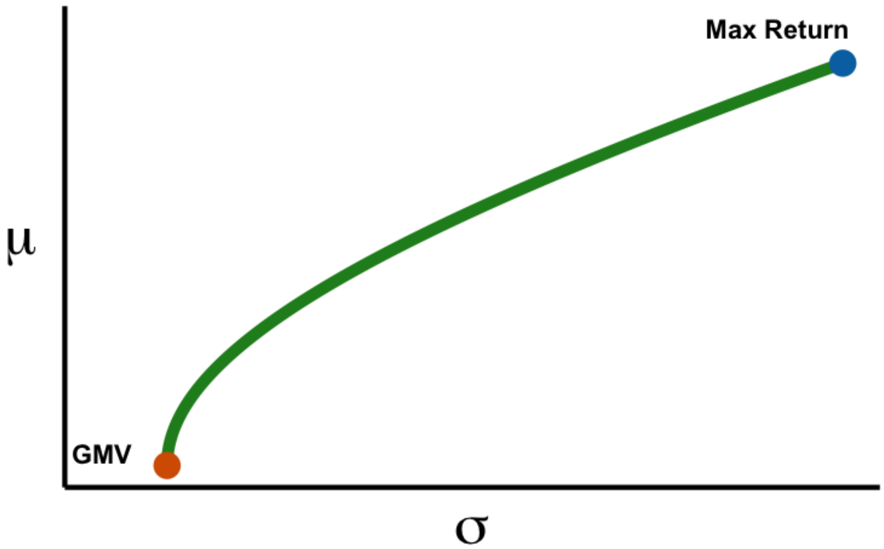
UNIVERSITY OF COPENHAGEN

DEPARTMENT OF MATHEMATICAL SCIENCES



Project Outside Course Scope (PUK)

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Investment Report

Best Pensions

30th of October 2025

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Contents

1 Introduction and Notation

This report is based exclusively on four Fama–French datasets, structured as illustrated in Figure 1. The data cover both the European and US markets, each subdivided into two portfolio constructions based on firm size and momentum characteristics. The sample period spans August 2004 to December 2024.

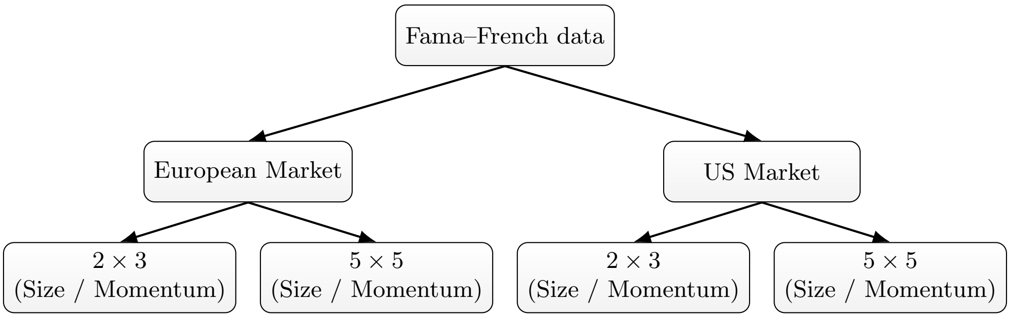


Figure 1: *Fama–French data for European and US markets with 2×3 and 5×5 Size–Momentum portfolios.*

Each market dataset is provided in two portfolio structures. The first, a 2×3 split, classifies firms by median size and momentum percentiles (30th and 70th). The second, a 5×5 grid, sorts firms into quintiles for both size and momentum, resulting in 25 portfolios. The portfolio labels for both classifications are summarized in Tables 1 and 2.

	Momentum				
Size	**Lo/1**	**2**	**3**	**4**	**Hi/5**
Small/1	SMALL LoPRIOR	ME1 PRIOR2	ME1 PRIOR3	ME1 PRIOR4	SMALL HiPRIOR
2	ME2 PRIOR1	ME2 PRIOR2	ME2 PRIOR3	ME2 PRIOR4	ME2 PRIOR5
3	ME3 PRIOR1	ME3 PRIOR2	ME3 PRIOR3	ME3 PRIOR4	ME3 PRIOR5
4	ME4 PRIOR1	ME4 PRIOR2	ME4 PRIOR3	ME4 PRIOR4	ME4 PRIOR5
Big/5	BIG LoPRIOR	ME5 PRIOR2	ME5 PRIOR3	ME5 PRIOR4	BIG HiPRIOR

Table 1: *Structure of the* 5×5 *Size–Momentum portfolios. Each cell represents one of 25 portfolios formed at the intersection of Size and Momentum quintiles.*

	Momentum		
Size	**Lo/1**	**2**	**Hi/3**
Small/1	SMALL LoPRIOR	ME1 PRIOR2	SMALL HiPRIOR
Big/2	BIG LoPRIOR	ME2 PRIOR2	BIG HiPRIOR

Table 2: *Structure of the* 2×3 *Size–Momentum portfolios. Each cell represents one of six portfolios formed at the intersection of Size and Momentum groups.*

To ensure full transparency and replicability, all factor returns were reconstructed manually from the 2×3 datasets rather than relying on pre-calculated series from the Fama–French library. This approach renders the report fully self-contained. As all Fama–French datasets are given in US dollars, returns were converted to euros for the EUR-based investor using the corresponding EUR/USD exchange rates.

Throughout the report, statistical significance refers to a level of 5% unless otherwise stated.

2 Factor Analysis

Factor investing has long been a central theme in financial research, demonstrating strong historical performance. This analysis re-examines whether the selected factors continue to explain expected returns in contemporary markets. The factors under consideration, chosen at the client's request, are:

1. *Momentum factor* (MOM)
2. *Size factor* (SMB)
3. *Market factor* (MKT)

Although the market factor was not explicitly requested, it is included to capture overall market movements and ensure model completeness. Since Best Pensions is EUR-based, the primary focus is on performance from a EUR-based investor's perspective. Nevertheless, results for a USD-based investor are included to facilitate direct comparison with the original Fama–French study (Fama & French, 1993). The factors are defined based on Table 2 as follows:

$$\begin{aligned} \text{SMB} &= \frac{1}{3}\left(\text{SMALL LoPRIOR} + \text{ME1 PRIOR2} + \text{SMALL HiPRIOR}\right) \\ &- \frac{1}{3}\left(\text{BIG LoPRIOR} + \text{ME2 PRIOR2} + \text{BIG HiPRIOR}\right), \end{aligned}$$

$$\begin{aligned} \text{MOM} &= \frac{1}{2}\left(\text{SMALL HiPRIOR} + \text{BIG HiPRIOR}\right) \\ &- \frac{1}{2}\left(\text{SMALL LoPRIOR} + \text{BIG LoPRIOR}\right) \end{aligned}$$

Conceptually, both SMB and MOM represent zero-cost long/short portfolios. The SMB factor is long 1/3 in each of the three small-firms portfolios and short 1/3 in each of the three large-firms portfolios. Similarly, the MOM factor goes long 1/2 in each high-momentum portfolio and short 1/2 in each low-momentum portfolio.

2.1 Existence of SMB and MOM

We begin by assessing whether the SMB and MOM factors continue to exhibit explanatory power consistent with the original Fama–French framework (Fama & French, 1993) and the initial research on the MOM factor (Carhart, 1997). This initial analysis serves to verify the existence of these factors and their potential relevance for generating excess returns in contemporary markets.

2.1.1 USD-Based Investor in US Market

	USD-based – US Market		
Factor	**Mean**	**Std.**	**t-statistic**
MKT–RF	0.847	4.434	2.985
SMB	0.008	2.736	0.044
MOM	0.133	4.394	0.474

Table 3: *Summary statistics for factors in the US market for a USD-based investor. Values are rounded to three decimals.*

As shown in Table 3, neither the SMB nor MOM factor displays statistical significance, while the market factor remains statistically significant. These findings suggest that the size and momentum effects are not clearly present in the recent US market.

The market factor's average excess return of 0.85 is notably higher than the 0.43 reported in Fama–French (Fama & French, 1993), reflecting the strong growth of US equities over the past two decades. When broad market performance is consistently high, dispersion in individual stock returns tends to narrow, making it more difficult for cross-sectional factors such as SMB and MOM to generate excess returns. This dynamic likely contributes to the weaker performance of SMB and MOM in the sample period.

