Analysis of Algorithms

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Introduction

What is an algorithm?

An algorithm is a step-by-step well-defined procedure for performing some given task in a finite amount of time

What do we mean by analyzing an algorithm?

Figuring out theoretically (using math) how much **time** and **memory** it is going to take when implemented using any programming language

In this course

We will assume that an algorithm and a program written using it are equivalent from the analysis point of view

Nobody likes slow and memory-hungry softwares!

Speed

Problem

Generate **really long** strings made up of English language characters

Approach A: uses `String`

```java
public static String generateLongStringA(int length) {
   String password = "";

   for(int i = 0; i < length; i++)
      password += randomLetter();

   return password;
}
```

Approach B: uses `StringBuilder`

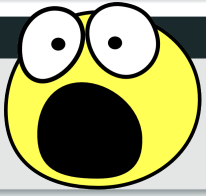
```java
public static String generateLongStringB(int length) {
   StringBuilder password = new StringBuilder();

   for(int i = 0; i < length; i++)
      password.append( randomLetter() );

   return password.toString();
}
```

Speed

```java
long startA = System.currentTimeMillis();
generateLongStringA(200000);
long timeTakenA = System.currentTimeMillis() - startA;

System.out.println("Time taken by A: " + timeTakenA + " ms" );

long startB = System.currentTimeMillis();
generateLongStringB(200000);
long timeTakenB = System.currentTimeMillis() - startB;

System.out.println("Time taken by B: " + timeTakenB + " ms" );

System.out.println("Speedup: " + (double)timeTakenA/timeTakenB);
```

Output in one run (result varies slightly every time)

```
Time taken by A: 6726 ms
Time taken by B: 33 ms
Speedup: 203.8181818181818
```



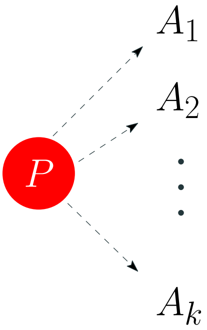
☞ **Tip**: for **`toString`** methods use **`StringBuilder`** instead of **`String`**

Reason

We will come back to the discussion of `StringBuilder` later this semester...

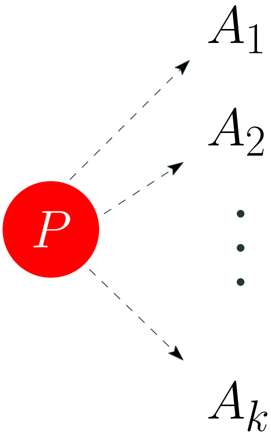
The most challenging aspect of coding

Given a computational problem P, there may exist quite a few algorithms, say, $A_1, A_2, \ldots, A_k$, for solving P. Now, which algorithm among these ones will be one of the fastest when implemented using a programming language?



A super-stressed coder!

"Gosh, which one am I going to use!"



A super-stressed coder!

“Gosh, which one am I going to use!”

An easy-peasy answer

I am going to implement all of them and find out the best ones

The real world situation

- ☞ Nobody has this amount of time unless you are doing **Algorithm Engineering and Experiments** for one problem (may even take months)
- ☞ Even if you do, which datasets are you going to use to judge the implementations? For complex algorithms, finding appropriate datasets is a hassle
- ☞ Computational experiments are always hardware and software dependent

Common concerns

- Why my program is taking so long?
- Why it is running out of memory?
- I am unsure if my program will run to completion within a reasonable time for every dataset! What should I do?

A good solution to all these problems

Theoretical approach

Use math to analyze algorithms/programs so that we can get away from time-consuming experiments

Does it work?

Yes, it does in most cases and will work everywhere in this course

What is it?

Analysis of algorithms (completely theoretical, no machines needed!)

Let a, b, n, m be real numbers. Then,

- $a^n \times a^m = a^{n+m}$
  Example: $2^3 \times 2^{10.5} = 2^{13.5}$
- $a^n / a^m = a^{n-m}$
  Example: $2^{30}/2^{10} = 2^{20}$, $6^3/6^{10} = 6^{-7}$
- $(a^n)^m = a^{nm}$
  Example: $(5^{30})^9 = 5^{270}$
- $a^0 = 1$, when $a \neq 0$
  Example: $(-27.18)^0 = 1$

The ***logarithm*** of a positive real number x with respect to base b is the exponent by which b must be raised to yield x. The logarithm of x to base b is denoted by $\log_b x$.

