Recursion

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Recursion is a technique for solving a computational problem where the final solution to the problem is constructed using the solutions of smaller subproblems, obtained recursively.

Factorial

For a non-negative integer n, we define $n!$ (read as n factorial) as:

$$n! = 1 \times 2 \times \ldots \times n$$

Factorial can also be defined recursively as:

$$n! = \begin{cases} 1 & \text{if } n = 0, 1 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Expressing using functions we obtain:

$$f(n) = \begin{cases} 1 & \text{if } n = 0, 1 \\ n \cdot f(n-1) & \text{otherwise} \end{cases}$$

Recursive code

$$f(n) = \begin{cases} 1 & \text{if } n = 0, 1 \\ n \cdot f(n-1) & \text{otherwise} \end{cases}$$

```java
public class Factorial {
   public static long factorial(int n) {
      if( n < 0 )
         throw new IllegalArgumentException("n must non-negative!");
      else if( n == 0 || n == 1 ) // base cases
         return 1;
      else
         return n * factorial(n-1); // recursive call
   }

   public static void main(String[] args) {
      System.out.println( factorial(5) );
   }
}
```

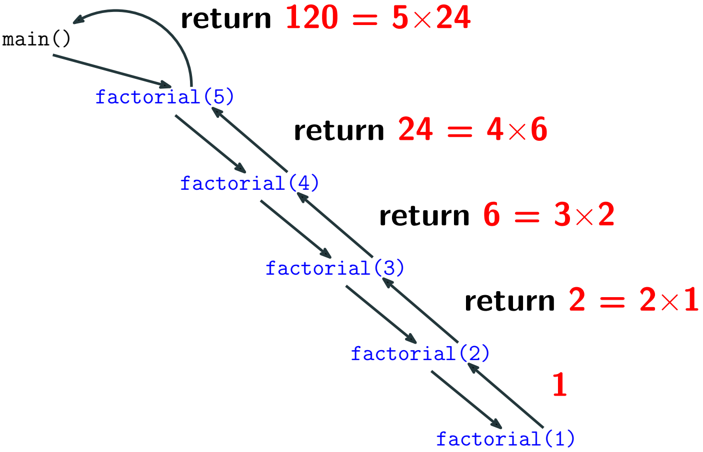
Recursive code

```
public class Factorial {
   public static long factorial(int n) {
      if( n < 0 )
         throw new IllegalArgumentException("n must non-negative!");
      else if( n == 0 || n == 1 ) // base cases
         return 1;
      else
         return n * factorial(n-1); // recursive call
   }

   public static void main(String[] args) {
      System.out.println( factorial(5) );
   }
}
```

Every recursive method contains the following two things:

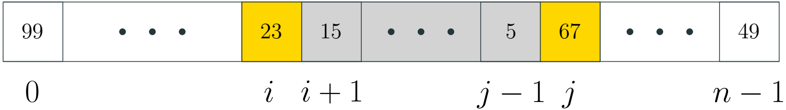
1. **Base case(s)**. the case(s) for which we know how to calculate the answer without recursion; at least one base case is always required; every possible chain of recursive calls must eventually reach a base case.
2. **Recursive call(s)**. these are the calls to the current method. Each recursive call should be defined so that it makes progress towards a base case.

Illustration

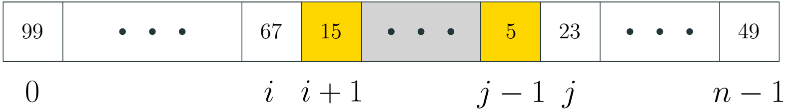


☞ The system uses a **stack** in the background to run recursive code

How to recursively reverse the subarray that starts at index i and ends at index j?



Swap $A[i]$ and $A[j]$



Reverse the subarray $A[i+1 \ldots j-1]$

Reversing an array

```java
import java.util.Arrays;

public class ReverseArray{
   public static void reverseArray(int[] A, int i, int j) {
        if (i > j)
          throw new IllegalArgumentException("i <= j is required.");

        int hold = A[i];
        A[i] = A[j];
        A[j] = hold;

        if( i + 1 < j - 1 )
           reverseArray(A, i + 1, j - 1); // recursive call
   }

   public static void main(String[] args) {
      int[] arr = {10, 20, 30, 40, 50};
      reverseArray(arr,0,arr.length-1);
      System.out.print(Arrays.toString(arr));
   }
}
```