2.1.2 EUR-Based Investor

We now examine the cases most relevant for Best Pensions: a EUR-based investor in the US market and in the European market. Changing the investor's currency base requires adjusting the risk-free rate accordingly. The EUR risk-free rate is derived from the 1-month zero-coupon rate $z(1/12)$, estimated using the parametric Svensson model. The monthly continuously compounded rate is computed as

$$1 + \mathrm{RF}_{EU} = \frac{1}{\mathrm{disc}(1/12)} = e^{z(1/12)/12},$$

where disc(1/12) denotes the one-month discount factor, from which the monthly return RF_{EU} follows.

	EUR-based − US Market			**EUR-based − EU Market**		
Factor	**Mean**	**Std.**	**t-statistic**	**Mean**	**Std.**	**t-statistic**
MKT-RF	0.933	4.127	3.533	0.583	4.175	2.180
SMB	0.003	2.741	0.016	0.086	1.813	0.738
MOM	0.153	4.382	0.546	0.770	3.598	3.344

Table 4: *Summary statistics for key risk factors in the US and European markets for a EUR-based investor. Values are rounded to three decimals.*

As reported in Table 4, the results are notably stronger than for the USD-based case in Table 3. The MOM factor displays improved performance in both markets and attains statistical significance in the European market, where it even exceeds the market factor in both mean return and t-statistic. This significance would likely persist after adjusting for multiple testing.

The market factor's mean excess return is lower in Europe than in the US, consistent with the relatively slower equity growth observed in European markets over the period. Overall, these findings indicate that, for EUR-based investors, factor exposures, particularly to momentum, remain relevant and may enhance portfolio performance, especially within the European market.

2.2 Portfolio-Level Regression

To evaluate the explanatory power of the selected factors, we extend the analysis following the Fama–French framework (Fama & French, 1993) by conducting regression analyses at the portfolio level.

The previous analysis employed a 2×3 market structure, where each portfolio represented an intersection of size and momentum categories. To enable a more detailed assessment, we now use the 5×5 structure defined in Table 1.

The regression model for each portfolio is specified as:

$$R(t) - \mathrm{RF}(t) = a + b[\mathrm{RM}(t) - \mathrm{RF}(t)] + s\,\mathrm{SMB}(t) + m\,\mathrm{MOM}(t) + e(t),$$

where the portfolio's excess return is modeled as a function of the excess market return, the SMB, and MOM factors along with a residual term $e(t)$.

Although the SMB factor exhibits limited excess return in Tables 3 and 4, it is retained to account for its potential contribution in explaining cross-sectional returns.

2.2.1 USD-Based Investor in US Market

As an update of the original Fama–French analysis (Fama & French, 1993), Table 5 reports regression coefficients and R^2 values for the 5×5 size-momentum portfolios in the US market.

	b					**s**				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	1.10	0.87	0.84	0.84	1.02	1.33	1.07	0.94	1.01	1.24
2	1.13	0.97	0.91	0.94	1.14	1.09	0.93	0.91	0.95	1.07
3	1.11	1.02	0.96	0.96	1.15	0.75	0.63	0.64	0.72	0.83
4	1.18	1.04	0.97	0.98	1.14	0.54	0.33	0.33	0.31	0.47
Big/5	1.23	0.94	0.92	0.97	1.10	-0.18	-0.14	-0.11	-0.10	-0.05^*
	m					R^2				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	-0.67	-0.27	-0.10	0.05*	0.28	0.93	0.95	0.93	0.90	0.91
2	-0.82	-0.33	-0.09	0.05	0.34	0.96	0.96	0.95	0.94	0.93
3	-0.79	-0.36	-0.16	0.04*	0.38	0.93	0.95	0.95	0.94	0.93
4	-0.86	-0.39	-0.12	0.09	0.42	0.90	0.94	0.94	0.93	0.88
Big/5	-0.88	-0.47	-0.16	0.15	0.41	0.86	0.93	0.93	0.91	0.88

Table 5: *Regression coefficients and R^2 values for the 5×5 size-momentum portfolios in the US market (USD-based investor). Values are rounded to two decimals. Red R^2 cells indicate significant intercepts.* $(*)$ *denotes insignificant coefficients.*

Despite its limited excess return, the SMB factor contributes meaningfully to model fit. The market factor (b) is stable around one and significant across all portfolios, as expected, since all portfolios are broadly exposed to market movements.

The size coefficient (s) declines monotonically with portfolio size, ranging from 1.33 for the smallest firms to -0.18 for the largest, with only one insignificant estimate. Similarly, the momentum coefficient (m) increases with momentum, ranging from -0.88 to 0.42. Interestingly, portfolios in the fourth momentum quintile show the weakest exposure to this factor.

The R^2 values are typically above 0.88, thus indicating strong explanatory power. While high R^2 values can sometimes suggest overfitting, the inclusion of only three explanatory variables (one of which is essential) mitigates this concern. Finally, only few intercepts differ significantly from zero, suggesting that most systematic variation in returns is well captured by the three-factor model.

2.2.2 EUR-Based Investor in US Market

We now consider the first client-relevant case: a EUR-based investor in the US market. The objective is to assess whether the three-factor model established above remains robust when returns are expressed in euros. Accordingly, we replicate the previous regression analysis to evaluate the model's validity under this currency framework.

	b					s				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	1.07	0.91	0.87	0.88	1.02	1.34	1.05	0.93	0.99	1.25
2	1.11	0.98	0.93	0.97	1.12	1.11	0.92	0.89	0.94	1.08
3	1.12	1.00	0.97	0.98	1.11	0.76	0.64	0.64	0.71	0.85
4	1.13	1.01	0.97	0.98	1.11	0.56	0.34	0.34	0.31	0.49
Big/5	1.18	0.96	0.95	0.97	1.07	-0.15	-0.15	-0.12	-0.10	-0.04^*
	m					R^2				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	-0.69	-0.25	-0.08	0.08	0.28	0.92	0.95	0.92	0.89	0.90
2	-0.83	-0.33	-0.08	0.06	0.32	0.96	0.96	0.94	0.94	0.92
3	-0.80	-0.37	-0.15	0.05	0.36	0.92	0.94	0.94	0.94	0.91
4	-0.88	-0.40	-0.11	0.10	0.40	0.89	0.93	0.93	0.92	0.87
Big/5	-0.91	-0.46	-0.14	0.15	0.39	0.84	0.92	0.92	0.90	0.86

Table 6: *Regression coefficients and R^2 values for the 5×5 size-momentum portfolios in the US market (EUR-based investor). Values are rounded to two decimals. Red R^2 cells indicate significant intercepts.* $(*)$ *denotes insignificant coefficients.*

Although the model structure is unchanged, Table 6 shows only minor differences after converting returns to euros. The market coefficient (b) remains close to one and significant across portfolios, confirming that returns move largely in line with the US market regardless of currency.

The size coefficients (s) follow the same monotonic pattern as in the USD-based case, declining from 1.34 among the smallest portfolios to -0.15 for the largest. Despite the absence of a meaningful SMB premium in mean returns, including the factor improves model fit, reflected in the uniformly high R^2 values. Only the largest, high-momentum portfolio shows an insignificant size loading, suggesting that size effects fade completely in that segment.

Momentum coefficients (m) behave as theoretically expected, increasing with momentum rank from roughly -0.9 for low-momentum portfolios to 0.4 for high-momentum portfolios.

Overall, the R^2 values remain between 0.86 and 0.96, indicating that the model continues to explain most cross-sectional variation in excess returns after currency conversion. While high explanatory power can raise concerns about overfitting, the inclusion of only three factors makes this unlikely. The behavior of the intercepts mirrors the USD-based case, suggesting that the model remains stable and valid for a EUR-based investor in the US market.

