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## Bachelor Thesis

in Applied Computer Science

**Topic:** A Constrained CYK Instances Generator:  
Implementation and Evaluation

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ABC

## **Abstract**

The abstract of this thesis will be found here.

## **Zusammenfassung**

Hier steht die Zusammenfassung dieser Bachelorarbeit.

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# 1 Introduction

## 1.1 Motivation

The starting point of this thesis is to get a tool to automatically generate a suitable 4-tuple *exercise* = (*grammar*, *word*, *parse table*, *derivation tree*), that is used to test if the students have understood the way of working of the CYK algorithm.

Various implementations and small online tools of the Cocke-Younger-Kasami (CYK) algorithm can be found [XXX]. Nevertheless it is required to automatically generate suitable *exercises*, that afterwards can be modified as wanted. This is the reason an own implementation has been made. It is also a task to find a more clever algorithm to automatically generate *exercises* with a high chance of being suitable as an exam exercise.

## 1.2 Context Free Grammar

### Definition 1. Context Free Grammar (CFG)

We define a CFG as the 4-tuple  $G = (V, \Sigma, S, P)$ :

- $V$  is a finite set of variables.
- $\Sigma$  is an alphabet
- $S$  is the start symbol and  $S \in V$ .
- $P$  is a finite set of rules:  $P \subseteq V \times (V \cup \Sigma)^*$ .

It is valid that  $\Sigma \cap V = \emptyset$ .

### Definition 2. CFG with restrictions

A CFG  $G = (V, \Sigma, S, P)$  is in CNF iff.:

- $P \subseteq V \times (V^2 \cup \Sigma)^*$ .

Throughout this thesis a grammar is always synonymous with Definition 2. For further convenience the following default values are always true:

- $V = \{A, B, \dots\}$
- $(V^2 \cup \Sigma)^* = \{a, b, \dots\} \cup \{AA, AB, BB, BA, BS, AC, \dots\}$

A rule consists out of a left hand side element (lhse) and a right hand side element (rhse). Example:  $lhse \rightarrow rhse$  applied to  $A \rightarrow c$  and  $B \rightarrow AC$  means that  $A$  and  $B$  are a *lhse* and  $c$  and  $AC$  are a *rhse*.

**Definition 3. Word  $w$  and language  $L(G)$** 

Word  $w$  and language  $L(G)$ :

- $w \in \Sigma^* = \{w_0, w_1, \dots, w_j\}$ .
- A language  $L(G)$  over an alphabet  $\Sigma$  is a set of words over  $\Sigma$ .

Moreover in the context of talking about sets, a set is always described beginning with an upper case letter, while one specific element of a set is described beginning with a lower case letter. Example: A "Pyramid" is a set consisting of multiple "Cell"s, whereas a *Cell* is again a subset of the set of variables "V". A "cellElement" is one specific element of a "Cell". (For further reasoning behind this example see chapter XXX "help data structure")

**1.3 General approaches**

Two basic approaches, that may help finding a good algorithm are explained informally.

**1.3.1 Forward Problem & Backward Problem**

The Forward Problem and the Backward Problem are two ways as how to determine if  $w \in L(G)$ .

**Definition 4. Forward Problem ( $G \xrightarrow{\text{derivation}} w$ )**

Input: Grammar  $G$  in CNF.

Output: Derivation  $d$  that shows implicitly  $w \subseteq L$ .

It is called Forward Problem, if you are given a grammar  $G$  and form a derivation from its root node to a final word  $w$ . The final word  $w$  is always element of  $L(G)$ .

**Definition 5. Backward Problem = Parsing ( $w \stackrel{?}{\subseteq} L(G)$ )**

Input:  $w$  and a grammar  $G$  in CNF.

Output:  $w \subseteq L(G) \implies$  derivation  $d$ .

If you are given a word  $w$  and want to determine if it is element of  $L(G)$ , it is called Backward Problem or parsing.

**1.3.2 Parsing Bottom-Up & Top-Down**

There are again two ways to classify the approach of parsing.

**Definition 6. Bottom-Up parsing**

Bottom-Up parsing means to start parsing from the leaves up to the root node.

"Bottom-Up parsing is the general method used in the Cocke-Younger-Kasami(CYK) algorithm, which fills a parse table from the "bottom up"[Duda 8.6.3 page 426].

**Definition 7. Top-Down parsing**

Top-Down parsing means to start parsing from the node down to the leaves.

"Top-Down parsing starts with the root node and successively applies productions from  $P$ , with the goal of finding a derivation of the test sentence  $w$ ." [XXX] (The so called test sentence is synonymous to an word  $w$ .) Reasonably criteria to guide the choice of which rewrite rule to apply could include to begin the parsing at the first (left) or last (right) character of the word  $w$  [XXX][Duda 8.6.3 page 428]

**1.4 Data Structure Pyramid**

To be able to describe the way of working of the different algorithms easier the help data structure *Pyramid* will be defined – note that *Pyramid* starts with upper case and therefore is a set). But before that:

**Definition 8.  $[i, j]$** 

$[i, j] := \{i, i + 1, \dots, j - 1, j\} \subseteq \mathbb{N}_{\geq 0}$ .

