Chapter 1: Exercises

Exercise 1: caches

Given: a L_1 cache of 32 Kbyte, 8-way set associative, cache line size of 64 Byte. Consider a matrix-vector multiplication A * x (algorithm A) of a **256 x 256** matrix using double precision elements. The matrix is stored in column-major format. The matrix is not preloaded in cache memory.

What is the L_1 cache read miss rate + explain.

Exercise 2: caches

Given: a L_1 cache of 32 Kbyte, 8-way set associative, cache line size of 64 Byte. Consider a matrix-vector multiplication A * x (algorithm A) of a **256 x 255** matrix using double precision elements. The matrix is stored in column-major format. The matrix is not preloaded in cache memory.

What is the L_1 cache read miss rate + explain.

Exercise 3: caches

Given: a L_1 cache of 32 Kbyte, 8-way set associative, cache line size of 64 Byte. Consider a matrix-vector multiplication A * x (algorithm A) of a **255 x 256** matrix using double precision elements. The matrix is stored in column-major format. The matrix is not preloaded in cache memory.

What is the L_1 cache read miss rate + explain.

Exercise 4: caches

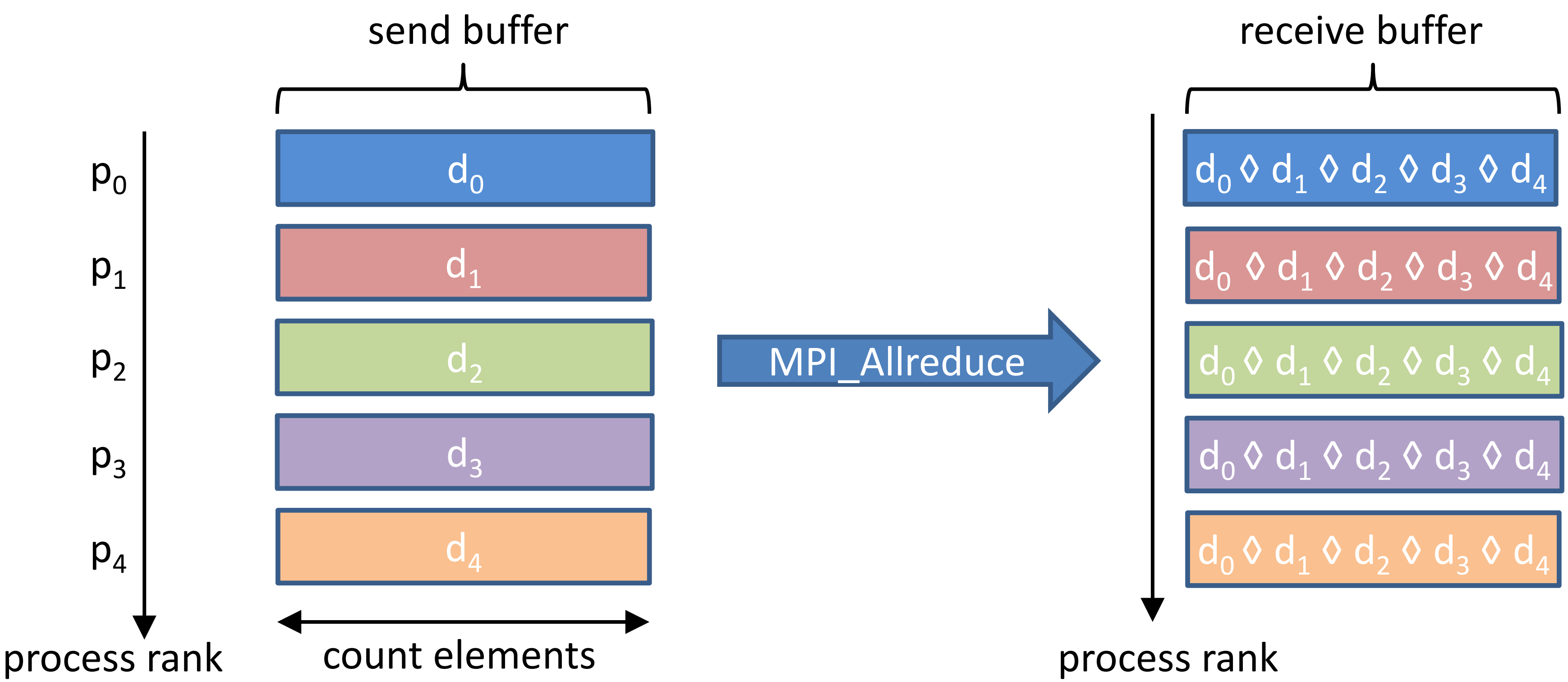
Given: a L_1 cache of 32 Kbyte, 8-way set associative, cache line size of 64 Byte. Consider a matrix-vector multiplication A * x (algorithm A) of a **64 x 64** matrix using double precision elements. The matrix is stored in column-major format. The matrix is not preloaded in cache memory.