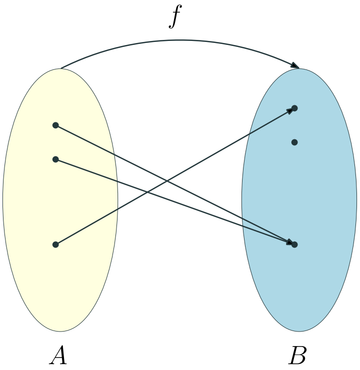
$$\log_2 1024 = 10 \text{ since } 2^{10} = 1024$$

$$\log_{10} 100 = 2 \text{ since } 10^2 = 100$$

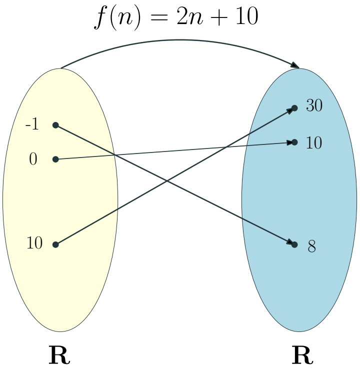
In this course, we will use **base-2** logarithms ($b = 2$) mostly

Definiton

Given two sets A, B, a function f from A to B is a rule that associates every element of A to an element of B



☞ A, B can be infinite sets



The function $f(n) = 2n + 10$ is shown pictorially

☞ R is an infinite set

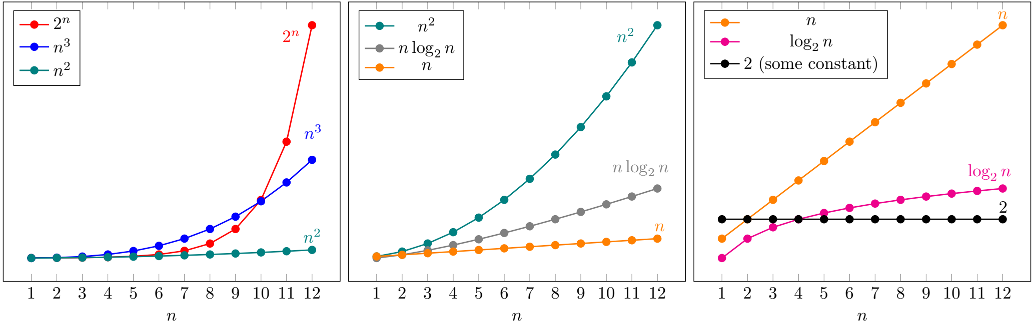
Upper bounds

Since we are interested in worst-case time analysis of algorithms, we will use functions like $T(n) \leq 2n + 10$, etc. to derive upper bounds on algorithm runtimes

The 7 types of functions we are interested in

1. **Constant.** $T(n) \leq k$, for some positive constant k
   Examples: $T(n) \leq 19.87$; $T(n) \leq 10$, etc.
2. **Logarithmic.** $T(n) \leq k \log n$, for some positive constant k
   Examples: $T(n) \leq 5 \log_2 n$; $T(n) \leq 19 \log_{10} n$, etc.
3. **Linear.** $T(n) \leq kn$, for some positive constant k
   Examples: $T(n) \leq 6n$; $T(n) \leq 2^{100}n$, etc.
4. **Linearithmic.** $T(n) \leq kn \log n$, for some positive constant k
   Examples: $T(n) \leq 100n \log_2 n$; $T(n) \leq 16n \log_{10} n$, etc.
5. **Quadratic.** $T(n) \leq kn^2$, for some positive constant k
   Examples: $T(n) \leq 100n^2$; $T(n) \leq \sqrt{2}n^2$, etc.
6. **Cubic.** $T(n) \leq kn^3$, for some positive constant k
   Examples: $T(n) \leq 8999n^3$; $T(n) \leq \sqrt{99}n^3$, etc.
7. **Exponential.** $T(n) \leq k \cdot 2^n$, for some positive constant k
   Examples: $T(n) \leq 10 \cdot 2^n$; $T(n) \leq 9^{99} \cdot 2^n$, etc.

Growth rates comparisons of the 7 functions



What are we trying to accomplish?

- Assume that we have an algorithm A
- Denote the input size by n
- Find out the function $T(n)$, such that A takes $T(n)$ units of time when executed on any input of size n
- $T(n)$ is a theoretical upper bound on the runtime; a.k.a. the **time complexity** of the algorithm A

One can also do the same for the additional space needed by A. This is known as the **space complexity** of A. We will talk about this later.