**Definition 9.  $Cell_{i,j}$** 

$Cell_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\}$

Now *Pyramid* can be defined as following:

**Definition 10. *Pyramid***

$Pyramid := \{Cell_{i,j} \mid i \in [0, i_{max}], j \in [0, j_{max,i}], i_{max} = |w| - 1, j_{max,i} = i_{max} - i\}$ .

The following is the visual representation of a *Pyramid* that additionally has written the word  $w$  above it:



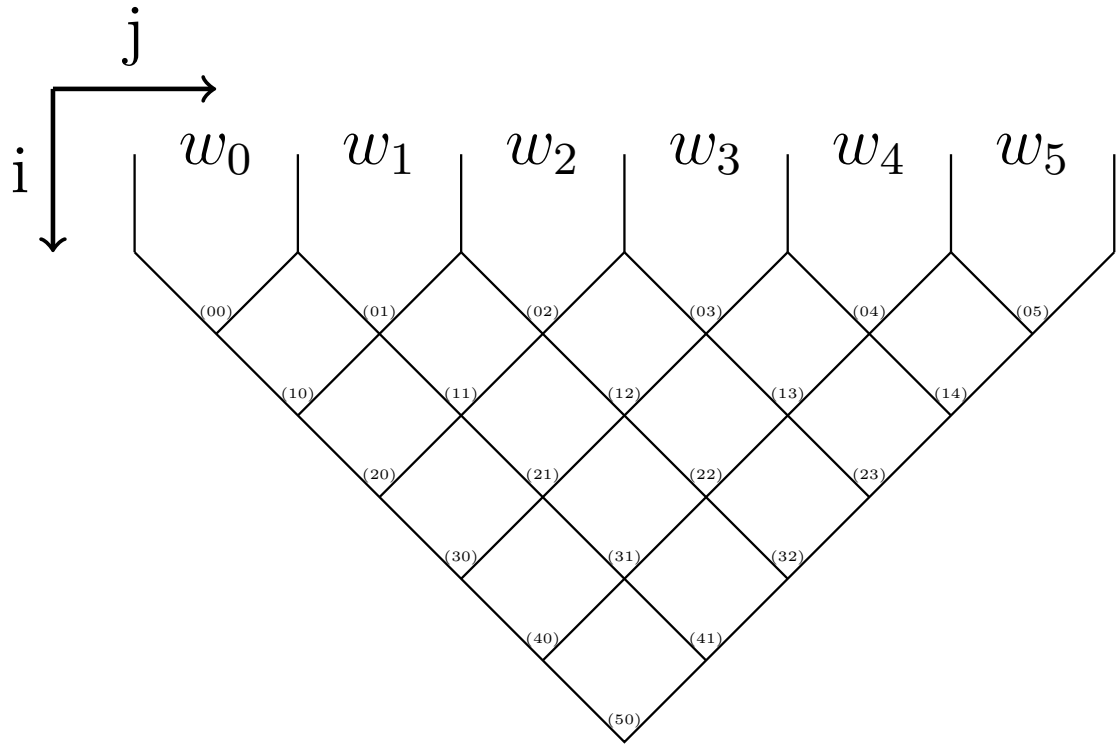


Figure 1: Visual representation of a *Pyramid* with the word  $w$  above it.

**Definition 11.** *CellDown*, *CellUpperLeft* and *CellUpperRight*

Let there be a  $Cell_{i,j}$  then the following is true:

- $CellDown = Cell_{i,j}$ .
- $CellUpperLeft = Cell_{i-1,j}$ .
- $CellUpperRight = Cell_{i-1,j+1}$ .

## 1.5 Cocke-Younger-Kasami Algorithm

The Cocke-Younger-Kasami Algorithm (CYK) has been developed independently in the 1960s by Itiroo Sakai, John Cocke, Tadao Kasami, Jacob Schwartz and Daniel Younger that uses the principle of dynamic programming. [wiki and the four sources] The description of the algorithm follows [TI Hofmann] adjusted to the help data structure *Pyramid*.

**Algorithm 1: CYK****Input:** Grammar  $G = (V, \Sigma, S, P)$  and word  $w \in \Sigma^* = \{w_0, w_1, \dots, w_j\}$ **Output:**  $\text{true} \Leftrightarrow w \in L(G)$ 

```

1  $Pyramid = \emptyset;$ 
2 for  $j := 0 \rightarrow i_{max}$  do
3    $Pyramid \cup Cell_{0,j} = \{(X, j) \mid X \rightarrow w_j\}$ 
4 end
5 for  $i := 1 \rightarrow i_{max}$  do
6   for  $j := 0 \rightarrow j_{max,i}$  do
7     for  $k := i - 1 \rightarrow 0$  do
8        $Pyramid \cup Cell_{i,j} = \{X \mid X \rightarrow YZ, Y \in Cell_{k,j}, Z \in$ 
9          $Cell_{i-k-1,k+j+1}\};$ 
10    end
11  end
12  $wInL = false;$ 
13 if  $(S, i) \in Cell_{i_{max},0}$  then
14    $wInL = true;$ 
15 end
16 return  $wInL;$ 

```

Line 2: First row.

Line 5: All rows except the first.

Line 6: All cells in each row.

