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Topic: A Constrained CYK Instances Generator:
Implementation and Evaluation

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ABC

Abstract

Every year, lecturer in the field of theoretical computer science or an related one face the task to create an exam exercise that tests if their students have understood the way of working of the Cocke-Younger-Kasami algorithm. Various implementations and small online tools of the CYK algorithm can be found, but none actually assists during the process of creating a exercise.

Therefore various algorithms to generate specifically suitable exercises have been designed and compared through their success rates. The different approaches for these algorithms involve the uniform randomly distribution of elements and the general Bottom-Up and Top-Down parsing approaches.

A GUI tool to automatically generate these exam exercises has been implemented. Its functionality contains that input parameters such as the count of variables, the count of terminals and the size of the word can be given. Suitable exam exercises are generated and one can be chosen for further modification and creation of the final exam exercise.

Zusammenfassung

Jedes Jahr stehen Dozenten der theoretischen Informatik oder eines verwandten Bereiches vor der Aufgabe Klausuraufgaben zu erstellen, die prüfen ob ihre Studenten die Arbeitsweise des Cocke-Younger-Kasami-Algorithmus verstanden haben. Verschiedene Implementierungen und kleinere Online-Tools des CYK-Algorithmus gibt es bereits, aber Keines unterstützt beim Prozess des Erstellen einer Aufgabe.

Verschiedene Algorithmen wurden zuerst entworfen, um genau passende Aufgaben zu generieren und wurden anschließend auch miteinander über ihre Erfolgsrate verglichen. Die unterschiedlichen Ansätze für die Algorithmen beinhalten das gleichmäßig zufällige Verteilen von Elementen und die allgemeinen Ansätze des Bottom-Up und Top-Down Parsings.

Es wurde ein GUI-Tool implementiert um automatisch Klausuraufgaben zu generieren. Die Funktionalität des Tools beinhaltet, dass Eingabewerte wie die Anzahl der Variablen, die Anzahl der Terminale und die Wortlänge gemacht werden können. Geeignete Klausuraufgaben werden automatisch generiert von denen Eine für weitere Modifikation und letztendlich für die Klausuraufgabenerstellung ausgewählt wird.

Contents

Abstract	5
1 Introduction	6
1.1 Motivation	6
1.2 Context Free Grammar	6
1.3 General approaches of parsing	7
1.4 Data Structure Pyramid	8
1.5 Cocke-Younger-Kasami Algorithm	9
2 Algorithms	11
2.1 Sub modules	11
2.2 Dice rolling the distributions only	13
2.3 Dice rolling and Bottom-Up variant 1	14
2.4 Dice rolling and Bottom-Up variant 2	16
2.5 Split Top-Down and fill Bottom-Up	19
2.6 Split Top-Down and fill Top-Down	24
2.7 Evaluation of Algorithms	29
2.7.1 Success Rates	29
2.7.2 Problem space exploration	31
2.7.3 Comparison of stopping criteria	31
2.7.4 Comparison of the algorithms	32
3 GUI Tool: CYK Instances Generator	33
3.1 Overview GUI	33
3.1.1 Working with the program	33
3.2 Exam Exercises	34
3.3 Scoring Model	35
3.4 Parsing input with ANTLR	36
3.5 Other matters	37

1 Introduction

1.1 Motivation

Every year, lecturer in the field of theoretical computer science or an related one face the task to create the 4-tuple exam *exercise* = (*grammar*, *word*, *parse table*, *derivation tree*) that tests if their students have understood the way of working of the Cocke-Younger-Kasami (CYK) algorithm. For it *exercises* need to be created which is a bit of a time consuming task that can only be done in $O(n^3)$ [5].

Various implementations and small online tools of the CYK algorithm can be found ¹ ² ³ but none actually assists during the process of creating an exercise.

Therefore algorithms are needed to generate specifically suitable exercises with a high chance of success. Also a GUI tool, that allows automatic generation of the suitable exam exercises and further modification, is required. Therefore an separate solution is implemented.

1.2 Context Free Grammar

Firstly, we define a Context Free Grammar (CFG) as follows:

Definition 1. Context Free Grammar (CFG)

A CFG is a 4-tuple $G = (V, \Sigma, S, P)$:

- V is a finite set of variables.
- Σ is an alphabet
- S is the start symbol and $S \in V$.
- P is a finite set of rules: $P \subseteq V \times (V \cup \Sigma)^*$.

It holds: $\Sigma \cap V = \emptyset$.

Secondly, we define a CFG with restrictions (CFGR) as:

Definition 2. CFG with restrictions (CFGR)

A CFG $G = (V, \Sigma, S, P)$ is a CFGR iff:

- $P \subseteq V \times (V^2 \cup \Sigma)$.

Throughout this thesis a grammar is always synonymous with Definition 2. Note that a CFGR is not necessarily in chomsky normal form (CNF) because it is still possible that there are unreachable variables – from the starting variable – or useless rules. For further convenience the following default values are always assumed in this thesis:

¹CYK online tool: <http://lxmls.it.pt/2015/cky.html>

²CYK parser implementation: <http://jflap.org/tutorial/grammar/cyk/index.html>

³CYK algorithm implementation in Java: <https://github.com/ajh17/CYK-Java>

- $V = \{A, B, \dots\}$
- $(V^2 \cup \Sigma) = \{AA, AB, BB, BA, BS, AC, \dots\} \cup \{a, b, \dots\}$

A rule consists of a left hand side element (lhse) and a right hand side element (rhse). Example: $lhse \rightarrow rhse$ applied to $A \rightarrow c$ and $B \rightarrow AC$ means that A and B are a *lhse* and c and AC are a *rhse*. Elements of V^2 are often referred to as variable compounds.

While talking about a word w or a language $L(G)$ Definition 3 holds:

Definition 3. Word w and language $L(G)$

- Word: $w = w_0 \cdot w_1 \cdot \dots \cdot w_j$ and $w \in \Sigma^*$.
- Language: $L(G)$ over an alphabet Σ is a set of words over Σ .

Moreover in the context of talking about sets, a set is always described beginning with an upper case letter, while one specific element of a set is described beginning with a lower case letter. Example: A "Pyramid" is a set consisting of multiple "Cell"s, whereas a *Cell* is again a subset of the set of variables "V". A "cellElement" is one specific element of a "Cell". (For further reasoning behind this example see chapter 1.4)

1.3 General approaches of parsing

Next, the basic approach that may help finding a good algorithm is explained informally like in [1]. At first, parsing is described in general and afterwards its two characteristics are explained.

Definition 4. Backward Problem = Parsing ($w \stackrel{?}{\subseteq} L(G)$)

Input: w and a grammar G .

Output: $w \subseteq L(G) \implies$ derivation d .

If you are given a word w and want to determine if it is element of $L(G)$, it is called parsing, which is also the basis of the Cocke-Younger-Kasami algorithm.

After having defined what parsing in general is, it is important to know the two different ways of parsing, that will act as an idea provider for the algorithms.

Bottom-Up parsing

Bottom-Up parsing means to start parsing from the leaves up to the root node.

Actually, Bottom-Up parsing is the method used in the Cocke-Younger-Kasami algorithm, which fills the parse table from the "bottom up" [1].

Bottom-up parsing starts by recognizing the words smallest sub words before its mid-size sub words, and leaving the largest overall word as the last.

Top-Down parsing

Top-Down parsing means to start parsing from the root node down to the leaves.

"Top-Down parsing starts with the root node and successively applies productions from P , with the goal of finding a derivation of the test sentence w ." [1] (The so called test sentence is synonymous to an word w .) [1].

1.4 Data Structure Pyramid

To be able to describe how the different algorithms work in a simpler way, the help data structure *Pyramid* will be defined – note that *Pyramid* starts with upper case and therefore is a set.

Definition 5. *Pyramid*

$Pyramid := \{Cell_{i,j} \mid i \in [0, i_{max}], j \in [0, j_{max,i}], i_{max} = |w|-1, j_{max,i} = i_{max}-i\}$
 where $Cell_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\}$ denotes the contents of the j 's cell in row i
 and $[i, j] := \{i, i+1, \dots, j-1, j\} \subseteq \mathbb{N}_{\geq 0}$.

The cell $Cell_{i_{max},0}$ is called the root of such a *Pyramid* and Figure 1.4 is the visual representation of one.

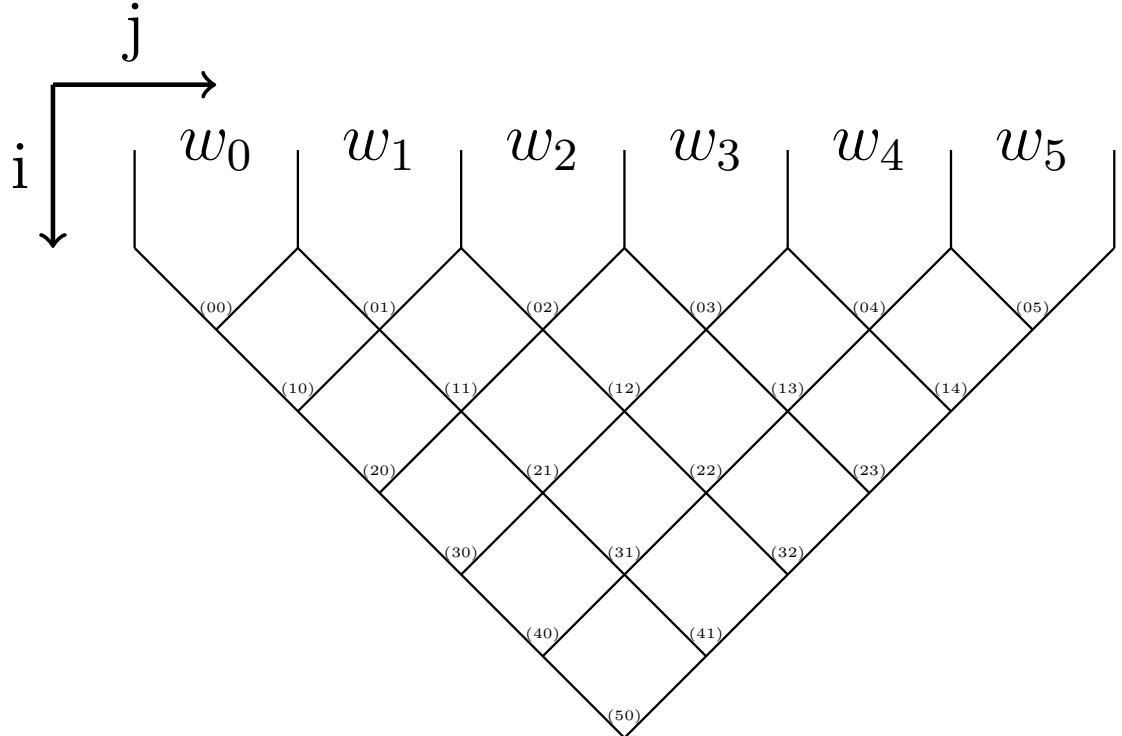


Figure 1: Visual representation of a *Pyramid* with the word w written above it.