2.2.3 EUR-Based Investor in European Market

Finally, we test whether the three-factor model retains its explanatory power in a different setting from the original Fama–French analysis (Fama & French, 1993) by applying it to the European market.

	b					s				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	1.06	0.83	0.84	0.86	0.97	1.23	0.89	0.85	0.89	0.98
2	1.07	0.99	0.98	0.99	1.11	0.99	0.82	0.84	0.87	1.01
3	1.13	1.04	1.02	1.01	1.16	0.73	0.63	0.65	0.70	0.71
4	1.08	1.05	0.99	1.07	1.13	0.24	0.30	0.33	0.45	0.46
Big/5	1.08	0.98	0.94	0.98	1.06	-0.24	-0.14	-0.22	-0.27	-0.14
	m					R^2				
Size/Momentum	Lo/1	2	3	4	Hi/5	Lo/1	2	3	4	Hi/5
Small/1	-0.46	-0.11	0.06	0.16	0.30	0.96	0.94	0.93	0.92	0.92
2	-0.71	-0.16	0.01*	0.22	0.40	0.96	0.95	0.95	0.94	0.94
3	-0.65	-0.19	−0.02*	0.23	0.48	0.96	0.95	0.94	0.92	0.92
4	-0.69	-0.20	0.03*	0.26	0.52	0.93	0.95	0.92	0.91	0.91
Big/5	-0.80	-0.35	0.01*	0.33	0.59	0.95	0.94	0.94	0.93	0.89

Table 7: *Regression coefficients and R^2 values for the 5×5 size-momentum portfolios in the European market (EUR-based investor). Values are rounded to two decimals. Red R^2 cells indicate significant intercepts. $(*)$ denotes insignificant coefficients.*

Table 7 reports the regression results for a EUR-based investor in the European market. As in the previous cases, the market coefficient (b) is close to one and consistent across portfolios, confirming that European equity portfolios largely move in line with the overall market. This stability supports the model's validity and highlights the dominant role of the common market factor in explaining returns.

The size coefficients (s) follow the expected monotonic pattern, declining as firm size increases. Smaller portfolios show strong positive exposure to the size factor, while the largest portfolios exhibit weak or insignificant loadings. Similar to the US case, the results align with theoretical expectations and indicate that firm size remains a relevant explanatory variable.

Momentum coefficients (m) increase systematically with momentum rank, turning from negative in low-momentum portfolios to positive among high-momentum portfolios. This consistent pattern confirms that the momentum effect is clearly present in the European market.

The R^2 values are uniformly high, ranging from roughly 0.89 to 0.96, indicating that the model explains most variation in portfolio excess returns. However, nine of the twenty-five intercepts are significantly different from zero, suggesting that some variation remains unexplained in the European sample. Despite this, the model retains sufficiently strong overall explanatory power.

In summary, the three-factor model demonstrates stability and robustness across both regions and currencies, maintaining explanatory power even within the European market context.

2.3 Preliminary Investment Advice

Following the baseline analysis, we provide preliminary investment guidance for Best Pensions. As shortselling is not permitted by the client, and since the SMB and MOM factors are constructed as long/short portfolios, this advice serves only as a conceptual basis for the subsequent allocation analysis. Because Best Pensions is EUR-based, only the EUR-based cases are considered.

Table 4 showed varying performances across factors, with the European MOM factor standing out as the strongest performer apart from the US market factor. The recommendations aim to balance return potential with diversification benefits. Based on this reasoning, SMB (US) is excluded due to its negligible excess return, while SMB (EU) is retained for its diversification value, despite its low performance.

The resulting factor universe is defined by the monthly expected excess returns, μ^e, in percentage and the covariance matrix, Σ. Additionally, the monthly return is compounded to present the corresponding yearly excess return. The order follows Table 4: US market, US SMB, European market, European SMB, and European MOM.

$$\mu^e = \begin{pmatrix} 0.933 \\ 0.153 \\ 0.583 \\ 0.086 \\ 0.770 \end{pmatrix}, \quad \mu^e_{\text{Yearly}} = \begin{pmatrix} 11.793 \\ 1.853 \\ 7.221 \\ 1.033 \\ 9.645 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 17.03 & -5.32 & 13.28 & -0.37 & -4.07 \\ -5.32 & 19.20 & -7.20 & -0.86 & 12.72 \\ 13.28 & -7.20 & 17.43 & -0.02 & -6.74 \\ -0.37 & -0.86 & -0.02 & 3.29 & -0.15 \\ -4.07 & 12.72 & -6.74 & -0.15 & 12.95 \end{pmatrix}$$

The covariance matrix exhibits both positive and negative relationships between factors, indicating potential for diversification. Negative covariances suggest that downturns in some factors may be offset by gains in others, helping to stabilize returns. While this also moderates peak performance, it supports a more resilient portfolio profile.

Overall, the five selected factors appear relevant for Best Pensions, especially the two market factors and the European momentum factor, while the remaining factors provide useful diversification benefits.

2.4 Long-Only Strategies

As shortselling is not permitted, the factor framework must be adapted to reflect long-only exposures. This constraint can be addressed by retaining the market factors while redefining the remaining factors into long-only versions. Specifically, we introduce the *Small Cap* (SC) and *Tech Stocks* (TS) factors as follows:

$$\begin{aligned} \text{Small Cap} &= \frac{1}{3}\left(\text{SMALL LoPRIOR} + \text{ME1 PRIOR2} + \text{SMALL HiPRIOR}\right) \\ \text{Tech Stocks} &= \text{BIG HiPRIOR} \end{aligned}$$

Under these definitions, the portfolios previously held short positions in are now excluded, resulting in pure long strategies.

We then replicate the analysis from Table 4. As the market factors remains unchanged, it is omitted from Table 8 below.

	EUR-based – US Market			**EUR-based – EU Market**		
Factor	**Mean**	**Std.**	**t-statistic**	**Mean**	**Std.**	**t-statistic**
SC-RF	0.948	6.030	2.454	0.672	4.624	2.270
TS-RF	0.985	4.305	3.574	0.741	3.977	2.910

Table 8: *Summary statistics for long-only SMB and MOM factors in the US and EU markets for a EUR-based investor. Values are rounded to three decimals.*

It is important to distinguish between the original and redefined factors. The original factors were market-neutral, constructed as long/short portfolios of value-weighted returns. Their performance thus represented returns aside from the market. In contrast, the reformed factors are purely long positions and therefore not market-neutral. Consequently, the two versions are not directly comparable, as the reformed factors inherently include part of the market exposure, which the original did not.

This difference is reflected in the change in factor correlations. Note, in both Tables 4 and 8, returns are expressed in excess of the risk-free rate (RF), and the correlations below are based on the corresponding underlying observations.

Panel A: Original factors

1.00	0.34	-0.29	0.77	-0.05	-0.27
0.34	1.00	-0.25	0.29	0.29	-0.20
-0.29	-0.25	1.00	-0.39	-0.11	0.81
0.77	0.29	-0.39	1.00	0.00	-0.45
-0.05	0.29	-0.11	0.00	1.00	-0.02
-0.27	-0.20	0.81	-0.45	-0.02	1.00

Panel B: Reformed factors

1.00	0.89	0.92	0.77	0.70	0.72
0.89	1.00	0.77	0.73	0.72	0.64
0.92	0.77	1.00	0.67	0.59	0.74
0.77	0.73	0.67	1.00	0.93	0.90
0.70	0.72	0.59	0.93	1.00	0.82
0.72	0.64	0.74	0.90	0.82	1.00

Table 9: *Correlation matrices for the three factors across the US and European markets. Panel A reports the original factors, while Panel B shows correlations for the reformed long-only factors. The order of rows and columns follows Tables 4 and 8. Values are rounded to two decimals.*

Table 9 reveals a substantial increase in correlations among the reformed factors, consistent with expectations. Because these long-only factors retain shared market exposure, multicollinearity would make traditional regression analysis unreliable. Running such regressions would likely yield unstable coefficients and poor model fit, as several factors would capture overlapping market effects.

2.4.1 Revised Factor Investment Advice

Based on the preceding discussion, we revise our investment recommendations to align with the long-only constraint imposed by Best Pensions. The factor definitions have been adjusted accordingly, as described above. Drawing on the results in Tables 4, 8, and 9, we conclude that all factors are relevant for inclusion in the investment universe.