What is the L_1 cache read miss rate + explain.

Chapter 2: Exercises

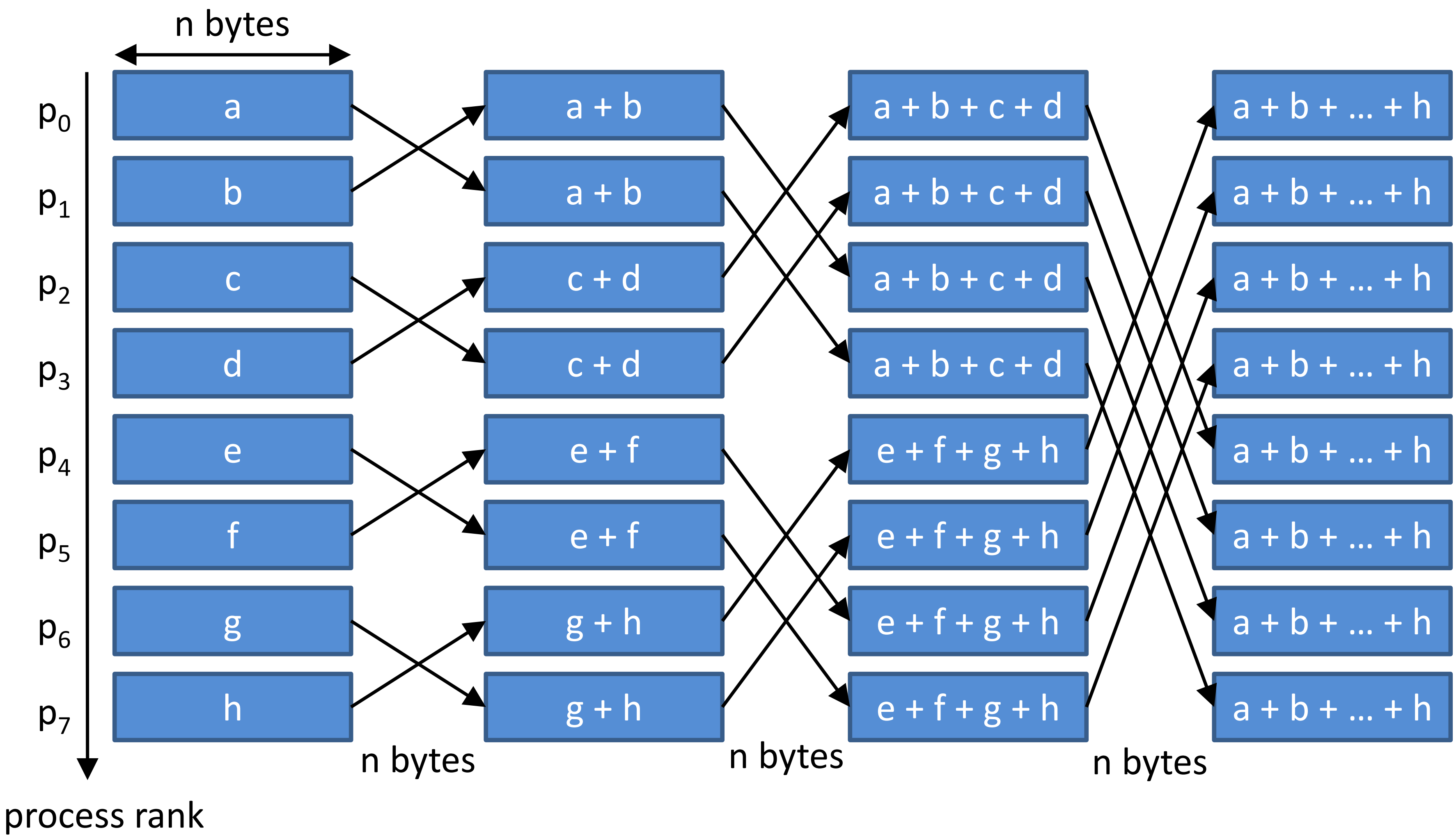
Allreduce algorithm

```
MPI_Allreduce(void *sendbuf, void *recvbuf, int count,
              MPI_Datatype dataType, MPI_Op op, MPI_Comm comm)
```