The two important aspects of an algorithm

- ☞ **Time complexity**. how much time as a function of the input size n does an algorithm takes to give the desired result? In this case, we are interested in counting the number of steps instead of real-world clock time, although these two are related.
- ☞ **Space complexity**. how much extra space (apart from the input and the expected output) as a function of the input size n does an algorithm need to get executed successfully?

Note that both are important in this age of Big Data!

Primitive operations

Definition

Basic computations performed by an algorithm which take constant amount of time on a fixed machine; such operations are largely independent from programming languages and hence can be used for theoretical analyses

Some examples

- `a = b + c;`
- `i++;`
- `arr[k] = 0;`
- `arr[k] = s[i] + t[j];`
- `Counter wallet = new Counter();`
- `return arr;`
- `boolean b = (i > j);`

  ⋮

Be careful!

Not every statement that ends with a semi-colon is a primitive operation! In that case, we need to analyze them separately.

Not primitive operations

- `Arrays.sort(...);`
- `findMax(someArray);`

  ⋮

So what?

- The **runtime** of a program *is proportional* to the number of primitive operations it has; larger input means more such operations must be executed
- Clearly, depending on n (input size), the number of such operations will vary; larger n means higher runtime
- Let $T(n)$ denote the theoretical runtime of a given algorithm (the total number of primitive operations executed by it)
- In algorithm analysis, we aim to obtain an upper-bound for $T(n)$ by estimating the number of primitive operations executing by the algorithm under investigation
- Know that $a \leq b$ is pronounced as 'a is at most b'

Example

```
1  public static double findMax(double[] array) {
2     int n = array.length;          // c_2 = 1
3     double maxSoFar = array[0];    // c_3 = 1
4
5     for(int i = 1; i < n; i++)     // c_5 = n
6        if( array[i] > maxSoFar )   // c_6 = n - 1
7           maxSoFar = array[i];     // c_7 <= n - 1
8
9     return maxSoFar;               // c_9 = 1
10 }
```

We assume that every primitive operation or a constant number of contiguous primitive operations takes 1 unit of time. Let the total cost of line i be c_i. In our case, c_i is the number of times line i is executed. Then,

$$\begin{aligned} T(n) &\leq c_2 + c_3 + c_5 + c_6 + c_7 + c_9 \\ &\leq 1 + 1 + n + (n-1) + (n-1) + 1 \\ &= 3 + 3n - 2 \\ &= 3n + 1 \leq 3n + n = 4n \end{aligned}$$

So, we conclude that $T(n) \leq 4n$.

What does this mean?

☞ It means that the worst-case wall-clock runtime $W(n)$ of this method is proportional to $4n$. In other words,

$$W(n) = m \cdot T(n) \leq m \cdot 4n,$$

for some positive constant m.

- If you are using a speedy machine, m is smaller than when you are using a slower machine
- But for a fixed machine m remains the same

Another example

```java
void doBubbleSort(int[] array) {  // demo: https://visualgo.net/en/sorting
   int n = array.length;                          // c_2 = 1

   for (int i = 0; i < n - 1; i++) {              // c_4 = n
      for (int j = 0; j < n - i - 1; j++) {       // c_5 < n^2
         if (array[j] > array[j + 1])    {        // c_6 < n^2
            int hold = array[j];                  // c_7 < n^2
            array[j] = array[j + 1];              // c_8 < n^2
            array[j + 1] = hold;                  // c_9 < n^2
         }
      }
    }
}
```

$$
\begin{aligned}
T(n) &\le c_1 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 \\
&\le 1 + n + n^2 + n^2 + n^2 + n^2 + n^2 \\
&= 5n^2 + n + 1 \\
&\le 5n^2 + n^2 + n^2 \\
&\le 7n^2
\end{aligned}
$$

So, we conclude that $T(n) \le 7n^2$.

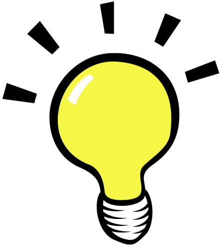
Growth rates and runtime

Class of $T(n)$	**How does $T(n)$ look like?**	**Real-world impact**
Constant	$T(n) \leq k$	Best possible speed!
Logarithmic	$T(n) \leq k \cdot \log n$	Crazy-fast
Linear	$T(n) \leq k \cdot n$	Super-fast
Linearithmic	$T(n) \leq k \cdot n \log n$	Very much acceptable in practice
Quadratic	$T(n) \leq k \cdot n^2$	Very slow in practice
Cubic	$T(n) \leq k \cdot n^3$	Do not expect to finish anytime soon
Exponential	$T(n) \leq k \cdot 2^n$	You will surely get fired!