Line 2: All possible cell combinations for each cell.

Line 14: True iff  $Cell_{i_{max},0}$  contains the start variable.

## 1.6 Success Rates

Success Rates ( $SR$ ) are used to compare the algorithms accounting to their performance of the different requirements.  $N \in \mathbb{N}$  is the count of all generated grammars of the examined algorithm.

**Success Rate:** An generated *exercise* contributes to the Success Rate ( $SR$ ) iff it contributes to the SR-Producibility, to the SR-Cardinality-Rules and to the SR-Pyramid at the same time.

It holds:  $SR = n/N$ , whereas  $n$  is the count of *exercises* that fulfil the requirements in this case.

**Success Rate Producibility:** An generated *exercise* contributes to the SR-Producibility iff the CYK algorithm's output (Algorithm 1) is true.

It holds:  $\text{SR-Producibility} = p/N$ , whereas  $p$  is the count of *exercises* that fulfil the requirement.

**Success Rate Cardinality-Rules** An generated *exercise* contributes to the SR-Cardinality-Rules iff the grammar has got less than a certain amount of productions. It is true:  $\text{SR-Cardinality-Rules} = cr/N$ , whereas  $cr$  is the count of *exercises* that fulfil this requirement.

**Success Rate Pyramid** An generated *exercise* contributes to the SR-Pyramid iff the following conditions are met:

1. At least one cell forces a right cell combination.
2. There are less than a certain amount of variables in the entire pyramid, per default 100.
3. There are less than a certain amount of variables in each cell of the pyramid, per default 3.

It holds:  $\text{SR-Pyramid} = p/N$ , whereas  $p$  is the count of *exercises* that fulfil the requirements above.

While checking 1., 2. and 3. a simplification of  $\text{Cell}_{i,j}$  is done:

$\text{Cell}_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\} \longrightarrow \text{Cell}_{i,j} \subseteq V$  See the following example:

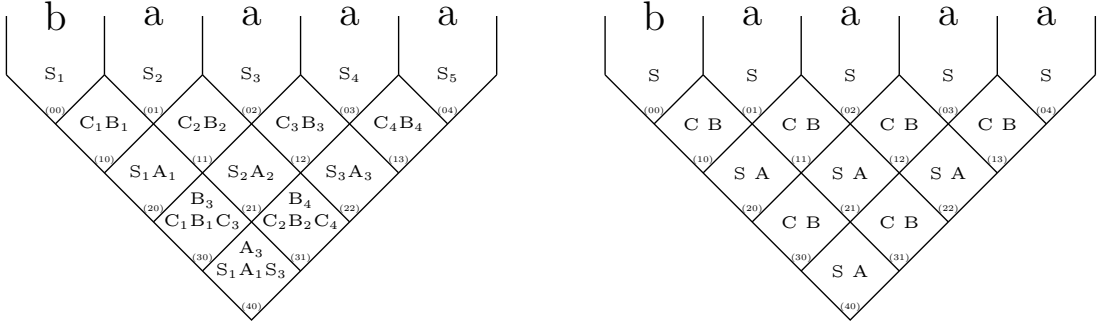


Figure 2: The simplification of cells in a pyramid in more detail.

For more detail to how a cell forces a right cell combination (1.) see the following algorithm. Note that a right cell combination can only be forced of cells with index  $i > 1$ .

**Algorithm 2:** checkForceCombinationPerCell**Input:**  $CellDown$ ,  $CellUpperLeft$ ,  $CellUpperRight \subseteq V$ ,  $P \subseteq V \times (V^2 \cup \Sigma)$ **Output:**  $true \iff$  at least one variable  $\in CellDown$  forces

```

1  $VarsForcing = \emptyset$ ; //  $VarsForcing \subseteq V$ 
2  $VarComp = \{xy \mid x \in CellUpperLeft \wedge y \in CellUpperRight\}$ ;
3 foreach  $v \in CellDown$  do
4    $Prods = \{p \mid p \in P \wedge p = (v_1, rhse_1) \wedge v_1 = v\}$ ;
5    $Rhse = \{rhse \mid p \in Prods \wedge p = (v_1, rhse_1) \wedge rhse_1 = rhse\}$ ;
6   if  $Rhse \not\subseteq VarComp$  then
7      $VarsForcing = VarsForcing \cup v$ ;
8   end
9 end
10 return  $|VarsForcing| > 0$ ;

```

Line 4: Get all rules of  $P$  that have  $v$  on their left side.Line 5: Get the rhse of each element of  $Prods$ .Line 6: If no  $rhse \in Rhse$  can be found in  $VarComp$ , then this variables forces, concluding that this cell as a hole forces.

As seen in Figure 3 the variables in  $Cell_{2,0}$  and in  $Cell_{2,1}$  force each a right cell combination. In both cases  $VarComp = \{SS\}$ . The variable  $v = C$  doesn't have  $SS$  as one of its rhses. Therefore the variable  $C$  forces.  $Cell_{3,0}$  doesn't force because  $VarComp = \{CC\}$  and the variable  $v = S$  has  $CC$  as its rhse. Remember that cells with index  $i \leq 1$  can't force at all.