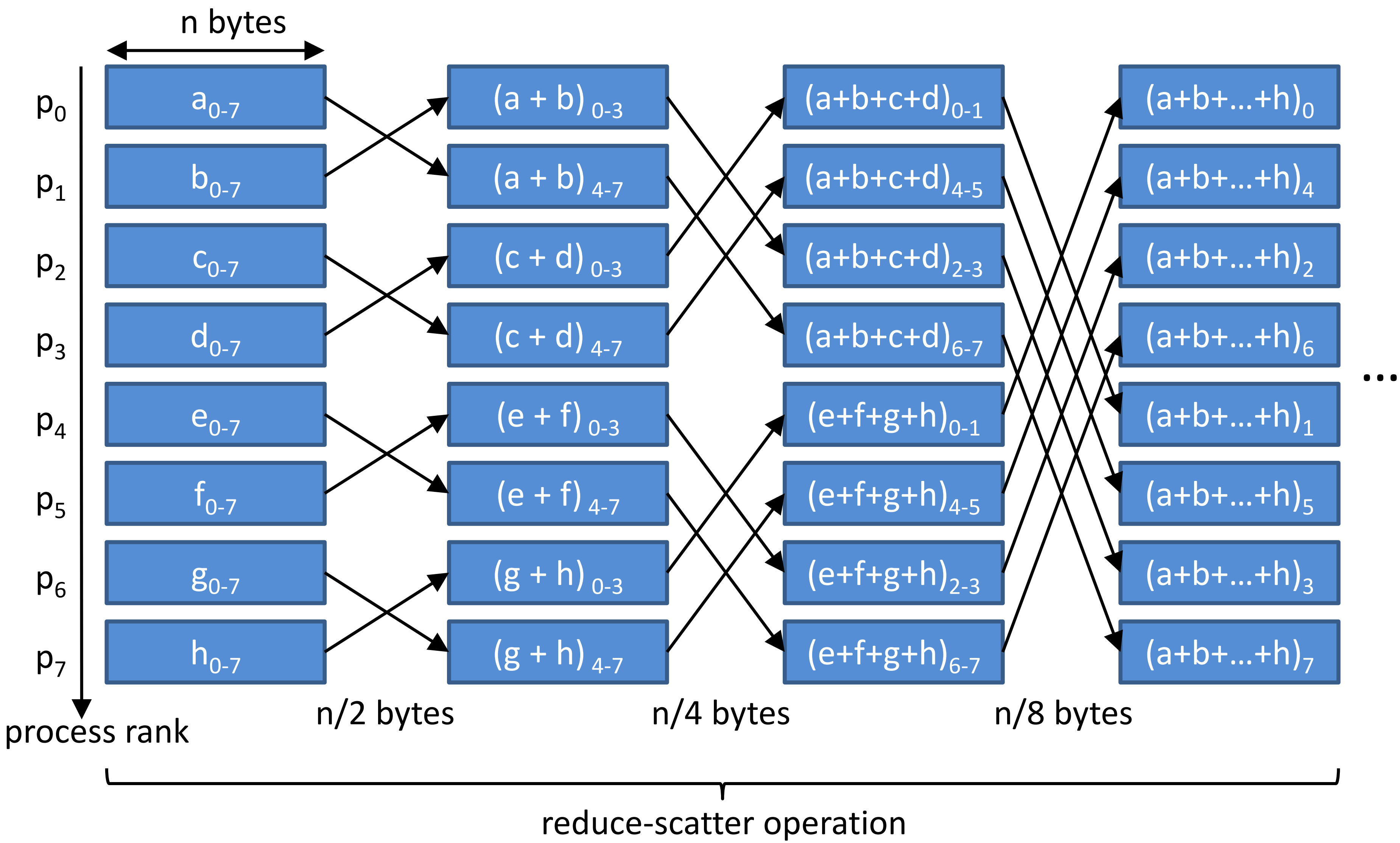
Q: Develop an algorithm for the allreduce routine. Derive the time complexity for that algorithm. Assume a fully non-blocking (duplex) network.