1.5 Cocke-Younger-Kasami Algorithm

The Cocke-Younger-Kasami Algorithm (CYK) was independently developed in the 1960s by Itiroo Sakai [2], John Cocke and Jacob Schwartz [3], Tadao Kasami [4] and Daniel Younger [5].

The idea is to find all possible derivations of each subword starting with size one and to consecutively use this information to find all possible derivations with a larger size of the subword up to the size of w . Finally it is checked whether $w \in L(G)$ through the presence of the start variable in the root of the pyramid..

The description of the algorithm follows the source [6] adjusted to the data structure *Pyramid*. Later on it can be seen, that the CYK algorithm can be used as a basis to find good algorithms.

Algorithm 1: CYK	
Input: Grammar $G = (V, \Sigma, S, P)$ and word $w \in \Sigma^* = \{w_0, w_1, \dots, w_j\}$ Output: $\text{true} \Leftrightarrow w \in L(G)$	
1	$Pyramid = \emptyset;$
2	for $j := 0 \rightarrow i_{max}$ do
3	$Pyramid \cup = \{(X, j+1) \mid X \rightarrow w_j\};$ // Fills cells $Cell_{0,j}$
4	end
5	for $i := 1 \rightarrow i_{max}$ do
6	for $j := 0 \rightarrow j_{max,i}$ do
7	for $k := i-1 \rightarrow 0$ do
8	$Pyramid \cup \{(X, k) \mid X \rightarrow YZ, Y \in Cell_{k,j}, Z \in Cell_{i-k-1, k+j+1}\};$ // Fills cells $Cell_{i,j}$??k?? XXX and $Y \in Cell_{k,j}$ is wrong like in circled D
9	end
10	end
11	end
12	if $(S, i) \in Cell_{i_{max},0}$ then
13	return true;
14	end
15	return false;
<hr/> Line 2: First row. Line 5: All rows except the first. Line 6: All cells in each row. Line 7: All possible cell combinations for each cell. Line 13: True iff $Cell_{i_{max},0}$ contains the start variable.	

During the execution of the CYK algorithm the parsing table is filled as shown in Figure 2. At first the row with index $i = 0$ is filled after Line 2 to Line 4 of the CYK algorithm, i.e. a $Cell_{0,j}$ will contain the variable if it has the terminal w_j as its *rhse*. Then for

each row i every cell with ascending index j is looked at. Every possible combination of sub words for a cell are taken into account, i.e. for $Cell_{4,1}$ there are the combinations of $(Cell_{0,1}, Cell_{3,2})$, $(Cell_{1,1}, Cell_{2,3})$, $(Cell_{2,1}, Cell_{1,4})$ and $(Cell_{3,1}, Cell_{0,5})$. Applying Line 8 for example to the cell combination $(Cell_{2,1}, Cell_{1,4})$ it leads to $X \rightarrow AC$ and because the compound variable AC is *rhse* of the variable S the $Cell_{4,1}$ contains the element $(S, 4)$.

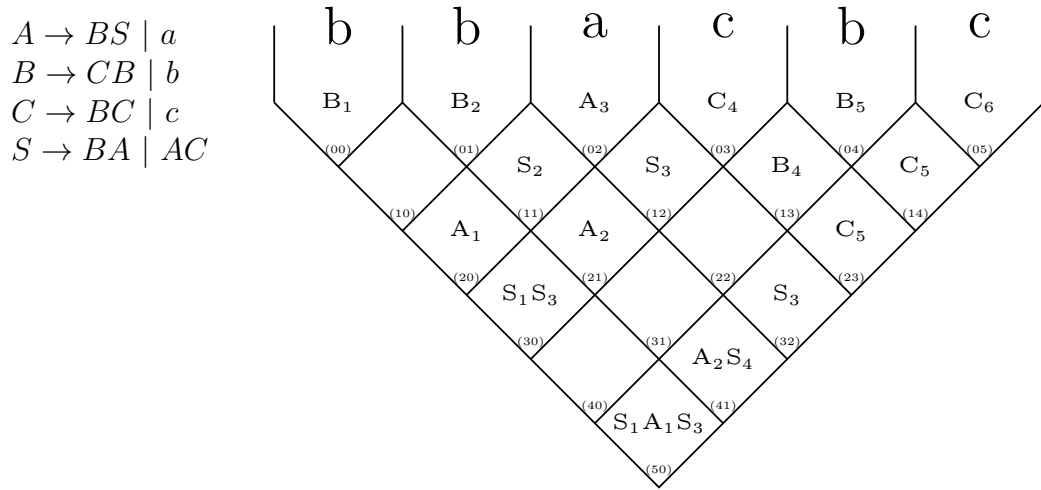


Figure 2: The CYK algorithm fills the cells of the pyramid during execution (Line 3 and Line 8).

2 Algorithms

2.1 Sub modules

Sub modules are parts of the algorithms that are denoted circled with (A), (B), (C), (D) and (E). They are procedures that should be explained in more detail for a better understanding of the way of working of algorithms in the following chapters. (E) is explained not until Chapter 2.4 because it is needed only there.

Distribute(Σ, V) (A) and Distribute(V^2, V) (B):

The difference between (A) and (B) is that one time Σ and the other time V^2 are distributed. But in both cases a uniform random subset of the *Rhse* is taken and again uniform randomly distributed over the set of available variables V . While distributing the terminals there exists at least one rule for every terminal used in the word w . The specifics of how they are distributed are described in the following algorithm:

Algorithm 2: Distribute	
Input: $V, Rhse \subseteq V^2$ or $Rhse \subseteq \Sigma$	
Output: Set of productions $P \subseteq V \times V^2$ or $P \subseteq V \times \Sigma$	
1	foreach $rhse \in Rhse$ do
2	<i>choose n uniformly randomly in $[i, j]$; // $i \in \mathbb{N}, j \in \mathbb{N}$</i>
3	$V_{add} :=$ <i>uniform random subset of size n from V;</i>
4	$P \cup \{(v, rhse) \mid v \in V_{add}, rhse \in Rhse\};$
5	end
6	return P ;

Stopping Criteria (C):

Two kinds of stopping criteria have been used to determine whether an algorithm should terminate early on because an already suitable exercise has been found.

- stop if more than half of the pyramid cells are not empty any more
- stop if the root of the pyramid is not empty any more

Both stopping criteria are compared in short in Chapter 2.7.

CalculateSubsetForCell(Pyramid, i, j) (D):

This procedure is needed to determine all possible compound variables out of all possible cell combinations for one specific cell. It works kind of analogous from Line 7 to Line 9 of the CYK algorithm (Algorithm 1).

Algorithm 3: CalculateSubsetForCell	
Input: $Pyramid, i \in \mathbb{N}, j \in \mathbb{N}$	
Output: $CellSet \subseteq V^2$	
<pre> 1 $CellSet = \emptyset;$ 2 for $k := i - 1 \rightarrow 0$ do 3 $CellSet \cup \{YZ \mid X \rightarrow YZ, Y \in Cell_{k,j}, Z \in Cell_{i-k-1,k+j+1}\};$ 4 end 5 return $CellSet;$ </pre>	

In the following situation a rule is added to $Cell_{3,0}$ while using Algorithm 3.

Grammar:
 $A \rightarrow AB \mid a$
 $B \rightarrow SC \mid b$
 $C \rightarrow AB$
 $S \rightarrow BA \mid AA$

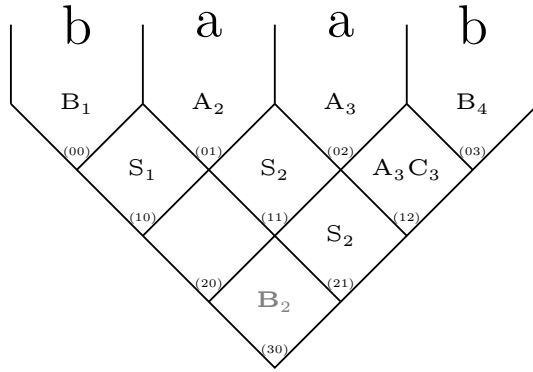


Figure 3: Example of Algorithm 3 while applying it on $Cell_{3,0}$ via adding the rule $B \rightarrow SC$.

The calculation of $CellSet$ for $Cell_{3,0}$ results in $\{SA, SC, BS\}$, whereas SA and SC stem from $Cell_{1,0}$ together with $Cell_{1,2}$ and BA comes from $Cell_{0,0}$ together with $Cell_{2,1}$. Now if either one of the rules $lhse \rightarrow SA$, $lhse \rightarrow SC$ or $lhse \rightarrow BS$ is added to the grammar, then $lhse \in Cell_{3,0}$. Here the rule $B \rightarrow SC$ has been added and finally $(B, 2)$ is element of $Cell_{3,0}$.

In general if for one $Cell_{i,j}$ a rule like $lhse \rightarrow cs$ with $cs \in CellSet$ (Line 3) is added then automatically $Cell_{i,j}$ won't be empty any more.

2.2 Dice rolling the distributions only

We start off by a primitive way of generating grammars, which will be the lower boundary while comparing the algorithms. Note that later on in Chapter 2.7.1 it is described what "performing better" means in the context of this thesis.

Algorithm 4: DiceRollOnlyCYK	
Input: Word $w \in \Sigma^*$	
Output: Set of productions P	
1	$P = \emptyset; \quad // \quad P \subseteq V \times (V^2 \cup \Sigma)$
2	$P = \text{Distribute}(\Sigma, V); \quad \textcircled{\text{A}}$
3	$P \cup \text{Distribute}(V^2, V); \quad \textcircled{\text{B}}$
4	return P ;

The algorithm DiceRollOnly (Algorithm 4) distributes terminals Σ to at least one *lhse*, but a compound variable V^2 must not be distributed at all. Note that for each terminal of $\Sigma = \{a, b\}$ at least one rule like $lhse \rightarrow a$ and $lhse \rightarrow b$ is generated. But for each possible compound variable $V^2 = \{AA, AB, AC, AS, BB, BC, BS, CC, CS, SS\}$ it is possible that only a smaller subset like $\{AA, BA, CC, SC\}$ is distributed so that only rules like $lhse \rightarrow AA$, $lhse \rightarrow BA$, $lhse \rightarrow CC$ and $lhse \rightarrow SC$ exist.