Observations are made when n (size of the input) is large

Deeper thoughts

- Say an algorithm A has runtime $T(n) \leq k \cdot n^2$
- What happens when the input size is doubled?
- $\dfrac{k \cdot (2n)^2}{k \cdot n^2} = \dfrac{k \cdot 4n^2}{k \cdot n^2} = 4$
- This means runtime may quadruple on some inputs!
- But if $T(n) \leq k \cdot n$, then $\dfrac{k \cdot 2n}{k \cdot n} = 2$ (quite expected, right?)
- What if $T(n) \leq k \cdot 2^n$?
- $\dfrac{k \cdot 2^{2n}}{k \cdot 2^n} = 2^n$ (is not even bounded by a constant!)



- Say you want to sort a million integers
- Quite common these days!
- Insertion sort takes roughly 70 hours on a ‘slow’ machine
- Merge sort takes roughly 40 seconds on the same machine
- Insertion sort: $T(n) \leq k \cdot n^2$ (quadratic)
- Merge sort: $T(n) \leq k \cdot n \log n$ (linearithmic)
- If the machine is 100x faster, then it is 40 minutes vs less than 0.5 seconds

Moral of the story: theoretical analyses maybe hard but super-helpful in practice!

Facts about $T(n)$

- $T(n)$ can be simplified if we just consider the highest order term without its coefficient; we denote it by h
- h for a given $T(n)$ is defined considering the growth rates of all the terms in $T(n)$
- **Example**: $T(n) \leq 10n^2 + 3n + 99$ (in this case, $h = n^2$)
- $T(n) \leq 10n^2 + 3n^2 + 99n^2 = 112n^2$
- **Another example**: $T(n) \leq 22n \log n + 56$ (in this case, $h = n \log n$)
- $T(n) \leq 22n \log n + 56n \log n = 78n \log n$
- **Yet another example**: $T(n) \leq 99 \cdot 2^n + 66n^3 + n + 100$ (in this case, $h = 2^n$)
- $T(n) \leq 99 \cdot 2^n + 66 \cdot 2^n + 2^n + 2^n = 167 \cdot 2^n$
- $T(n)$ can always be simplified to a form $T(n) \leq c \cdot h$, for some constant positive constant c
- This brings us to the famous **Big-Oh** notation used by the programmers around the globe no matter what programming language they are using!

Big-Oh

- The constant c does not matter since h is enough to determine the class of the algorithm we are analyzing; so just drop c
- $T(n) \leq 112n^2$ is thus written as $O(n^2)$
- $T(n) \leq 78n \log n$ is thus written as $O(n \log n)$
- $T(n) \leq 167 \cdot 2^n$ is thus written as $O(2^n)$
- **For a particular problem P, there may exist many algorithms for P which have the same runtime when expressed using Big Oh; in that case, they all are considered to be equivalent for solving P from efficiency perspective**
- An easy method for determining the Big-Oh of $T(n)$: *drop the lower-order terms and then drop the coefficient of the highest order term*
- Examples: $T(n) \leq 5n^2 + 6n + 21 = O(n^2)$;
  $T(n) \leq 99 \cdot 2^n + 66n^3 + n + 100 = O(2^n)$
- In other words, given $T(n)$, we can simply write $T(n) = O(h)$ where the highest order term in $T(n)$ without its coefficient

Runtimes using Big-Oh

Class of $T(n)$	How it looks like?	Expressing $T(n)$ using Big-Oh
Constant	$T(n) \leq k$	$O(1)$
Logarithmic	$T(n) \leq k \cdot \log n$	$O(\log n)$
Linear	$T(n) \leq k \cdot n$	$O(n)$
Linearithmic	$T(n) \leq k \cdot n \log n$	$O(n \log n)$
Quadratic	$T(n) \leq k \cdot n^2$	$O(n^2)$
Cubic	$T(n) \leq k \cdot n^3$	$O(n^3)$
Exponential	$T(n) \leq k \cdot 2^n$	$O(2^n)$