Summing up an array

Recursive idea

To add the numbers inside the subarray $A[0]$ to $A[i]$, first **recursively** add the numbers inside the subarray $A[0]$ to $A[i-1]$ and then add the number $A[i]$ to the result.

```java
public class ArraySummer {

   public static int add(int[] A, int i) {
      if ( i < 0 )
         throw new IllegalArgumentException("i should be non-negative.");
      else if( i == 0 )
         return A[0];
      else
         return add(A, i-1) + A[i]; // recursive call
   }

   public static void main(String[] args) {
      int[] arr = {10, 20, 30, 40, 50};
      System.out.print( add(arr,arr.length-1) );
   }
}
```

Binary search

- Given a **sorted** array A of n items, how fast can you search a given element?
- One can search by scanning A from left to right (**linear search**), but this takes $O(n)$ time
- Can we do it faster? Use the fact that the array is already sorted
- Yes, we can using binary search; runs in $O(\log n)$ time

Recursive algorithm (asssumption: A is sorted)

- If the target equals $A[\text{mid}]$, then we have found the target!
- If the target is less than $A[\text{mid}]$, search recursively in the left half
- Othewise, search recursively in the right half

2	4	5	7	8	9	12	**14**	17	19	22	25	27	28	33	37
low							mid								high

2	4	5	7	8	9	12	14	17	19	22	**25**	27	28	33	37
								low			mid				high

2	4	5	7	8	9	12	14	17	**19**	22	25	27	28	33	37
								low	mid	high					

2	4	5	7	8	9	12	14	17	19	**22**	25	27	28	33	37
										low = mid = high					

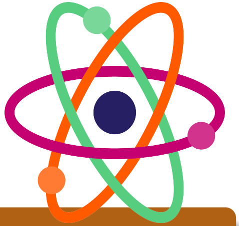
Searching for **22** in the array

Code

```java
public class BinarySearch {
   public static boolean binarySearchRec(int[] A, int target, int low, int high) {
      if( low > high )
         return false;
      else {
         int mid = (low + high) / 2; // mid takes the floor of (low + high) / 2
         if( target == A[mid] )         return true;
         else if( target < A[mid] )     return binarySearchRec(A, target, low, mid - 1 );  // recursive call
         else                           return binarySearchRec(A, target, mid + 1, high ); // recursive call
      }
   }

   public static boolean binarySearch(int[] A, int target) {
      return binarySearchRec(A, target, 0, A.length-1);
   }

   public static void main(String[] args) {
      int[] A = {2,4,5,7,8,9,12,14,17,19,22,25,27,28,33,37};
      System.out.println(binarySearch(A,22)); // prints true; search successful
      System.out.println(binarySearch(A,21)); // prints false; search unsuccessful
   }
}
```

Time complexity

- At every recursive call, we discard approximately half of the array
- Also, at every recursive call, we do constant amount of work – $O(1)$
- Let m be the number of recursive calls made
- At every recursive call, array size gets halved
- After m recursive calls, array size equals $n/2^m$
- In the worst case, we stop when $n/2^m = 1 \implies 2^m = n$
- Taking log of both sides we obtain, $m = \log_2 n = O(\log n)$
- **Time complexity.** $O(\log n) \times O(1) = O(\log n)$



Fun fact

Number of atoms in this universe: $10^{80} \approx 2^{266}$

Even if we have a dataset as large as this, binary search will make just $\log(2^{266}) = 266 \cdot \log_2 2 = 266 \cdot 1 = 266$ recursive calls in the worst case!

Suggested excercise

Write a non-recursive (iterative) binary search

For a given value of n, we need to print a string made up of $n-1$ **comp**s, **computing**, and $n-1$ **ting**s; here are few examples for you...

n	**Output**
1	`computing`
2	`compcomputingting`
3	`compcompcomputingtingting`
4	`compcompcompcomputingtingtingting`
5	`compcompcompcompcomputingtingtingtingting`

Code

```java
public class RecursiveStringPrinter {

   public static String printer(int n) {
      if( n <= 0)      return null;
      else if( n == 1) return "computing";
      else             return "comp" + printer(n-1) + "ting";
   }

   public static void main(String[] args) {
      System.out.print(printer(5));
   }
}
```

Self-referential classes

```java
private static class Node<E> {
   private E element;
   private Node<E> prev, next; // defined recursively

   // ...
}
```

