University of Bayreuth

Institute for Computer Science

Bachelor Thesis

in Applied Computer Science

Topic:	A Constrained CYK Instances Generator: Implementation and Evaluation
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ABC

Abstract

The abstract of this thesis will be found here.

Zusammenfassung

Hier steht die Zusammenfassung dieser Bachelorarbeit.

Contents

1 Introduction

1.1 Motivation

The starting point of this thesis is to get a tool to automatically generate a suitable 4-tuple *exercise* = (*grammar, word, parse table, derivation tree*), that is used to test if the students have understood the way of working of the CYK algorithm.

Various implementations of the Cocke-Younger-Kasami (CYK) algorithm can be found [XXX]. Nevertheless it is required to automatically generate suitable *exercise*s, that afterwards can be modified as wanted. This is the reason an own implementation has been made. It is also a task to find a more clever algorithm to automatically generate *exercise*s with a high chance of being suitable as an exam exercise.

1.2 Context Free Grammar in Chromsky Normal Form

Definition 1. Context Free Grammar (CFG)
We define a CFG as the 4-tuple $G = (V, \Sigma, S, P)$:

- V is a finite set of variables.

- Σ is an alphabet

- S is the start symbol and $S \in V$.

- P is a finite set of rules: $P \subseteq V \times (V \cup \Sigma)^*$.

It is valid that $\Sigma \cap V = \emptyset$.

Definition 2. CFG in Chromsky Normal Form (CNF)
A CFG $G = (V, \Sigma, S, P)$ is in CNF iff.:

- $P \subseteq V \times (V \cup \Sigma)^*$.

Regarding this thesis a grammar is always synonymous with Definition 2. For further convenience the following default values are always true:

- $V = \{A, B, ...\}$

- $(V^2 \cup \Sigma)^* = \{a, b, ...\} \cup \{AA, AB, BB, BA, BS, AC, ...\}$

Definition 3. Word w and language $L(G)$
Word w and language $L(G)$:

- $w \in \Sigma^* = \{w_0, w_1, ..., w_j\}$.

- A language $L(G)$ over an alphabet Σ is a set of words over Σ .

Moreover in the context of talking about sets, a set is always described beginning with an upper case letter, while one specific element of a set is described beginning with a lower case letter. Example: A "*Pyramid*" is a set consisting of multiple "*Cell*"s, whereas a *Cell* is again a subset of the set of variables "*V*". A "*cellElement*" is one specific element of a "*Cell*". (For further reasoning behind this example see chapter XXX "help data structure")

1.3 General approaches

Two basic approaches, that may help finding a good algorithm are explained informally.

1.3.1 Forward Problem & Backward Problem

The Forward Problem and the Backward Problem are two ways as how to determine if $w \in L(G)$.

> **Definition 4. Forward Problem** ($G \xrightarrow{derivation} w$)
>
> Input: Grammar G in CNF.
>
> Output: Derivation d that shows implicitly $w \subseteq L$.

It is called Forward Problem, if you are given a grammar G and form a derivation from its root node to a final word w. The final word w is always element of $L(G)$.

> **Definition 5. Backward Problem = Parsing** ($w \overset{?}{\subseteq} L(G)$)
>
> Input: w and a grammar G in CNF.
>
> Output: $w \subseteq L(G) \implies$ derivation d.

If you are given a word w and want to determine if it is element of $L(G)$, it is called Backward Problem or parsing.

1.3.2 Parsing Bottom-Up & Top-Down

There are again two ways to classify the approach of parsing.

> **Definition 6. Bottom-Up parsing**
>
> Bottom-Up parsing means to start parsing from the leaves up to the root node.

"Bottom-Up parsing is the general method used in the Cocke-Younger-Kasami(CYK) algorithm, which fills a parse table from the "bottom up""[Duda 8.6.3 page 426].

> **Definition 7. Top-Down parsing**
>
> Top-Down parsing means to start parsing from the node down to the leaves.

