.

2 Algorithms

2.1 Sub modules

Sub modules are parts of the algorithms that are denoted circled like (A), (B), (C), (D) and (E). They are procedures that should to be explained in more detail a little bit for a better understanding of the way of working of the following algorithms.

Distribute$(\Sigma,\ V)$ **(A) and Distribute**$(V^2,\ V)$ **(B):**
The difference between (A) and (B) is that one time Σ and the other time V^2 are distributed. While distributing the terminals there exists at least one rule for every terminal used in the word w. The specifics of how they are distributed are the same in both cases as described in the following algorithm:

Algorithm 3: Distribute

Input: $Rhse \subseteq\ (V^2 \cup \Sigma),\ V$
Output: Set of productions $P \subseteq V \times (V^2 \cup \Sigma)$

```
1 foreach rhse ∈ Rhse do
2 |   choose n uniform randomly in [i, j];  // i ∈ ℕ,  j ∈ ℕ
3 |   V_add := uniform random subset of size n from V;
4 |   P ∪ {(v, rhse) | v ∈ V_add,  rhse ∈ Rhse};
5 end
6 return P;
```

Stopping Criteria (C):
Two kinds of (C) have been used. One is that it is true iff more than half of the pyramid cells are not empty and the other one is that there is at least one variable in the tip of the pyramid. It is to be taken in consideration that the latter is somewhat dependent on the count possible variables as seen in [XXX] (chapter problem space analysis).

CalculateSubsetForCell(Pyramid, i, j) (D):
This procedure works kind of analogous from Line 7 to Line 9 of the CYK algorithm. For one $Cell_{i,j}$ every possible cell combination is looked at, i.e. if a rule like $lhse \rightarrow cs$ with $cs \in CellSet$ is added then automatically $Cell_{i,j}$ won't be empty any more.

Algorithm 4: CalculateSubsetForCell

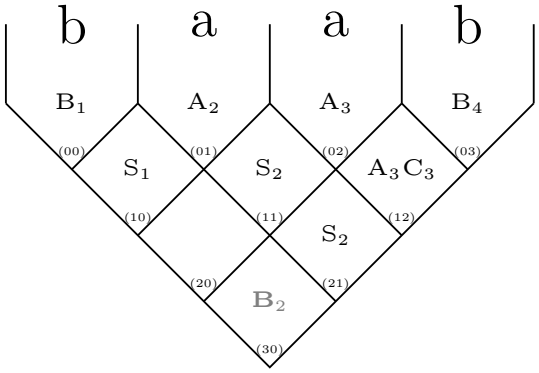
Input: $Pyramid,\ i \in \mathbb{N},\ j \in \mathbb{N}$

Output: $CellSet \subseteq V^2$

```
CellSet = ∅;
for k := i − 1 → 0 do
    CellSet ∪ {YZ | X ⟶ YZ, Y ∈ Cell_{k,j}, Z ∈ Cell_{i−k−1,k+j+1}};
end
return CellSet;
```

In the following situation a rule should be added so that $Cell_{3,0}$ won't be empty:

Grammar:
$A \to AB \mid a$
$B \to \mathbf{SC} \mid b$
$C \to AB$
$S \to BA \mid AA$



Then the calculation of CellSet for $Cell_{3,0}$ results in $\{SA,\ SC,\ BS\}$, whereas SA and SC stem from $Cell_{1,0}$ together with $Cell_{1,2}$ and BA is from $Cell_{0,0}$ together with $Cell_{2,1}$. Now if either one of the rules $lhse \to SA$, $lhse \to SC$ or $lhse \to BS$ is added to the grammar, then $lhse \in Cell_{3,0}$. Here the rule $\mathbf{B \to SC}$ has been added and now $(B, 2)$ is element of $Cell_{3,0}$.

Choose one xy with probability depending on row (E) :

There is the set $RowSet \subseteq \{(XY, i) \mid X, Y \in V \wedge i \in \mathbb{N}\}$ and one xy is chosen out of it as following: Firstly the $RowSet$ is compressed, i.e. every tuple with the same xy will be merged to its lowest i. See the example: $RowSet = \{(AB, 3),\ (AB, 1),\ (AB, 5),\ ...\}$ will become $RowSet = \{(AB, 1),\ ...\}$. Afterwards all elements of $RowSet$ will be placed in the $RowMultiSet$ that can contain multiple equivalent elements. Now each element of $RowMultiSet$ will be weighted according to their i. That means that elements like $(AB, 1)$ will only occur one time though elements like $(BC, 3)$ will occur three times and so on. Example: $RowMultiSet = \{(AB, 1),\ (BC, 3),\ ...\}$ becomes $RowMultiSet = \{(AB, 1),\ (BC, 3),\ (BC, 3),\ (BC, 3),\ ...\}$. Now one element will be uniform randomly picked out of this weighted $RowMultiSet$ like $xy = BC$.