Order-of-growth classifications

- **Constant**. The algorithm under consideration has a constant (can be terribly big) number of operations; runtime is not dependent on input size
  - Printing out the first integer in A
  - Finding the maximum and minimum elements in a sorted array A
  - Finding the Euclidean distance between two given points (x_1, y_1) and (x_2, y_2)

  We say that the algorithm runs in constant time or $O(1)$ time

- **Logarithmic**. The algorithm under consideration has a runtime of $O(\log n)$ and is barely slower than a constant-time algorithm
  - Binary search on a sorted array of size n

  We say that the algorithm has a logarithmic runtime

Order-of-growth classifications

- **Linear.** The algorithm under consideration spends a constant amount of time processing each piece of input data, or is based on a single loop and has a runtime of $O(n)$
  - Finding the largest integer in an array A of length n
  - Given a set A of n points, find the farthest point in A from a given point not in A

  We say that the algorithm has a linear runtime

- **Linearithmic.** The running time of the algorithm under consideration has a runtime of $O(n \log n)$
  - Merge sort an array of n comparable items

  We say that the algorithm has a linearithmic runtime

Order-of-growth classifications

- **Quadratic.** The running time of the algorithm under consideration has a runtime of $O(n^2)$; usually has two nested loops each of which iterates for n times approximately

  - Bubble sort an array of n comparable items
  - Insertion sort an array of n comparable items
  - Selection sort an array of n comparable items

  We say that the algorithm has a quadratic runtime

- **Cubic.** The running time of the algorithm under consideration has a runtime of $O(n^3)$; usually has three nested loops each of which iterates for n times approximately

  - Multiply two $n \times n$ matrices in a naive way

  We say that the algorithm has a cubic runtime

Order-of-growth classifications

- **Exponential.** The running time of the algorithm under consideration has a runtime of $O(2^n)$ or even worse
  - Print out all possible subsets of an n-element set

We say that the algorithm has an exponential runtime

What to find the runtime without doing math?

1. Find out the most costly statement/step; the one that is executed most number of times among all the statements/steps. Let the cost of that statement/step be $t(n)$ and h be its highest order term without its constant coefficient

   ☞ *Approximating $t(n)$ is okay but over-approximation is certainly a bad idea!*

2. Runtime of the algorithm is $O(h)$

Figuring out runtime just by eyeballing

```
1  public static double findMax(double[] array) {
2     int n = array.length;
3     double maxSoFar = array[0];
4
5     for(int i = 1; i < n; i++)
6        if( array[i] > maxSoFar )
7           maxSoFar = array[i];
8
9     return maxSoFar;
10 }
```

The most costly steps are $5, 6, 7$ and each of them run approximately n times each. So, the whole algorithm runs in $O(n)$ time.

Figuring out runtime just by eyeballing

```
1  void doBubbleSort(int[] array) {
2     int n = array.length;
3
4     for (int i = 0; i < n-1; i++) {
5        for (int j = 0; j < n-i-1; j++) {
6           if (array[j] > array[j+1])    {
7              int hold = array[j];
8              array[j] = array[j+1];
9              array[j+1] = hold;
10          }
11       }
12    }
13 }
```

The most costly steps are $5, 6$ and each of them run approximately n^2 times each.
So, the whole algorithm runs in $O(n^2)$ time.

☞ Visualize sorting algorithms

https://visualgo.net/en/sorting

Figuring out runtime just by eyeballing

```
1 void somemethod(int[] items) {
2    int result = 0;
3    for (int i = 0; i < items.length; i++)
4       for (int j = 0; j < 100; j++)
5          result += items[i];
6
7    System.out.println(result);
8 }
```

The most costly steps are $4, 5$ and each of them run approximately $100n$ times each. So, the whole algorithm runs in $O(n)$ time.

Now from problem to code

Problem: used in finance to keep tracking of rising and falling profit averages

You are given an array X of n numbers. For every $0 \leq i \leq n-1$, you need to find out: $A[i] = \dfrac{X[0] + X[1] + \ldots + X[i]}{i+1}$. These are called **prefix averages** of an array.

