Hashing

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More on implementing maps

- Arrays and Lists? Too inefficient! Linear runtimes
- Plain BSTs? Too inefficient! Linear runtimes
- RB-trees? Good choice! Logarithmic runtimes are guaranteed

Can we do better in practice?

Answer

HASH TABLES

☞ With hash-tables, it is not possible to obtain a sorted sequence of the records in $O(n)$ time like BSTs.

Warm up (the simplest possible case)

INDEX/KEY	0	1	2	3	4	5	6	7	8	9	10
Value	D		Z			C	Q				

Assume that we have $n \leq 11$ records to maintain where the keys are in the range $[0, 10]$

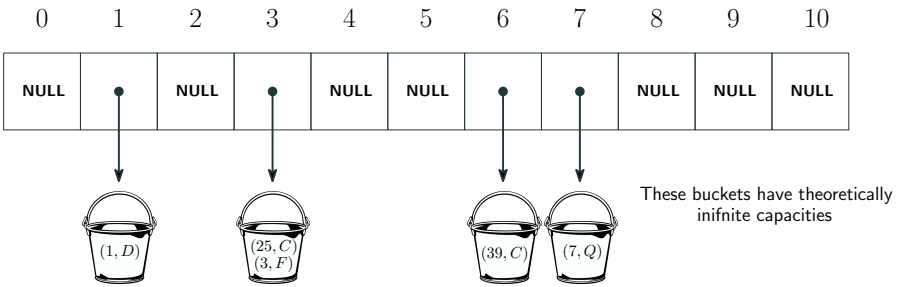
Insertions, deletions, look-ups can be executed in $O(1)$ time each since an array of length $N = 11$ can be used for implementing the table

The situation

- Let n be the number of records stored and N be the number of possible keys
- What if N is really large, say in the order of millions and n much less than N?
- **Example.** for integer keys, $N = 2^{31} - 1 = 2,147,483,647$; but, n is most cases is much less than $2^{31} - 1$. Are we still going to use an array of size $2,147,483,647$? Probably not a good idea. Space wastage may be severe. Storing such an array will require $(2^{31} - 1) \times 1$ byte ≈ 2.147 GB of space (assuming 1 byte is enough to store a character)!

A space-efficient solution

Map the keys to the set of array indices using some function (a.k.a. hash function). Every index can hold more than one records (a bucket of records). In this case, more than N records can be maintained!



$h(\textbf{key}) = \textbf{key} \bmod 11$

For any key k, $0 \leq h(k) \leq 10$

The record (k, v) is put in the bucket at index $h(k)$

What if the keys are not integers?

1. Convert the non-integer key to an integer using some function h_1; after applying h_1, we get an integer $h_1(k)$; the function h_1 is known as the **hashcode**
2. Next, map $h_1(k)$ to an array index using another function h_2 known as the **compression function**

The record (k, v) maps to the index $h_2(h_1(k))$

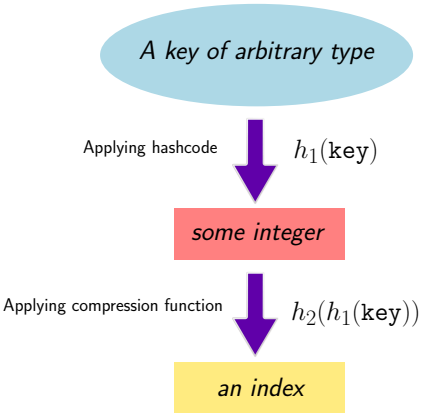
Example

$h_1(\texttt{"Doctor Strange"}) = 1938383$

$h_2(1938383) = 1938383 \bmod N$, where N is the size of the array used

☞ In the previous example, $N = 11$

The hashcode and the compression function



Hash function

$h = h_2(h_1(k))$

The array plus the hash function is called **hash-table**

The main idea behind hashing

It is *unlikely* that two different records (k_i, v_i) and (k_j, v_j), where $k_i \neq k_j$ will map to the same bucket in the hash table when the size of the table N is sufficiently large and the hash function is chosen appropriately. Consequently, it is unlikely that the buckets will be crowded.

If the buckets are not crowded, *searches*, *insertions*, and *deletions* would run fast in practice.

Hashcodes

- Based on the type of keys we are using, one can design various kinds of hash-codes
- Desired properties of hash codes:
  - ☞ If two keys k_i, k_j are different, then the two corresponding outputs of hashcode should be different

$$k_i \neq k_j \implies h_1(k_i) \neq h_1(k_j)$$

  - ☞ Should be very fast to compute
- In Java, the `Object` class (super-class of every Java class) defines the `hashCode()` method using the object's memory address
- This means the `hashCode()` method can be invoked on any object!
- If two objects are equal according to the optional `equals` method of the class, then calling the `hashCode` method on each of the two objects must produce the same integer result.