Solution 1: butterfly communication scheme



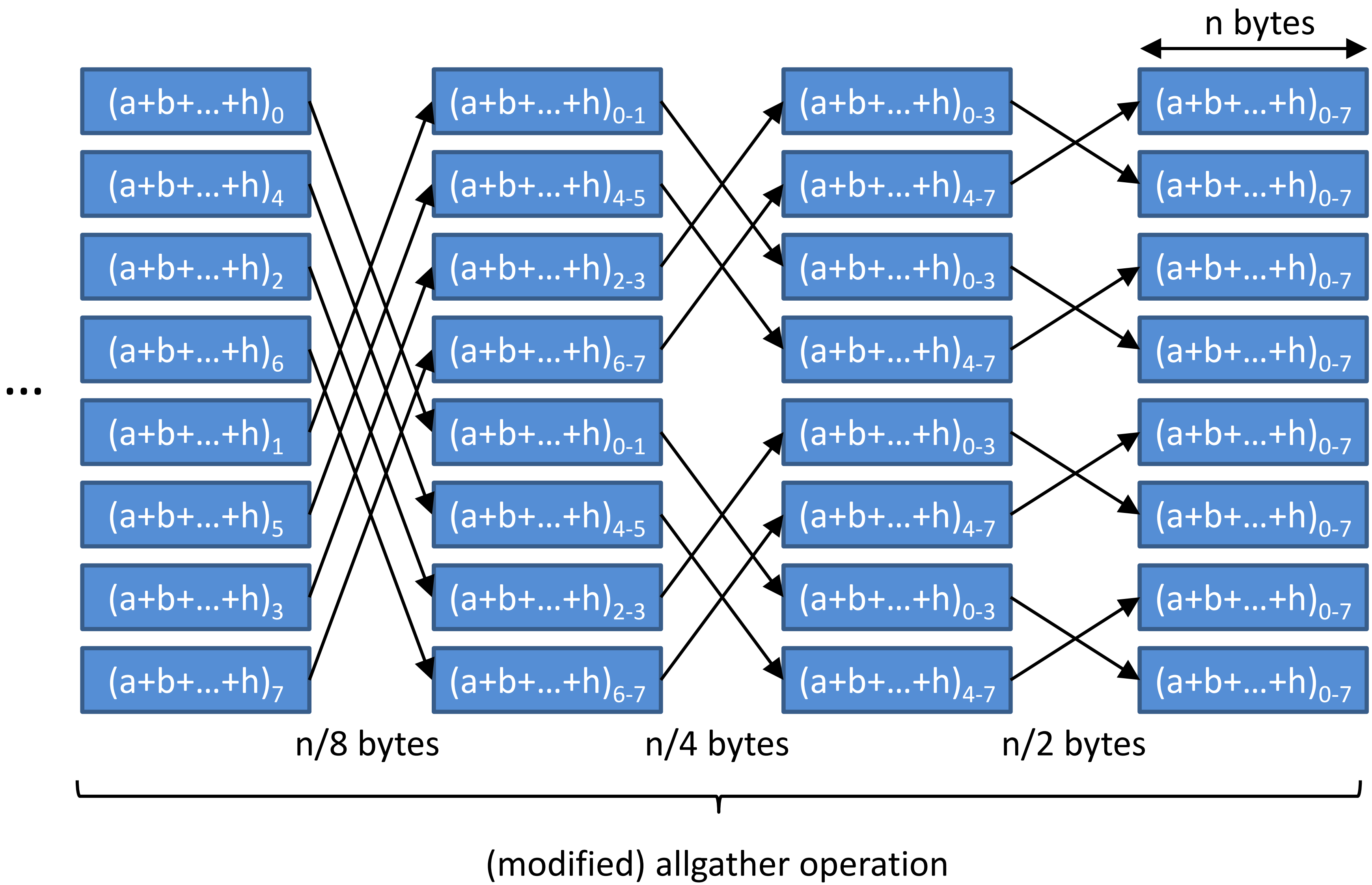
$$T_{sol1} = (\alpha + \beta n + \gamma n) \log_2 P \quad \text{(assume } P = 2^i\text{)}$$

Solution 2: reduce-scatter + allgather



$$T_{sol2} = \alpha \log_2 P + (\beta n + \gamma n)(1-1/P) + \ldots$$

Solution 2: reduce-scatter + allgather (cont'd)



$$T_{sol2} = 2\alpha \log_2 P + (2\beta n + \gamma n)(1-1/P) \qquad \text{(assume } P = 2^i\text{)}$$