A **self-referential class** contains an instance variable that refers to another object of the same class type

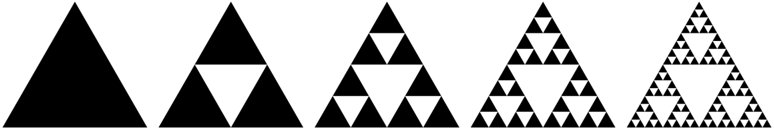
Using recursion for linked-lists

```java
public class DoublyLinkedList<E> implements Iterable<E>{
   // other methods, variables, classes
   public String print() {
      return (printRecursive(head)).toString();
   }

   private StringBuilder printRecursive(Node<E> n) {
      if( n == null )
         return new StringBuilder();

      StringBuilder s = new StringBuilder(n.element.toString() + " ");
      s.append(printRecursive(n.next));
      return s;
   }
   // other methods, variables, classes
}
```

Fractals

What are fractals? `https://en.wikipedia.org/wiki/Fractal`

Fascinating geometric figures that can be drawn recursively



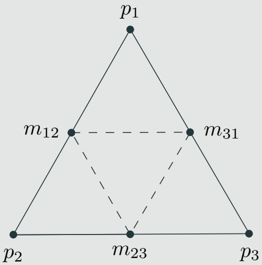
Sierpiński triangle (source: Wikipedia)

Wacław Sierpiński (source: Wikipedia)

Pseudo-code

Sierpiński triangle (source: Wikipedia)

```
private static void drawTriangles(Graphics g, int d, Point p1, Point p2, Point p3) {
   if (d == 0) {  // depth is 0, draw the triangle; base case
      Polygon P = new Polygon();
      P.addPoint(p1.x,p1.y); P.addPoint(p2.x,p2.y); P.addPoint(p3.x,p3.y);
      g.fillPolygon(P); // draws a filled triangle
      return;
   }

   Point m12 = midpoint(p1,p2);
   Point m23 = midpoint(p2,p3);
   Point m31 = midpoint(p3,p1);

   // Draw 3 Sierpinski triangles recursively of depth d-1
   drawTriangles(g, d - 1, p1, m12, m31); // recursive call 1
   drawTriangles(g, d - 1, m12, p2, m23); // recursive call 2
   drawTriangles(g, d - 1, m31, m23, p3); // recursive call 3
}
```



Merge sort

Merge sort

- Merge sort runs in $O(n \log n)$ time
- It uses a linear-time algorithm known as **merging** for sorting the input
- Let us begin by understanding what is meant by merging two sequences ...

Given two **sorted** sequences S_1, S_2, how fast can you **merge** them into one final sorted sequence S?

S_1	244	311	478					
S_2	324	415	499	505	666			
S	244	311	324	415	478	499	505	666

Assume that S_1 has k_1 elements and S_2 has k_2 elements
Clearly, S has $k_1 + k_2$ elements

We need to do it in $O(k_1 + k_2)$ time

S_1	244	311	478					
S_2	324	415	499	505	666			
S								

Merging two sorted sequences

S_1	<u>244</u>	311	478					
S_2	<u>324</u>	415	499	505	666			
S								

Merging two sorted sequences

S_1	244	311	478					
S_2	324	415	499	505	666			
S	244							

Merging two sorted sequences

S_1	244	311	<u>478</u>					
S_2	<u>324</u>	415	499	505	666			
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Merging two sorted sequences

S_1	244	311	<u>478</u>					
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Merging two sorted sequences

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Merging two sorted sequences

S_1	244	311	478					
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S	244	311	324	415	478	499	505	666

Merging takes time proportional to the total number of blue cursors movements, and they move $k_1 + k_2$ times to the right in total. Further, at each cursor movement, we spend $O(1)$ time for comparing two elements, sending an element to S, and incrementing a blue pointer.