Although some factors yield lower excess returns, they still contribute valuable diversification benefits by allowing to reduce overall portfolio variance. In the US market, all factors exhibit high mean excess returns but differ notably in volatility, with SC-RF (US) showing the highest variance. In contrast, both mean returns and standard deviations are lower in the European market, making these factors suitable for stabilizing portfolio performance.

Table 9 further shows that correlations between European and US factors are generally weaker than within each market, implying that cross-market diversification is achievable. While SC-RF (EU) offers relatively low returns, it is also the least correlated with US factors, justifying its inclusion from a diversification standpoint.

Overall, we recommend retaining all six factors, as each contributes either to return potential or to portfolio stability. The expected excess returns and covariance matrix are summarized below.

$$\mu^e_{\text{long}} = \begin{pmatrix} 0.933 \\ 0.948 \\ 0.985 \\ 0.583 \\ 0.672 \\ 0.741 \end{pmatrix}, \quad \mu^e_{\text{long,Yearly}} \begin{pmatrix} 11.793 \\ 8.369 \\ 12.482 \\ 7.221 \\ 8.369 \\ 9.260 \end{pmatrix} \quad \Sigma_{\text{long}} = \begin{pmatrix} 17.03 & 22.04 & 16.23 & 13.28 & 13.28 & 11.84 \\ 22.04 & 36.16 & 19.86 & 18.24 & 20.01 & 15.22 \\ 16.23 & 19.86 & 18.44 & 11.95 & 11.74 & 12.66 \\ 13.28 & 18.24 & 11.95 & 17.43 & 17.95 & 14.92 \\ 13.28 & 20.01 & 11.74 & 17.95 & 21.22 & 15.01 \\ 11.84 & 15.22 & 12.66 & 14.92 & 15.01 & 15.74 \end{pmatrix}.$$

3 Applications of Recommended Investment Universe

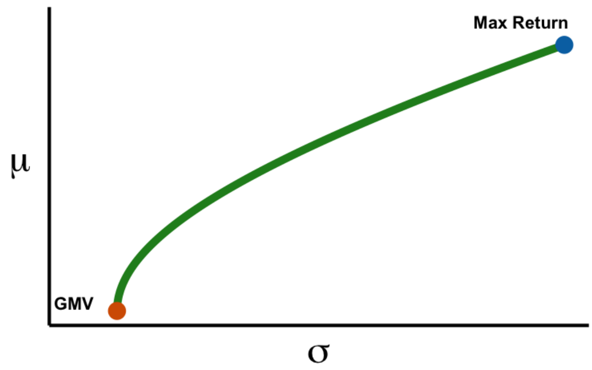
Several frameworks exist for constructing optimal investment strategies from a given set of assets or factors. In this section, we focus on two well-established approaches: *Mean-Variance Optimization* (Markowitz, 1952) and *Risk Parity* (Dalio, 2010).

The Mean-Variance approach seeks to minimize portfolio variance for a specified level of expected return, subject to relevant constraints. It is most widely recognized for defining the *Efficient Frontier* and the *Global Minimum-Variance Portfolio* (GMV). In contrast, the Risk Parity method allocates capital to balance risk contributions across distinct risk factors, ensuring that each factor contributes equally to total portfolio volatility, at least within the structure of the selected market model.

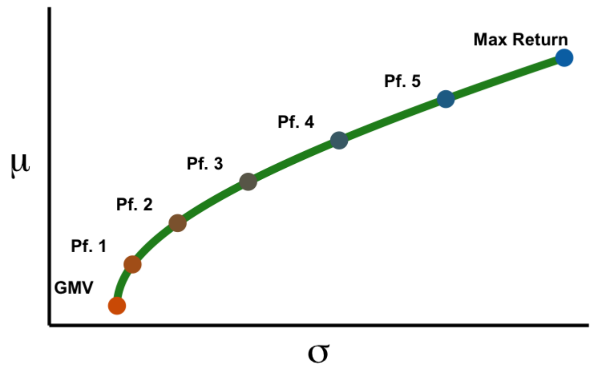
Using the investment universe derived in the preceding section, both methods are implemented explicitly on the available data. To obtain realistic performance estimates, we conduct an out-of-sample analysis. Each strategy is rebalanced monthly using a rolling three-year estimation window. The first portfolio is established at the end of August 2007 and subsequently updated at the end of every month until November 2024, achieving its final return at the end of December 2024.

3.1 Mean-Variance Optimization

Initially we examine two portfolios from the Mean-Variance framework: one that rebalances monthly to the GMV portfolio, and another that rebalances to the *Maximum Expected Return* portfolio.



(a) *GMV and Max Return portfolios.*



(b) *Efficient frontier with five intermediate portfolios.*

Figure 2: *Hypothetical illustrations of the efficient frontier. The left panel shows the GMV and Maximum-Return portfolios, while the right panel includes five intermediate portfolios (pf. in short) evenly spaced along the frontier. Axes represent expected return (μ) and standard deviation (σ).*

Conceptually, this corresponds to computing the efficient frontier each month and rebalancing to the portfolios represented by the two points in Figure 2a.

Over the sample period, the GMV portfolio achieved a total return of 332%, with an average monthly return of 0.78% and a volatility of 3.91%. In comparison, the maximum-return portfolio produced a total return of 255%, with an average monthly return of 0.73% and a volatility of 4.9%.

As expected, the GMV portfolio entails lower volatility, however, it is notable that it also delivered a higher realized return. This suggests that a more conservative allocation can outperform strategies that pursue short-term return maximization.

Figure 3 presents the cumulative performance of the two optimized strategies relative to European equity between September 2007 and December 2024. Both portfolios declined sharply during the 2008 financial crisis but subsequently recovered steadily. From 2014 onward, the GMV portfolio increasingly outperformed the maximum-return portfolio, ending the period with the highest cumulative gain.

Overall, both optimized portfolios outperformed the broad European equity market, illustrating the practical value of systematic portfolio optimization for enhancing long-term performance.

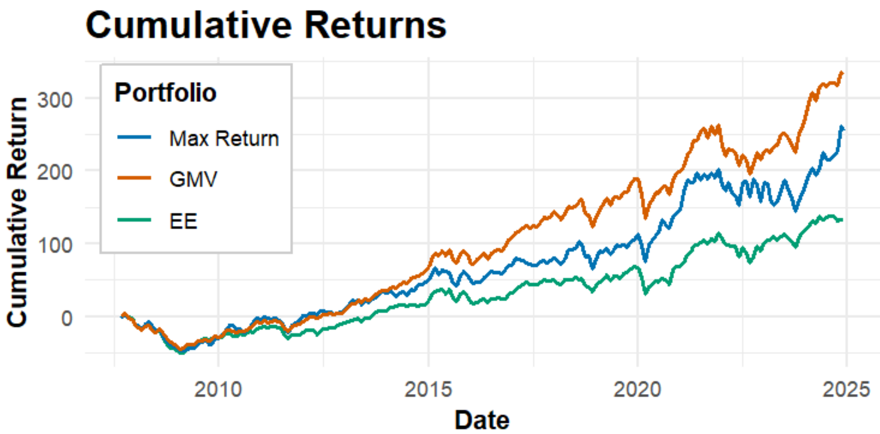


Figure 3: *Cumulative returns of the minimum-variance (GMV, orange), maximum-return (Max Return, blue) and European Equity (EE, green) portfolios from September 2007 to December 2024.*

The analysis is now extended to the entire efficient frontier by introducing five intermediate portfolios, denoted Portfolios 1 through 5. In the dynamic out-of-sample framework, the construction process at each step begins by identifying the expected returns of the minimum-variance and maximum-return portfolios, as shown in Figure 2a. The target returns for the five intermediate portfolios are then defined as equally spaced points between these two extremes. Each portfolio is obtained by minimizing variance subject to its target expected return, thereby forming a representative discretization of the efficient frontier, as illustrated in Figure 2b. The intermediate portfolios are evenly spaced in terms of expected return rather than standard deviation.

Figure 4 extends the previous analysis by including these additional portfolios. Consistent with Figure 3, the portfolios based on lower expected returns achieved the highest realized cumulative returns over the sample period.