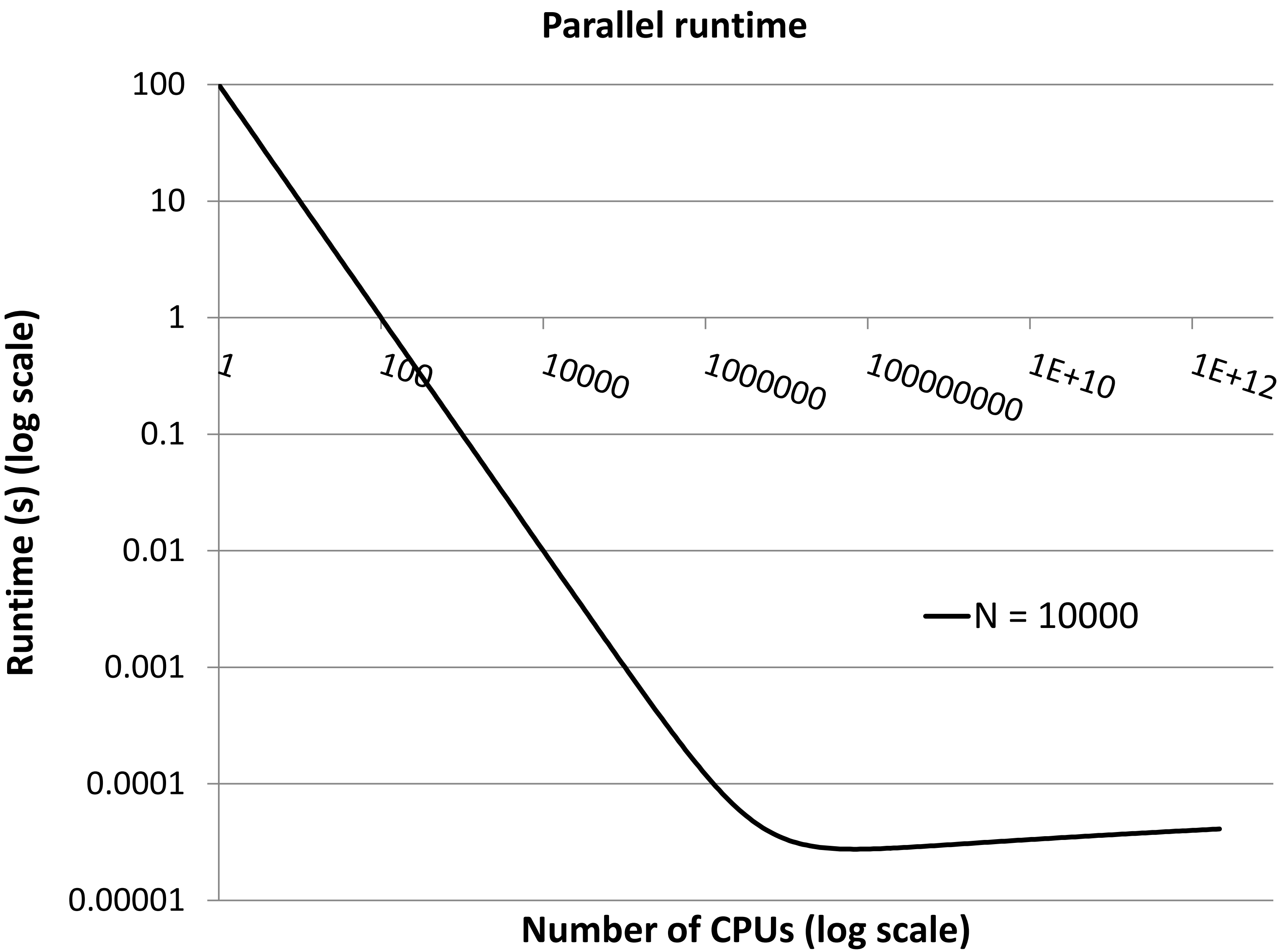
Exercise 2: Sendrecv

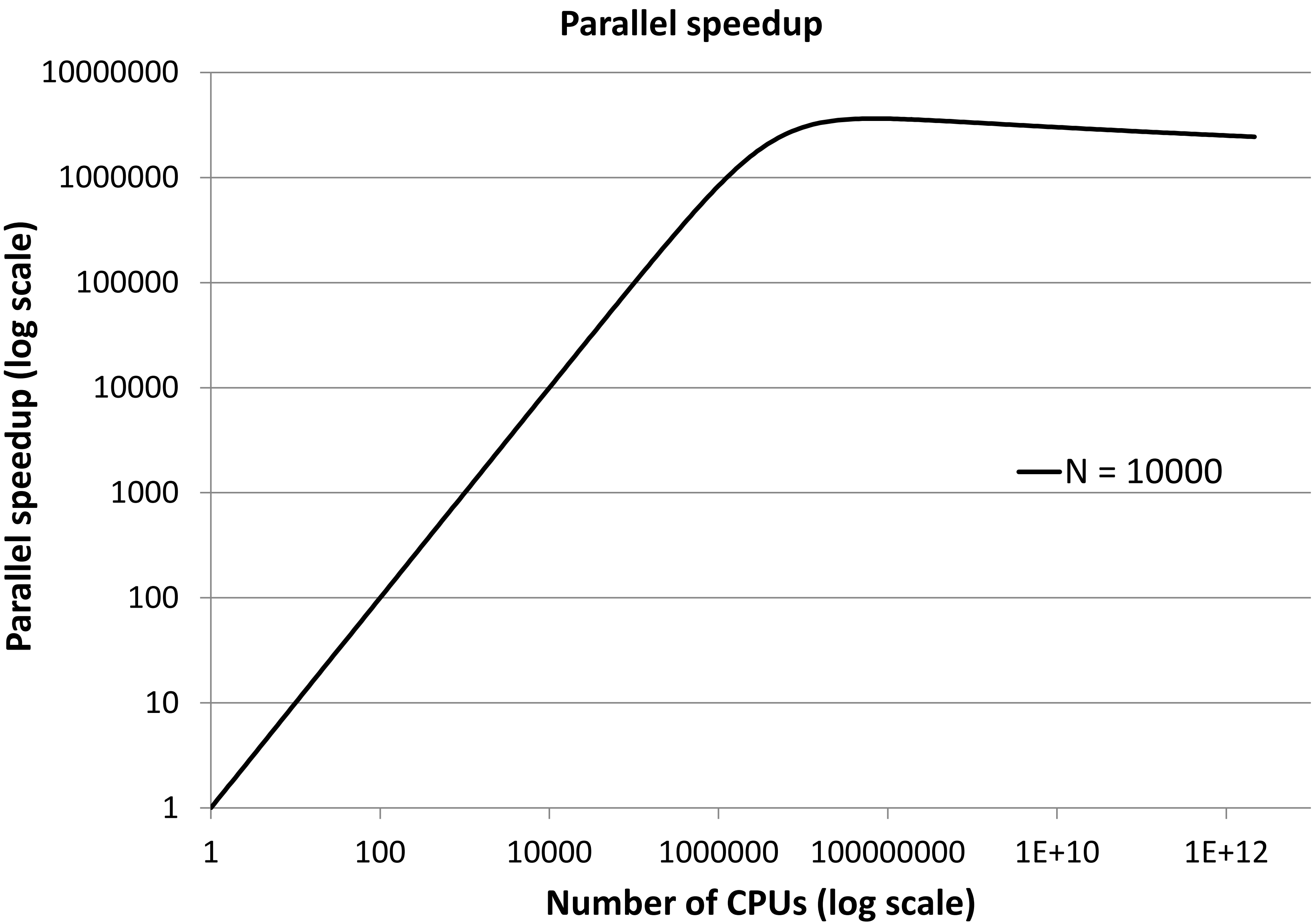
How would you implement **MPI_Sendrecv** so that it will never deadlock? What other benefits might you expect compared to using a blocking send / receive ?

Exercise 3: Speedup and efficiency

Given: $T_1(N) = N^2$; $T_P(N) = N^2/P + \log_2 P$

- What is the speedup ?
- What is the efficiency ?
- For fixed N, what is the limit for large P ?
- For fixed P, what is the limit for large N ?


Parallel runtime
100
10
1
0.1
0.01
0.001
0.0001
0.00001
Runtime (s) (log scale)
1
100
10000
1000000
100000000
1E+10
1E+12
N = 10000
Number of CPUs (log scale)


Parallel speedup
10000000
1000000
100000
10000
1000
100
10
1
Parallel speedup (log scale)
N = 10000
1
100
10000
1000000
100000000
1E+10
1E+12
Number of CPUs (log scale)

Exercise 4: Isoefficiency

Given: **$T_1(N) = N$; $T_P(N) = N/P + \log_2 P$**

If we increase the P by a factor of k, by what factor do we need to increase N in order to maintain constant efficiency?