$RowSet = \{(AB, 3), (AB, 1), (AB, 5), ...\}$ // compress
$RowSet = \{(AB, 1), ...\}$ // place into RowMultiSet
$RowMultiSet = \{(AB, 1), (BC, 3), ...\}$ // weight elements
$RowMultiSet = \{(AB, 1), (BC, 3), (BC, 3), (BC, 3), ...\}$ // pick element
$xy = BC$

Figure 4: Short example of the procedure E.

2.2 DiceRollOnlyCYK

This is a naive way of generating grammars, which will be the lower boundary while comparing the algorithms. Each future algorithm should have a higher score than this algorithm or otherwise it would be worse, than simple dice rolling the distribution of terminals (Line 2) and compound variables (Line 3).

Algorithm 5: DiceRollOnlyCYK

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1 $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$
2 $P = Distribute(\Sigma,\ V)$; (A)
3 $P \cup Distribute(V^2,\ V)$; (B)
4 **return** P;

A terminal Σ is distributed to at least one $lhse$, but a compound variable V^2 must not be distributed at all. This means that for each terminal of $\Sigma = \{a, b\}$ there exists at least one rule like $lhse \to a$ and $lhse \to b$ and for each possible compound variable $V^2 = \{AA,\ AB,\ AC,\ AS,\ BB,\ BC,\ BS,\ CC,\ CS, SS\}$ it is possible that only a smaller subset like $\{AA,\ BA,\ CC,\ SC\}$ is distributed so that only rules like $lhse \to AA$, $lhse \to BA$, $lhse \to CC$ and $lhse \to SC$ exist.

Grammar after Line 2:	Grammar after Line 3:
$C \to a$	$C \to BA \mid AA \mid a$
$B \to b$	$B \to b$
	$S \to CC \mid SC$

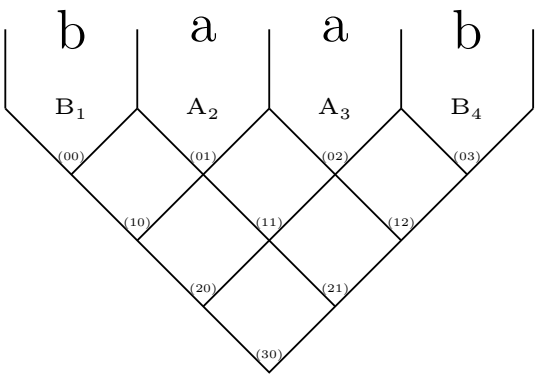
Figure 5: Short example of Algorithm 5.

2.3 BottomUpDiceRollVar1

This algorithm uses the Bottom-Up approach (Chapter 1.3) whereby the parsing table is filled starting from the leaves in direction to the root node.
The basic idea behind this algorithm is to guide the choice of rules while distributing the compound variables V^2. In Algorithm 5 it can happen that the terminals are distributed to the variables A and B and Algorithm 5 completely discards this fact during

the distribution of the compound variables. In a situation as seen below it can happen

Grammar:
$A \to a$
$B \to b$



that rules like $lhse \to CC$ or $lhse \to SC$ are added, that obviously not directly help to fill the parsing table and bloat the grammar with useless rules. More reasonably rules to add would be $lhse \to BA$, $lhse \to AA$ and $lhse \to AB$.
Algorithm 6 takes this up: After distributing the terminals (Line 6) the updated parsing table (Line 12) is always taken into consideration while choosing (Line 10) variable compounds and to finally add them (Line 11). I.e. for each chosen cell a *CellSet* is calculated, that only contains reasonably variable compounds. Now only variable compounds are added that directly help to fill the parsing table.