"Top-Down parsing starts with the root node and successively applies productions from P, with the goal of finding a derivation of the test sentence w." [XXX] (The so called test sentence is synonymous to an word w.) Reasonably criteria to guide the choice of which rewrite rule to apply could include to begin the parsing at the first (left) or last (right) character of the word w [XXX][Duda 8.6.3 page 428]

1.4 Data Structure Pyramid

To be able to describe the way of working of the different algorithms easier the help data structure *Pyramid* will be defined – note that *Pyramid* starts with upper case and therefore is a set). But before that:

Definition 8. $[i, j]$

$[i,\ j] := \{i,\ i+1, ..., j-1,\ j\} \subseteq \mathbb{N}_{\geq 0}$.

Definition 9. $Cell_{i,j}$

$Cell_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\}$

Now *Pyramid* can be defined as following:

Definition 10. *Pyramid*

$Pyramid := \{Cell_{i,j} \mid i \in [0,\ i_{max}],\ j \in [0,\ j_{max,i}],\ i_{max} = |w|-1,\ j_{max,i} = i_{max}-i\}$.

The following is the visual representation of a *Pyramid* that additionally has written the word w above it:
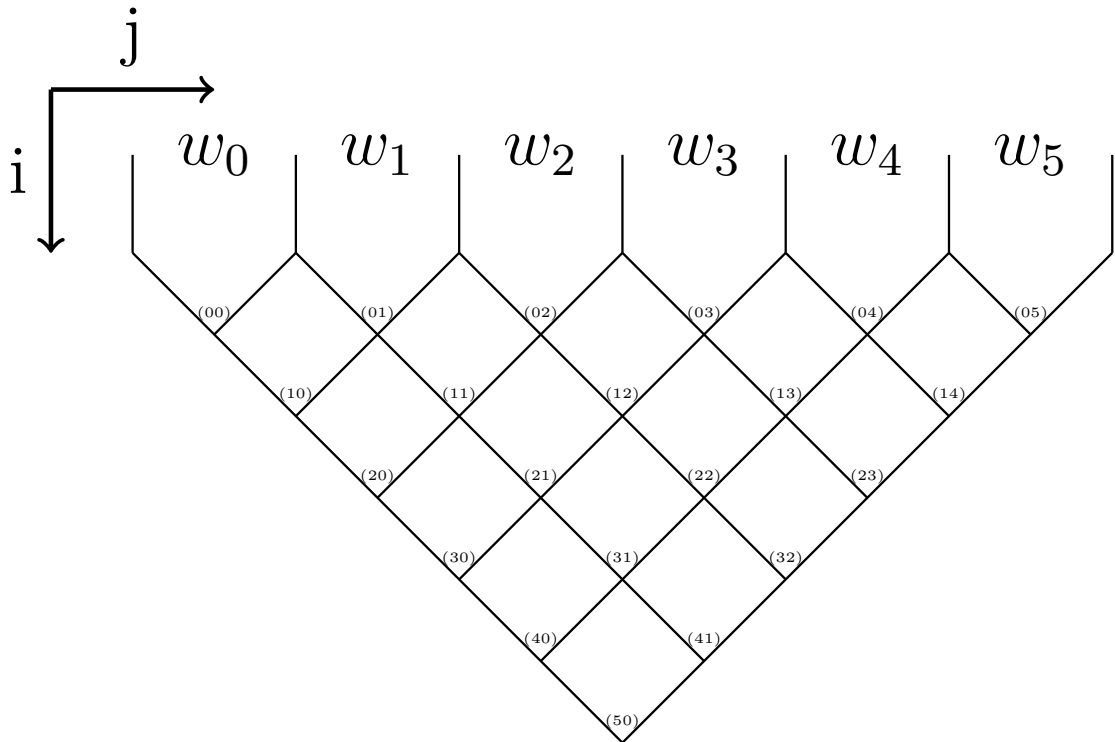

Figure 1: Visual representation of a *Pyramid* with the word w above it.

> **Definition 11.** *CellDown, CellUpperLeft and CellUpperRight*
>
> Let there be a $Cell_{i,j}$ then the following is true:
>
> - $CellDown = Cell_{i,j}$.
>
> - $CellUpperLeft = Cell_{i-1,j}$.
>
> - $CellUpperRight = Cell_{i-1,j+1}$.