Summary statistics of the portfolios are reported in Table 10. Portfolio 1 exhibits both the highest realized return and the lowest volatility among all portfolios. More broadly, portfolios constructed with lower target variance tend to generate higher cumulative returns. This outcome likely reflects their more conservative allocation profile, which provides greater stability during downturns compared to the high-return portfolio. In several instances, the maximum-return portfolio effectively concentrated on a single asset, the one with the highest expected return. While such a strategy can deliver short-term gains, it also exposes the investor to large losses, as evidenced by its substantially higher volatility.

Statistic/Portfolio	**EE**	**GMV**	**1**	**2**	**3**	**4**	**5**	**Max μ**	**Risk Parity**
Cumulative return (%)	132	332	349	331	322	326	317	255	340
Mean return (% p.m.)	0.63	0.78	0.8	0.77	0.76	0.74	0.76	0.73	0.81
Volatility (% p.m.)	4.37	3.91	3.84	3.86	3.98	4.17	4.45	4.90	4.25
Sharpe-Ratio (p.m.)	0.15	0.24	0.26	0.28	0.29	0.30	0.31	0.27	0.22

Table 10: *Summary statistics of the out-of-sample performance for the investment strategies. "GMV" and "Max μ" denotes the mean-variance portfolios with minimum expected variance and highest expected return, and portfolios 1-5 are the intermediate portfolios. "EE" refers to European Equity.*

The statistics in Table 10 summarize performance across the full sample period. While these aggregate values provide a useful overview, understanding the time evolution of returns is equally important for drawing practical investment conclusions. The cumulative, or pathwise, behavior of the portfolios is therefore examined in Figure 4. Note that the Risk Parity strategy is included in the Table 10 for completeness but will be discussed separately in a later subsection.

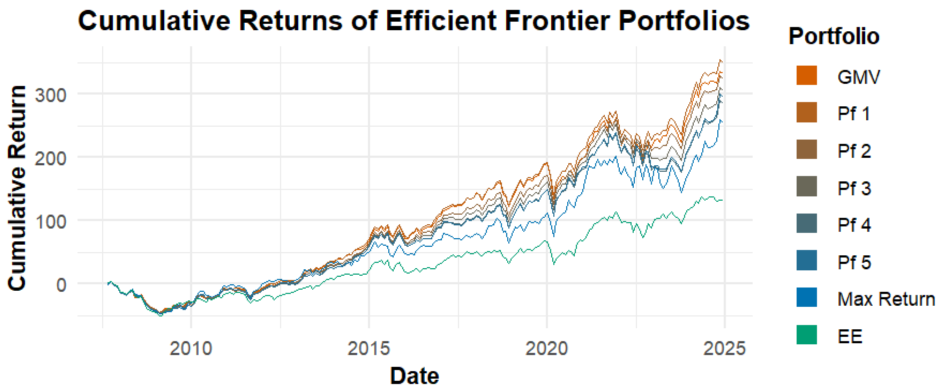


Figure 4: *Cumulative returns for 7 portfolios on the efficient frontier. "GMV" and "Max Return" are the global minimum variance and the maximum expected returns portfolios. Pf 1-5 are the 5 portfolios with expected returns between "GMV" and "Max Return".*

Before interpreting the cumulative performance in Figure 4, it is important to emphasize that these results are based on a single realized sample and therefore remain very dependent on the underlying market dynamics. However, since the strategies are evaluated using an out-of-sample framework, the analysis provides the most realistic approximation of how each strategy might perform in real time.

As shown in Figure 4, the maximum-return portfolio exhibits pronounced fluctuations compared to the other portfolios. While the cumulative paths generally move together, the degree of smoothness increases progressively from the maximum-return portfolio toward the minimum-variance portfolio. This pattern highlights the trade-off between stability and aggressiveness: investors can achieve smoother performance by accepting modestly lower expected returns, or conversely, pursue higher expected returns at the cost of substantially greater volatility.

In conclusion, Portfolio 1 appears to be the most suitable option for Best Pensions. It achieves the strongest realized performance within the mean-variance framework while maintaining relatively low volatility.

3.2 Risk Parity

As an alternative to the mean-variance framework, we consider a risk parity portfolio, one that allocates capital such that each risk factor contributes equally to the portfolio's total risk. This approach aims to achieve greater resilience during periods of market stress, such as the 2008 financial crisis.

Formally, the portfolio weights are defined as in (Anderson et al., 2012) and are given by

$$w_i = \frac{1/\sigma_i}{\sum_{j=1}^{n} 1/\sigma_j},$$

where w_i denotes the weight assigned to factor i and σ_i its volatility. Factors with higher volatility therefore receive smaller allocations, ensuring that risk exposure is balanced across all components.

To enable a direct comparison with the mean-variance strategies, the same dynamic out-of-sample procedure is applied to the risk parity portfolio. Figure 5 presents the cumulative performance, where the risk parity portfolio is shown in purple, Portfolio 1 from the mean-variance analysis in orange, and the European equity in green.

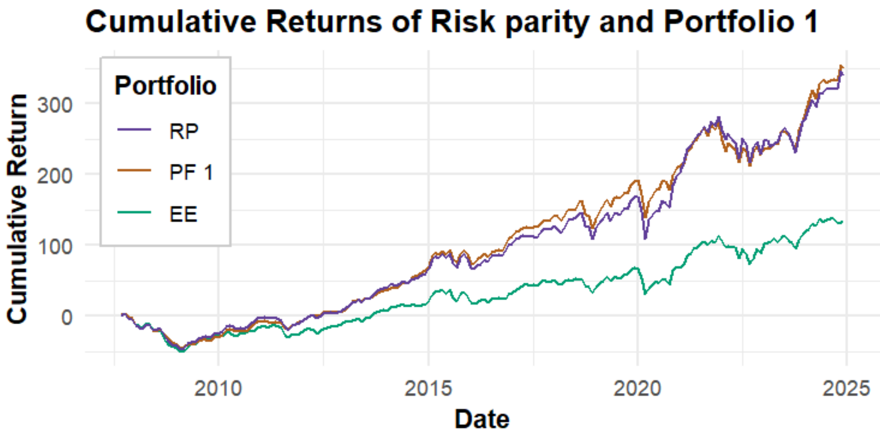


Figure 5: *Cumulative returns of the risk parity portfolio (RP), Portfolio 1 (PF 1) from the mean-variance optimization and European Equity (EE).*

As shown in Figure 5, the performance of the risk parity portfolio closely tracks that of Portfolio 1 for most of the sample period. Portfolio 1 achieves a marginally higher return between 2015 and 2021 and consequently ends the period with a slightly greater cumulative gain by December 2024. Both strategies experience similar losses during the 2008 financial crisis, indicating that the risk parity approach did not provide additional downside protection in that event. However, given that the crisis affected the entire equity market and the portfolio is fully invested in equity by design, such losses are not unexpected. Overall, the two strategies exhibit remarkably similar performance, suggesting that both approaches deliver comparable long-term outcomes under the given constraints.

As noted earlier, Table 10 also reports the results for the risk parity portfolio, allowing for a direct comparison with the mean-variance strategies. Figure 5 demonstrated that both approaches yield similar cumulative outcomes, though Table 10 reveals that the risk parity portfolio exhibits slightly higher volatility. While this evidence is based on a single realized sample and should not be interpreted as a general result, it nonetheless suggests that the mean-variance approach, specifically Portfolio 1, offers a more favorable balance between return and risk under the present data. Accordingly, we retain the recommendation Portfolio 1 as the preferred investment strategy for Best Pensions, supported by the statistical evidence in Table 10 and the observed performance patterns in Figure 5.

3.3 Consequences of No Shortselling Allowed

In portfolio construction, permitting short positions enhances flexibility and improves the efficiency of attainable risk-return combinations. Best Pensions, however, restricts its strategies to long-only positions. Although this constraint simplifies implementation, it limits optimization potential and typically leads to less favorable risk-return trade-offs. Shortselling expands the feasible investment universe by allowing negative portfolio weights, thereby increasing diversification opportunities. It also enhances efficiency, enabling investors to achieve higher expected returns for a given level of risk or, conversely, lower risk for a desired return, effectively shifting the efficient frontier outward.