Grammar after Line 2:	Grammar after Line 3:
$C \rightarrow a$	$C \rightarrow BA \mid AA \mid a$
$B \rightarrow b$	$B \rightarrow b$
	$S \rightarrow CC \mid SC$

Figure 4: Shortend overview of an example of Algorithm 4 as described before.

2.3 Dice rolling and Bottom-Up variant 1

Another approach to design an algorithm is after the Bottom-Up approach (Chapter 1.3) in which the parsing table is filled starting from the leaves in direction of the root node.

The basic idea is to guide the choice of rules while distributing the compound variables V^2 . In Algorithm 4, the naive approach, it can happen that the terminals are distributed to the variables A and B and Algorithm 4 completely discards this fact during the distribution of the compound variables (see Figure 5, the middle part of the example).

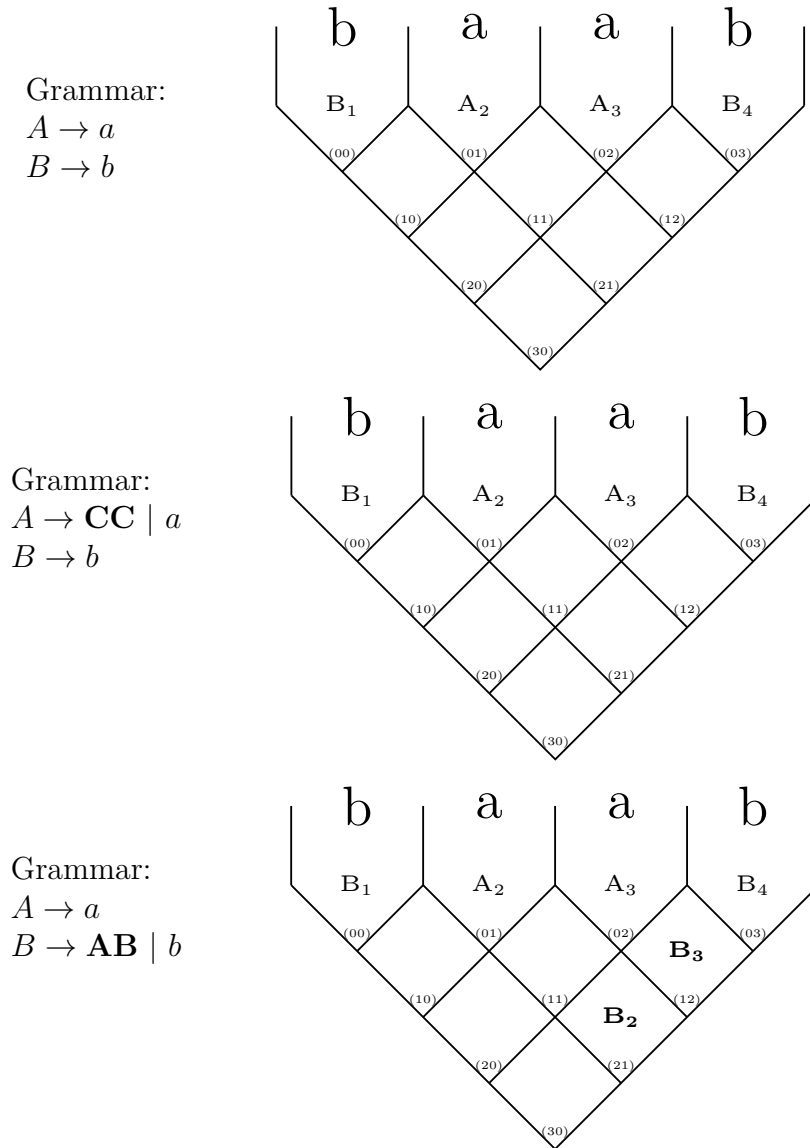


Figure 5: Example of disregarding the already added rules. Top: starting situation. Middle: Unfortunate adding of rules that doesn't help to fill the parsing table and can happen in Algorithm 4. Bottom: Good adding of rules as intended in Algorithm 5 that helps filling.

If rules like $lhse \rightarrow CC$ or $lhse \rightarrow SC$ are added they don't directly help to fill the parsing table and bloat the grammar with useless rules (see Figure 5, the middle part of the example). More reasonable rules to add would be $lhse \rightarrow BA$, $lhse \rightarrow AA$ or $lhse \rightarrow AB$ (see Figure 5, the bottom part of the example).

Algorithm 5 continues on this idea: After distributing the terminals (Line 2) the updated parsing table (Line 12) is always taken into consideration while calculating (Line 10) variable compounds and to finally add a part of them (Line 11) in form of rules to the grammar. I.e. for each chosen cell a *CellSet* (Line 10) is calculated, that only contains reasonable variable compounds. This way only variable compounds are added that directly help to fill the parsing table.

Algorithm 5: BottomUpDiceRollVar1	
Input: Word $w \in \Sigma^*$	
Output: Set of productions P	
1	$P = \emptyset; \quad // \quad P \subseteq V \times (V^2 \cup \Sigma)$
2	$P = \text{Distribute}(\Sigma, V); \quad \textcircled{A}$
3	$\text{Pyramid} = \text{CYK}(G, w);$
4	for $i := 1$ to i_{\max} do
5	$J = \{0, \dots, j_{\max} - 1\}; \quad // \quad J \subseteq \mathbb{N}$
6	$\text{CellSet} = \emptyset; \quad // \quad \text{CellSet} \subseteq V^2$
7	while $ J > 0$ do
8	choose one $j \in J$ uniform randomly;
9	$J = J \setminus \{j\};$
10	$\text{CellSet} = \text{CalculateSubsetForCell}(\text{Pyramid}, i, j); \quad \textcircled{D}$
11	$P \cup \text{Distribute}(\text{CellSet}, V); \quad \textcircled{B}$
12	$\text{Pyramid} = \text{CYK}(G, w);$
13	if <i>stopping criteria met</i> \textcircled{C} then
14	return $P;$
15	end
16	end
17	end
18	return $P;$
<hr/> Line 3: Fills the $i=0$ row of the pyramid. Line 9: A cell is visited only once.	

2.4 Dice rolling and Bottom-Up variant 2

While examining Algorithm 5 via its log file (Figure 6) it can be seen that already a very small number of rules in the grammar is sufficient so that the stopping criteria \textcircled{C} is met – the cells that indirectly decide what rules to add are mostly from row one ($i = 1$) and sometimes if at all from row two ($i = 2$).

Final cell worked with Index: 1,2
 Final cell worked with Index: 1,0
 Final cell worked with Index: 1,6
 Final cell worked with Index: 1,0
 Final cell worked with Index: 1,2
 Final cell worked with Index: 1,3
 Final cell worked with Index: 2,4

Figure 6: Digest of log files of Algorithm 5 with $|V| = 4$ and $|\Sigma| = 2$.

This again leads to a further improvement idea to introduce a row dependent *threshold_i* (Line 9) which helps that more cells with $i \geq 2$ are chosen – what possibly leads to more diverse grammars being generated. The diversity, in context of the procedure BottomUpDiceRollVar1 (Algorithm 5), is somewhat too restricted to the *lhses* that have one of the terminals as its *rhse*. Most of the rules that are part of the grammar will contain one of these *lhses* as explained in Figure 5. This is caused by the basic idea of Algorithm 5 but also due to the relatively small number of rules that are added to the grammar altogether.

Further diversification is achieved through the usage of \textcircled{E} (Line 10; for a detailed explanation see at the end of this chapter), i.e. variable compounds that already have been used in a row with low index i are at a disadvantage to be picked again.

As seen in Figure 7 rules with BA and AA are added to the variables B and A in Grammar1. For Grammar2 instead the rule $B \rightarrow SS$ is added that contributes to a better diversity compared to Grammar1.

Grammar0:	Grammar1:	Grammar2:
$C \rightarrow BA \mid AA \mid a$	$C \rightarrow BA \mid AA \mid a$	$C \rightarrow BA \mid AA \mid a$
$B \rightarrow b$	$B \rightarrow BA \mid AA \mid b$	$B \rightarrow SS \mid b$
$S \rightarrow CC \mid SC$	$S \rightarrow BA \mid AA \mid CC \mid SC$	$S \rightarrow CC \mid SC$

Figure 7: Example for better diversity. Starting point is Grammar0. Grammar2 is of better diversity than Grammar1.

Algorithm 6: BottomUpDiceRollVar2**Input:** Word $w \in \Sigma^*$ **Output:** Set of productions P

```

1  $P = \emptyset$ ; //  $P \subseteq V \times (V^2 \cup \Sigma)$ 
2  $RowSet = \emptyset$ ; //  $RowSet \subseteq \{(xy, i) \mid x, y \in V \wedge i \in \mathbb{N}\}$ 
3  $P = Distribute(\Sigma, V)$ ; (A)
4  $Pyramid = CYK(G, w)$ ;
5 for  $i := 1$  to  $i_{max}$  do
6   for  $j := 0$  to  $j_{max} - i$  do
7      $RowSet \cup \{(xy, i) \mid xy \in CalculateSubsetForCell(Pyramid, i, j)\}$ ; (D)
8   end
9   while  $threshold_i$  not reached do
10    choose one  $xy$  from  $(xy, i) \in RowSet$  uniform randomly with
        probability depending on  $i$ ; (E)
11     $P \cup Distribute(xy, V)$ ; (B)
12     $Pyramid = CYK(G, w)$ ;
13    if stopping criteria met (C) then
14      return  $P$ ;
15    end
16  end
17 end
18 return  $P$ ;

```

Line 4: Fills the $i=0$ row of the pyramid.

Choose one xy from $(xy, i) \in RowSet$ uniform randomly with probability depending on row i (E) :

At some point a decision needs to be made about what rule $lhse \rightarrow xy$ with $xy \in V^2$ will be added to the grammar. Depending on which xy is chosen the influence on the entire pyramid varies. Some xy only change the parsing table in one of its later rows ($i \gg 1$) but other xy even change it in one of the first rows. If there is a change in one of the first rows it is more likely that the entire pyramid will be filled with more elements. Now the task of choosing rules to add, that only change the pyramid in one of the later rows, with a higher probability than the others is tackled with (E).

The approach here only makes sense together with (D) in which all possible compound variables are calculated that help to fill one specific cell. $RowSet \subseteq \{(xy, i) \mid x, y \in V \wedge i \in \mathbb{N}\}$ whereas the xy are calculated with (D) and i is the row number of the specific cell.