1.5   Cocke-Younger-Kasami Algorithm

The Cocke-Younger-Kasami Algorithm (CYK) has been developed independently in the 1960s by Itiroo Sakai, John Cocke, Tadao Kasami, Jacob Schwartz and Daniel Younger that uses the principle of dynamic programming. [XXX]

The description of the algorithm follows [TI Hofmann] adjusted to the help data structure *Pyramid*.

Algorithm 1: CYK

Input: Grammar $G = (V,\ \Sigma,\ S,\ P)$ and word $w \in \Sigma^* = \{w_0,\ w_1,\ ...,\ w_j\}$

Output: true $\Leftrightarrow w \in L(G)$

1 $Pyramid = \emptyset$;

2 **for** $j := 0 \to i_{max}$ **do**

3 $\quad$ $Pyramid \cup Cell_{0,j} = \{(X, j) \mid X \longrightarrow w_j\}$

4 **end**

5 **for** $i := 1 \to i_{max}$ **do**

6 $\quad$ **for** $j := 0 \to j_{max,i}$ **do**

7 $\quad\quad$ **for** $k := i - 1 \to 0$ **do**

8 $\quad\quad\quad$ $Pyramid \cup Cell_{i,j} = \{X \mid X \longrightarrow YZ,\ Y \in Cell_{k,j},\ Z \in Cell_{i-k-1,k+j+1}\}$;

9 $\quad\quad$ **end**

10 $\quad$ **end**

11 **end**

12 $wInL = false$;

13 **if** $(S, i) \in Cell_{i_{max},0}$ **then**

14 $\quad$ $wInL = true$;

15 **end**

16 **return** $wInL$;

Line 2: First row.
Line 5: All rows except the first.
Line 6: All cells in each row.
Line 7: All possible cell combinations for each cell.
Line 14: True iff $Cell_{i_{max},0}$ contains the start variable.

1.6 Success Rates

Success Rates (SR) are used to compare the algorithms accounting to their performance of the different requirements. Let $N \in \mathbb{N}$ be the overall count of all generated grammars of the examined algorithm.

Write down if the cellsVar or the cellsVarK are used.
Mention the basic connection between the success rates.

Overall Success Rate An generated *exercise* contributes to the Overall Success Rate (SR) iff it contributes to the Success Rate Producibility (SRP), to the Success Rate Grammar Constraints (SRG) and to the Success Rate Pyramid Word Constraints ($SRPW$) at the same time.
It holds: $SR = n/N$, whereas n is the count of *exercises* that fulfil the requirements in this case.

Success Rate Producibility An generated *exercise* contributes to the SRP iff the CYK algorithm's output is true.
It holds: $SRP = p/N$, whereas p is the count of *exercises* that fulfil the requirements in this case.

Success Rate Grammar Constraints An generated *exercise* contributes to the SRG iff the following conditions are met:

- grammar has got less than a certain amount of productions.

It holds: $SRG = g/N$, whereas g is the count of *exercises* that fulfil the requirements in this case.

Success Rate Word Pyramid Constraints An generated *exercise* contributes to the $SRWP$ iff the following conditions are met:

- A certain amount of cells force a right cell combination.

- There are less than a certain amount of variables in the entire pyramid.

- There are less than a certain amount of variables in each cell of the pyramid.

It holds: $SRWP = wp/N$, whereas wp is the count of *exercises* that fulfil the requirements in this case.

Fore more detail of fore right cell combination see here:

Algorithm 2: checkForceCombinationPerCell

Input: $CellDown \subseteq V,\ CellUpperLeft \subseteq V,\ CellUpperRight \subseteq V,\ P \subseteq V \times (V^2 \cup \Sigma)$

Output: $true \iff at\ least\ one\ variable\ forces$

1   $VarsForcing \subseteq V$;

2   $VarComp = \{xy \mid x \in CellUpperLeft\ \wedge\ y \in CellUpperRight\}$;

3   **foreach** $v \in CellDown$ **do**

4     $Prods = \{p \mid p \in P\ \wedge\ p = (v_1, rhse_1)\ \wedge\ v == v_1\}$;