Grammar :

$$C \rightarrow CS \mid a \mid b$$

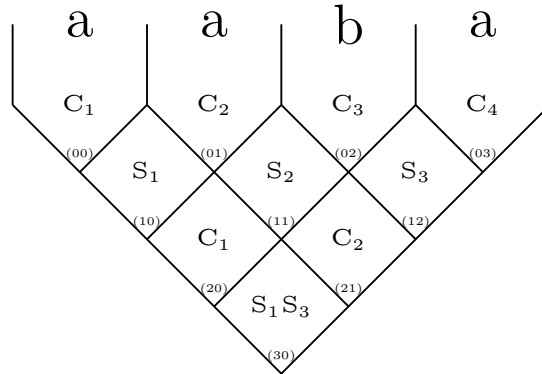
$$S \rightarrow CC$$


Figure 3: Example grammar and pyramid for the application of Algorithm 2.

## 2 Algorithms

### 2.1 Sub modules

Sub modules are parts of the algorithms that are denoted circled like (A), (B), (C), (D) and (E). They are procedures that should to be explained in more detail a little bit for better understanding of the way of working of the algorithms.

**Distribute( $\Sigma, V$ )(A) and Distribute( $V^2, V$ )(B):**

The difference between (A) and (B) is that one time  $\Sigma$  and the other time  $V^2$  are distributed. The specifics of how they are distributed are the same in both cases as described in the following algorithm:

<b>Algorithm 3:</b> Distribute	
<b>Input:</b> $Rhse \subseteq (V^2 \cup \Sigma), V$	
<b>Output:</b> Set of productions $P \subseteq V \times (V^2 \cup \Sigma)$	
1	<b>foreach</b> $rhse \in Rhse$ <b>do</b>
2	<i>choose <math>n</math> uniform randomly in <math>[i, j]</math>; // <math>i \in \mathbb{N}, j \in \mathbb{N}</math></i>
3	$V_{add} :=$ <i>uniform random subset of size <math>n</math> from <math>V</math>;</i>
4	$P \cup \{(v, rhse) \mid v \in V_{add}, rhse \in Rhse\};$
5	<b>end</b>
6	<b>return</b> $P$ ;

**Stopping Criteria (C):**

Two kinds of (C) have been used. One is that it is true iff more than half of the pyramid cells are not empty and the other one is that there is at least one variable in the tip of the pyramid. It is to be taken in consideration that the latter is somewhat dependent on the count possible variables as seen in [XXX].

**ChooseXYDependingOnIFFromRowSet (D):**

$RowSet \subseteq \{(XY, i) \mid X, Y \in V \wedge i \in \mathbb{N}\}$

Compression of the RowSet like: (AB,3) and (AB,1) -> (AB,1) -> RowSetCompressed  
 rowListWeighted = add i times XY to rowListWeighted. XXX

**CalculateSubsetForCell(Pyramid, i, j) (E):**

This works kind of analogous from Line 7 to Line 9 of the CYK algorithm. For one  $Cell_{i,j}$  every possible cell combination is looked at, i.e. if a rule like  $lhse \rightarrow cs$  with  $cs \in CellSet$  is added then automatically  $Cell_{i,j}$  won't be empty any more.

**Algorithm 4:** CalculateSubsetForCell**Input:**  $Pyramid$ ,  $i \in \mathbb{N}$ ,  $j \in \mathbb{N}$ **Output:**  $CellSet \subseteq V^2$ 

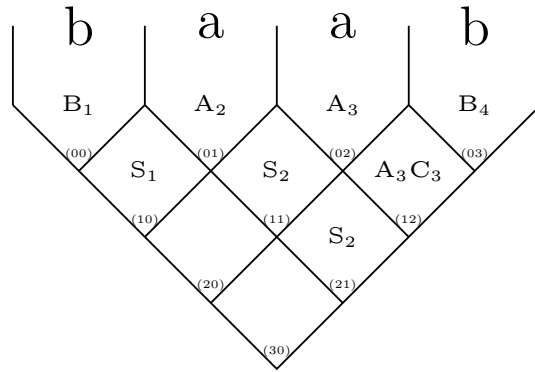
```

1  $CellSet = \emptyset$ ;
2 for  $k := i - 1 \rightarrow 0$  do
3    $CellSet \cup \{X \mid X \rightarrow YZ, Y \in Cell_{k,j}, Z \in Cell_{i-k-1,k+j+1}\}$ ;
4 end
5 return  $CellSet$ ;

```

If there is the following situation and a rule should be added so that  $Cell_{3,0}$  won't be empty:

Grammar:  
 $A \rightarrow AB \mid a$   
 $B \rightarrow b$   
 $C \rightarrow AB$   
 $S \rightarrow BA \mid AA$



Calculation of  $CellSet$  for  $Cell_{3,0}$  results in  $\{SA, SC, BS\}$ , whereas  $SA$  and  $SC$  stem from  $Cell_{1,0}$  together with  $Cell_{1,2}$  and  $BA$  is from  $Cell_{0,0}$  together with  $Cell_{2,1}$ . Now if either one of the rules  $lhse \rightarrow SA$ ,  $lhse \rightarrow SC$  or  $lhse \rightarrow BS$  is added to the grammar, then  $lhse \in Cell_{3,0}$ .