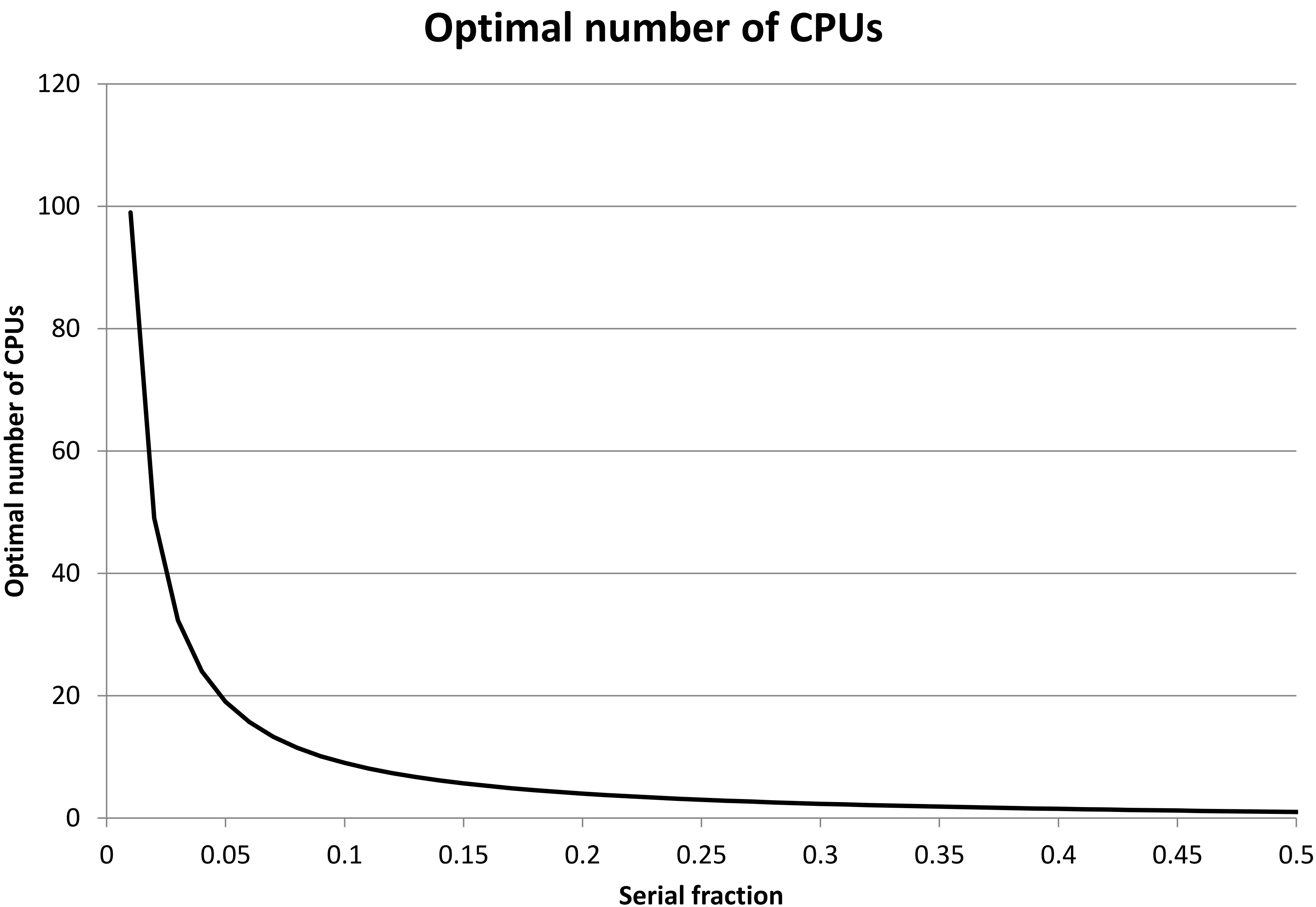
Exercise 5: Amdahl's law

The financial cost C_f for renting compute power on a cluster is proportional to the product of the wall clock time T_P and the number of CPU cores allocated, i.e. **$C_f = P \,.\, T_P$**

Derive an expression for the optimum number of CPUs (P_{opt}) that minimizes the cost function **$C = C_f \,.\, T_P$**
Assume that T_P can be expressed by Amdahl's law.