5     $Rhses = \{rhse \mid p \in Prods\ \wedge\ p = (v_1, rhse_1)\ \wedge\ rhse == rhse_1\}$;

6   **if** $Rhses \nsubseteq VarComp$ **then**

7       $VarsForcing = VarsForcing \cup v$;

8   **end**

9   **end**

10   **return** $|VarsForcing| > 0$;

---

2 Algorithms

Does the output $P \subseteq V \times (V^2 \cup \Sigma)$ imply that G is in CNF? CNF does only have useful variables [TI script Def. 8.3 page 210] vs. $P \subseteq V \times (V^2 \cup \Sigma)$.

More of a problem is that the set P is not necessarily in CNF. It is possible that there are unreachable variables – from the starting variable.

2.1 Algorithm sub modules

Sub modules are parts of the algorithms that are denoted with (A), (B), ... They are noteworthy procedures that need to be explained in more detail for a better understanding of the way of working of the algorithms.

Write down the default values used for this.

2.1.1 Distribute A & B

Algorithm 3: Distribute

    Input: $Rhse \subseteq (V^2 \cup \Sigma)$, V

    Output: Set of productions P

1 $i \in \mathbb{N}$, $j \in \mathbb{N}$;

2 **foreach** $rhse \in Rhse$ **do**

3   *choose n uniform randomly in $[i, j]$;*

4 $V_{add} := uniform\ random\ subset\ of\ size\ n\ from\ V$;

5 $P = P \cup \{(v, rhse) \mid v \in V_{add},\ rhse \in Rhse\}$;

6 **end**

7 **return** P;

Algorithm 3 isn't needed anymore for the descriptions of the basic idea of the algorithm. It will be a module later on while tweaking the algorithms.

2.1.2 Stopping Criteria C

It is fulfilled if more than half of the pyramid cell are not empty. This is an independent criteria.

    Stop if any variable is in the tip cell of the pyramid. This is dependent on the count of different available variables.

2.1.3 ChooseXYDependingOnIFromRowSet D

$RowSet \subseteq \{(XY, i) \mid X, Y \in V \land i \in \mathbb{N}\}$

Compression of the RowSet like: (AB,3) and (AB,1) -> (AB,1) –> RowSetCompressed
rowListWeighted = add i times XY to rowListWeighted.

2.2 DiceRollOnlyCYK

2.2.1 Basic Idea

This is a very naive way of generating grammars, which will be the starting point for our algorithms to be found. Each future algorithm must have a higher score than this algorithm or otherwise it would be worse, than simple dice rolling the distribution of terminals and compound variables with removing the not contributing productions afterwards.

2.2.2 Algorithm

Algorithm 4: DiceRollOnlyCYK

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1   $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$

2   $P = Distribute(\Sigma, \ V)$; Ⓐ

3   $P = P \cup Distribute(V^2, \ V)$; Ⓑ

4   $Pyramid = CYK(G, \ w)$;

5   $P = DeleteUnneccessaryRules(P, w, Pyramid)$;

6   **return** P;

Line 13: ?Removes all production that don't contribute, but unreachable productions still possible.?
Contributing production iff *useful* i.e. it appears in some derivation of some terminal string from the start symbol AND *producing* i.e. it is needed for this parsing table.

As seen in table **??** the algorithm shows a relatively low success rate for producibility. This can be explained with ... .

Something about what can be improved in another attempt or the next attempt.

2.3 BottomUpDiceRollVar1

2.3.1 Basic Idea

This algorithm uses the Bottom-Up approach where the parsing table is filled starting from the leaves. An extension compared to algorithm **??** is that productions are only added as long as the the stopping criteria isn't met.