## 2.2 DiceRollOnlyCYK

This is a naive way of generating grammars, which will be the lower boundary while comparing the algorithms. Each future algorithm should have a higher score than this algorithm or otherwise it would be worse, than simple dice rolling the distribution of terminals  $\textcircled{A}$  and compound variables  $\textcircled{B}$ .

**Algorithm 5:** DiceRollOnlyCYK**Input:** Word  $w \in \Sigma^*$ **Output:** Set of productions  $P$ 

```

1  $P = \emptyset$ ; //  $P \subseteq V \times (V^2 \cup \Sigma)$ 
2  $P = Distribute(\Sigma, V); \textcircled{A}$ 
3  $P \cup Distribute(V^2, V); \textcircled{B}$ 
4 return  $P$ ;

```

A terminal  $\Sigma$  is distributed to at least one  $lhse$ , but a compound variable  $V^2$  must not

be distributed at all.

For each terminal of  $\Sigma = \{a, b\}$  there exists at least one rule like  $lhse \rightarrow a$  and  $lhse \rightarrow b$  and for each possible compound variable  $V^2 = \{AA, AB, AC, AS, BB, BC, BS, CC, CS, SS\}$  it is possible that only a small subset like  $\{AA, BA, CC, SC\}$  is distributed so that rules like  $lhse \rightarrow AA$ ,  $lhse \rightarrow BA$ ,  $lhse \rightarrow CC$  and  $lhse \rightarrow SC$  exist.

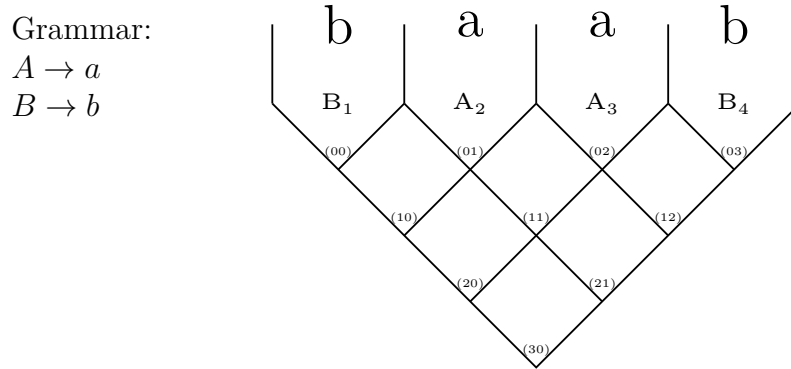
Grammar after Line 2:	Grammar after Line 3:
$C \rightarrow a$	$C \rightarrow BA \mid AA \mid a$
$B \rightarrow b$	$B \rightarrow b$
	$S \rightarrow CC \mid SC$

Figure 4: Short example of Algorithm 5.

### 2.3 BottomUpDiceRollVar1

This algorithm uses the Bottom-Up approach (Chapter 1.3) whereby the parsing table is filled starting from the leaves in direction to the root node.

The idea behind this algorithm is to guide the choice of rules while distributing the compound variables  $V^2$ . Because in Algorithm 5 it can happen that the terminals are distributed to the variables  $A$  and  $B$  and Algorithm 5 completely discards this fact while distributing the compound variables. In a situation as seen below it can happen



that rules like  $lhse \rightarrow CC$  or  $lhse \rightarrow SC$  are added, that obviously not directly help to fill the parsing table and bloat the grammar with useless rules. More reasonably rules to add would be  $lhse \rightarrow BA$ ,  $lhse \rightarrow AA$  and  $lhse \rightarrow AB$ .

Algorithm 6 takes this up: After distributing the terminals  $\textcircled{A}$  the parsing table is always taken into consideration while choosing variable compounds to add. I.e. for each chosen cell a *CellSet* is calculated, that only contains reasonably variable compounds. Now only variable compounds are added that directly help to fill the parsing table.

**Algorithm 6:** BottomUpDiceRollVar1

**Input:** Word  $w \in \Sigma^*$   
**Output:** Set of productions  $P$

```

1  $P = \emptyset$ ; //  $P \subseteq V \times (V^2 \cup \Sigma)$ 
2  $P = \text{Distribute}(\Sigma, V)$ ; (A)
3  $\text{Pyramid} = \text{CYK}(G, w)$ ;
4 for  $i := 1$  to  $i_{\max}$  do
5    $J = \{0, \dots, j_{\max} - 1\}$ ; //  $J \subseteq \mathbb{N}$ 
6    $\text{CellSet} = \emptyset$ ; //  $\text{CellSet} \subseteq V^2$ 
7   while  $|J| > 0$  do
8     choose one  $j \in J$  uniform randomly;
9      $J = J \setminus \{j\}$ ;
10     $\text{CellSet} = \text{CalculateSubsetForCell}(\text{Pyramid}, i, j)$ ; (E)
11     $P \cup \text{Distribute}(\text{CellSet}, V)$ ; (B)
12     $\text{Pyramid} = \text{CYK}(G, w)$ ;
13    if stopping criteria met (C) then
14      return  $P$ ;
15    end
16 end
17 return  $P$ ;

```

Line 2: Fills the  $i=0$  row of the pyramid.