With this *RowSet* the choice can be influenced regarding the row number i : Firstly the *RowSet* is compressed, i.e. every tuple with the same xy will be merged to its lowest i , as following: $RowSet = \{(AB, 3), (AB, 1), (AB, 5), \dots\}$ will become $RowSet = \{(AB, 1), \dots\}$. Afterwards all elements of *RowSet* will be placed in the *RowMultiSet* that can contain multiple equivalent elements. Now each element of *RowMultiSet* will be weighted according to their i . That means that elements like $(AB, 1)$ will only occur one time though elements like $(BC, 3)$ will occur three times and so on: $RowMultiSet = \{(AB, 1), (BC, 3), \dots\}$ becomes $RowMultiSet = \{(AB, 1), (BC, 3), (BC, 3), (BC, 3), \dots\}$. Now one element will be chosen uniformly randomly out of this weighted *RowMultiSet*. In the example in Figure 18 this results in $xy = BC$.

```

RowSet = {(AB, 3), (AB, 1), (AB, 5), ...}           // compress
RowSet = {(AB, 1), ...}                             // place into RowMultiSet
RowMultiSet = {(AB, 1), (BC, 3), ...}               // weight elements
RowMultiSet = {(AB, 1), (BC, 3), (BC, 3), (BC, 3), ...} // pick element
xy = BC

```

Figure 8: Shortened example of the procedure E as before in the text.

2.5 Split Top-Down and fill Bottom-Up

Until now we have only discussed algorithms that purely use the Bottom-Up approach, so another way is to make use of the Top-Down approach in combination with the Bottom-Up approach.

The idea here is to first distribute the terminals (Line 2 of Algorithm 7 SplitThenFill) and then to uniformly randomly generate a predefined structure of the derivation tree (Line 4 of Algorithm 7 and in general Algorithm 8 SplitThenFillRec) Top-Downwards and then again to fill the parsing table Bottom-Upwards accordingly to fill this derivation tree. The structure of the derivation tree for instance can look as follows:

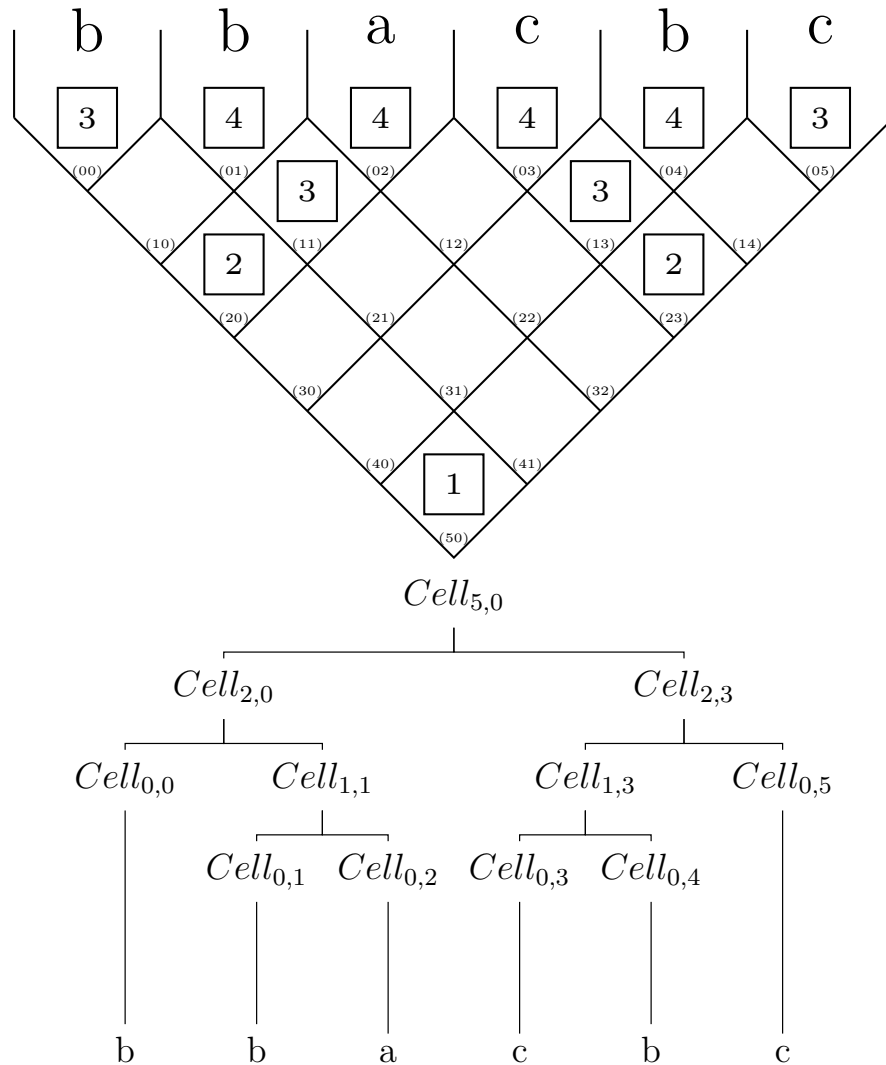


Figure 9: Example derivation tree structure. Top: Shown in the pyramid, the numbers correspond to the depth in the tree. Down: Shown as a derivation tree.

As the name of the algorithms implies only after completely generating the structure of the derivation tree (splitting of the word in subwords) the rules are added to the grammar that help filling the cells occurring in the derivation tree.

Now every time before adding a new rule (Algorithm 8 SplitThenFillRec Line 15) the

already available information regarding the other rules is used to determine if a new rule is needed to fill this node of the derivation tree (Line 12 of Algorithm 8 SplitThenFillRec).

Algorithm 7: SplitThenFill	
Input: Word $w \in \Sigma^*$	
Output: Set of productions P	
1	$P = \emptyset; \quad // \quad P \subseteq V \times (V^2 \cup \Sigma)$
2	$P = \text{Distribute}(\Sigma, V); \quad \textcircled{\text{A}}$
3	$Sol = (P_{Sol}, \text{Cell}_{i_{max},0}); \quad // \quad P_{Sol} \subseteq P \quad \wedge \quad \text{Cell}_{i_{max},0} \in \text{Pyramid}$
4	$Sol = \text{SplitThenFillRec}(P, w, i_{max}, 0);$
5	return $P_{Sol};$
Line 2: Fills the $i=0$ row of the pyramid.	

For this algorithm it is important to mention that while using $\textcircled{\text{B}}$ (Line 15 of Algorithm 8) a variable compound is added to at least one $lhse$. For every element of $vc \in \text{VarComp}$ (Line 14 of Algorithm 8) there exists at least one rule $lhse \rightarrow vc$.

Algorithm 8: SplitThenFillRec	
Input: $P_{in} \subseteq V \times (V^2 \cup \Sigma), \quad w \in \Sigma^*, \quad i, j \in \mathbb{N}$	
Output: $(P, \text{Cell}_{i,j})$	
1	$P = P_{in};$
2	if $i = 0$ then
3	return $(P, \text{Cell}_{i,j});$
4	end
5	<i>choose one m uniform randomly in $[j + 1, j + i];$</i>
6	$(P, \text{Cell}_l) = \text{SplitThenFillRec}(P, w, (m - j - 1), j);$
7	$(P, \text{Cell}_r) = \text{SplitThenFillRec}(P, w, (j + i - m), m);$
8	$\text{Pyramid} = \text{CYK}(G, w);$
9	if <i>stopping criteria met</i> $\textcircled{\text{C}}$ then
10	return $(P, \text{Cell}_{i,j});$
11	end
12	if $\text{Cell}_{i,j} = \emptyset$ then
13	$\text{VarComp} = \text{uniform random subset from } \{vc \mid v \in \text{Cell}_l \wedge$
14	$c \in \text{Cell}_r\}$ <i>with</i> $ \text{VarComp} \geq 1;$
15	$P \cup \text{Distribute}(\text{VarComp}, V); \quad \textcircled{\text{B}}$
16	end
17	return $(P, \text{Cell}_{i,j});$

The same example tree structure as in Figure 2.5 is used in the following example – each number represents the recursion depth of its subtree:

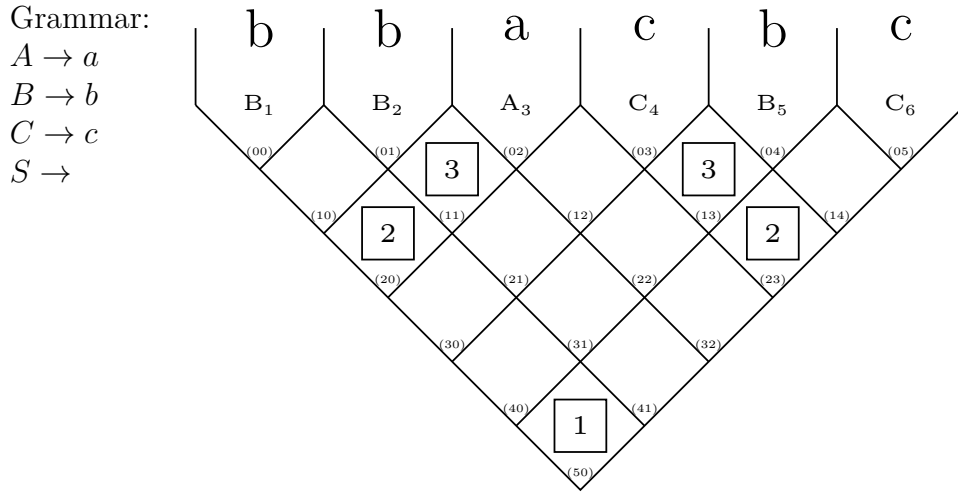


Figure 10: Illustration of Algorithm 7 SplitThenFill part 1 after adding $A \rightarrow a$, $B \rightarrow b$ and $C \rightarrow c$.