2.3.2 Algorithm

Algorithm 5: BottomUpDiceRollVar1

   Input: Word $w \in \Sigma^*$
   Output: Set of productions P

1 $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$
2 $P = Distribute(\Sigma,\ V)$; Ⓐ
3 $Pyramid = CYK(G,\ w)$;
4 **for** $i := 1$ **to** i_{max} **do**
5     $J = \{0,\ ... \ ,\ j_{max} - 1\}$; // $J \subseteq \mathbb{N}$
6     $CellSet = \emptyset$; // $CellSet \subseteq V^2$
7   **while** $|J| > 0$ **do**
8         $choose\ one\ j \in J\ uniform\ randomly$;
9         $J = J \setminus \{j\}$;
10        $CellSet = CalculateSubsetForCell(Pyramid,\ i,\ j)$;
11        $P = P \cup Distribute(CellSet,\ V)$; Ⓑ
12        $Pyramid = CYK(G,\ w)$;
13        $P = DeleteUnneccessaryRules(P, w, Pyramid)$;
14   **if** $stopping\ criteria\ met$ Ⓒ **then**
15   **return** P;
16   **end**
17   **end**
18 **end**
19 **return** P;

Line 2: Fills the i=0 row of the pyramid.
Line 8: A cell is only visited only once.
Note: Maybe modify algorithm to also work with the threshold.

A relatively small number of productions is already sufficient to completely fill the parsing table. This can be seen if one does take look at the log-file where the final cell that has been worked with is denoted. A good chosen stopping criteria allows a higher success rate.

2.4 BottomUpDiceRollVar2

2.4.1 Basic Idea

As seen in algorithm 5 a small number of productions is sufficient to make the parsing table quite full. If an cell is nearer to the leaves its chance to be in the set of one of the calculated sub sets for a cell is higher. Therefore you could introduce a bias that favours cells with an higher index i to allow different cell combinations.

Berechnung von RowSet für alles Restlichen Zellen in der Zeile!!!!!??????????!!!!!!! Bisher nur für alle bisher Verwendeten.

2.4.2 Algorithm

Algorithm 6: BottomUpDiceRollVar2

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1   $P = \emptyset$;   // $P \subseteq V \times (V^2 \cup \Sigma)$

2   $RowSet = \emptyset$;   // $RowSet \subseteq \{(XY, i) \mid X, Y \in V \wedge i \in \mathbb{N}\}$

3   $P = Distribute(\Sigma,\ V)$; (A)

4   $Pyramid = CYK(G,\ w)$;

5   **for** $i := 1$ **to** i_{max} **do**

6   **for** $j := 0$ **to** $j_{max} - i$ **do**

7       $RowSet = RowSet \cup \{(XY, i) \mid XY \in$
       $CalculateSubsetForCell(Pyramid,\ i,\ j)\}$;

8   **end**

9   **while** $threshold_i$ *not reached* **do**

10   *choose one xy out of* $(XY,\ i) \in RowSet$ *uniform randomly with*
       probability depending on i; (D)

11       $P = P \cup Distribute(xy,\ V)$; (B)

12       $Pyramid = CYK(G,\ w)$;

13       $P = DeleteUnneccessaryRules(P, w, Pyramid)$;

14   **if** *stopping criteria met* (C) **then**

15   **return** P;

16   **end**

17   **end**

18   **end**

19 **return** P;

Line 2: Fills the i=0 row of the pyramid.

Line 7: $(AB, 1), (AB, 2), (BC, 3)... \in sub \rightarrow$ multiple occurrences of AB are allowed here yet.

Note Line 9: threshold is reached iff more than half of the cells of one row aren't empty.

2.5 SplitThenFill

2.5.1 Basic Idea

The basic idea for this algorithm is to uniform randomly generate a predefined structure of the derivation tree that helps adding the "right" productions. You always update the pyramid after adding one production to the grammar. This is also some kind of BottumUp approach - Bottom Up: The parse table is filled relatively evenly. All information regarding the upper cells are available and can be used. Similar to the CYK Algorithm approach.

It is important to distribute the varComp exactly to one var.

2.5.2 Algorithm

Algorithm 7: SplitThenFillPrep

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1 $P = \emptyset$; // $P \subseteq V \times (V^2 \cup \Sigma)$

2 $P = Distribute(\Sigma, V)$; Ⓐ

3 $Sol = (P_{Sol}, Cell_{i,j})$; // $P_{Sol} \subseteq P \ \wedge \ Cell_{i,j} \in Pyramid$

4 $Sol = SplitThenFill(P, w, i_{max}, 0)$;

5 **return** P_{Sol};

Line 2: Fills the i=0 row of the pyramid.