Algorithm 6: BottomUpDiceRollVar1

Input: Word $w \in \Sigma^*$

Output: Set of productions P

```
1  P = ∅;  // P ⊆ V × (V² ∪ Σ)
2  P = Distribute(Σ, V); (A)
3  Pyramid = CYK(G, w);
4  for i := 1 to i_max do
5      J = {0, ... , j_max − 1};  // J ⊆ ℕ
6      CellSet = ∅;  // CellSet ⊆ V²
7      while |J| > 0 do
8          choose one j ∈ J uniform randomly;
9          J = J \ {j};
10         CellSet = CalculateSubsetForCell(Pyramid, i, j); (D)
11         P ∪ Distribute(CellSet, V); (B)
12         Pyramid = CYK(G, w);
13         if stopping criteria met (C) then
14             return P;
15         end
16     end
17 end
18 return P;
```

Line 4: Fills the i=0 row of the pyramid.
Line 8: A cell is only visited only once.

2.4 BottomUpDiceRollVar2

While examining the Algorithm 6 via its log files [XXX] it can be seen that already a very small number of rules in the grammar is sufficient so that the stopping criteria (C) is met – the cells that indirectly decide what rules to add are mostly from row one ($i = 1$) and sometimes if at all from row two ($i \leq 2$).
This again leads to another idea to introduce a row dependent $threshold_i$ (Line 9) that helps that more cells with $i \geq 2$ are chosen – what possibly can lead to more diverse (= homogeneity criteria, see Chapter 3.1) grammars. The diversity, in context of Algorithm 6, is somewhat too restricted to the $lhse$s that have one of the terminals as its $rhse$. Most of the rules that are part of the grammar will contain one these $lhse$s. This is due to the basic idea of Algorithm 6 but also due to the relatively small number of rules in the grammar.
Further diversification is achieved through the usage of (E) (Line 10). Variable compounds that already have been used in a row with low index i are at a disadvantage to be picked again as described in Algorithm 4.
As seen in Figure 6 rules with the same $rhse = BA$ *and* AA have been added to the variables B and A in Grammar1. For Grammar2 instead the rule $B \rightarrow SS$ was added that contributes to a better diversity.

Grammar0:	Grammar1:	Grammar2:
$C \rightarrow BA \mid AA \mid a$	$C \rightarrow BA \mid AA \mid a$	$C \rightarrow BA \mid AA \mid a$
$B \rightarrow b$	$B \rightarrow BA \mid AA \mid b$	$B \rightarrow SS \mid b$
$S \rightarrow CC \mid SC$	$S \rightarrow BA \mid AA \mid CC \mid SC$	$S \rightarrow CC \mid SC$

Figure 6: Goal of better diversity: Starting point is Grammar0. Grammar2 is of better diversity than Grammar1.

Algorithm 7: BottomUpDiceRollVar2

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1 $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$

2 $RowSet = \emptyset$; // $RowSet \subseteq \{(XY, i) \mid X, Y \in V \wedge i \in \mathbb{N}\}$

3 $P = Distribute(\Sigma,\ V)$; (A)

4 $Pyramid = CYK(G,\ w)$;

5 **for** $i := 1$ ***to*** i_{max} **do**

6 **for** $j := 0$ ***to*** $j_{max} - i$ **do**

7 $RowSet \cup \{(XY, i) \mid XY \in CalculateSubsetForCell(Pyramid,\ i,\ j)$ (D)$\}$;

8 **end**

9 **while** $threshold_i$ *not reached* **do**

10 *choose one xy out of* $(XY,\ i) \in RowSet$ *uniform randomly with probability depending on* i; (E)

11 $P \cup Distribute(xy,\ V)$; (B)

12 $Pyramid = CYK(G,\ w)$;

13 **if** *stopping criteria met* (C) **then**

14 **return** P;

15 **end**

16 **end**

17 **end**

18 **return** P;

Line 4: Fills the i=0 row of the pyramid.

2.5 SplitThenFill

The idea here is to first distribute the terminals (Line 2 of Algorithm 8) and then to uniform randomly generate a predefined structure of the derivation tree (Line 4 of Algorithm 2 and in general Algorithm 9) Top-Downwards and then again to fill the parsing table Bottom-Upwards accordingly to this derivation tree. The reason behind its name comes from the fact that only after completely generating the structure (splitting of the word in subwords) of the derivation tree then the rules are added to the grammar that allows further filling of the parsing table.
Every time before adding a new rule (Algorithm 9 Line 14) the already available information regarding the other rules is used to determine if a new rule is needed to fill this node of the derivation tree (Line 12 of Algorithm 9).

