Asset Allocation 2025 Case Study

The fictive pension fund *Best Pensions* is a European pension provider. The plan offers a series of target date funds (TDFs), each fund guaranteeing a minimum investment return of 0%, that is, clients are guaranteed at least to get their money back. Each customer of *Best Pensions* can therefore target a minimum retirement cashflow by allocating their contributions across TDFs to engineer a pension to their preferences.

At any point in time, each TDF holds an Active Portfolio and a Reserve Portfolio. The reserve portfolio is a portfolio of zero-coupon bonds maturing at the expiration date of the TDF. Over time, the TDF will buy more ZC bonds but always with the same maturity. The active portfolio is invested 100% in European Equity.

At time *t*, when a contribution, c_t, is added to a TDF expiring at time T*, Best Pensions* buys more ZC bonds and more equity proportional to the current market values, MVA_t and MVR_t, of the Active and the Reserve portfolios, respectively, hence, $c_t^R = c_t MVR_t/(MVA_t + MVR_t)$ is invested in (more) ZC bonds and $c_t^A = c_t MVA_t/(MVA_t + MVR_t)$ is invested in (more) stocks

From clients' perspectives, the allocation, c_t^R, to the Reserve Portfolio ensures an additional minimum income at expiry of the TDF of N_t. The minimum *payout* at expiry therefore is $\sum_{t<T} N_t$ plus the active portfolio, MVA_T, which possibly is zero but most likely is not.

In fact, the ratio MVA_t/MVR_t tends to drift upwards over time whereby the guaranteed component is diluted. Therefore, if the funded ratio, $L_t = (MVR_t + MVA_t)/MVR_t$, exceeds a trigger level, $L_{trigger} = 130\%$, then the TDF is internally rebalanced to a target funded ratio, $L_{target} = 125\%$, by buying more ZC bonds and selling equity, hence, increasing the guaranteed payment to clients. We will refer to this mechanism as *tie-in*. At the inception of new TDFs, *Best Pensions* splits the inaugural contributions 80/20 to the Reserve Portfolio and Active Portfolio, respectively, to start the TDF at L_{target}.

Lately, *Best Pensions* has been heavily criticized for the guarantee: Critics have claimed that the guarantee leads to (too) low allocations to equity resulting in inferior pensions compared to other pension designs. The Executive Board therefore decided to form an Expert Commission to evaluate the current investment strategy and provide advice to the Executive Board as to changes to the investment strategy.

The Executive Board believes strongly in the value of guarantees in pensions and is not willing to materially deviate from the current product design. Furthermore, Best Pensions cannot walk away from the current stock of guarantees so any change to investment strategy must honour existing guarantees.

The Executive Board is aware, though, that the management of the Active Portfolio is rather simplistic and is therefore open to consider a more modern factor approach. In addition, the Executive board is open to changes to the rebalancing mechanism between Active and Reserve portfolios if this improves the outcome to clients but will not accept neither shorting nor leverage in either portfolio.

The Executive Board is keen on the advice to be honest and impartial and therefore decided to hire your team – a group of younger academics and investment experts – all of you with an established reputation of creativity and objectivity.

Request for Advice

The Executive Board requires you to structure your advice according to the following topics:

1) Factor Design

The Executive Board is aware of the progress in factor allocation and is particularly interested in the momentum factor, MOM, and potentially also in the size factor, SMB, but it is sceptical if the promise holds for a EUR-based investor. Furthermore, the original analysis could be outdated, so the Executive Board wants an updated analysis on the existence of MOM and SMB.

a) Based on the methodology developed in FF93, present a comparative analysis of the market factor, MKT, the momentum factor, MOM, and the size factor, SMB, for the period September 2004 for December 2024 of the following three cases:
   i) A USD- based investor investing in US Equity.
   ii) A EUR-based investor investing in US Equity (unhedged).
   iii) A EUR-based investor investing in European Equity

b) Based on your analysis, discuss which of the six factors are relevant to a EUR-based investor (cases ii and iii) and present your preliminary conclusion on which factors are relevant to Best Pensions.
   i) Summarize your recommendation by illustrating the expected (surplus) return and co-variance matrix of the recommended factor universe.