Algorithm 8: SplitThenFill

Input: $P_{in} \subseteq V \times (V^2 \cup \Sigma), \ w \in \Sigma^*, \ i, j \in \mathbb{N}$

Output: $(P, Cell_{i,j})$

1 $P = P_{in}$;

2 **if** $i = 0$ **then**

3   **return** $(P, Cell_{i,j})$;

4 **end**

5 *choose one m uniform randomly in* $[j + 1, \ j + i]$;

6 $(P, Cell_l) = SplitThenFill(P, w, (m - j - 1), j)$;

7 $(P, Cell_r) = SplitThenFill(P, w, (j + i - m), m)$;

8 $Pyramid = CYK(G, w)$;

9 **if** *stopping criteria met* Ⓒ **then**

10   **return** $(P, Cell_{i,j})$;

11 **end**

12 **if** $Cell_{i,j} = \emptyset$ **then**

13     $Vc = uniform \ random \ subset \ from \ \{vc \mid v \in Cell_l \ \wedge \ c \in$
       $Cell_r\} \ with \ |Vc| \geq 1$;

14     $P = P \cup Distribute(Vc, V)$; Ⓑ

15 **end**

16 **return** $(P, Cell_{i,j})$;

The stopping criteria is met if the tip of the pyramid is not empty. It is a valid approach because if this cell is not empty it means that there is a chance of being able to generate the word. To add further productions only results in a grammar that has to many productions with its pyramid having to many variables.

2.6 SplitAndFill

2.6.1 Basic Idea

It is dependent on the length of the word.

It is important that the terminals and the varcomps are distributed to exactly one var. The stopping criteria will be that each cell with index i = 0 must be not empty. Now there is a second option to fill the parse table:

1. Top Down: The parse table is filled quiet unevenly. You don't have all information available. Think about adding a production for the node cell: You can add a production so that its producing cells fill the node cell, but you don't know what actually would be the best to fill in these producing cells because they themselves aren't looked at yet. This problem is kept until the last depth of the recursion, where the cells in row $i = 0$ are taken into account. Only starting there you know what variables actually produce the terminals.

   Maybe solution: For the Top Down approach, don't assume that the terminals are already distributed over the V. Distribute the terminals over the variables in an ideal way that fits your already generated productions best.

The problem is that we have as much productions as splits in the derivation tree exist. The productions count can be reduced via merging duplicate productions and via reducing the split count in the tree.

Merging productions means: If there are A –> BS and C –> BS then only one Production of these two can remain.

2.6.2 Algorithm

Algorithm 9: SplitAndFillPrep

Input: Word $w \in \Sigma^*$

Output: Set of productions P

1 $P = \emptyset;$ // $P \subseteq V \times (V^2 \cup \Sigma)$

2 $Sol = (P_{Sol}, \ v);$ // $P_{Sol} \subseteq P$

3 $Sol = SplitAndFill(P, \ w, \ i_{max}, \ 0);$

4 *Merge productions with the same variableCompound in P_{Sol};*

5 **return** $P_{Sol};$

Algorithm 10: SplitAndFill

Input: $P_{in} \subseteq V \times (V^2 \cup \Sigma), \ w \in \Sigma^*, \ i, j \in \mathbb{N}$

Output: $(P, \ v)$

1 $P = P_{in};$

2 **if** $i = 0$ **then**

3 $\quad$ **return** $(P \cup (v, \ w_j), \ v_{lhse});$

4 **end**

5 *choose one m uniform randomly in $[j + 1, \ j + i]$;*

6 $(P, \ v_l) = SplitAndFill(P, \ w, \ (m - j - 1), \ j);$

7 $(P, \ v_r) = SplitAndFill(P, \ w, \ (j + i - m), \ m);$

8 **if** $i = i_{max}$ **then**

9 $\quad$ **return** $(P \cup (S, \ v_l v_r), \ v);$

10 **end**

11 **return** $(P \cup (v, \ v_l v_r), \ v);$

2.7 Comparision of Algorithms

Algorithm	SR	SRP	SRG	SRWP
DiceRollOnly	10 %	24 %	87 %	53 %
BotomUpVar1	18 %	53 %	88 %	48 %
BotomUpVar2	22 %	47 %	93 %	62 %
SplitThenFill	26 %	39 %	96 %	78 %
SplitAndFill	15 %	100 %	99 %	15 %

Table 1: Comparison of the success rates of the algorithms

Finding of ideal parameter for each algorithm.