So, the time complexity amounts to $(k_1 + k_2) \times O(1) = O(k_1 + k_2)$

Merge sort

- It is a recursive **divide and conquer** sorting algorithm
- Runs in $O(n \log n)$ time (faster than Insertion, Bubble, and Selection sorts)

The algorithm

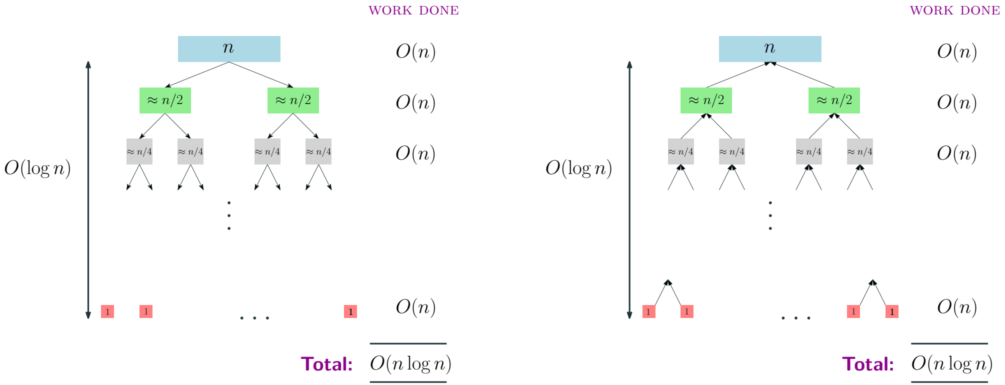
Let the input be denoted by S

1. **Divide**. **Split** the array into two halves S_1, S_2
2. **Conquer**.
    1. **Recursively** sort the left half S_1
    2. **Recursively** sort the right half S_2
3. **Combine**. **Merge** the two sorted halves S_1, S_2 into S

`https://opendsa-server.cs.vt.edu/embed/mergesortAV`

Try: **`85 24 63 45 17 31 96 50 67 88 11`**

Time complexity of merge sort



Left: total time spent to create the two halves S_1, S_2 at every recursive call;
Right: total time spent for merging.

☞ Merge sort runs in $O(n \log n) + O(n \log n) = 2 \times O(n \log n) = O(n \log n)$ time

Space complexity

Space complexity

Space complexity of an algorithm refers to the amount extra space the algorithm needs (apart from the input) for its execution.

- To find space complexity, focus on the additional defined data structures (arrays, stacks, queues, lists, etc.) whose sizes are dependent on n. For recursive code, also consider the stack depth of the call stack.
- Count the total number of data elements stored in those data structures in the worst case
- Let s be total number of such data elements
- Space complexity is $O(s)$
- If no such data structures are used, space complexity is $O(1)$ (constant amount of extra space is used)

- The space complexity of the `ExpresssionChecker` implementation is $O(n)$ where n is the number of symbols since it uses a stack whose size is n in the worst case
- The space complexity of bubble sort/insertion sort/selection sort is $O(1)$ since they use just a constant amount of extra space (size independent of n) for maintaining a bunch of variables
- Let us say a method uses a doubly linked list having at most n nodes and a bunch of variables for processing. The space complexity of the method is $O(n)$
- If a method uses a linked list of size n and a $n \times n$ matrix of size n^2. The space complexity of the method amounts to $O(n) + O(n^2) = O(n^2)$

Space complexity of merge sort

Merge sort

- For creating the two subsequences S_1, S_2 we need $O(n/2) + O(n/2) = O(n)$ extra space.
- So, the total amount of extra space needed by a series of recursive call from the root to a leaf of the recursion tree also amounts to $O(n)$ since

$$O(n)+O\left(\frac{n}{2}\right)+O\left(\frac{n}{4}\right)+\ldots+O(1)=O\left(n+\frac{n}{2}+\frac{n}{4}+\ldots+1\right)=O(2n)=O(n)$$

- For recursion, a stack is needed of size $O(\log n)$
- Total space complexity: $O(n)+O(\log n)=O(n)$

The `Comparable` interface in Java

https://docs.oracle.com/en/java/javase/17/docs/api/java.base/java/lang/Comparable.html

Why you should use the `Comparable` interface?

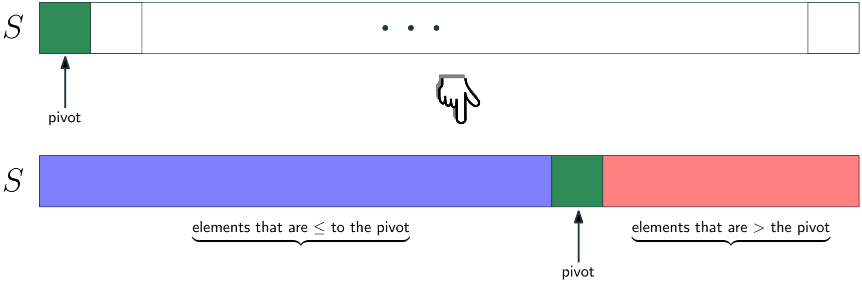
If we ever need to compare two objects of a class, there must be a comparison method for the class. This interface forces the class to define such a method if it is not already defined inside it. For the wrapper classes such as **`Integer`**, **`Double`**, **`Character`** etc. comparison methods are already defined. Generic sorting methods in Java use the comparison method for sorting by comparisons.