After adding the terminals to the grammar (Line 2 in Algorithm 7 SplitThenFill) now the recursion step at $Cell_{1,1}$ is taken on. Now $Cell_l = \{B_2\}$ and $Cell_r = \{A_3\}$ and therefore $VarComp = \{BA\}$. Adding the rule $S \rightarrow BA$ leads to the following *Pyramid*:

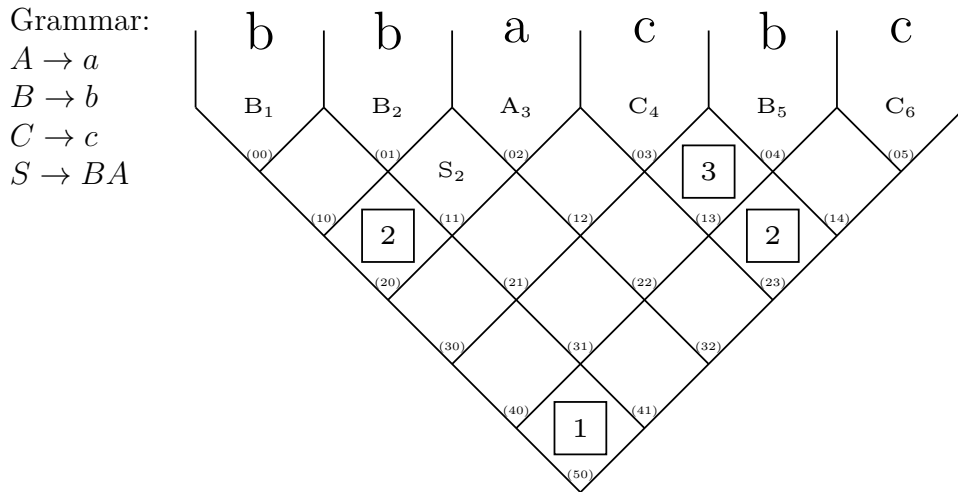


Figure 11: Illustration of Algorithm 7 SplitThenFill part 2 after adding $S \rightarrow BA$.

The next recursion step happens in $Cell_{2,0}$. Now $Cell_l = \{B_1\}$ and $Cell_r = \{S_2\}$. Analogously the rule $A \rightarrow BS$ is added to the grammar:

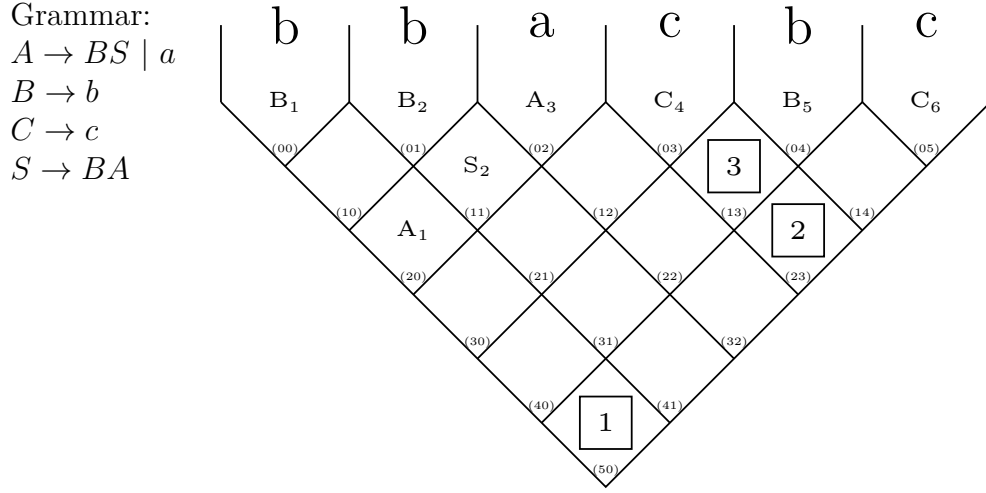


Figure 12: Illustration of Algorithm 7 SplitThenFill part 3 after adding the rule $A \rightarrow BS$.

The next two analogous steps are described in Figure 13 and in Figure 14.

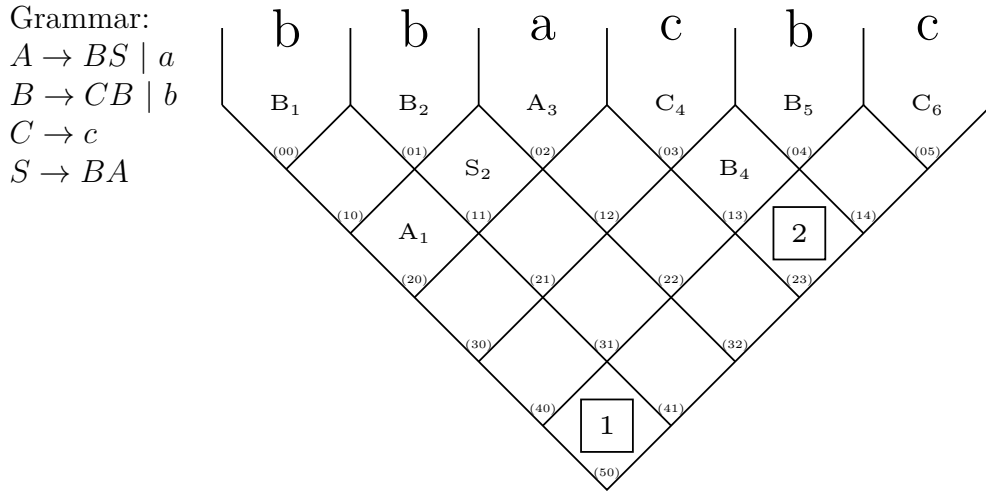


Figure 13: Illustration of Algorithm 7 SplitThenFill part 4. The recursion step in $Cell_{1,3}$ is resolved by adding the rule $B \rightarrow CB$.

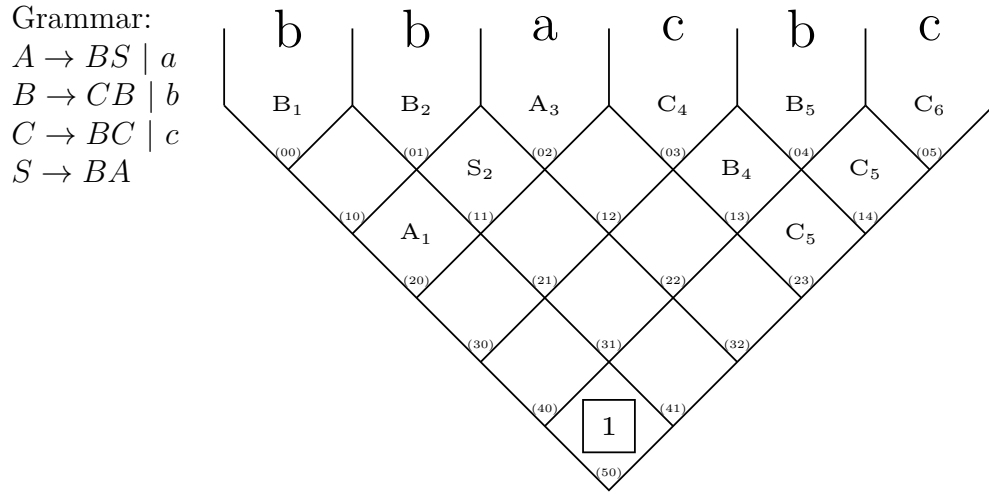


Figure 14: Illustration of Algorithm 7 SplitThenFill part 5. The recursion step in $Cell_{2,3}$ is resolved by adding the rule $C \rightarrow BC$.

Finally the last recursion step that decides on the content of the root cell is shown in Figure 15:

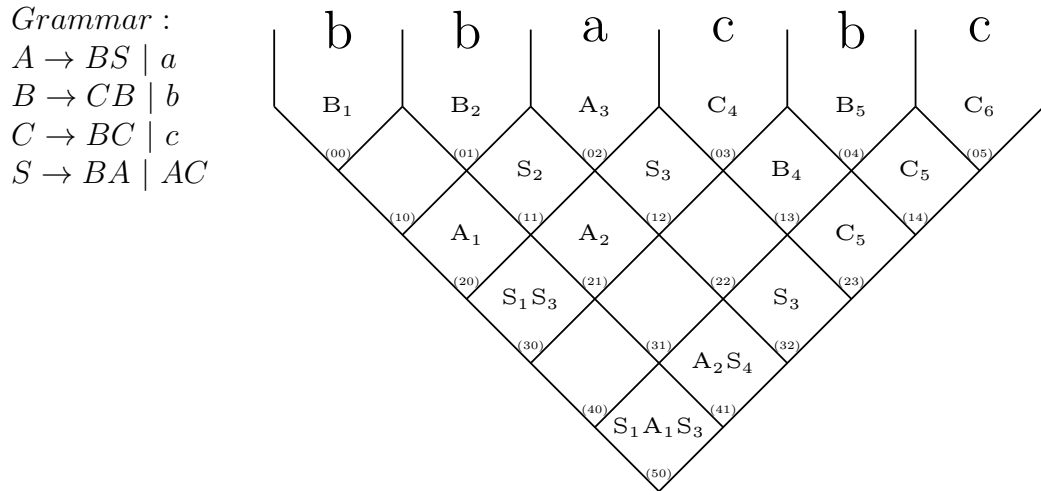


Figure 15: Illustration of Algorithm 7 SplitThenFill part 6. The recursion step in $Cell_{5,0}$ is resolved by adding the rule $S \rightarrow AC$.

2.6 Split Top-Down and fill Top-Down

After defining an algorithm that uses the combination of the Bottom-Up approach and the Top-Down approach (Algorithm 7) a step further is to find an algorithm that only makes use of the Top-Down approach.

This algorithm again generates a predefined structure of the derivation tree Top-Downwards. Every time a node of the structure of the derivation tree has been decided on, a rule is immediately added to the grammar – therefore the name SplitAndFill, which is like "split for a node and then directly add a rule so that the node is then filled with at least one variable".

Note that the count of rules in the grammar is dependent on the count of nodes in the derivation tree and a terminal is distributed to only one variable. While resolving the last recursion step (Line 12) of Algorithm 10 SplitAndFillRec the start variable will be in the root of the pyramid that always leads to $w \in L(G)$.

Algorithm 9: SplitAndFill	
Input: Word $w \in \Sigma^*$	
Output: Set of productions P	
1	$P = \emptyset; \quad // \quad P \subseteq V \times (V^2 \cup \Sigma)$
2	$Sol = (P_{Sol}, v); \quad // \quad P_{Sol} \subseteq P$
3	$Sol = SplitAndFillRec(P, w, i_{max}, 0);$
4	return $P_{Sol};$
Line 2: v can be any random element $v \in V$.	

Algorithm 10: SplitAndFillRec**Input:** $P_{in} \subseteq V \times (V^2 \cup \Sigma)$, $w \in \Sigma^*$, $i, j \in \mathbb{N}$ **Output:** (P, v)

```

1  $P = P_{in}$ ;
2 if  $i = 0$  then
3   if terminal  $w_j$  not distributed yet then
4     return  $(P \cup \{(v, w_j)\}, v_{lhse})$ ;
5   end
6   return  $(P, v_{lhse})$ ;
7 end
8 choose one  $m$  uniform randomly in  $[j + 1, j + i]$ ;
9  $(P, v_l) = \text{SplitAndFillRec}(P, w, (m - j - 1), j)$ ;
10  $(P, v_r) = \text{SplitAndFillRec}(P, w, (j + i - m), m)$ ;
11 if  $i = i_{max}$  then
12   return  $(P \cup \{(S, v_l v_r)\}, S)$ ;
13 end
14 return  $(P \cup \{(v, v_l v_r)\}, v)$ ;

```

Line 4 and Line 6: There is the rule $v_{lhse} \rightarrow w_j$, then v_{lhse} is the variable on the left side of the one rule that has the terminal w_j as its rhse. Line 4 and Line 14: v is a random element $v \in V$.