3 CLI Tool

Write much of this stuff in the appendix.

3.1 Scoring Model

Only valid ResultSamples are given a score. Parameters to be scored:

- RightCellCombinationsForcedCount

- maxSumOfVarsInPyramidCount

- maxNumberOfVarsPerCellCount

- maxSumOfProductionsCount

Maybe add a diversity criterion = homogeneity of the cells to the scoring matrix.

Parameter	Points					
	2	4	6	8	10	-100
cellCombinationsForced	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
sumVarsInPyramid	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
maxVarsPerCell	[5,5]	[4,4]	[1,1]	[3,3]	[2,2]	>5
sumProductions	[1,2]	[3,4]	[5,6]	[9,10]	[7,8]	>10
homogeneity	[ ]	[ ]	[ ]	[ ]	[ ]	>10
maxVarKsPerCell	[ ]	[ ]	[ ]	[ ]	[ ]	>10

Table 2: Scoring of the different parameter values

Based on table 2 each result sample is scored. Out of the #??? best result samples one can choose.
The result will be normalized to the maximum possible points -> range 0.0 to 1.0.

3.2 Short Requirements Specification

Generating the latex code and storing it in .tex-file. Then converting the .tex-file to .pdf-file via:
Runtime rt = Runtime.getRuntime();
Process pr = rt.exec("pdflatex mydoc.tex");
Process pr = rt.exec("pdflatex mydoc.tex");
Process pr = rt.exec("pdflatex mydoc.tex");
The triple invocation of LaTeX is to ensure that all references have been properly resolved and any page layout changes due to inserting the references have been accounted for. [http://www.arakhne.org/autolatex/]
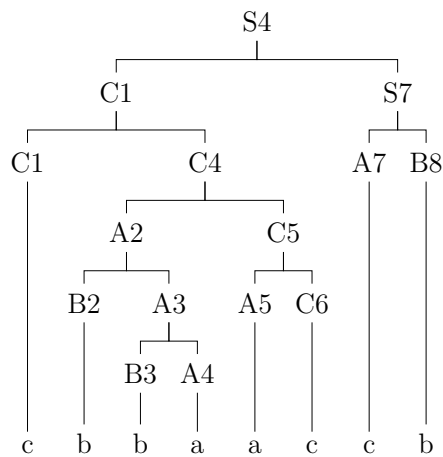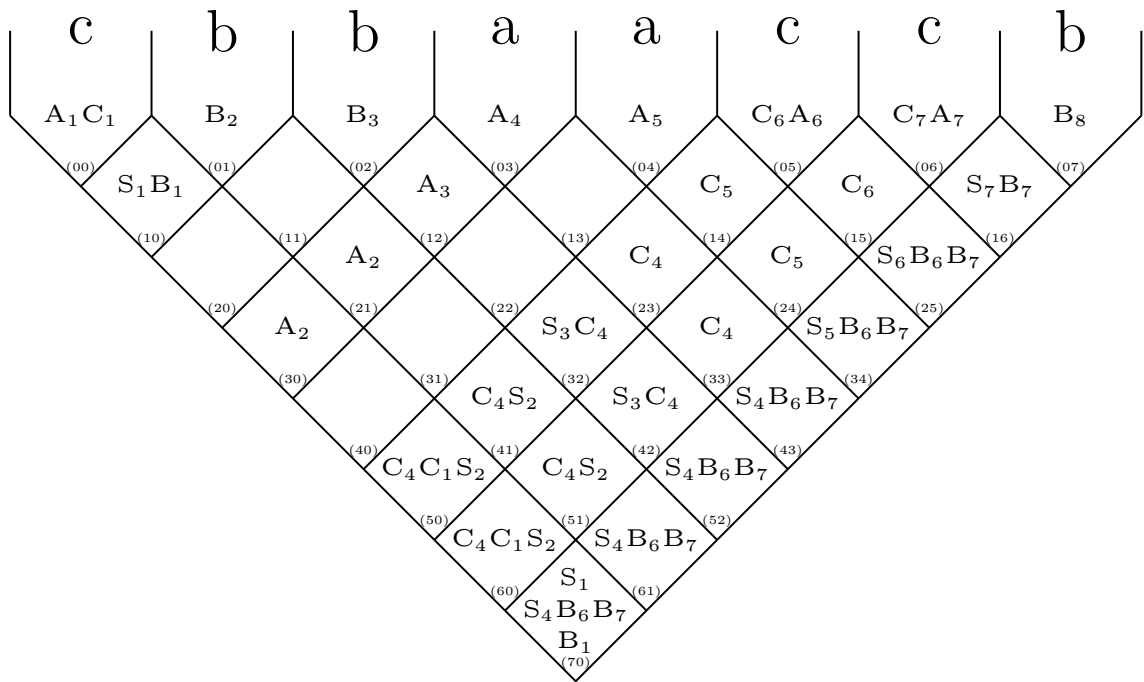
3.2.1 Exam Exercises