Example

Input: $X = \{10, 0, 1, 5, 99, 3\}$ (here $n = 6$)

$A[0] = (10)/1 = 10$
$A[1] = (10 + 0)/2 = 5$
$A[2] = (10 + 0 + 1)/3 \approx 3.67$
$A[3] = (10 + 0 + 1 + 5)/4 = 4$
$A[4] = (10 + 0 + 1 + 5 + 99)/5 = 23$
$A[5] = (10 + 0 + 1 + 5 + 99 + 3)/6 \approx 19.67$

Output: $A = \{10, 5, 3.67, 4, 23, 19.67\}$

Approach A

Idea

Compute every $A[i]$ from scratch

```
1  public static double[] findPrefixAveragesA(double[] x) {
2     int n = x.length;
3     double[] a = new double[n];
4
5     for(int i = 0; i < n; i++) {
6        double total = 0;
7
8        for(int j = 0; j <= i; j++)
9           total += x[j];
10
11       a[i] = total/(i+1);
12    }
13    return a;
14 }
15
```

Lines 8, 9 are the most expensive ones; they are executed approximately n^2 times each; so overall runtime is $O(n^2)$

Approach B

Idea

$$A[i] = \frac{\text{the sum in the numerator when } A[i-1] \text{ was calculated plus } X[i]}{i+1};$$

We save additions using this observation!

```java
1  public static double[] findPrefixAveragesB(double[] x) {
2     int n = x.length;
3     double[] a = new double[n];
4
5     double total = 0;
6
7     for(int i = 0; i < n; i++) {
8        total += x[i];
9        a[i] = total/(i+1);
10    }
11    return a;
12 }
```

Lines 7, 8, 9 are the most expensive ones; they are executed approximately n times each; so overall runtime is $O(n)$

Experiment

```java
Random generator = new Random();
int n = 100000;
double[] testArray = new double[n];
for( int i = 0; i < n; i++)
   testArray[i] = generator.nextInt(n);

long startA = System.currentTimeMillis();
findPrefixAveragesA(testArray);
long timeTakenA = System.currentTimeMillis() - startA;

System.out.println("Time taken by approach A (runs in O(n^2) time): " + timeTakenA + " ms" );

long startB = System.currentTimeMillis();
findPrefixAveragesB(testArray);
long timeTakenB = System.currentTimeMillis() - startB;

System.out.println("Time taken by B (runs in O(n) time): " + timeTakenB + " ms" );

System.out.println("Speedup: " + (double)timeTakenA/timeTakenB);
```

Output in one run (result varies slightly every time)

```
Time taken by approach A (runs in O(n^2) time): 4454 ms
Time taken by approach B (runs in O(n) time): 3 ms
Speedup: 1484.6666666666667
```

Theoretically, why there is no faster algorithm?

Answer

We need to scan the whole array. Such a scan already takes cn steps, for some constant $c > 0$. This means a faster algorithm does not exist for the prefix sum problem!

What happens in real life?

1. You get a computational problem P
2. You find multiple algorithms for solving P
3. Having the abstract ideas and/or pseudo-codes of these algorithms and using pencil + paper, you find out their theoretical runtimes
4. Implement the algorithm that appears best (has the best theoretical runtime)

☞ In some cases, you may need to find out their **space complexities** too

Sorting faster

Problem

Sort an array of length n, where the array elements are taken from the set $\{0, 1, 2, 3, 4\}$.

Question

What's the fastest algorithm you can design for this problem?

Solution

Straightforward solution: runs in $O(n \log n)$ time

```
Arrays.sort(arr1);
System.out.println("Time taken by Arrays.sort(): " + (System.currentTimeMillis() - start) + " ms" );
```

A faster solution: runs in $O(n)$ time

```
// count the number of 0s, 1s, 2s, 3s, and 4s
int[] count = new int[5];
for(int i = 0; i < n; i++)
   count[arr2[i]]++;

// put 0s first, then 1s, and then 2s, 3s, and 4s
int pos = 0;
for(int i = 0; i < 5; i++)
   for(int j = 0; j < count[i]; j++)
      arr2[pos++] = i;

System.out.println("Time taken by our approach: " + (System.currentTimeMillis() - start) + " ms" );
```

Speed comparison, $n = 1,000,000$

```
Time taken by Arrays.sort(): 27 ms
Time taken by our approach: 11 ms
```

Real-world problem

Given a long list of IP addresses, determine if they are unique, meaning no two IP addresses are identical.

The brute-force approach

Idea

Take the no-brainer approach; take one address at a time and scan the whole array to see if it is present elsewhere in the input array

Takes $O(n^2)$ time

```
public static boolean bruteForce(String[] addresses) {
   for( int i = 0; i < addresses.length-1; i++ )
      for( int j = i+1; j < addresses.length; j++ ){
         if( i != j && addresses[i].equals(addresses[j]))
            return false;
      }
   return true;
}
```

Smart approach: sort and check

Idea

Sort the whole array first. If there are duplicates, those must appear contiguously in the array.