c) MOM and SMB are long vs short portfolios, but the Executive Board has decided not to allow shorting in the Active Portfolio (nor the Reserve Portfolio).
   i) Revise you recommendation in 1b) if only allowed to invest in the 'long' portfolios of MOM and SMB, respectively. We will refer to 'long' MOM as 'Tech Stocks' and 'long' SMB as 'Small Cap' (see hints).
   ii) Summarize your recommendation by illustrating the expected (surplus) return and co-variance matrix of the recommended investment universe.

d) Based on your recommendation 1c), perform an out-of-sample analysis of historical returns of
   i) A rolling mean-variance optimized allocation where expected surplus return and the covariance matrix is based on the previous 3 years.
      (1) First, discuss the minimum variance and the maximum expected return portfolios.
      (2) Extend your analysis to the full efficient frontier and conclude which point is best suited for Best Pensions.
   ii) A rolling risk-parity optimized allocation where the volatility estimate of each factor is calculated from the previous 3 years.

   In either case, portfolios should be fully invested and shorting is not allowed.

e) Based on your findings in 1d) which investment strategy do you propose for the Active Portfolio?

f) Compare your findings in 1b) and 1e) and discuss (and quantify) the cost of not allowing shorting in the Active Portfolio. Take into consideration how to size allocation to MOM and SMB (long/short portfolios requires no cash). You are allowed to allocate to cash (RF) but borrowing is not allowed.

2) CPPI & Tie-In

The criticism of *Best Pensions* is that the plan holds too little equity leading to inferior pension outcomes. Within the current design, the Executive Board expects the allocation to equity will increase on average, if the Active Portfolio is allowed to follow a Constant Proportion Portfolio Insurance (CPPI) strategy and/or if the trigger and target levels to tie-in are modified. You are therefore asked to perform a historical analysis of the accumulation of guaranteed payout of a hypothetical 10-year TDF.

a) Present a baseline analysis of the empirical distribution of the accrual of guarantee in the current system.
b) Based on your advice in the previous sections compare the empirical distribution of guaranteed payout for a 10Y TDF over the data period for different CPPI multipliers for the current levels of tie-in if the Active Portfolio is not allowed to be leveraged.
    i) Expand your analysis for different levels to trigger tie-in (currently 130%) and after tie-in (currently 120%) levels and present your results.
c) From the historical analysis, which CPPI multiplier (*m*), trigger and target levels do you recommend for a revised product design of *Best Pensions* to provide the best pension outcomes.
    i) You are allowed also to revise your portfolio advice 1e) for the Active Portfolio.
d) The historical analysis is heavily influenced by *Quantitative Easening (QE)*, a prolonged period in which interest rates were kept unnaturally low by the actions of the European Central Bank. Looking forward – and assuming *QE* was an outlier – would you maintain your advice of 2c)? If so, argue why you maintain your advice. If not, argue why not and modify your advice above accordingly. You are expected to base your response on both quantitative as well as qualitative arguments.

Data Sources

In your report, you are allowed to use the following data (ONLY!):

Kenneth R French Data Library:

- Monthly US Stock Market Sorts (USD)
    - Fama/French 3 Factors
    - 6 Portfolios Formed on Size and Momentum (2x3)
    - 25 Portfolios Formed on Size and Momentum (5 x 5)
- Monthly European Stock Market Sorts (Notice, in USD!)
    - 6 European Portfolios Formed on Size and Momentum (2 x 3)
    - 25 European Portfolios Formed on Size and Momentum (5 x 5)

European Central Bank:

- Foreign Exchange
    - Time series
- Yield Curves
    - All Years - euro area (very large file – consider using the smaller Current year – euro area to develop your code)
    - Yield Curve Technical Note (Svensson yield curve formula)
        - For the zero-coupon rate, use the formula for the spot rate *z(TTM)*.
        - Use continuous compounding for discount factors, $disc(TTM) = e^{-TTM \cdot z(TTM)}$, cf. footnote 5.
- The first observation of yield curves is 6 September 2004. We will treat this first observation as 30 August 2004 for an added month of observations.

It is advisable to read up on the details of each dataset.

General Assumptions

- A month is counted from the last observation the previous month to last observation current month
    - For example, October 2004 spans 30 September 2004 for 29 October 2004.
- A month is always 1/12 year regardless of the actual observation period.