Table 9 shows that long-only portfolios cannot offset exposures through negatively correlated assets, whereas allowing short positions enables such counterbalancing and thus more efficient risk reduction.

To assess the practical impact of this constraint, we now extend the analysis by allowing investing in the SMB and MOM factors, in addition to the two market factors. The same three-year rolling out-of-sample framework is applied to ensure comparability with the long-only case.

Previously, the optimal long-only portfolio corresponded to Portfolio 1 from the mean-variance optimization. The goal was to compare the long/short and long-only strategies under the constraint of matching the variance of Portfolio 1 and maximizing the expected return.

The portfolio composition was determined using the same mean-variance approach as in mean-variance optimization. As SMB and MOM are zero-cost portfolios we can achieve an expected return as high as we want. This will result in a proportionally high volatility. To balance this, we have added an extra constraint, for each euro invested in shorting, we have to allocate α in the bank as collateral, for $\alpha \in [0, 1]$. This caps the amount we can short to $\frac{1}{\alpha}$. This minimizes losses if the strategy does not perform and is often required in practice.

It was not possible to match the variance of Portfolio 1 exactly, instead we determined 22 strategies on the efficient frontier, and for each month selected the weights from the strategy with variance closest to that of Portfolio 1, always picking one with variance lower than Portfolio 1. This ensures that we always have a lower variance, close to the variance of Portfolio 1.

Statistic	**Portfolio 1**	$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0.25$
Mean expected excess return (% p.m.)	0.75	1.53	1.64	1.66
Δ Mean expected excess return (% p.m.)	-	0.78	0.89	0.91
Cumulative return (%)	349	335	631	651
Mean return (% p.m.)	0.8	0.78	1.08	1.09
Δ Mean return (% p.m.)	-	-0.02	0.28	0.29
Volatility (% p.m.)	3.84	3.7	4.8	4.79
Mean Sharpe-ratio	0.26	0.49	0.51	0.48

Table 11: *Summary statistics for comparison between the investment strategy used in portfolio 1 (long strategy), and three different shorting strategies. α determines the collateral we have in the bank for each euro we short. Δ denotes the difference between the shorting strategies and Portfolio 1. Mean expected returns and the difference are calculated using the prior data. Cumulative return, mean return and volatility are calculated using the realized gains. p.m. means the values are given per month.*

Table 11 summarizes the results of three shorting strategies for different α. Most shorting strategies had a substantially higher expected return than Portfolio 1. The realized gains for the period shows that the strategies with $\alpha = 0.5$ and 0.25 outperformed Portfolio 1 by 282-302 pp, at the cost of slightly higher volatility of 0.95-0.96 pp. The strategy with $\alpha = 1$ performed similarly to Portfolio 1.

A primary advantage of shortselling strategies is their potential to act as a hedge during market downturns. This characteristic was empirically observed in the three analyzed shortselling strategies. Specifically, whereas the long-only strategies yielded negative returns during the 2008 financial crisis, all three shortselling strategies generated positive returns over the same period.

Note that these shortselling strategies subsequently experienced a significant performance depreciation in the period following the crisis. This depreciation is attributed to the model's construction, which utilized a three-year historical period to determine portfolio weights. This methodology proved to be slow to adapt to the rapid market recovery. Consequently, the strategies maintained short positions on appreciating assets, incurring substantial losses as the market rebounded. Despite large losses, the cumulative returns of the shortselling strategies remained superior to the long-only strategies.

All together, the cost of not allowing shorting is a lower return for the same volatility and a less flexible strategy, that performs worse in bad market periods. In this particular sample the investor who did not permit short positions would only achieve approximately half the gain of the investor who did allow for short positions.

4 A CPPI Approach

Critics argue that Best Pensions offer inferior products compared to other pension designs because their guaranteed returns limit equity exposure, leading to lower overall performance. Despite this, Best Pensions remain committed to maintaining the guarantee while seeking increased equity allocation. A promising approach is the *Constant Proportion Portfolio Insurance* (CPPI) method (Black & Perold, 1992), which relies on a given wealth (W), multiplier (m), trigger and target levels ($L_{\text{trigger}}, L_{\text{target}}$). The portfolio is initially divided between an active asset (typically equity) and a reserve asset (zero-coupon bond) according to the target level and a multiplier related to risk aversion and equity exposure. The reserve asset roughly forms the floor, representing the minimum wealth level - if wealth reaches this floor, all assets shift to zero-coupon bonds until maturity. Conversely, strong equity performance may lift wealth to the trigger level, prompting a rebalancing that reduces equity exposure and raises the floor to protect accrued gains.

This is analyzed in a general case of European equity as the active asset, and then extended to our recommended investment strategy from previous section. Finally, we experiment with CPPI parameters as well as consider the consequences of *Quantitative Easening* during the sample period.

4.1 Baseline Analysis for Accrual of Guarantees

In the following a baseline analysis on the CPPI method will be conducted with parameters given as: $W = 100€$, $m = 1$, $L_{\text{trigger}} = 125\%$ and $L_{\text{target}} = 130\%$. Rebalancing happens whenever $(A_t+R_t)/F_t > L_{\text{trigger}}$ where A_t, R_t and F_t are the active portfolio value, the reserve portfolio value and the floor. For $m = 1$ the floor is equal to the reserve portfolio value. The analysis is presented in terms of the guarantees produced by 10 year TDFs starting from September 2007 through December 2014.

Figure 6 presents the guarantees of TDFs plotted against start date of the fund. The green line represents the European equity market, which stays well above 100 at all times, supporting that this strategy allows Best Pensions to give a guarantee of 0% at least historically. It fluctuates a fair amount until 2012 from when it declines monotonically. This can be explained by the increasing prices in zero-coupon bonds equivalently lower risk-free rates. This reduces the initial guarantees as seen in the blue line.

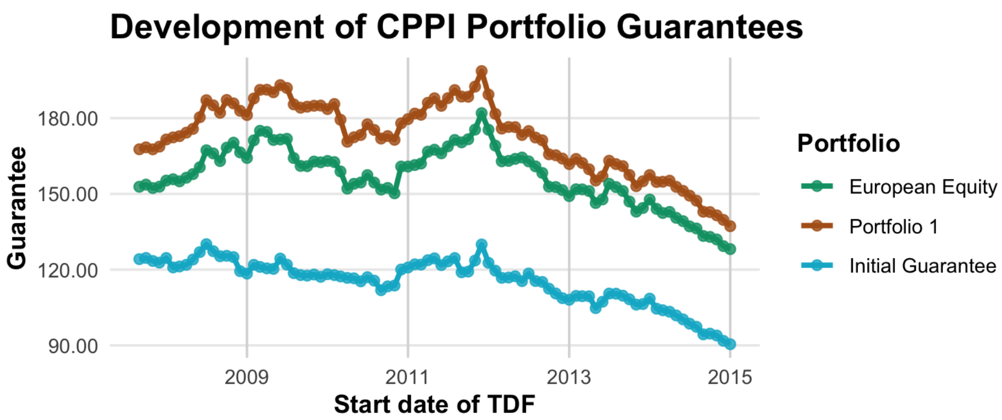
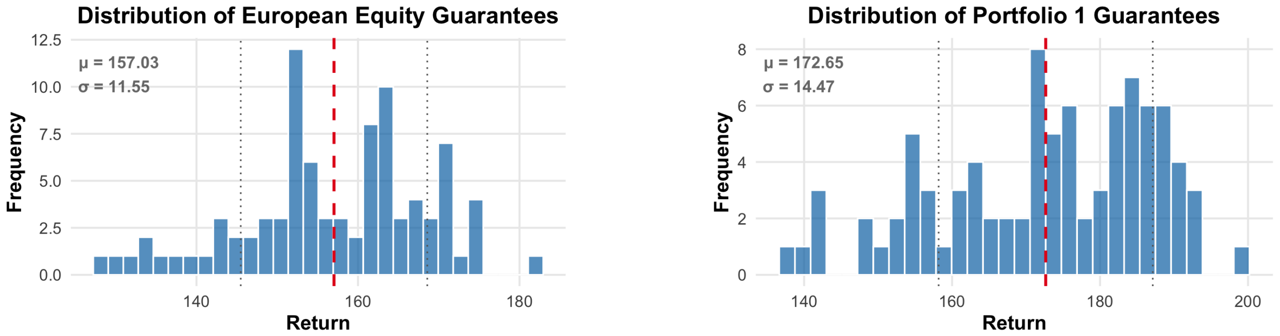


Figure 6: *Development of guarantees generated by two different active assets for the CPPI approach. The baseline case of European equity is given as the green line, Portfolio 1 as the brown line and for comparison the blue line is the initial guarantee, G_0, for the TDF starting at that date.*

Figure 6 also shows that for this particular period with the choice of parameters that Portfolio 1 performs better than the European equity market. This was expected since Portfolio 1 is an optimizing strategy on a broader market that includes European equity.