Line 8: A cell is only visited only once.



## 2.4 BottomUpDiceRollVar2

While examining the Algorithm 6 via its log files [XXX] it can be seen that already a very small number of rules in the grammar is sufficient so that the stopping criteria  $\textcircled{C}$  is met –the cells that indirectly decide what rules to add are mostly from row one ( $i = 1$ ) and sometimes if at all from row two ( $i \leq 2$ ).

This again leads to another idea to introduce a row dependent  $threshold_i$  that helps that more cells with  $i \geq 2$  are chosen, what possibly can lead to more diverse grammars. The diversity, in context of Algorithm 6, is somewhat too restricted to the  $lhse$ s that have one of the terminals as its  $rhse$ . Most of the rules that are part of the gramamr will contain one these  $lhse$ s. This is due to the basic idea of Algorithm 6 but also due to the small number of rules.

Algorithm 7: BottomUpDiceRollVar2	
<b>Input:</b> Word $w \in \Sigma^*$ <b>Output:</b> Set of productions $P$	
1	$P = \emptyset$ ; // $P \subseteq V \times (V^2 \cup \Sigma)$
2	$RowSet = \emptyset$ ; // $RowSet \subseteq \{(XY, i) \mid X, Y \in V \wedge i \in \mathbb{N}\}$
3	$P = Distribute(\Sigma, V)$ ; $\textcircled{A}$
4	$Pyramid = CYK(G, w)$ ;
5	<b>for</b> $i := 1$ <b>to</b> $i_{max}$ <b>do</b>
6	<b>for</b> $j := 0$ <b>to</b> $j_{max} - i$ <b>do</b>
7	$RowSet \cup \{(XY, i) \mid XY \in CalculateSubsetForCell(Pyramid, i, j)\}$ ; $\textcircled{E}$
8	<b>end</b>
9	<b>while</b> $threshold_i$ not reached <b>do</b>
10	choose one $xy$ out of $(XY, i) \in RowSet$ uniform randomly with probability depending on $i$ ; $\textcircled{D}$
11	$P \cup Distribute(xy, V)$ ; $\textcircled{B}$
12	$Pyramid = CYK(G, w)$ ;
13	<b>if</b> stopping criteria met $\textcircled{C}$ <b>then</b>
14	<b>return</b> $P$ ;
15	<b>end</b>
16	<b>end</b>
17	<b>end</b>
18	<b>return</b> $P$ ;
<hr/> Line 2: Fills the $i=0$ row of the pyramid. Line 7: $(AB, 1), (AB, 2), (BC, 3) \dots \in sub \rightarrow$ multiple occurrences of $AB$ are allowed here yet. Note Line 9: threshold is reached iff more than half of the cells of one row aren't empty.	

## 2.5 SplitThenFill

The basic idea for this algorithm is to uniform randomly generate a predefined structure of the derivation tree that helps adding the "right" productions. You always update the pyramid after adding one production to the grammar. This is also some kind of BottumUp approach - Bottom Up: The parse table is filled relatively evenly. All information regarding the upper cells are available and can be used. Similar to the CYK Algorithm approach.

It is important to distribute the varComp exactly to one var.

<b>Algorithm 8:</b> SplitThenFillPrep	
<b>Input:</b> Word $w \in \Sigma^*$	
<b>Output:</b> Set of productions $P$	
<b>1</b>	$P = \emptyset; \quad // \quad P \subseteq V \times (V^2 \cup \Sigma)$
<b>2</b>	$P = Distribute(\Sigma, V); \quad \textcircled{A}$
<b>3</b>	$Sol = (P_{Sol}, Cell_{i,j}); \quad // \quad P_{Sol} \subseteq P \quad \wedge \quad Cell_{i,j} \in Pyramid$
<b>4</b>	$Sol = SplitThenFill(P, w, i_{max}, 0);$
<b>5</b>	<b>return</b> $P_{Sol};$
Line 2: Fills the i=0 row of the pyramid.	

<b>Algorithm 9:</b> SplitThenFill	
<b>Input:</b> $P_{in} \subseteq V \times (V^2 \cup \Sigma)$ , $w \in \Sigma^*$ , $i, j \in \mathbb{N}$	
<b>Output:</b> $(P, Cell_{i,j})$	
1	$P = P_{in};$
2	<b>if</b> $i = 0$ <b>then</b>
3	<b>return</b> $(P, Cell_{i,j});$
4	<b>end</b>
5	<i>choose one <math>m</math> uniform randomly in <math>[j + 1, j + i];</math></i>
6	$(P, Cell_l) = SplitThenFill(P, w, (m - j - 1), j);$
7	$(P, Cell_r) = SplitThenFill(P, w, (j + i - m), m);$
8	$Pyramid = CYK(G, w);$
9	<b>if</b> <i>stopping criteria met</i> $\textcircled{C}$ <b>then</b>
10	<b>return</b> $(P, Cell_{i,j});$
11	<b>end</b>
12	<b>if</b> $Cell_{i,j} = \emptyset$ <b>then</b>
13	$Vc = \text{uniform random subset from } \{vc \mid v \in Cell_l \wedge c \in Cell_r\}$ <i>with</i> $ Vc  \geq 1;$
14	$P \cup Distribute(Vc, V); \textcircled{B}$
15	<b>end</b>
16	<b>return</b> $(P, Cell_{i,j});$

The stopping criteria is met if the tip of the pyramid is not empty. It is a valid approach because if this cell is not empty it means that there is a chance of being able to generate the word. To add further productions only results in a grammar that has too many productions with its pyramid having too many variables.