Hints

1) Factor Design

a) Data

i) US Stock Market

(1) Risk-Free Rate (USD RF): Use 'Fama/French 3 Factors' (column RF)

(2) Portfolio sorts: Use '6 Portfolios Formed on Size and Momentum (2 x 3)' and '25 Portfolios Formed on Size and Momentum (5 x 5)'

(3) Calculate (entire) stock market return from '6 Portfolios Formed on Size and Momentum' by appropriately weighting 'Average Value Weighted Returns – Monthly' by 'Number of Firms in Portfolio' and 'Average Market Cap.

ii) Europe Stock Market

(1) EUR/USD exchange rates: Use 'Time Series' column USD.

(a) Calculate monthly returns of holding USD if you an EUR-based investor. The file provides the price of 1€ in USD but Best Pensions is EUR-based … think!

(2) EUR zero-coupon (ZC) rates: Read Svensson parameters $(BETA0, BETA1, BETA2, BETA3, TAU1, TAU2)$ from 'All Years – euro area' and use formula *z(TTM)* in 'Yield Curve Technical Note' to calculate zero-coupon rates.

(3) Risk-Free Rate (EUR RF): Use 1-month (1/12 year) zero-coupon rate, $ZC_{1/12} = z\left(\frac{1}{12}\right)$.

(a) Convert to period return using the formula $RF = e^{ZC/12} - 1$ (why?)

(4) Portfolio sorts (in USD): Use '6 European Portfolios Formed on Size and Momentum (2 x 3)' and '25 European Portfolios Formed on Size and Momentum (5 x 5)'

(a) Adjust portfolio returns by monthly change in USD exchange rate, cf. above. Use discrete convention.

(5) Calculate (entire) European stock market return from '6 European Portfolios Formed on Size and Momentum' by appropriately weighting 'Average Value Weighted Returns – Monthly' by 'Number of Firms in Portfolio' and 'Average Firm Size' using the currency correction of the previous point.

b) You are expected to perform linear regressions

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + mMOM(t) + e(t)$$

similar to FF93 for the three cases. Use USD RF for case i) and EUR RF for cases ii) and iii). Factors are defined as

$$\begin{aligned} SMB \; = \; & 1/3('SMALL\ LoPRIOR' + 'ME1\ PRIOR2' + 'SMALL\ HiPRIOR') \\ & -1/3('BIG\ LoPRIOR' + 'ME2\ PRIOR2' + 'BIG\ HiPRIOR\text{^}') \end{aligned}$$

and

$$\begin{aligned} MOM = \; & 1/2('SMALL\ HiPRIOR' + 'BIG\ HiPRIOR\text{^}') \\ & -1/2('SMALL\ LoPRIOR' + 'BIG\ LoPRIOR') \end{aligned}$$

c) For the market factor, MKT, use the time-series constructed above. MOM and SMB are long/short portfolios. Convert to long-only as
   i) 'Tech Stocks' (MOM): Use 'BIG HiPRIOR' from the two '6 Portfolios ...' datafiles
   ii) 'Small Cap' (SMB): Use the market cap weighted returns of 'SMALL LoPRIOR','ME1 PRIOR2', and 'SMALL HiPRIOR' from the two '6 Portfolios ...' datafiles.

   You will end up with 6 long-only time-series for US and European stock markets for a EUR-based investor.

d) By August 2007 you have 36 monthly observations of previous returns.
   i) Mean Variance
      (1) From these 36 observations, you are expected to
         (a) Calculate portfolio weights by mean-variance optimization subject to no shorting constraints (portfolio weights must be non-negative).
         (b) Calculate the portfolio return for the subsequent month from these portfolio weights.
      (2) Time is now October 2007. Repeat (1) and (2) for the previous 36 months and calculate the subsequent return.
      (3) Repeat until the final month of 2024.
   ii) Risk Parity
      (1) From the 36 observations,
         (a) Calculate the volatility, σ_i, of each of the factors.
         (b) Calculate portfolio weights, w_i, by the Risk Parity principle by solving $\forall\, i,j$: $w_i\sigma_i = w_j\sigma_j$ subject to $\sum_i w_i = 1$ and $\forall\, i$: $w_i \geq 0$.
      (2) Repeat ...
e) You are expected to compare strategies similar to Anderson, Bianchi and Goldberg (2012).
f) Use the factor time-series constructed above for MKT, MOM, and SMB in cases ii) and iii) – six in total.