According to this algorithm, only productions corresponding to the tree structure are added to the grammar. For illustration purposes, the pyramid is also shown to reflect the immediate changes of the added rules to the pyramid in the Figures 16 to 21. Again the predefined derivation tree structure of Figure 2.5 is used.

Grammar:

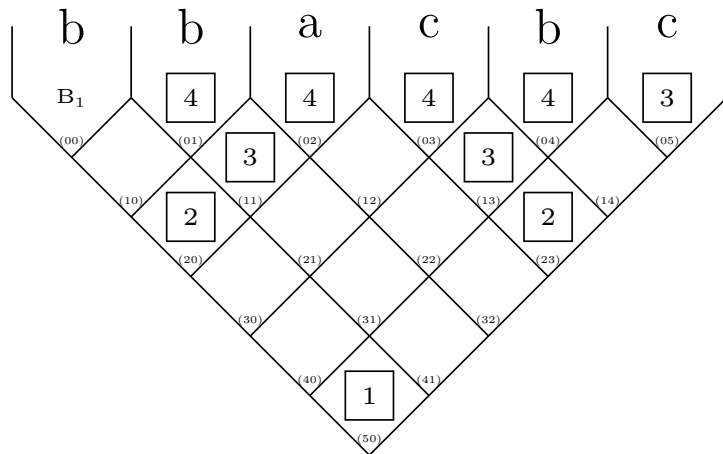
 $A \rightarrow$ $B \rightarrow b$ $C \rightarrow$ $S \rightarrow$ 

Figure 16: Illustration of Algorithm 9 part 1. To resolve the recursion step that fills $Cell_{0,0}$ the rule $B \rightarrow b$ is added.

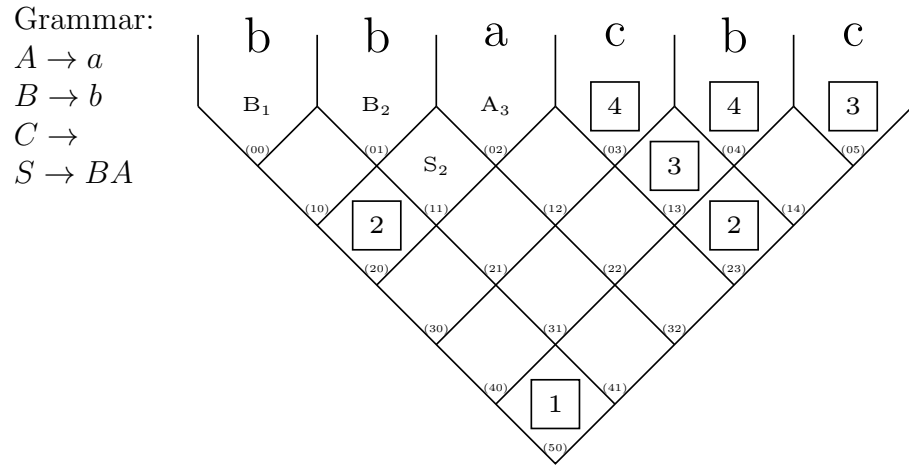


Figure 17: Illustration of Algorithm 9 part 2. Resolving the recursion step that fills $Cell_{0,1}$ no rule is added because a rule $lhse \rightarrow b$ already exists. To fill $Cell_{0,2}$ the rule $A \rightarrow a$ is added. Regarding $Cell_{1,1}$ the rule $S \rightarrow BA$ is added.

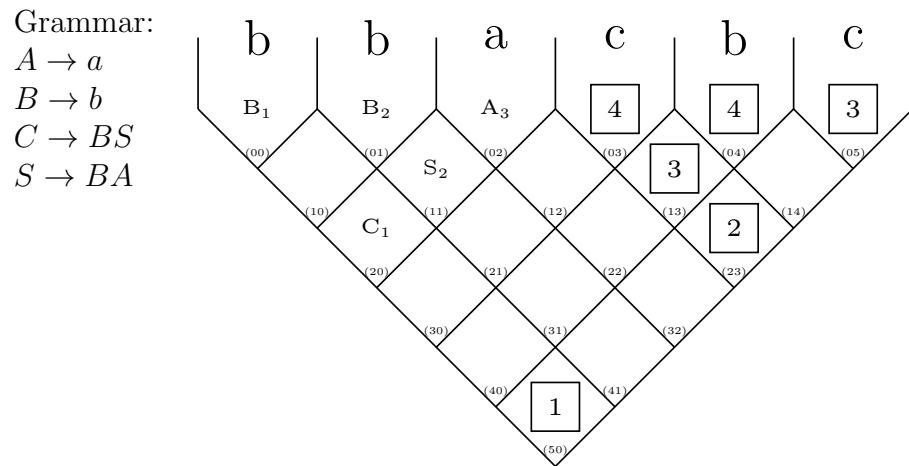


Figure 18: Illustration of Algorithm 9 part 3. Filling the $Cell_{2,0}$ the rule $C \rightarrow BS$ is added.

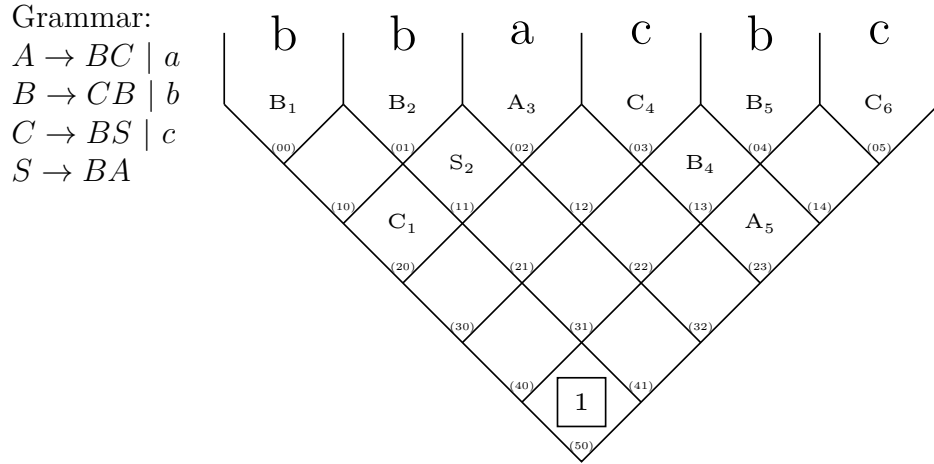


Figure 19: Illustration of Algorithm 9 part 4. Analogously the other cells are filled. $Cell_{0,3}$ is responsible for the rule $C \rightarrow c$, $Cell_{0,4}$ doesn't cause a rule because again there already is the rule $B \rightarrow b$, $Cell_{1,3}$ contributes for the rule $B \rightarrow CB$, $Cell_{0,5}$ does not add a rule because of $C \rightarrow c$ and to fill $Cell_{2,3}$ the rule $A \rightarrow BC$ is added.

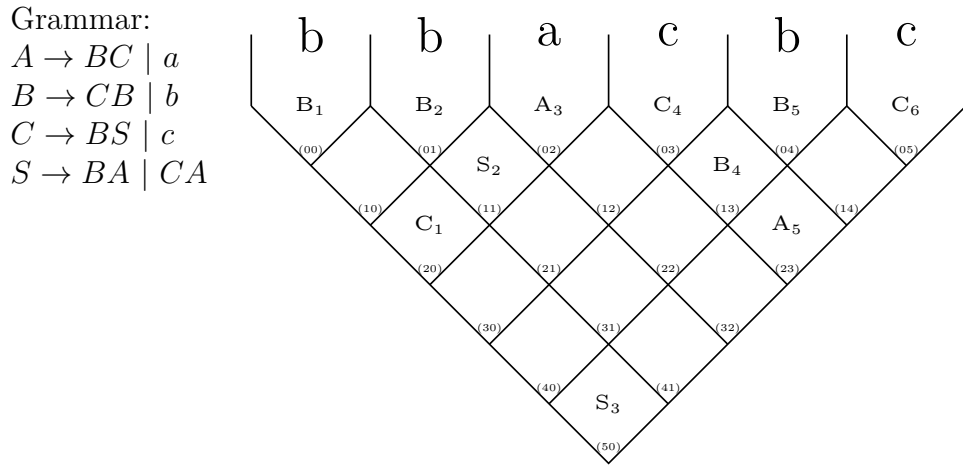


Figure 20: Illustration of Algorithm 9 part 5. Finally, to fill the cell in the root a rule must be added that has the start variable as its *lhse* that guarantees $w \in L(G)$. Here the rule $S \rightarrow CA$ is added.

Grammar:
 $A \rightarrow BC \mid a$
 $B \rightarrow CB \mid b$
 $C \rightarrow BS \mid c$
 $S \rightarrow BA \mid CA$

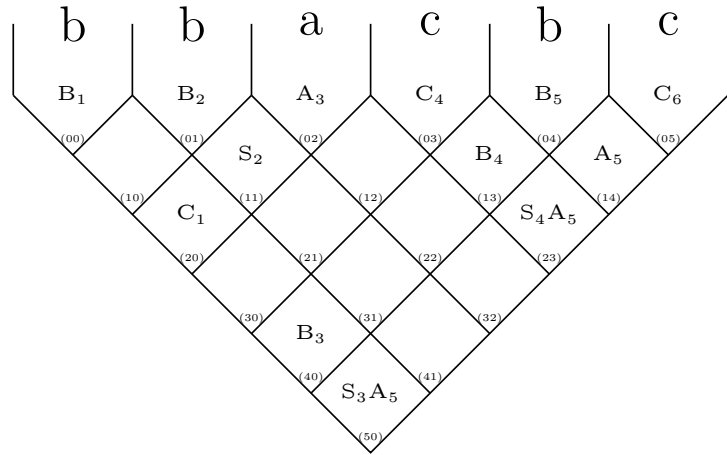


Figure 21: Illustration of Algorithm 9 part 6. With part 5 of the example the algorithm is finished. In comparison to Figure 20 the complete parsing table looks like above.

2.7 Evaluation of Algorithms

2.7.1 Success Rates

Now that different algorithms have been described to generate exam *exercises* it is of interest to compare them. Therefore Success Rates are used on the algorithms according to their performance of the different requirements for an exam *exercise*. Here $N \in \mathbb{N}^+$ is the sample size of all generated grammars of the examined algorithm. Before defining a overall Success Rate (*SR*) three other Success Rates set the basis for it.