Algorithm 8: SplitThenFill

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1 $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$

2 $P = Distribute(\Sigma,\ V)$; (A)

3 $Sol = (P_{Sol},\ Cell_{i,j})$; // $P_{Sol} \subseteq P \ \wedge \ Cell_{i,j} \in Pyramid$

4 $Sol = SplitThenFill(P,\ w,\ i_{max},\ 0)$;

5 **return** P_{Sol};

Line 2: Fills the i=0 row of the pyramid.

For this algorithm it is important to mention that while using (B) (Line 14 of Algorithm 9) a variable compound is added to at least one $lhse$. For every element of Vc (Line 13 of Algorithm 9) there exists at least one rule $lhse \rightarrow vc$ with $vc \in Vc$.
The construction of the derivation tree for instance is done as following – each number represents the recursion depth of its subtree:

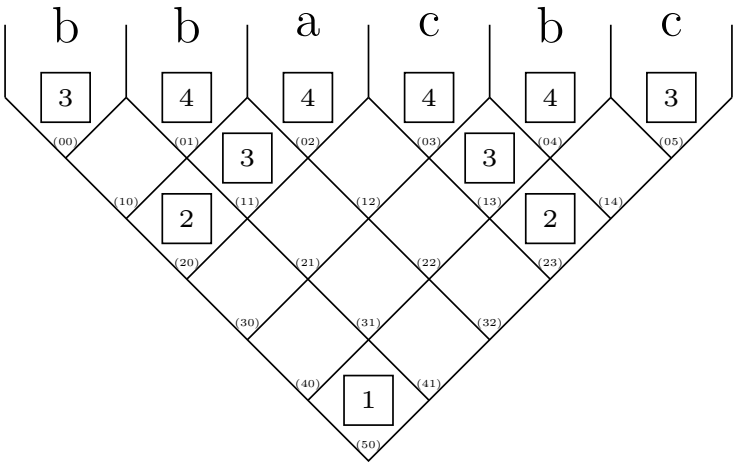


Figure 7: Example derivation tree structure.

Algorithm 9: SplitThenFillRecursion

Input: $P_{in} \subseteq V \times (V^2 \cup \Sigma),\ w \in \Sigma^*,\ i, j \in \mathbb{N}$

Output: $(P,\ Cell_{i,j})$

1 $P = P_{in}$;
2 **if** $i = 0$ **then**
3 | **return** $(P,\ Cell_{i,j})$;
4 **end**
5 *choose one m uniform randomly in* $[j+1,\ j+i]$;
6 $(P,\ Cell_l) = SplitThenFill(P,\ w,\ (m-j-1),\ j)$;
7 $(P,\ Cell_r) = SplitThenFill(P,\ w,\ (j+i-m),\ m)$;
8 $Pyramid = CYK(G,\ w)$;
9 **if** *stopping criteria met* (C) **then**
10 | **return** $(P,\ Cell_{i,j})$;
11 **end**
12 **if** $Cell_{i,j} = \emptyset$ **then**
13 | $Vc = uniform\ random\ subset\ from\ \{vc \mid v \in Cell_l\ \wedge\ c \in Cell_r\}\ with\ |Vc| \geq 1$;
14 | $P \cup Distribute(Vc,\ V)$; (B)
15 **end**
16 **return** $(P,\ Cell_{i,j})$;

The same example tree structure is used as seen in Figure 5.

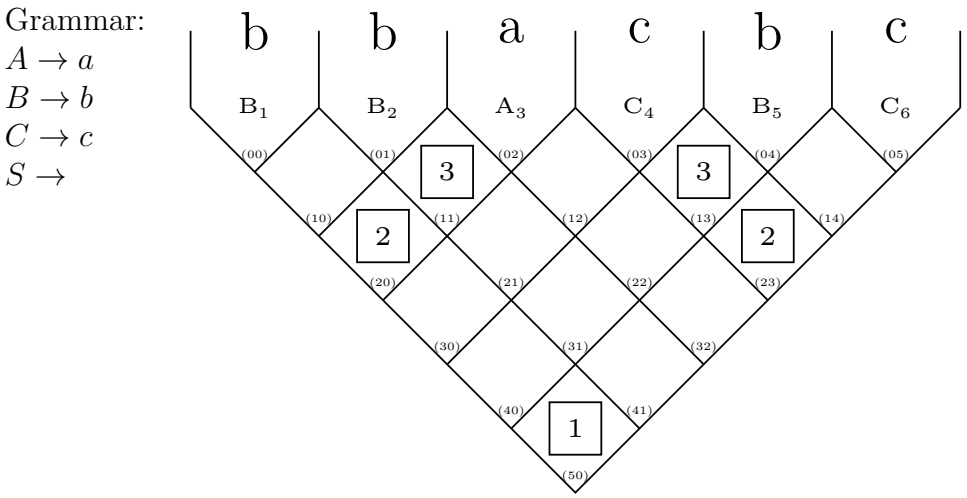


Figure 8: Illustration of Algorithm 8 part 1.