Illustration

Java's hashcode for Strings

Let $s = s_0 s_1 \ldots s_{n-1}$, where every s_i is a character

$h_1(s) = (\texttt{ASCII}(s_0) \times 31^{n-1}) + (\texttt{ASCII}(s_1) \times 31^{n-2}) + \ldots + (\texttt{ASCII}(s_{n-1}) \times 31^0)$

```
public class HashCodeDemo {
   public static void main(String[] args) {
      String s1 = "UNF is FUN";
      String s2 = "FUN is UNF";
      String s3 = "UNF iss FUN";

      System.out.print(s1.hashCode() + " ");
      System.out.print(s2.hashCode() + " ");
      System.out.print(s3.hashCode());
   }
}
```

Output

```
120001564 63052472 -499412747
```

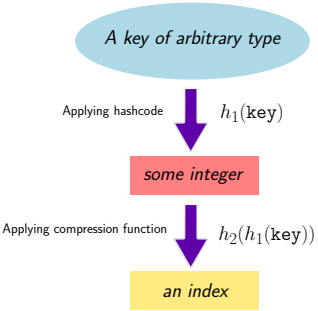
- The built-in Java classes such as `String`, `Integer`, `Double`, etc. redefine this function; see Java's documentation to see the precise mathematical functions

```
Double d1 = 101.98;
System.out.print(d1.hashCode() + " ");

d1 = 101.981;
System.out.print(d1.hashCode() + " ");

d1 = -101.981;
System.out.print(d1.hashCode() + " ");
```

Output

```
296942503 -195025192 1952458456
```

Compression function



The popular compression function

$$h_2(x) = x \bmod N$$

where N is the size of the table.

☞ Make sure that $x \geq 0$, otherwise, $h_2(x) < 0$, making it useless for array indexing!

What is a collision?

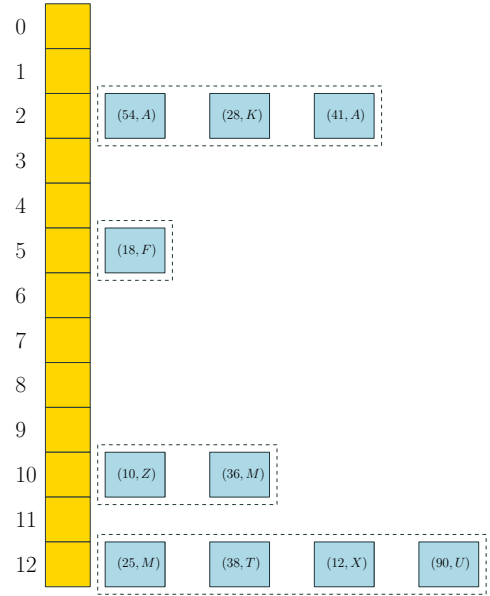
We say that two records (k_i, v_i) and (k_j, v_j) are said to have **collided** if $h(k_i) = h(k_j)$. This means both got mapped to the same array index.

What to do if multiple records map to the same index?

- Choice 1: **Separate chaining** (uses buckets, very popular); number of records can be way more than the size of the array
- Choice 2: **Open addressing** (does not use buckets, much less popular); number of records cannot be more than the size of the array

Separate Chaining

Separate chaining: use a container (linked-list/ArrayList) at every index



Load-factor of hash-tables

$$\lambda = \frac{\text{the number of records present in the map } (n)}{\text{size of the array } (N)}$$

☞ $\lambda < 1$ is always desired. This implies no container is probably overcrowded, resulting in fast operation speeds. In Java's implementation of hash-tables, by default, load-factors are never allowed to exceed **0.75**. If the load-factor exceeds 0.75, the size of the array is increased and all the records present in the map are re-inserted (every one of them). We are allowed to choose a different value for the load-factor.

☞ In the beginning, when the table is empty, the number of buckets is set to **16** in Java's implementation of hash-tables.

put(k, v): inserts the record (k, v) into the map

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is already present in L having key k;
4. If such a record is present in L, the new record (k, v) cannot be inserted;
5. Otherwise, insert the record into L;
6. Compute the current load factor λ;
7. If λ exceeds **0.75**, execute **rehash()**;

Takes $O(n^2)$ time in the worst-case (when rehashing is needed). If rehashing is not required, it takes $O(n)$ time in the worst-case and $O(1)$ time on average.