☞ The comparison method must be named **`compareTo`**, as declared inside the **`Comparable`** interface

☞ `obj1.compareTo(obj2) < 0` if `obj1` is **less than** `obj2`;
`obj1.compareTo(obj2) == 0` if `obj1` is **equals** `obj2`;
`obj1.compareTo(obj2) > 0` if `obj1` is **greater than** `obj2`;

See the class `MergeSort`

Quick sort



The algorithm

Denoted the input by S

1. Select the first element in S; call it **pivot**
2. Find the elements in S that are less than equal to pivot and send them to the left part of S and the ones that are greater than the pivot to the right part of S
3. Put the pivot at the appropriate location in S, meaning put it at the location where it would appear if S is sorted

An example

first								last
44	75	23	43	55	12	64	77	33
up								down

44	75	23	43	55	12	64	77	33
	up							down

44	33	23	43	55	12	64	77	75
				up	down			

44	33	23	43	12	55	64	77	75
				down	up			

12	33	23	43	44	55	64	77	75
				down				

Pseudocode

1. $\texttt{pivot} = S[\texttt{first}]$, $\quad \texttt{up} = \texttt{first}$, $\quad \texttt{down} = \texttt{last}$
2. **do**
   - 2.1 Increment $\texttt{up}$ until $\texttt{up}$ selects the first element greater than the pivot value or $\texttt{up}$ has reached $\texttt{last}$
   - 2.2 Decrement $\texttt{down}$ untill $\texttt{down}$ selects the first element less than or equal to the pivot value or $\texttt{down}$ has reached $\texttt{first}$
   - 2.3 if $\texttt{up} < \texttt{down}$, exchange $S[\texttt{up}]$ and $S[\texttt{down}]$
3. **while** $\texttt{up}$ is to the left of $\texttt{down}$
4. Exchange $S[\texttt{first}]$ and $S[\texttt{down}]$

Partition

```java
private static <K extends Comparable<K>> void swapTheItemsAt(K[] S, int i, int j) {
   K hold = S[i];
   S[i] = S[j];
   S[j] = hold;
}

private static <K extends Comparable<K>> int partition(K[] S, int first, int last) {
   K pivot = S[first];
   int up = first, down  = last;

   do {
      while( (up < last) && (pivot.compareTo(S[up]) >= 0))
         up++;

      while( pivot.compareTo(S[down]) < 0)
         down--;

      if( up < down )
         swapTheItemsAt(S,up,down);

   }while(up < down);

   swapTheItemsAt(S,first,down);
   return down;
}
```

Quick sort

- It is another divide and conquer sorting algorithm
- Runs in $O(n^2)$ time (explained next)

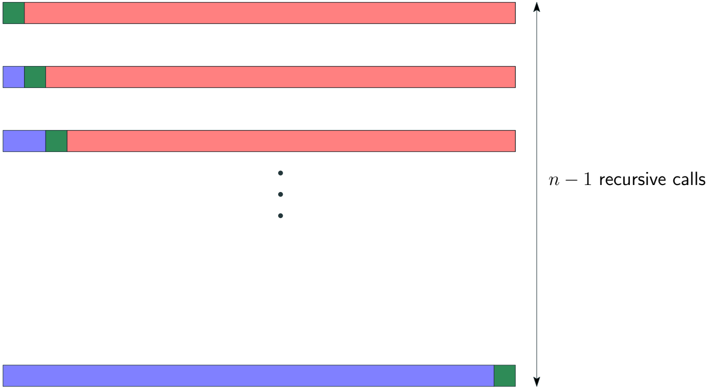
The algorithm

Let the input be denoted by $S[\texttt{first}, \ldots, \texttt{last}]$

1. **Divide**. **Partition** the array so that the pivot item reaches its correct place in the array (its index is `pivIndex`)
2. **Conquer**.
   1. **Recursively** sort the subarray $\texttt{first}, \ldots, \texttt{pivIndex-1}$ (the subarray to the left of pivot)
   2. **Recursively** sort the subarray $\texttt{pivIndex+1}, \ldots, \texttt{last}$ (the subarray to the right of pivot)