Success Rate Producibility: A generated *exercise* contributes to the SR-Producibility iff the CYK algorithm's output (Algorithm 1) is true or in other words $w \in L(G)$.

SR-Producibility = p/N , where p is the count of *exercises* that fulfil the requirement.

Success Rate Cardinality-Rules: A generated *exercise* contributes to the SR-Cardinality-Rules iff the grammar has got less than a certain amount of productions.

SR-Cardinality-Rules = cr/N , where cr is the count of *exercises*.

Success Rate Pyramid: A generated *exercise* contributes to the SR-Pyramid iff the following conditions are met:

1. At least one cell forces to do a correct cell combination.
2. There are less than 100 variables in the entire pyramid.
3. There are less than 3 variables in each cell of the pyramid.

SR-Pyramid = p/N , where p is the count of *exercises* that fulfil the three requirements above. While checking 1., 2. and 3. a simplification of $Cell_{i,j}$ is done:

$Cell_{i,j} \subseteq \{(V, k) \mid k \in \mathbb{N}\} \longrightarrow Cell_{i,j} \subseteq V$, see Figure 22.

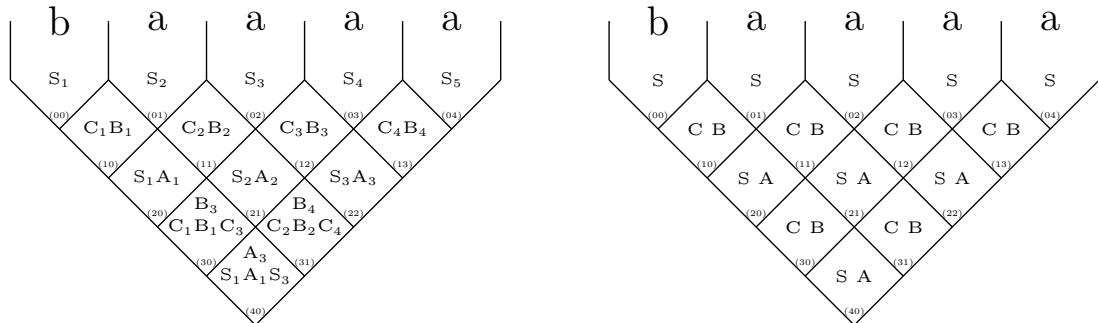


Figure 22: The simplification of cells in a pyramid.

The experience of professor Martens shows that usually most students easily find a pattern of how to fill the first two rows of the *pyramid* during the execution of the CYK algorithm but do more mistakes starting at row $i \geq 2$. The approach of only finding

patterns and not thoroughly understanding the algorithm is countered by Algorithm 11 `CheckForceCombinationPerCell`. Students often don't know what cell combinations need to be considered while filling one specific cell of the pyramid. They simply take the *UpperLeftCell* and the *UpperRightCell* and try to find rules in the grammar that match the resulting compound variables. More detail on how to force a correct cell combination (1.) see Algorithm 11. But note that a right cell combination can only be forced of cells with index $i > 1$.

Algorithm 11: <code>CheckForceCombinationPerCell</code>	
Input: $CellBottom, CellUpperLeft, CellUpperRight \subseteq V, P \subseteq V \times (V^2 \cup \Sigma)$	
Output: $true \iff VarsForcing > 0$	
1 $VarsForcing = \emptyset; \quad // \quad VarsForcing \subseteq V$	
2 $VarComp = \{xy \mid x \in CellUpperLeft \wedge y \in CellUpperRight\};$	
3 foreach $v \in CellDown$ do	
4 $Rhse = \{rhse \mid p \in P \wedge p = (v, rhse)\};$	
5 if $Rhse \not\subseteq VarComp$ then	
6 $VarsForcing = VarsForcing \cup v;$	
7 end	
8 end	
9 return $ VarsForcing > 0;$	
Note: $CellBottom = Cell_{i,j}$, $CellUpperLeft = Cell_{i-1,j}$ and $CellUpperRight = Cell_{i-1,j-1}$ Line 4: Get all rules of P that have v as the <i>lhse</i> and add their <i>rhse</i> to $Rhse$. Line 5: If no $rhse \in Rhse$ can be found in $VarComp$, then this variables forces, concluding that this cell as a hole forces.	

In the example in Figure 23, the variables in $Cell_{2,0}$ and in $Cell_{2,1}$ each force a right cell combination and in both cases $VarComp = \{SS\}$. The variable $v = C$ doesn't have SS as one of its *rhse*s and therefore the variable C forces. $Cell_{3,0}$ doesn't force because $VarComp = \{CC\}$ and the variable $v = S$ has CC as its *rhse*. Note again, that cells with index $i \leq 1$ can't force at all.

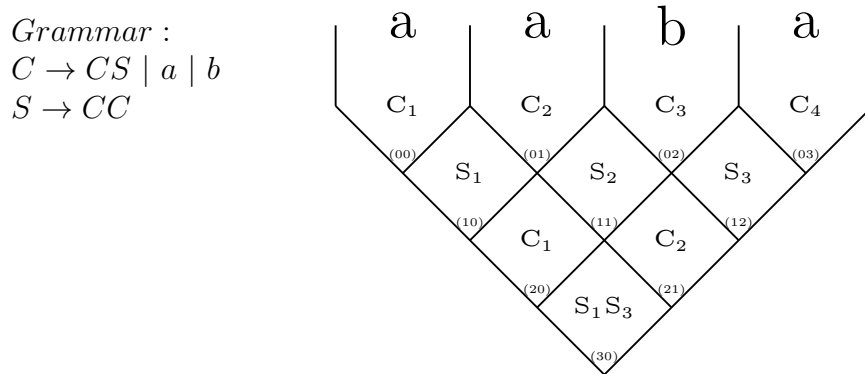


Figure 23: Application of Algorithm 11 onto an entire pyramid.

With the help of the three known Success Rates the overall Success Rate can be specified.

Success Rate: A generated *exercise* contributes to the Success Rate (SR) iff it contributes to the SR-Producibility, to the SR-Cardinality-Rules and to the SR-Pyramid at the same time.

It holds: $SR = n/N$, whereas n is the count of *exercises* that fulfil the requirement above in this case.

2.7.2 Problem space exploration

Now that there is a measure to compare the different algorithms it is of interest for which input values the different algorithms perform the best. Therefore a problem space exploration is done while the following ranges of the parameters are considered:

Input Values of the program:

- count of variables = [2; 8]
- count of terminals = [2; 8]
- size of word = [4; 11]

The resulting best SRs of each algorithm are used to decide which stopping criteria (C) gives more exam suitable *exercises* and finally the algorithms are compared to each other.

2.7.3 Comparison of stopping criteria

For all of the five algorithms the best SRs have been calculated for both stopping criteria and are shown in Table 1.

	MoreThanHalf	RootNotEmpty
DiceRollOnly	17%	18%
DiceRollVar1	18%	35%
DiceRollVar2	20%	37%
SplitThenFill	36%	36%
SplitAndFill	74%	74%

Table 1: Comparison of the two stopping criteria: Half of the cells in the pyramid are not empty and at least one variable is in the root of the pyramid. (N = 1024)

It can be seen that the stopping criteria does not really have a big impact on the SR.
... XXX

2.7.4 Comparison of the algorithms

It is interesting to see which input values of the algorithms are responsible for the good SR. They are shown in Table 2.

	SR	count of variables	count of terminals	size of word
DiceRollOnly	18%	3	3	9
DiceRollVar1	35%	3	3	8
DiceRollVar2	37%	3	3	8
SplitThenFill	36%	3	4	9
SplitAndFill	74%	3	3	8

Table 2: Comparison of the input values of each algorithm for with best SR. (N = 1024)

By comparing the five algorithms in Table 3 in more detail it is ... XXX

Algorithm	SR	Produci- bility	Cardinality- Rules	Pyramid			
					Force- Right	Vars- PerCell	VarsIn- Pyramid
DiceRollOnly	18%	22%	98%	48%	48%	100%	100%
BottomUpVar1	35%	55%	92%	66%	66%	100%	100%
BottomUpVar2	37%	56%	96%	69%	69%	100%	100%
SplitThenFill	36%	50%	93%	75%	76%	100%	99%
SplitAndFill	74%	100%	100%	74%	74%	100%	100%

Table 3: Comparison of the five algorithms with their best SR in more detail. (N = 1024)

3 GUI Tool: CYK Instances Generator

One of the goals of the thesis is to get a tool that assists in creating exam exercises. A Graphical User Interface (GUI) tool is preferred over a Command Line (CL) tool because of the ease of use and the better overview during the exercise creation.

3.1 Overview GUI

The developed tool consists of four major elements as marked in Figure 24.

1. Elementary input values can be given to the programm.
2. The status output of the programm is displayed.
3. Suitable exercises are calculated automatically and one can be selected.
4. The chosen exercise can be modified as wanted.

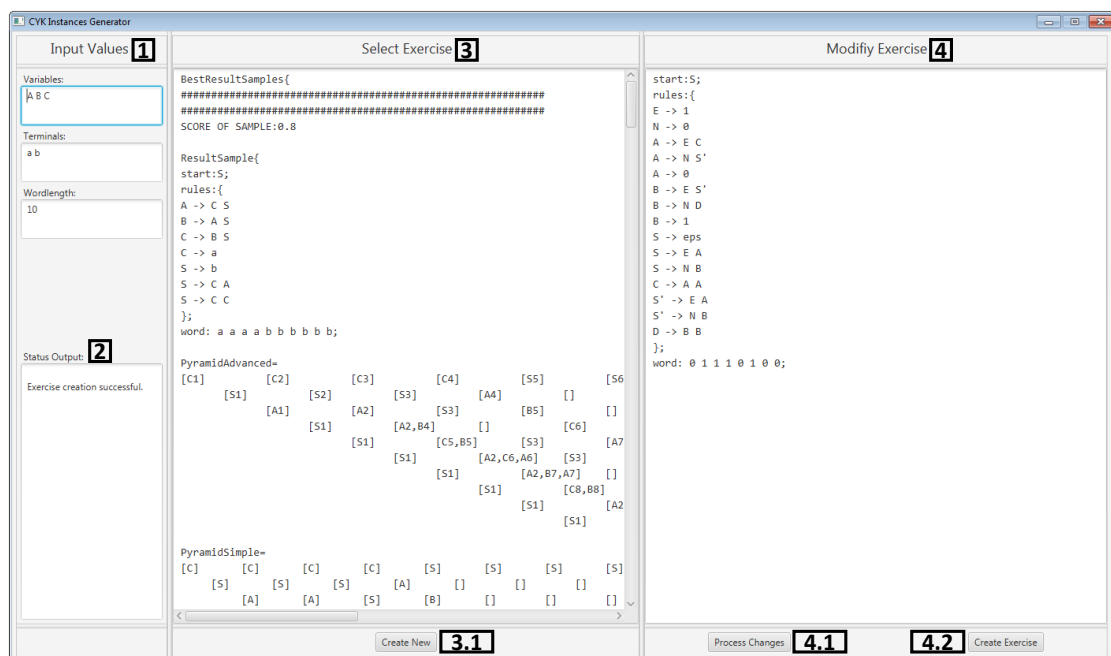


Figure 24: CYK Instances Generator.