It is interesting to analyze the behavior as a function of time, but if we naively consider each TDF to be different observations coming from the same pool of TDFs and guarantees, then we could analyze a histogram showing the distribution of accrual guarantees.



(a) *Empirical distribution of accrual guarantees of European Equity as active asset.*

(b) *Empirical distribution of accrual guarantees of Portfolio 1 as active asset.*

Figure 7: *Histograms of empirical distribution of accrual guarantees for the baseline case and Portfolio 1. Mean (μ) and standard deviation (σ) is annotated in the upper left corner. The mean is plotted as a red vertical dashed line with a dotted line to each side indicating the range of being one standard deviation from the mean.*

Figures 7a and 7b provides the necessary comparison of the baseline case and the case of Portfolio 1 as the active asset. Both distributions are slightly skewed towards higher returns than their means, and Portfolio 1 has the highest mean return. However, it also has the highest volatility, but by revisiting Figure 6 one can verify that Portfolio 1 guarantees are never below European equity guarantees in this sample period. This suggests that the increased volatility is a consequence of being able to hit higher returns, while sometimes, but less often, the guarantee can drop to the one from the European equity.

In summary, Portfolio 1 continues to provide good results superior to general European equity, but the superiority in the analyses from previous sections is not necessarily transferrable to CPPI superiority. Therefore, we seek to confirm that Portfolio 1 still remains as the best portfolio in the CPPI setup, and we seek to enhance its performance even more by possibly changing CPPI parameters. However, this is not done without thought and will be analyzed thoroughly in the sequel.

4.2 Choice of Multiplier, Trigger and Target Levels

It is preferable to analyze the problem for all three parameters simultaneously, but we did not find this feasible. Instead we adopted an approach of splitting the problem into three sub-problems, and analyze each sub-problem individually: **1)** Multiplier optimization, where we disregard the effect of the target and trigger levels, **2)** target level optimization and lastly, **3)** trigger level optimization.

The following parameter values was analyzed: A multiplier from 1 to 20, with increments of 0.5, target levels between $105 - 200\%$ and trigger levels between $110 - 230\%$. The analysis is based on the out-of-sample returns we achieved using our recommended investment strategy.

4.2.1 Multiplier optimization

The first step in the optimization process is the selection of the multiplier. The multiplier determines our exposure to equity, and thus, a higher multiplier increases the potential for larger gains in the market, and thus larger guarantees for the insured do to the tie-in process. However, increasing equity exposure also increases portfolio volatility.

To balance the opportunities and risk we adopt two selection criteria. The first criterion is the shortfall probability, which we define as the probability that the wealth of the portfolio falls below the floor. In theory, the wealth would never fall below the floor, but only hit the floor, however, due to the discrete nature of the analysis it might happen in this case, but we regard it as the same. We want to select a multiplier that is within an acceptable range, which we choose to be all multipliers for which the shortfall probability lies below 1%. Secondly, we will look at the minimum guarantee achieved for all TDFs in the sample period. This ensures that Best Pensions are still able to uphold their current guarantee in the new system.

The shortfall probability is estimated as the proportion of TDFs that resulted in the portfolio hitting the floor for each multiplier, target level and trigger level. To isolate the marginal effect of the multiplier, we average the outcomes over all target and trigger levels.

The estimated shortfall probabilities and the minimum guarantees are illustrated in Figure 8. The shortfall probability is low for low values of m, but for $m > 7$ we see a sharp increase until approximately 30%. From the shortfall probability, we achieve an acceptable range for the multiplier between $1 - 7$.

The minimum guarantee is observed to be highest for multipliers between $2 - 2.5$. As this is within our acceptable range and we want to achieve as high an exposure as possible to the market, we recommend a multiplier of 2.5.

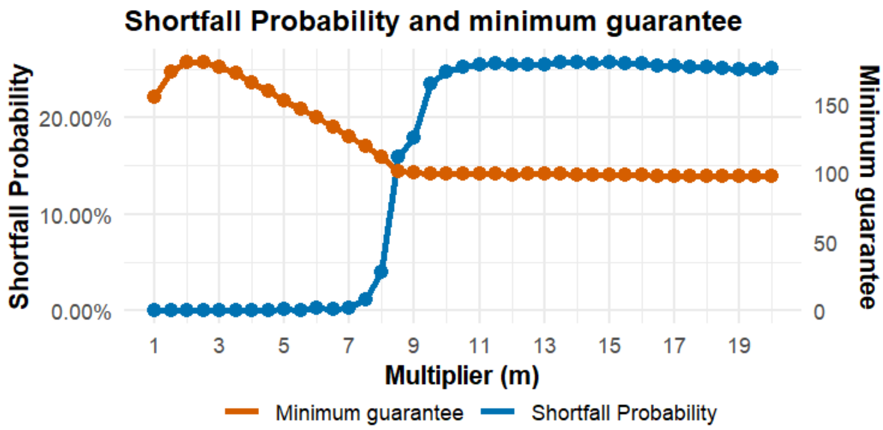


Figure 8: *Shortfall probability and the minimum guarantee (€) for different multipliers. The shortfall probability is defined as the probability that the wealth falls below the floor in the CPPI strategy.*

4.2.2 Target level optimization

Conditional upon the chosen multiplier of 2.5 we now analyse the effect of the target level. Given the primary objective of upholding the current guarantee of 0% return on the premiums, we will employ a conservative approach. The selection is done by focusing on the minimum initial guarantee, to ensure the guarantee is mostly funded from the start date. A low initial guarantee would expose Best Pensions to significant liability, as a poor market period early on in the TDFs lifecycle would result in the wealth of the portfolio hitting the floor before gains from the active portfolio can increase the guarantee. In these cases Best Pensions is not able to uphold the guarantee. Furthermore, we focus on the safety rating, which we define as the percentage of the TDFs with initial guarantee of 100 EUR, or greater. This ensures that Best Pensions can uphold their guarantee at the beginning of the TDFs in most cases.

To select the optimal target level we require that the minimum initial guarantee should be above 85 EUR. Table 12 shows that this is met for a target level below 130%. Additionally we want a safety rating above 90%, which holds for target level below 127%. From this analysis we recommend a target level of 127%, which has a safety rating of 91% and a minimum initial guarantee of 89 EUR.

Target	Safety rating (%)	Minimum initial guarantee (€)
113	100	100
125	92.1	90.4
127	91	89
130	87.6	87
163	0	69.4

Table 12: *Safety rating and minimum initial guarantee for different target levels. Safety rating is the percentage of the initial guarantees that lies above 100 EUR.*

4.2.3 Trigger Level

Following the selection of an optimal multiplier $m = 2.5$, and an optimal target level $L_{\text{Target}} = 127\%$ we proceed to analyze the effect of the trigger level. For this analysis we have used the frequency of rebalancing (tie-ins) and the terminal wealth at the end of the TDFs as performance metrics.

The gap between the trigger and target levels dictates the frequency of tie-ins. A trigger level close to the target level results in frequent tie-ins and thus secures the gains against market dips often. Conversely, a high trigger level reduces the frequency of tie-ins. This maintains a larger allocation of the wealth in the active portfolio, thereby preserving the potential for future gains. To balance these trade-offs we propose that an appropriate tie-in frequency is between $6 - 10$ for a 10-year TDF. Which corresponds to locking in the gains, on average, slightly more than once every 2 years.

Figure 9 presents the median number of tie-ins as a function of the trigger level. As expected, there is an inverse relationship, as the trigger level rises, the tie-in frequency falls.