☞ After rehashing, $\lambda \leq 0.75$

rehash(): rehashes the map (an internal method)

1. Let N be the size of the current array A;
2. Create a new array of empty containers A' whose size is $2N$;
3. Insert every record currently present in the map into the appropriate container in A' using the new compression function $h_2(x) = x \bmod 2N$ (before rehashing, it was $h_2(x) = x \bmod N$) but using the old hashcode h_1 (since the type of keys did not change). This step requires n $\texttt{put}(k, v)$ operations.

☞ Rehashing runs in $O(n^2)$ time in the worst case where n is the number of records currently present in the map

Rehashing illustration

```
Bucket 0: [96, Jim] -> [112, Peter]
Bucket 1:
Bucket 2:
Bucket 3: [99, Jack]
Bucket 4: [36, Rose]
Bucket 5:
Bucket 6: [22, Charles]
Bucket 7: [23, Alice]
Bucket 8: [40, Bob]
Bucket 9: [41, Matthew]
Bucket 10: [10, Tom]
Bucket 11: [11, Dorothy]
Bucket 12: [92, Eric]
Bucket 13:
Bucket 14: [62, Donald]
Bucket 15:
```

Load-factor $(\lambda) = \frac{n}{N} = \frac{12}{16} = 0.75$

```
Bucket 0: [96, Jim]
Bucket 1:
Bucket 2:
Bucket 3: [99, Jack]
Bucket 4: [36, Rose]
Bucket 5:
Bucket 6:
Bucket 7:
Bucket 8: [40, Bob]
Bucket 9: [41, Matthew]
Bucket 10: [10, Tom]
Bucket 11: [11, Dorothy]
Bucket 12 - 15:
Bucket 16: [112, Peter]
Bucket 17 - 21:
Bucket 22: [22, Charles]
Bucket 23: [23, Alice]
Bucket 24 - 27:
Bucket 28: [92, Eric]
Bucket 29:
Bucket 30: [62, Donald]
Bucket 31: [31, Toby]
```

After inserting `[31, Toby]`, $\lambda = \frac{n}{N} = \frac{13}{16} = 0.8125 > 0.75$;
rehashing is required; new $\lambda = \frac{n}{N} = \frac{13}{32} \approx 0.40625 \leq 0.75$

get(k): returns the value part of the record whose key is k

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is already present in L having key k;
4. If such a record is present in L, return its value part;
5. Otherwise, return `null`;

Takes $O(n)$ time in the worst-case (when most records map to the same bucket) but in practice takes $O(1)$ time on average

`remove`(k): removes the record having key k, if present

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is present in L having key k;
4. If it is present in L, remove and return the value part of the record (k, v);
5. Otherwise, return **`null`**;

Takes $O(n)$ time in the worst-case (when most records map to the same bucket) but in practice takes $O(1)$ time on average

Code

See the class `HashMapSeparateChaining`

In our implementation, the containers
are **singly linked-lists**

Worst-case scenario of separate chaining

- In the worst case, most of the records can map to the same index!
- Thus, insertion, deletion, and searching take $O(n)$ time in the worst-case if lists are used and $O(\log n)$ time if RB-trees are used
- In such cases, we do not get any advantage out of hashing
- However, if we are using good hash function, these extreme situations will almost never happen and we get super-speedy performance in the real-world

Java's `HashMap` vs `TreeMap`: insertion time comparison

n	`java.util.`**`HashMap`**	`java.util.`**`TreeMap`**
10	1	1
100	1	1
1000	1	3
10000	1	7
100000	17	19
1000000	97	167
10000000	815	2889

☞ Times are reported in milliseconds

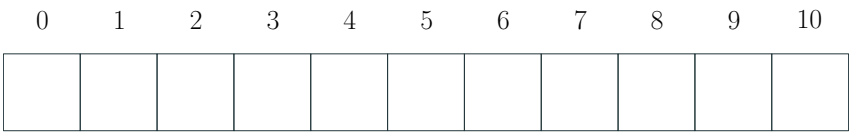
☞ In practice, `HashMaps` are more efficient in practice than `TreeMaps`

Open Addressing

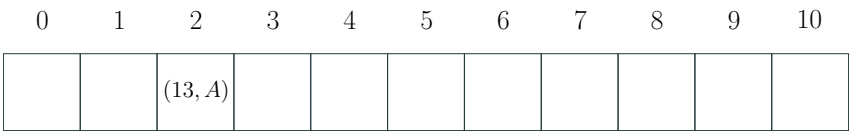
Open addressing

- Use an array of size N
- At every index, we can store at most one record
- Let us look at such a popular technique known as **Linear Probing**: if the spot is already occupied by some other record (collision), we consider the next available spot in the array by wrapping around