## 2.6 SplitAndFill

It is dependent on the length of the word.

It is important that the terminals and the varcomps are distributed to exactly one var. The stopping criteria will be that each cell with index  $i = 0$  must be not empty. Now there is a second option to fill the parse table:

1. Top Down: The parse table is filled quite unevenly. You don't have all information available. Think about adding a production for the node cell: You can add a production so that its producing cells fill the node cell, but you don't know what actually would be the best to fill in these producing cells because they themselves aren't looked at yet. This problem is kept until the last depth of the recursion, where the cells in row  $i = 0$  are taken into account. Only starting there you know what variables actually produce the terminals.

Maybe solution: For the Top Down approach, don't assume that the terminals are already distributed over the V. Distribute the terminals over the variables in an ideal way that fits your already generated productions best.

The problem is that we have as much productions as splits in the derivation tree exist. The productions count can be reduced via merging duplicate productions and via reducing the split count in the tree.

Merging productions means: If there are  $A \rightarrow BS$  and  $C \rightarrow BS$  then only one Production of these two can remain.

**Algorithm 10:** SplitAndFillPrep**Input:** Word  $w \in \Sigma^*$ **Output:** Set of productions  $P$ 

```

1  $P = \emptyset$ ; //  $P \subseteq V \times (V^2 \cup \Sigma)$ 
2  $Sol = (P_{Sol}, v)$ ; //  $P_{Sol} \subseteq P$ 
3  $Sol = SplitAndFill(P, w, i_{max}, 0)$ ;
4 Merge productions with the same variableCompound in  $P_{Sol}$ ;
5 return  $P_{Sol}$ ;

```

**Algorithm 11:** SplitAndFill**Input:**  $P_{in} \subseteq V \times (V^2 \cup \Sigma)$ ,  $w \in \Sigma^*$ ,  $i, j \in \mathbb{N}$ **Output:**  $(P, v)$ 

```

1  $P = P_{in}$ ;
2 if  $i = 0$  then
3   | return  $(P \cup (v, w_j), v_{lhse})$ ;
4 end
5 choose one  $m$  uniform randomly in  $[j + 1, j + i]$ ;
6  $(P, v_l) = SplitAndFill(P, w, (m - j - 1), j)$ ;
7  $(P, v_r) = SplitAndFill(P, w, (j + i - m), m)$ ;
8 if  $i = i_{max}$  then
9   | return  $(P \cup (S, v_l v_r), v)$ ;
10 end
11 return  $(P \cup (v, v_l v_r), v)$ ;

```

Table 3: My caption

Algorithm	SR	Produci- bility	Cardinality- Rules	Pyramid			
					Force- Right	Vars- PerCell	VarsIn- Pyramid
DiceRollOnly	09%			67%			
BottomUpVar1							
BottomUpVar2							
SplitThenFill							
SplitAndFill							

## 2.7 Comparision of Algorithms

Write about the standard configuration used.

Algorithm	SR	SR-Producibility	SR-Cardinality-Rules	SR-Pyramid
DiceRollOnly	09 %	24 %	88 %	39 %
BotomUpVar1	16 %	52 %	90 %	41 %
BotomUpVar2	19 %	47 %	93 %	53 %
SplitThenFill	24 %	40 %	97 %	67 %
SplitAndFill	11 %	100 %	69 %	15 %

Table 1: Comparison of the SRs of the algorithms. Stopping criteria root not empty.

Finding of ideal parameter for each algorithm.

Algorithm	SR	SR-Producibility	SR-Cardinality-Rules	SR-Pyramid
DiceRollOnly	09 %	23 %	88 %	38 %
BotomUpVar1	11 %	30 %	99 %	58 %
BotomUpVar2	13 %	26 %	99 %	66 %
SplitThenFill	24 %	40 %	97 %	67 %
SplitAndFill	11 %	100 %	70 %	14 %

Table 2: Comparison of the SRs of the algorithms. Stopping criteria more than half.

## 3 CLI Tool

Write much of this stuff in the appendix.

### 3.1 Scoring Model

Only valid ResultSamples are given a score. Parameters to be scored:

- RightCellCombinationsForcedCount
- maxSumOfVarsInPyramidCount
- maxNumberOfVarsPerCellCount
- maxSumOfProductionsCount

Maybe add a diversity criterion = homogeneity of the cells to the scoring matrix.

Parameter	Points					
	2	4	6	8	10	-100
cellCombinationsForced	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
sumVarsInPyramid	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
maxVarsPerCell	[5,5]	[4,4]	[1,1]	[3,3]	[2,2]	>5
sumProductions	[1,2]	[3,4]	[5,6]	[9,10]	[7,8]	>10
homogeneity	[ ]	[ ]	[ ]	[ ]	[ ]	>10
maxVarKsPerCell	[ ]	[ ]	[ ]	[ ]	[ ]	>10

Table 4: Scoring of the different parameter values

Based on table 4 each result sample is scored. Out of the #??? best result samples one can choose.