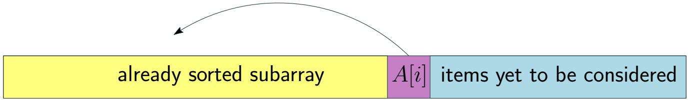
See the class `QuickSort`

Time and space complexities of quick sort



- When the array is sorted, at every recursive call, we find that all the other elements are bigger than the pivot! This is the worst case in fact
- So, we make $n-1=O(n)$ recursive calls; we spend $O(n)$ time for paritioning at every level
- Total time taken $O(n^2)$
- Space complexity: $O(n)$ since recursion depth can be at most $n-1$

Speed comparison



Insertion sort, needed for comparison; runs in $O(n^2)$ time

Input: An array A of n comparable elements

for $i = 1$ to $n - 1$ do

  Insert $A[i]$ at the proper spot within the sorted subarray $A[0], A[1], \ldots, A[i]$;

☞ If A is already sorted then every $A[i]$ is already in its correct position. As a result, every iteration of the for-loop runs in $O(1)$ time. Consequently, insertion sort takes $O(n)$ time when A is already sorted.

Applet. `https://visualgo.net/en/sorting`

Now see the class `SortingSpeedComparison`

Quick sort vs Merge sort, output in some run, $n = 1M$

```
n = 1,000,000
QuickSort (O(n^2)): 624 ms
MergeSort (O(n*log n)): 1264 ms
```

Quick sort ***beats*** *merge sort in practice on randomly ordered arrays despite having worse time complexity!*

Merge sort vs. Quick sort vs. Insertion sort

```
n  = 100,000
QuickSort (O(n^2)): 85 ms
MergeSort (O(n log n)): 124 ms
InsertionSort (O(n^2)): 19002 ms
```

Insertion sort is unusable in general when n is large. However, if the input was already sorted, it could finish in just 10 ms when $n = 100K$. In such cases, every iteration of the for-loop takes $O(1)$ time since $A[i]$ is already at its correct spot! As result, insertion sort takes $O(n)$ time on sorted inputs.

Quick sort performs terribly when the input is already sorted!

Output, $n = 10K$

```
n = 10,000
QuickSort (O(n^2)) on a random array: 6 ms
QuickSort on a sorted array: 200 ms
```

☞ *When the input is sorted, quick sort runs in quadratic time*

What happens when the input size is 50,000?

```
n = 50,000
Exception in thread "main" java.lang.StackOverflowError
  at recursion.QuickSort.recurseAndSort(QuickSort.java:12)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  at recursion.QuickSort.recurseAndSort(QuickSort.java:13)
  .
  .
  .
```

☞ *The call stack runs out of space since quick sort strives to make around $50K$ calls in this case!*

Avoiding `StackOverflowError` exception

How to avoid `StackOverflowError` exception?

- Make quick sort non-recursive by using a stack explicitly
- ```
  LinkedStack<Pair> stack = new LinkedStack<>();
  stack.push( new Pair(0,S.length-1) );

  while( !stack.isEmpty() ) {
     var currentPair = stack.pop();

     if( currentPair.leftIndex >= currentPair.rightIndex ) continue;

     int pivotIndex = partition(S, currentPair.leftIndex, currentPair.rightIndex);
     stack.push( new Pair(currentPair.leftIndex, pivotIndex - 1) );
     stack.push( new Pair(pivotIndex+1, currentPair.rightIndex) );
  }
  ```
- See the class `NonRecursiveQuickSort`
- The non-recursive version is still **slow** on **sorted** inputs but unlike the recursive version, it doesn't crash by throwing a `StackOverflowError` exception
- Now, `NonRecursiveQuickSort` can sort a $50K$-sized **sorted** input in 2608 ms
  ```

How to ‘almost’ avoid quadratic runtime in practice?

Speeding up quick sort in practice

- Quick sort slows down on sorted inputs because of the bad pivots which generate empty left chunks (all the other elements goes to the right chunks)

☞ **Idea.** choose pivots **randomly** instead of sticking to the first element every time

- It will be then **unlikely** a bad pivot is chosen every time a partition is executed
- It can be shown theoretically that this small change will result in $O(n \log n)$ behavior in practice (proof is out of scope)
- See the class `RandomizedQuicksort` (**recursive**)
-
```
Random generator = new Random();
int pivotIndex =  generator.nextInt(first, last + 1); // generate a random index for pivot selection
K pivot = S[pivotIndex];
```
- The randomized **recursive** version can sort a $50K$-sized sorted input in well under 30 ms (previously it took 2608 ms!)

Avoiding `StackOverflowError` exceptions

- Although very unlikely, bad pivots can still be chosen resulting in `StackOverflowError` exceptions since `RandomizedQuicksort` is recursive
- **Solution.** make it non-recursive
- See the class `NonRandomizedQuicksort`
- No more stack overflows and painful slowdowns on sorted datasets are unlikely!

Making quick sort faster in practice

Median of three heuristic

Use the median of the three items `S[first]`, `S[(first+last)/2]`, `S[last]` as the pivot. In this case, median is the second item of the sorted sequence of the above three items.

- In the randomized version, we choose pivots randomly
- Finding a pivot using random number generator is **slower** than computing the median of the above three items since only comparison operators and swapping can be used to select the median