4-tuples $exercise = (grammar,\ word,\ parse\ table,\ derivation\ tree)$ that needs to be printed.

$$A \rightarrow a \mid c \mid BA$$
$$B \rightarrow b \mid CB$$
$$C \rightarrow c \mid AC$$
$$S \rightarrow AB \mid BC$$

Parse table (CYK chart) for the word **c b b a a c c b**:

Length-1 cells (under each terminal):

c	b	b	a	a	c	c	b
$A_1 C_1$	B_2	B_3	A_4	A_5	$C_6 A_6$	$C_7 A_7$	B_8

Chart cells with position indices:

- (00) $S_1 B_1$
- (01) —
- (02) A_3
- (03) —
- (04) C_5
- (05) C_6
- (06) $S_7 B_7$
- (07) —
- (10) —
- (11) A_2
- (12) —
- (13) C_4
- (14) C_5
- (15) $S_6 B_6 B_7$
- (16) —
- (20) A_2
- (21) —
- (22) $S_3 C_4$
- (23) C_4
- (24) $S_5 B_6 B_7$
- (25) —
- (30) —
- (31) $C_4 S_2$
- (32) $S_3 C_4$
- (33) $S_4 B_6 B_7$
- (34) —
- (40) $C_4 C_1 S_2$
- (41) $C_4 S_2$
- (42) $S_4 B_6 B_7$
- (43) —
- (50) $C_4 C_1 S_2$
- (51) $S_4 B_6 B_7$
- (52) —
- (60) S_1 $S_4 B_6 B_7$
- (61) —
- (70) B_1

Derivation tree:

```
                          S4
              ┌───────────┴───────────┐
             C1                       S7
        ┌─────┴─────┐              ┌───┴───┐
       C1          C4             A7      B8
        │     ┌─────┴─────┐        │       │
        │    A2           C5       │       │
        │ ┌───┴───┐    ┌───┴───┐   │       │
        │ B2     A3   A5      C6   │       │
        │ │   ┌───┴───┐ │       │  │       │
        │ │  B3      A4 │       │  │       │
        │ │   │       │ │       │  │       │
        c b   b       a a       c  c       b
```

3.3 Overview - UML

UML-Diagramm showing the general idea of the implementation.

List noteworthy used libraries here, too.

Maybe some information out of the statistics tool of IntelliJ.

3.3.1 UML: More Detail 1

3.3.2 UML: More Detail 2

3.4 User Interaction

Here the specific must can do's are explained with short examples.

3.4.1 Use Case 1

3.4.2 Use Case i

References

[1] JSR 220: Enterprise Java Beans 3.0 `https://jcp.org/en/jsr/detail?id=220`, 09/09/2015