After adding the terminals to the grammar (Line 2 in Algorithm 8) now one must take on the recursion step at $Cell_{1,1}$. Now $Cell_l = \{B_2\}$ and $Cell_r = \{A_3\}$ and therefore $Vc = BA$. Now adding the rule $S \to BA$ leads to the following changes:

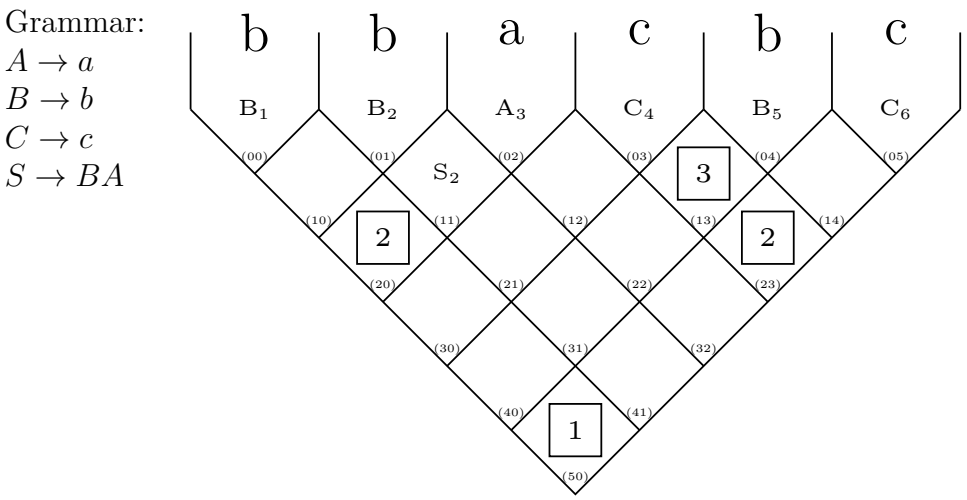


Figure 9: Illustration of Algorithm 8 part 2.

The next recursion step happens in $Cell_{2,0}$. Now $Cell_l = \{B_1\}$ and $Cell_r = \{S_2\}$. Analogously the rule $A \to BS$ is added to the grammar:

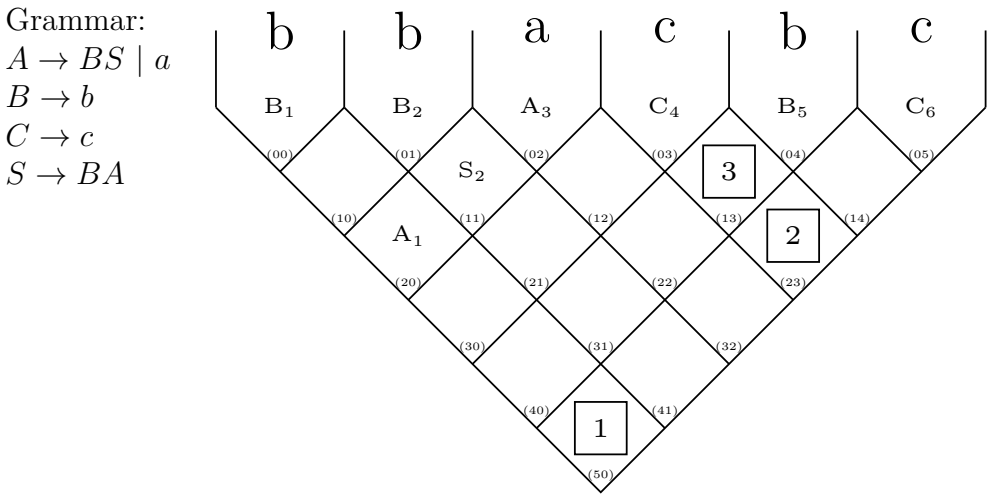


Figure 10: Illustration of Algorithm 8 part 3.