Example

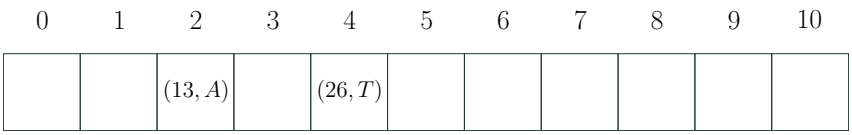


Insert $(13, A)$



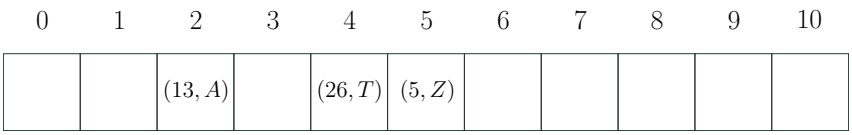
$$13 \texttt{ mod } 11 = 2$$

Insert $(26, T)$



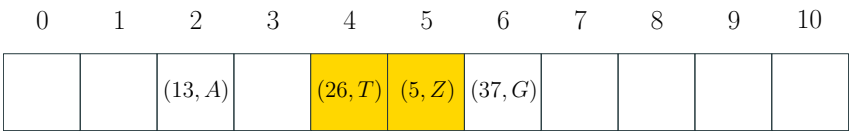
$$26 \texttt{ mod } 11 = 4$$

Insert $(5, Z)$



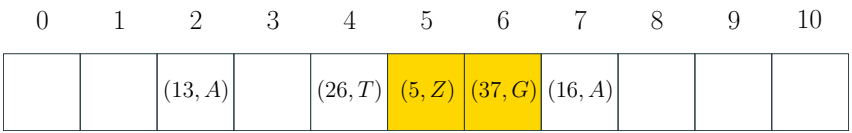
$$5 \texttt{ mod } 11 = 5$$

Insert $(37, G)$; collision!



$37 \mod 11 = 4$, linear probe and then put the record at index 6

Insert $(16, A)$; collision!



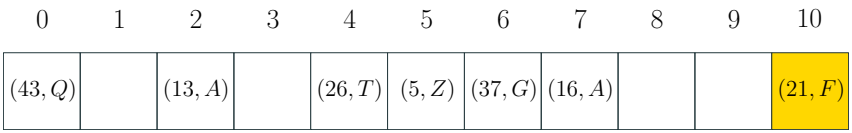
$16 \mod 11 = 5$, linear probe and then put the record at index 7

Insert $(21, F)$

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$	$(5, Z)$	$(37, G)$	$(16, A)$			$(21, F)$

$$21 \texttt{ mod } 11 = 10$$

Insert $(43, Q)$; collision!



$43 \mod 11 = 10$, linear probe by wrapping around and then put the record at index 0

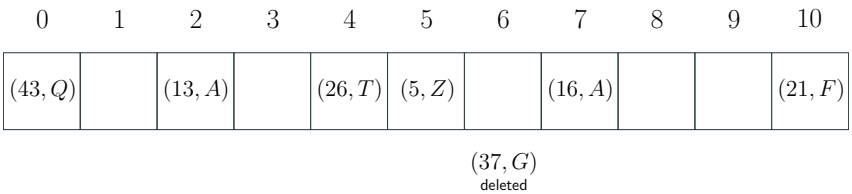
Operations

In the following, **DEFUNCT** is a special record $(\texttt{null}, \texttt{null})$ whose key and value are set to $\texttt{null}$

1. **Insertion.** Use the hash function on the key to obtain the target index; if the spot is taken by some other record, use wrap-around linear probing to find the next available spot; cells containing **DEFUNCT** are considered to be empty
2. **Searching.** Use the hash function on the key to obtain the target index; if the desired record is not present at the index, do a wrap-around linear probe to search the record. If an empty spot is encountered, the desired record is not present. However, skip over the cells containing **DEFUNCT**. In this case, cells containing **DEFUNCT** are not considered to be empty.
3. **Deletion.** Use the hash function on the key to obtain the target index; if the desired record is not present at the index, use wrap-around linear probing to locate the record. However, skip over the cells containing **DEFUNCT**. In this case, cells containing **DEFUNCT** are not considered to be empty. If located, delete the record from the cell and put a special symbol **DEFUNCT** in its place (see the example next to see why)

All these three operations takes $O(n)$ time in the worst-case

Why DEFUNCT is necessary?



Search for $(16, A)$ will fail if we put store null at index 6

Observations

- If we are using open addressing, the number of records that can be present in the array cannot exceed the array size since we are not using chaining
- But it is not the case in separate chaining
- Separate chaining consumes more space to maintain the containers at every index

Reading

Chapter 15 from

`https://opendsa-server.cs.vt.edu/ODSA/Books/Everything/html/index.html`