Based on the criterion of 6-10 tie-ins in a 10-year TDF, a suitable trigger level would lie between $132 - 136\%$.

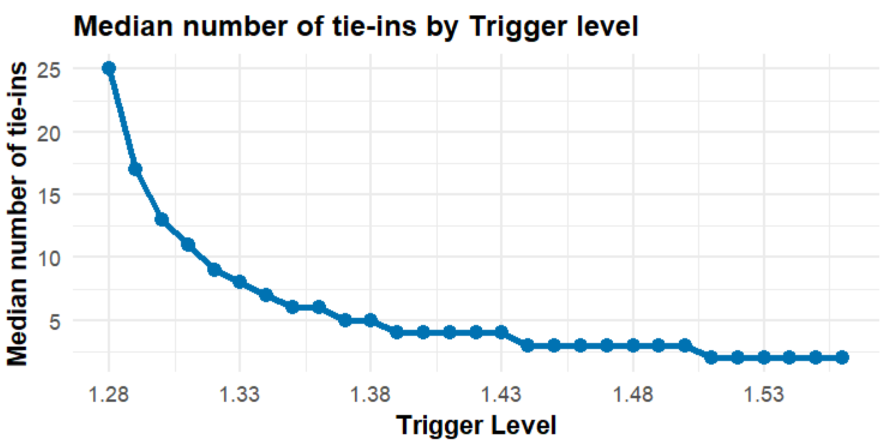


Figure 9: *Median frequency of tie-ins through all TDFs in the period for different trigger levels.*

To select the optimal trigger level from the acceptable range, the impact on the terminal wealth was analyzed. The analysis demonstrated that the effect on the terminal wealth for these trigger levels were negligible. Given this, we recommend using the median value, that is, a trigger level of 134%.

4.3 Recommendations for CPPI

Based on the sequential analysis of the multiplier, target level, and trigger level, the recommended parameters for the CPPI strategy are given in Table 13. This parameter configuration is designed to provide an optimal balance between the risk of the investment strategy and the opportunity of future gains. The most significant deviation from the baseline case is the recommended multiplier of 2.5. This change increases the proportion of wealth invested in the active asset, which is the primary driver for the increase in terminal wealth.

m	Target	Trigger
2.5	127%	134%

Table 13: *Recommended parameters to use in the CPPI strategy.*

Figure 10 presents the terminal guarantees achieved by our recommended CPPI strategy. The results shows that the proposed CPPI strategy yields a higher return across all TDFs compared to the current system. Our analysis demonstrates that the CPPI approach leads to an increase in the terminal guarantee for the insured. We therefore recommend that Best Pensions adopt this strategy going forward.

For completeness, we also calculated the guarantees from the recommended CPPI strategy, where the investment strategy used for the active asset was exchanged for another of the previously analyzed investment strategies. We found that the guarantees from the risk parity strategy were almost on par with Portfolio 1, and that all other strategies had sub-par guarantees. This led us to proceed with the recommendation of Portfolio 1.

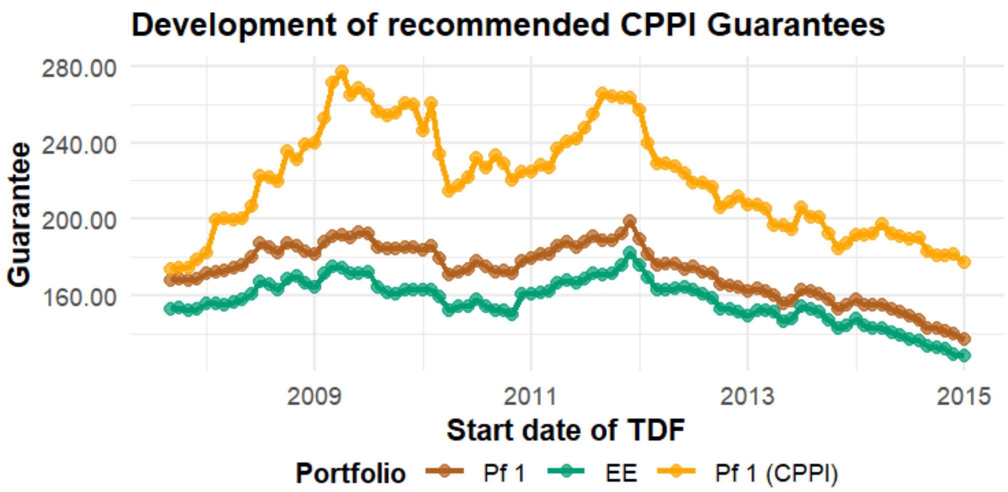


Figure 10: *Comparison of guarantees (€) with and without using the CPPI approach. The CPPI strategy was carried out with a multiplier of 2.5, target level of 127%, and trigger level of 134%.*

4.4 Impacts of Quantitative Easening

As a final remark, we note that the sample period was heavily influenced by *Quantitative Easing* (QE), which significantly lowered the risk-free rate and may limit the generalization of our results to more typical market conditions.

Between 2009 and 2021, QE policies kept risk-free rates artificially low, leading to inflated zero-coupon bond prices. In the CPPI framework, this translates into a lower initial guarantee, as illustrated in Figure 6. Consequently, the guarantee becomes more expensive to maintain, requiring stronger growth to reach the same terminal level as under normal rate conditions. This dynamic makes achieving even a 0% guaranteed return more difficult during periods of suppressed interest rates. As a result, our estimated parameters, particularly L_{target} and L_{trigger}, are most likely biased downward relative to what might be optimal in a more typical rate environment.

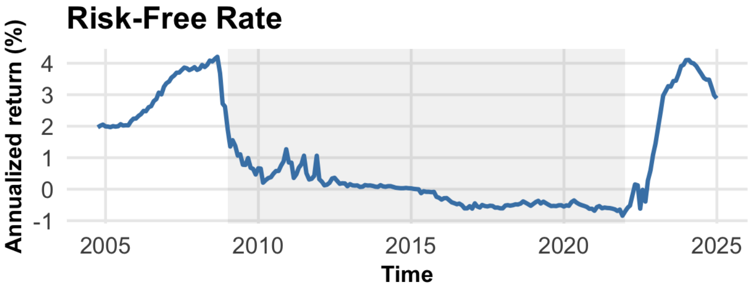


Figure 11: *Risk-free rate based on Svensson's model with empirically estimated parameters. Values are given on a monthly basis and annualized. The shaded area marks the period of Quantitative Easening.*

Figure 11 clearly shows the manipulation in the risk-free rate during the QE period, emphasizing that our sample is affected by this anomaly. However, recent data from 2022–2025 indicate a normalization toward higher and more volatile rates, which we consider more representative of future conditions.

To account for the QE bias, we conservatively adjust our parameter recommendations upward, as summarized in Table 14.

m	Target	Trigger
2.5	130%	137%

Table 14: *Recommended parameters to use in the CPPI strategy after correcting for QE bias.*

5 Conclusion

Following external criticism, Best Pensions became more open to investment approaches with greater equity exposure. However, the company also emphasized its commitment to maintaining portfolio guarantees, reflecting a strong belief in their long-term value.

To balance these objectives, the analysis focused on factor investing, examining the market, size, and momentum factors across both the US and European markets. Given the restriction against shortselling, the factors were reformulated into investable long-only strategies, all deemed relevant for Best Pensions. The resulting return series were then applied in an out-of-sample analysis to determine optimal portfolio allocations using both mean-variance optimization and risk parity.

The findings showed that a slightly more risk-oriented allocation than the global minimum-variance (GMV) portfolio, referred to as *Portfolio 1*, was best suited for Best Pensions. To evaluate the guarantee development alongside the equity allocation, the CPPI framework was applied, demonstrating that Portfolio 1 performed strongly while maintaining the guarantee effectively.

Based on these results, we recommend the following CPPI parameters for Best Pensions: $m = 2.5$, $L_{\text{target}} = 130\%$, and $L_{\text{trigger}} = 137\%$, adjusted to account for distortions in the risk-free rate caused by Quantitative Easening during the sample period, with Portfolio 1 serving as the active asset

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