Following up the the recursion step in $Cell_{1,3}$ is resolved by adding the rule $B \to CB$.

Grammar:
$A \to BS \mid a$
$B \to CB \mid b$
$C \to c$
$S \to BA$

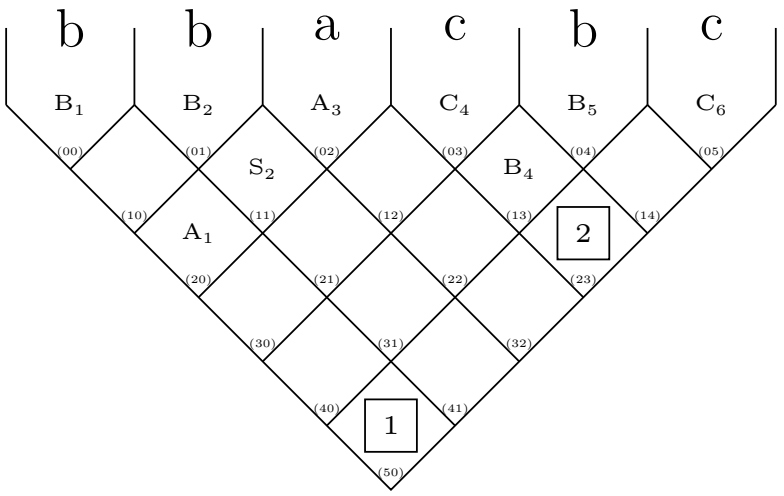


Figure 11: Illustration of Algorithm 8 part 4.

Grammar:
$A \to BS \mid a$
$B \to CB \mid b$
$C \to BC \mid c$
$S \to BA$

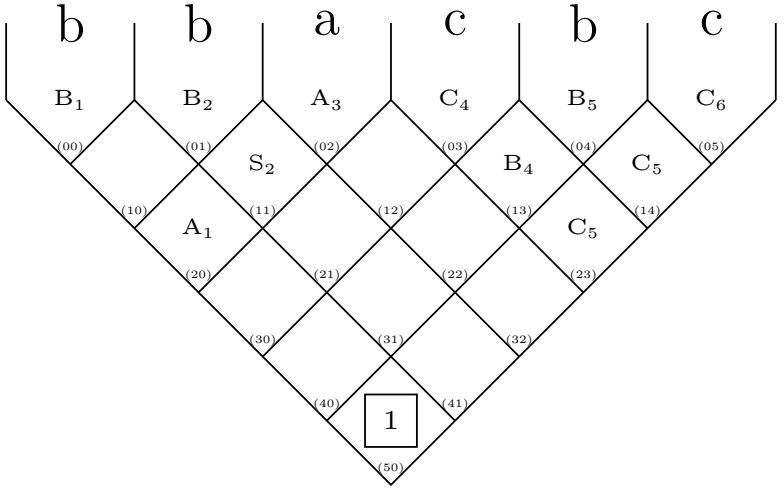


Figure 12: Illustration of Algorithm 8 part 5: After adding rule $C \to BC$.

$A \to BS \mid a$
$B \to b \mid CB$
$C \to c \mid BC$
$S \to BA \mid AC$

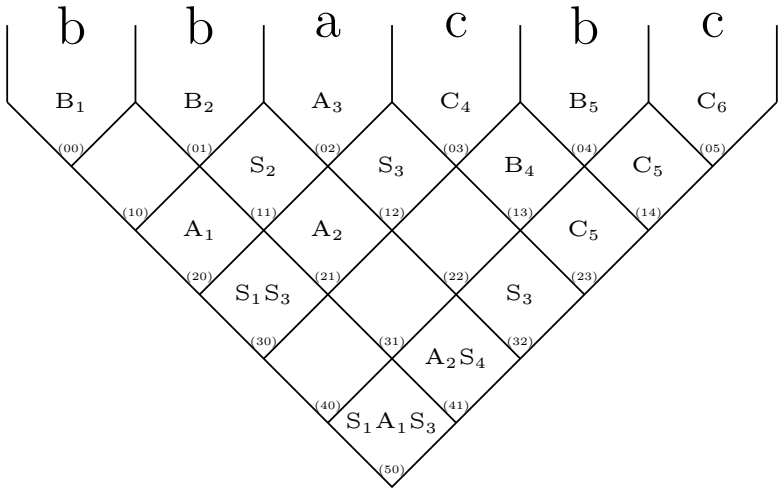


Figure 13: Illustration of Algorithm 8 part 5: After adding rule $S \to AC$.