Clicking the button 3.1 allows the creation of new suitable exercises with the given input values from area 1. Pressing button 4.1 processes the latest input given in area 4 to create a preview analogously to area 3 of how the created exercise would look like and finally button 4.2 creates the desired *exercise*. The *exercise* is created as a \LaTeX -code-file and a pdf-file. The \LaTeX -file is standalone compilable and allows further modification, whereas the pdf-file shows directly what the *exercise* looks like.

3.1.1 Working with the program

The application structure contains only the executable "bachelor_thesis_cyk.jar"-file and the folder named "exercise". The mechanics of the application is mostly self explana-

tory. Just note that after clicking button 4.2 "Create Exercise" a new "exerciseLatex.tex"-file and the corresponding "exerciseLatex.pdf"-file will be generated within it and any files with the same name are overridden.

3.2 Exam Exercises

A exam *exercise* is a 4-tuple $exercise = (grammar, word, parse\ table, derivation\ tree)$. The pdf-file output of the tool looks similar to this:

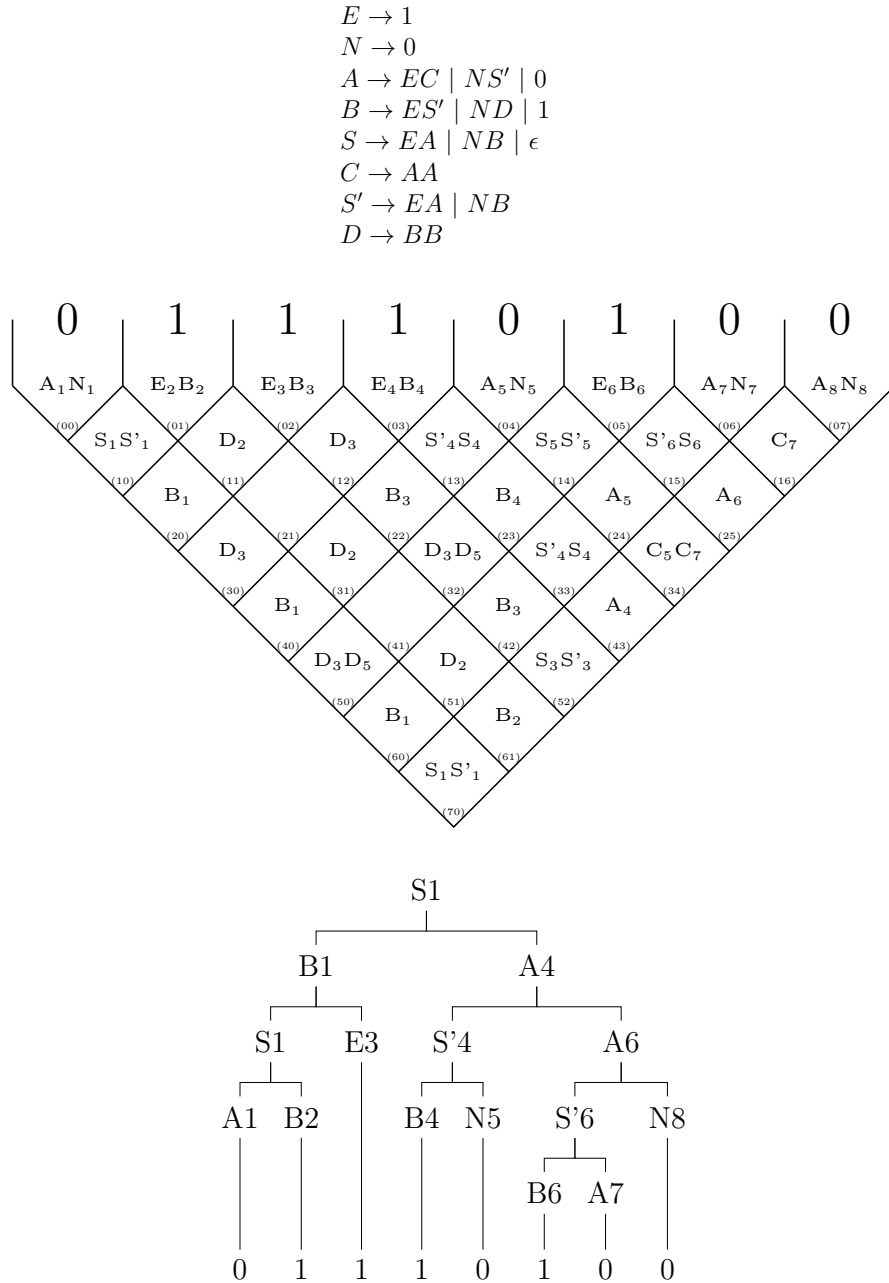


Figure 25: Example output for a *exercise*. Top: Context free grammar. Middle: A *pyramid* filled after the CYK algorithm. Bottom: Exemplary random derivation tree.

3.3 Scoring Model

Not every exercise is actually suitable for an exam. Therefore a scoring model is needed so that suitable exercise can be displayed in area 3 of the tool. Each exercise is given a score according to Table 4 and the parameters that influence the score are:

- `countRightCellCombinationsForced`, i.e. number of times a student is forced to make the right choice to fill the parsing table.
- `sumOfVarsInPyramid`, i.e. all variables in the pyramid.
- `countVarsPerCell`, i.e. maximum count of variables per cell.
- `sumOfRules`, i.e. all rules in the grammar.
- `countUniqueCells`, excluding row $i = 0$.

Parameter	Points					
	2	4	6	8	10	-100
<code>#cellCombinationsForced</code>	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
<code>sumVarsInPyramid</code>	[0,10]	[11,20]	[21,30]	[41,50]	[31,40]	>50
<code>#VarsPerCell</code>	[5,5]	[4,4]	[1,1]	[3,3]	[2,2]	>5
<code>sumOfRules</code>	[1,2]	[3,4]	[5,6]	[9,10]	[7,8]	>10
<code>countUniqueCells</code>	[3,3]	[4,4]	[5,5]	[6,6]	[7,7]	≤2

Table 4: Scoring of the different parameter values.

The score of each *exercise* is normalized to the maximum possible points so the maximum score is 1.0: $score = (\#Parameter \cdot 10)^{-1} \cdot \sum_{parameter} points$.

One negative score is already sufficient to avoid examples in area 3 with undesired properties.

3.4 Parsing input with ANTLR

The first step here is the tokenization of the input. After that with the help of the Grammar seen below a abstract syntax tree is generated out of which intern Java objects can be parsed.

The used grammar is a LL(k) grammar whereas each derivation step can be distinctly identified through the next k tokens.

ANTLR has been used because it enables a clear separation between the language definition and the object handling in the code.

In Figure 26 the rules of the grammar are seen and in Figure 27 its used tokens.

```
START: ('start');
RULES: ('rules');
ARROW: ('->');
WORD: ('word');

UPPERCASE: ('A'..'Z');
LOWER_CASE_OR_NUM: ('a'..'z' | '0'..'9');

OPEN_BRACE: '(';
CLOSE_BRACE: ')';
OPEN_BRACE_CURLY: '{';
CLOSE_BRACE_CURLY: '}';

SEMICOLON : ';';
COLON: (':');
WHITE_SPACE: ' ' | '\t';
NEWLINE: '\n';

SPECIALSYMBOL: ('\\');
```

Figure 26: Formal definition of the used ANTLR grammar tokens.

```

grammar Exercise;

exerciseDefinition: grammarDefinition NEWLINE
                    wordDefinition NEWLINE?;

grammarDefinition: NEWLINE* WHITE_SPACE* varStart WHITE_SPACE* NEWLINE
                    rules;

varStart: START COLON WHITE_SPACE* nonTerminal SEMICOLON;

rules: RULES COLON WHITE_SPACE* OPEN_BRACE_CURLY NEWLINE
        (singleRule NEWLINE)+
        CLOSE_BRACE_CURLY SEMICOLON;

singleRule: WHITE_SPACE* nonTerminal // A
            WHITE_SPACE* ARROW WHITE_SPACE* // ->
            terminal WHITE_SPACE* // a
            |
            WHITE_SPACE* nonTerminal // A
            WHITE_SPACE* ARROW WHITE_SPACE* // ->
            nonTerminal WHITE_SPACE+ nonTerminal WHITE_SPACE*;

wordDefinition: WORD COLON WHITE_SPACE* terminals WHITE_SPACE* SEMICOLON;

terminals: terminal
            |
            terminal WHITE_SPACE terminals;

nonTerminal: UPPERCASE+ SPECIALSYMBOL?;
terminal: LOWER_CASE_OR_NUM+;

```

Figure 27: Formal definition of the used ANTLR grammar rules.

3.5 Other matters

Technologies that have been used for programming are Github ⁴ with Sourcetree ⁵ for version control, Maven ⁶ for build management, IntelliJ ⁷ as the IDE, ANTLR ⁸ with ANTLRWorks for parsing input and JavaFX Scene Builder ⁹ to create the GUI.

Important used frameworks are: JUnit ¹⁰ for testing and Project Lombok ¹¹ to greatly reduce boilerplate code.

⁴Github: <https://github.com/>

⁵Sourcetree: <https://www.sourcetreeapp.com/>

⁶Maven: <https://maven.apache.org/>

⁷IntelliJ: <https://www.jetbrains.com/idea/>

⁸ANTLR: <http://www.antlr.org/>

⁹JavaFX Scene Builder: <http://gluonhq.com/products/scene-builder/>

¹⁰JUnit: <http://junit.org/junit4/>

¹¹Project Lombok: <https://projectlombok.org/features/all>

Altogether around 7100 lines of code have been written, of which 5400 are pure java code, 900 are comment lines and 800 are blank lines.

Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die Bachelorarbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt und die aus fremden Quellen direkt oder indirekt übernommenen Gedanken als solche kenntlich gemacht habe. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und noch nicht veröffentlicht.

Ort, Datum

Unterschrift

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