2) **Tie-In & Leverage**

Notation refers to slide 4 and 5 from Group 7's presentation of CPPI where possible.

a) Data
   i) Active Assets, A_t: European Equity returns and return series from your advice in 1e).
   ii) Reserve Asset, R_t: Zero-Coupon bond prices calculated from 'All Years - euro area'.
b) Historical analysis of accrual of guarantees – current system
   i) By August 2007, ($i = 0$), assume *Best Pensions* starts a new 10-year TDF and €100 is added as the first contribution
      (1) €80 (€100/F_{target}) is invested in the Reserve Portfolio and the remainder in the Active Portfolio, hence, the market values are $MVA_0 = €20$ and $MVR_0 = €80$, respectively.
      (2) The price of a zero-coupon bond is $P_{10}(0) = 0.644$, hence, the initial guarantee, N_o, is $G_0 = \frac{€80}{P_{10}(0)} = 124.2$.
      (3) The value, A_0, of the Active Assets is set to $A_0 = 1$, hence allocation to the active asset is, $\eta_0^A = 20$.
   ii) The next month ($i = 1$), the value of the Active Asset is $A_1 = (1 + a_i)A_0$, where a_i is the period return of the European Equity.
      (1) The market value of the Reserve Portfolio is $R_1 = N_0 e^{-(T-t_i)z_{T-t_i}(t_i)}$ since the portfolio consists of N_0 zero-coupon bonds now with market value $P_{T-t_i}(t_i) = e^{-(T-t_i)z_{T-t_i}(t_i)}$.
      (2) If $L_1 = \frac{(A_1+R_1)}{R_1}$ exceeds $L_{trigger}$, then the guarantee is increased to solve $A_1 + R_1 = L_{target} \cdot N_1 P_{T-t_i}(t_i)$.

iii) Roll forward 10 years to capture final guarantee, N_{120}.
iv) Repeat steps i)-iii) to capture historical distribution of guarantees.

c) Modify the algorithm of the previous point to allow for a CPPI strategy:
CPPI multiplier $m \geq 1$. Maximum leverage ratio $b \equiv 1$.
i) By August 2007
(1) The wealth, W_0, is €100.
(2) The floor, F_0 is €80.
(3) The cushion, $C_0 = W_0 - F_0$, is €20.
(4) Exposure $E_0 = m \cdot C_0$.
(5) The allocation to the Active Asset is $\eta_0^A = \min(mC_0, bW_o)/A_0$
(6) The guarantee is $G_0 = \frac{F_0}{P_{10}(0)} = 124.2$
(7) The Reserve Asset, R_0, is a 10-year zero-coupon bond with price $P_{10}(0)$.
(8) The allocation to the Reserve Asset is $\eta_0^R = (A_0 - E_0)/P_{10}(0)$
ii) Next month:
(1) Active Asset is $A_1 = (1 + a_i)A_0$, where a_1 is the period from 1e).
(2) Reserve Asset is $R_1 = P_{10-1/12}(1/12)$ where $P_{10-1/12}(1/12)$ is the value of a zero-coupon bond expiring in 9 years and 11 months.
(3) Wealth is $W_1 = \eta_0^A A_1 + \eta_0^R R_1$
(4) The floor is $F_1 = N_0 \cdot R_1$
(5) Funded ratio $L_1 = W_1/F_1$
(6) ...

Report Requirements

The report should be written as an investment advice addressed to the Executive Board of *Best Pensions*. The report must cover all points in the *Request for Advice* – but not necessarily in that order. Your ability to present your analysis, considerations, and conclusions to a professional recipient in a clear and concise manner is a main determinant in assessing the report.

The report is limited to a maximum of 20 pages including tables, figures, and appendices but excluding title page (separate page), table of contents (separate page), and references (separate page).

The report must be submitted in PDF format.

The format must satisfy

- Page size is A4
- Top/bottom margins: 3 cm or more
- Left/right margins: 2 cm or more
- Font size: 11pt or more.

In evaluating your report, we will simply stop reading pages exceeding the 20-page limit.

Your report should be uploaded to Digital Eksamen no later than 30 October 2025, 11.59PM along with your code. It is acceptable to provide a link for your code, fx. a GitHub repository, but the report must be submitted to Digital Eksamen.