```
int mid = (first + last)/2;
if (S[last].compareTo( S[first] ) < 0)   swapTheItemsAt(S, first, last);
if (S[mid].compareTo( S[first]) < 0)    swapTheItemsAt(S, mid, first);
if (S[last].compareTo( S[mid]) < 0 )    swapTheItemsAt(S, last, mid);

K pivot = S[mid]; // S[mid] is now the pivot since the median item is now present at S[mid]
```

- Also, this results in careful selection of pivots in practice
- Consequently, quick sort surprisingly faster!

Further optimization

- Insertion sort works very fast for small inputs
- We leverage insertion sort in the algorithm
- When array size is at most 50 (other small numbers may work as well), do not partition anymore, use insertion sort instead

See the class `MedOfThreeNonRecQuickSort`

Demonstration

$n = 10,000,000$, randomly generated integer array

```
n = 10,000,000
NonRecursiveRandomizedQuickSort (O(n log n) behavior expected): 6723 ms
MedOfThreeNonRecQuickSort (O(n log n) behavior expected): 4971 ms
MergeSort (O(n log n)): 5195 ms
Arrays.sort() (O(n log n)): 5435 ms
```

$n = 10,000,000$, sorted integer array

```
n = 10,000,000
NonRecursiveRandomizedQuickSort (O(n log n) behavior expected): 3654 ms
MedOfThreeNonRecQuickSort (O(n log n) behavior expected): 1498 ms
MergeSort (O(n log n)): 4411 ms
Arrays.sort() (O(n log n)): 161 ms
```

☞ Moral of the story: `Arrays.sort()` is hard to beat in general

$n = 10,000,000$, randomly generated integer array

```
n = 10,000,000
Arrays.sort() (O(n log n)): 5564 ms
Arrays.parallelSort() (O(n log n)): 488 ms
```

Suggested exercise

Implement the selection sort algorithm using the `Comparable` interface

https://en.wikipedia.org/wiki/Selection_sort

Timsort (optional, for algorithm lovers only)

Java uses **Timsort** for sorting an array of non-primitives
`https://en.wikipedia.org/wiki/Timsort`

Generic binary search

```
public class GenericBinarySearch {
   private static <K extends Comparable<K>> boolean binarySearchRec(K[] A, K target, int low, int high) {
      if( low > high )
         return false;
      else {
         int mid = (low + high) / 2; // mid takes the floor of (low + high) / 2
         if( target.equals(A[mid]) )      return true;
         else if( target.compareTo( A[mid] ) < 0 ) return binarySearchRec(A, target, low, mid - 1 ); // recursive call
         else return binarySearchRec(A, target, mid + 1, high );// recursive call
      }
   }

   public static <K extends Comparable<K>> boolean search(K[] A, K target) {
      return binarySearchRec(A, target, 0, A.length-1);
   }
}
```

The `Comparable` interface also helps us to implement a generic binary search

Recursion tips

- Make sure every chain of recursive calls eventually reach at least one base case
- Long chains of recursive calls can throw `StackOverflowError`; be careful!
- If such long chains cannot be avoided, make your code iterative (non-recursive)

https://opendsa-server.cs.vt.edu/ODSA/Books/Everything/html/RecIntro.html