What is the optimum P for s = 1% and s = 5%?

What about efficiency?

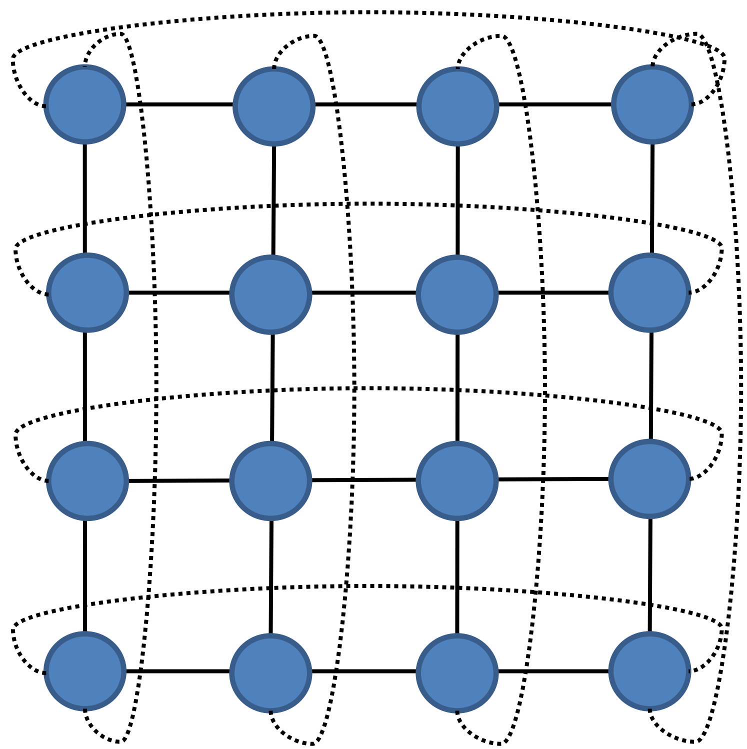

Optimal number of CPUs
120
100
80
60
40
20
0
Optimal number of CPUs
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Serial fraction

Exercise 6: Message Passing Interface

Write an MPI program that outputs data
(e.g. writing data to disc) in rank order (pseudocode)

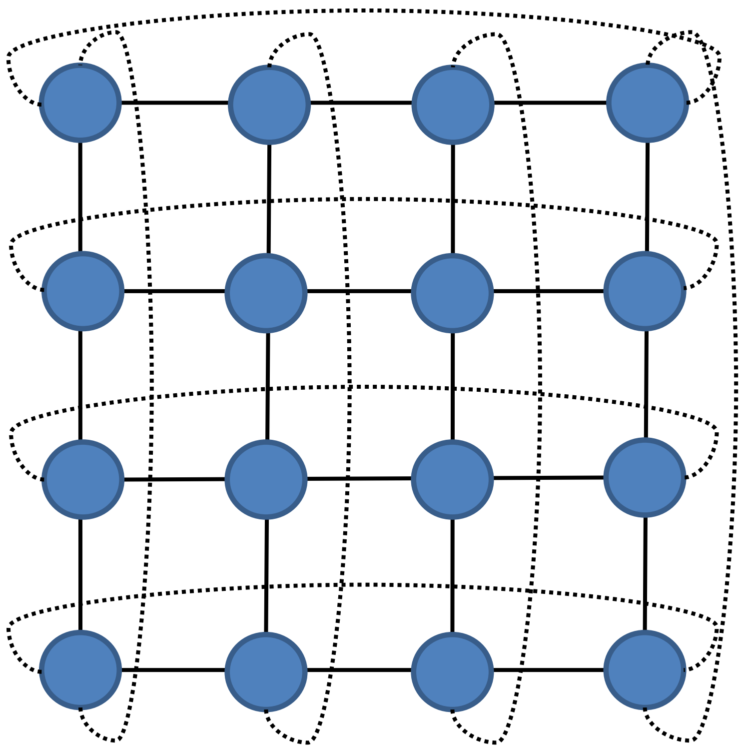
Exercise 7: Mesh networks

Given a square mesh network organized as √P x √P machines
Each machine can communicate to one of its four neighbors



What is the bisection bandwidth? What is the diameter?

Exercise 8: Mesh networks



Same mesh network. Derive a broadcast algorithm and derive time complexity.