2.6 SplitAndFill

Algorithm 8 creates the derivation tree structure Top-Downwards but adds rules to the grammar Bottom-Upwards. Another way to do this would be an attempt to do both Top-Downwards.
This algorithm therefore makes use of a part of the same idea as Algorithm 8. It also uses the concept of generating a predefined structure of the derivation tree Top-Downwards. The difference now is, that every time a node of the structure of the derivation tree has been decided on a rule is directly added to the grammar – therefore the name SplitAndFill, which is like "split for a node and then directly add a rule so that the node is filled with at least one variable".
Note that the count of rules in the grammar is dependent on the count of nodes in the derivation tree and while using (A) a terminal is distributed to exactly one variable.

Algorithm 10: SplitAndFill

Input: Word $w \in \Sigma^*$
Output: Set of productions P

$P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$
$Sol = (P_{Sol},\ v)$; // $P_{Sol} \subseteq P$
$Sol = SplitAndFill(P,\ w,\ i_{max},\ 0)$;
return P_{Sol};

Algorithm 11: SplitAndFillRecursion

Input: $P_{in} \subseteq V \times (V^2 \cup \Sigma),\ w \in \Sigma^*,\ i, j \in \mathbb{N}$
Output: $(P,\ v)$

$P = P_{in}$;
if $i = 0$ **then**
  return $(P \cup (v,\ w_j),\ v_{lhse})$;
end
choose one m uniform randomly in $[j+1,\ j+i]$;
$(P,\ v_l) = SplitAndFill(P,\ w,\ (m-j-1),\ j)$;
$(P,\ v_r) = SplitAndFill(P,\ w,\ (j+i-m),\ m)$;
if $i = i_{max}$ **then**
  return $(P \cup (S,\ v_l v_r),\ v)$;
end
return $(P \cup (v,\ v_l v_r),\ v)$;

2.7 Comparision of Algorithms

Write about the standard configuration used.

Algorithm	SR	SR-Producibility	SR-Cardinality-Rules	SR-Pyramid
DiceRollOnly	09 %	24 %	88 %	39 %
BotomUpVar1	16 %	52 %	90 %	41 %
BotomUpVar2	19 %	47 %	93 %	53 %
SplitThenFill	24 %	40 %	97 %	67 %
SplitAndFill	11 %	100 %	69 %	15 %

Table 1: Comparison of the SRs of the algorithms. Stopping criteria root not empty.

Finding of ideal parameter for each algorithm.

Algorithm	SR	SR-Producibility	SR-Cardinality-Rules	SR-Pyramid
DiceRollOnly	09 %	23 %	88 %	38 %
BotomUpVar1	11 %	30 %	99 %	58 %
BotomUpVar2	13 %	26 %	99 %	66 %
SplitThenFill	24 %	40 %	97 %	67 %
SplitAndFill	11 %	100 %	70 %	14 %

Table 2: Comparison of the SRs of the algorithms. Stopping criteria more than half.

Analysis of the problem space possibly visualized in heat maps.

Table 3: My caption

Algorithm	SR	Producibility	Cardinality-Rules	Pyramid			
					Force-Right	Vars-PerCell	VarsIn-Pyramid
DiceRollOnly	09%			67%			
BottomUpVar1							
BottomUpVar2							
SplitThenFill							
SplitAndFill							

3 CLI Tool

Write much of this stuff in the appendix.

3.1 Scoring Model

Only valid ResultSamples are given a score. Parameters to be scored:

- RightCellCombinationsForcedCount
- maxSumOfVarsInPyramidCount
- maxNumberOfVarsPerCellCount
- maxSumOfProductionsCount

Maybe add a diversity criterion = homogeneity of the cells to the scoring matrix.