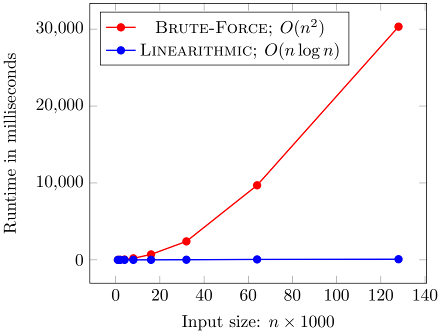
Takes $O(n \log n)$ time

```java
public static boolean linearithmic(String[] addresses) {
   Arrays.sort(addresses); // Takes O(n log n) time; uses the compareTo() method from the String class
   for( int i = 0; i < addresses.length - 1; i++) // this for loop runs in O(n) time
      if( addresses[i].equals(addresses[i+1]) )
         return false;
   return true;
}
```

Comparison

n	Brute-Force	Linearithmic
$1K$	9	3
$2K$	12	2
$4K$	49	3
$8K$	203	4
$16K$	714	7
$32K$	2406	20
$64K$	9697	61
$128K$	30314	88

Runtimes are shown in milliseconds



Experiment ran on a 2019 Macbook Pro 16

☞ *Depending on the array, the brute-force algorithm may terminate fast. For instance, when the first two elements in the array are identical, both the loops will iterate exactly once!*

The problem

Given an array A of n numbers and a number x, verify if there are two numbers $A[i]$ and $A[j]$ such that $A[i] + A[j] = x$ and $i \neq j$

Examples

$A = [99, 1, -3, 0, 22, 5, 6, 22, -101, 1, 17], x = 23$, **Answer.** `true`

$A = [99, 1, -3, 0, 22, 5, 6, 22, -101, 1, 17], x = 10$, **Answer.** `false`

The brute-force approach

Idea

Try all possible pairs of numbers from the array and check if there are two numbers in the array that satisfy the condition

Takes $O(n^2)$ time

```
public static boolean bruteForce(int[] A, int x) {
   for(int i = 0; i < A.length - 1; i++)
      for( int j = i + 1; j < A.length; j++)
         if( A[i] + A[j] == x )
            return true;
   return false;
}
```

Smart approach: sort and check

Idea

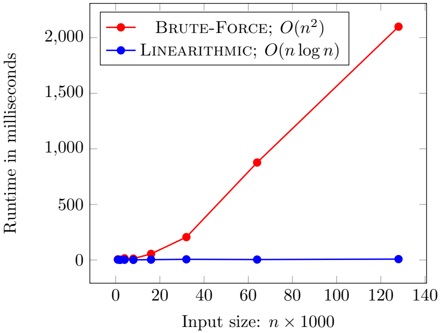
Sort the array and then check

Takes $O(n \log n)$ time

```
public static boolean linearithmic(int[] A, int x) {
   Arrays.sort(A);

   int left = 0, right = A.length-1;

   while( left < right )
      if( A[left] + A[right] > x )  right--; // sum is larger; decrement right
      else if( A[left] + A[right] < x )  left++; // sum is less; increment left
      else return true; // A[left] + A[right] == x; success!

   return false;
}
```

n	BRUTE-FORCE	LINEARITHMIC
$1K$	6	3
$2K$	2	1
$4K$	15	2
$8K$	12	1
$16K$	55	3
$32K$	206	6
$64K$	877	4
$128K$	2099	8

Runtimes are shown in milliseconds



Experiment ran on a 2019 Macbook Pro 16

☞ *Depending on the array, the brute-force algorithm may terminate fast. For instance, when the first two elements in the array equals x, both the loops will iterate exactly once!*

FOOD FOR THOUGHT

3-SUM problem (an extension of the 2-SUM problem)

Given an array A of n numbers and a number x, verify if there are three numbers $A[i], A[j], A[k]$ such that $A[i] + A[j] + A[k] = x$ and $i \neq j \neq k$

Brute-force

Takes $O(n^3)$ time

Question

Can you design a faster algorithm? Write a code for the algorithm and then experimentally compare it with the brute-force algorithm for large values of n.

Reading

Chapter 4 from

https://opendsa-server.cs.vt.edu/OpenDSA/Books/CS3/html/index.html