The result will be normalized to the maximum possible points -> range 0.0 to 1.0.

### 3.2 Exam Exercises

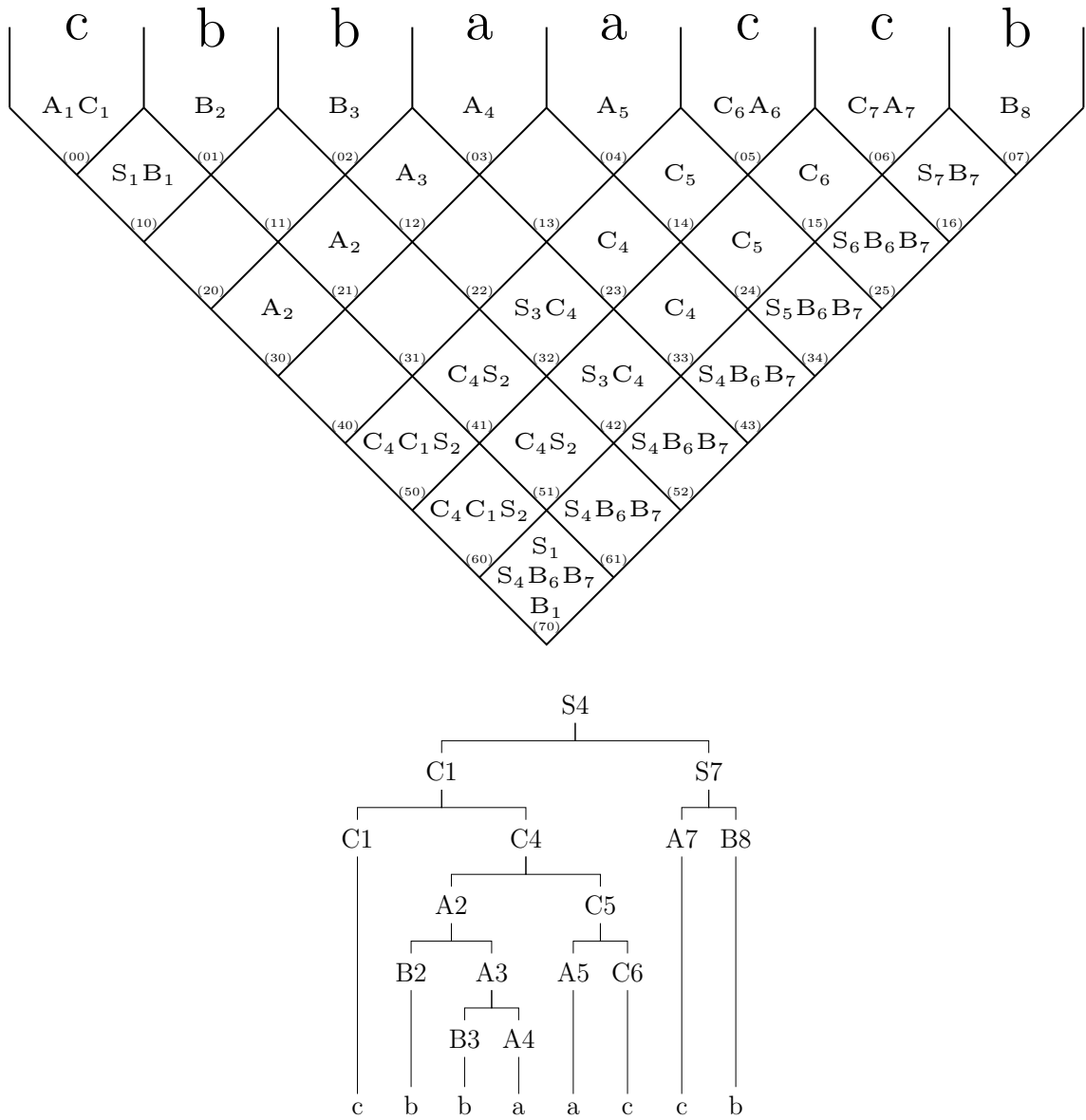
4-tuples *exercise* = (*grammar*, *word*, *parse table*, *derivation tree*) that needs to be printed.

$$A \rightarrow a \mid c \mid BA$$

$$B \rightarrow b \mid CB$$

$$C \rightarrow c \mid AC$$

$$S \rightarrow AB \mid BC$$



Does the output  $P \subseteq V \times (V^2 \cup \Sigma)$  imply that  $G$  is in oCNF? CNF does only have useful variables [TI script Def. 8.3 page 210] vs.  $P \subseteq V \times (V^2 \cup \Sigma)$ .

More of a problem is that the set  $P$  is not necessarily in CNF. It is possible that there are unreachable variables – from the starting variable.



### 3.3 Overview

UML-Diagramm showing the general idea of the implementation.

List noteworthy used libraries here, too.

Maybe some information out of the statistics tool of IntelliJ.

Show the stuff about Antler.

## References

- [1] JSR 220: Enterprise Java Beans 3.0 <https://jcp.org/en/jsr/detail?id=220>, 09/09/2015