Parameter	Points					
	2	4	6	8	10	-100
cellCombinationsForced	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
sumVarsInPyramid	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
maxVarsPerCell	[5,5]	[4,4]	[1,1]	[3,3]	[2,2]	>5
sumProductions	[1,2]	[3,4]	[5,6]	[9,10]	[7,8]	>10
homogeneity	[ ]	[ ]	[ ]	[ ]	[ ]	>10
maxVarKsPerCell	[ ]	[ ]	[ ]	[ ]	[ ]	>10

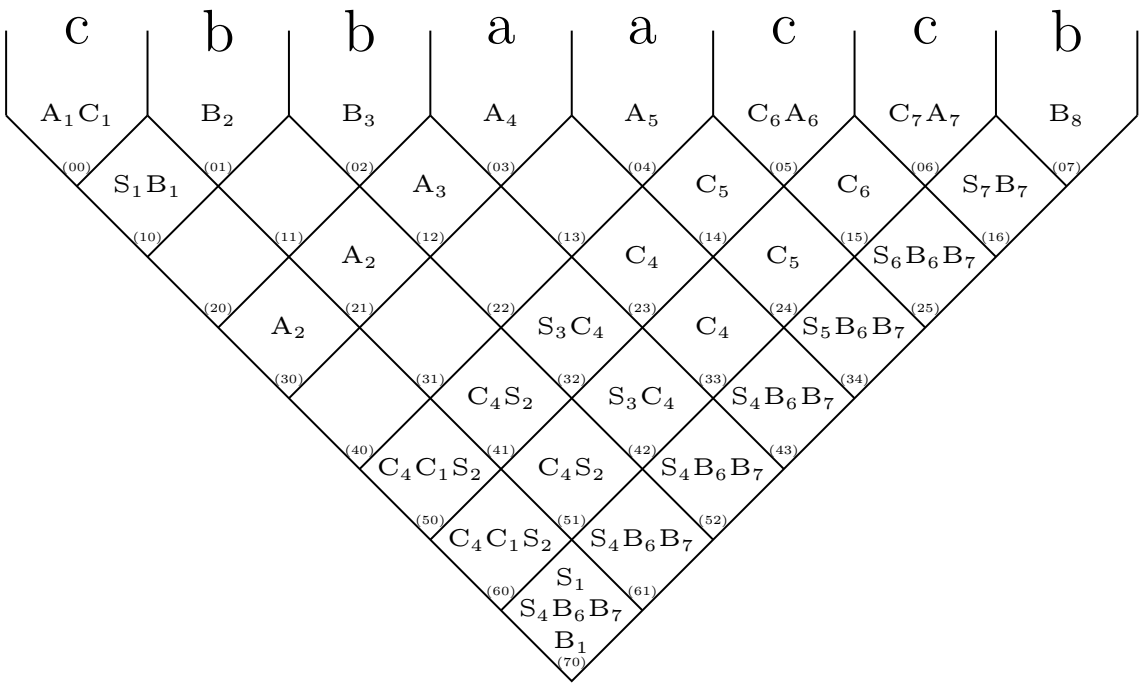
Table 4: Scoring of the different parameter values

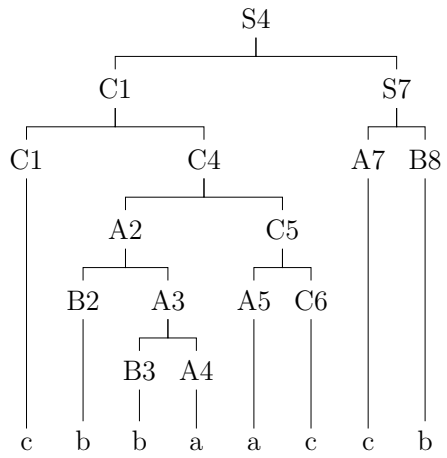
Based on table 4 each result sample is scored. Out of the #??? best result samples one can choose.
The result will be normalized to the maximum possible points -> range 0.0 to 1.0.

3.2 Exam Exercises

4-tuples $exercise = (grammar,\ word,\ parse\ table,\ derivation\ tree)$ that needs to be printed.

$$A \to a \mid c \mid BA$$
$$B \to b \mid CB$$
$$C \to c \mid AC$$
$$S \to AB \mid BC$$





Does the output $P \subseteq V \times (V^2 \cup \Sigma)$ imply that G is in oCNF? CNF does only have useful variables [TI script Def. 8.3 page 210] vs. $P \subseteq V \times (V^2 \cup \Sigma)$.
More of a problem is that the set P is not necessarily in CNF. It is possible that there are unreachable variables – from the starting variable.

3.3 Overview

UML-Diagramm showing the general idea of the implementation.
List noteworthy used libraries here, too.
Maybe some information out of the statistics tool of IntelliJ.
Show the stuff about Antler.

References

[1] JSR 220: Enterprise Java Beans 3.0 https://jcp.org/en/jsr/detail?id=220, 09/09/2015