Digitale Signalverarbeitung

Zusammenfassung

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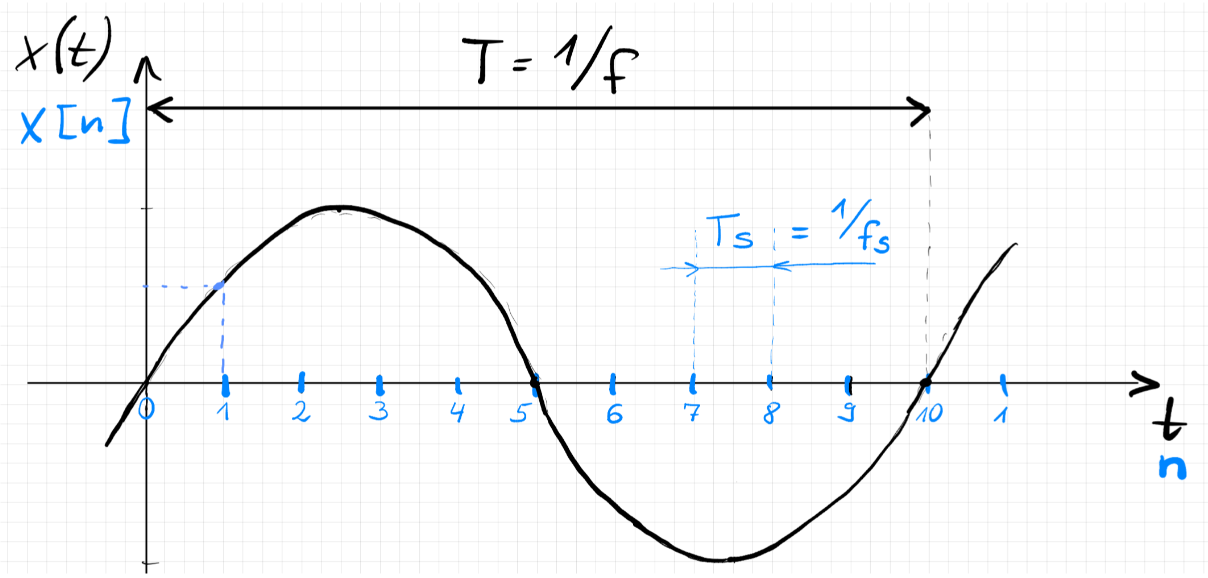
Digital Signals in the Time Domain

Signal Analysis

Sampling of Analog Signals

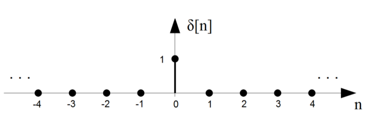
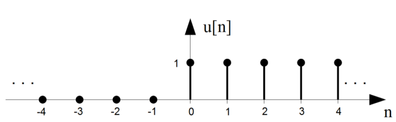
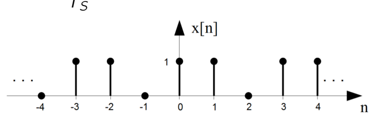
By sampling $x(t)$ in the interval of T_S we get the sequence of signal values $x[n]$ with $-\infty \leq n \leq +\infty$

$$x(n \cdot T_S) = x[n]$$



Signal	Property
causal	$x[n] = 0$ for $n < 0$
real	$x[n]$ Real
complex	Re & Im or Amplitude & Phase

Basic Digital Signals

unit impulse	unit step	periodical signal
$\delta[n] = \begin{cases} 0 : n \neq 0 \\ 1 : n = 0 \end{cases}$	$u[n] = \begin{cases} 0 : n < 0 \\ 1 : n \geq 0 \end{cases}$	$x[n] = x\left[n + \frac{T_0}{T_S}\right]$ with $\frac{T_0}{T_S} = k$
		

There is also a **complex hamonic** sequence with the period duration of $T_0 = \frac{1}{f_0}$

$$x[n] = \hat{X} \cdot e^{j2\pi f_0 n T_S}$$

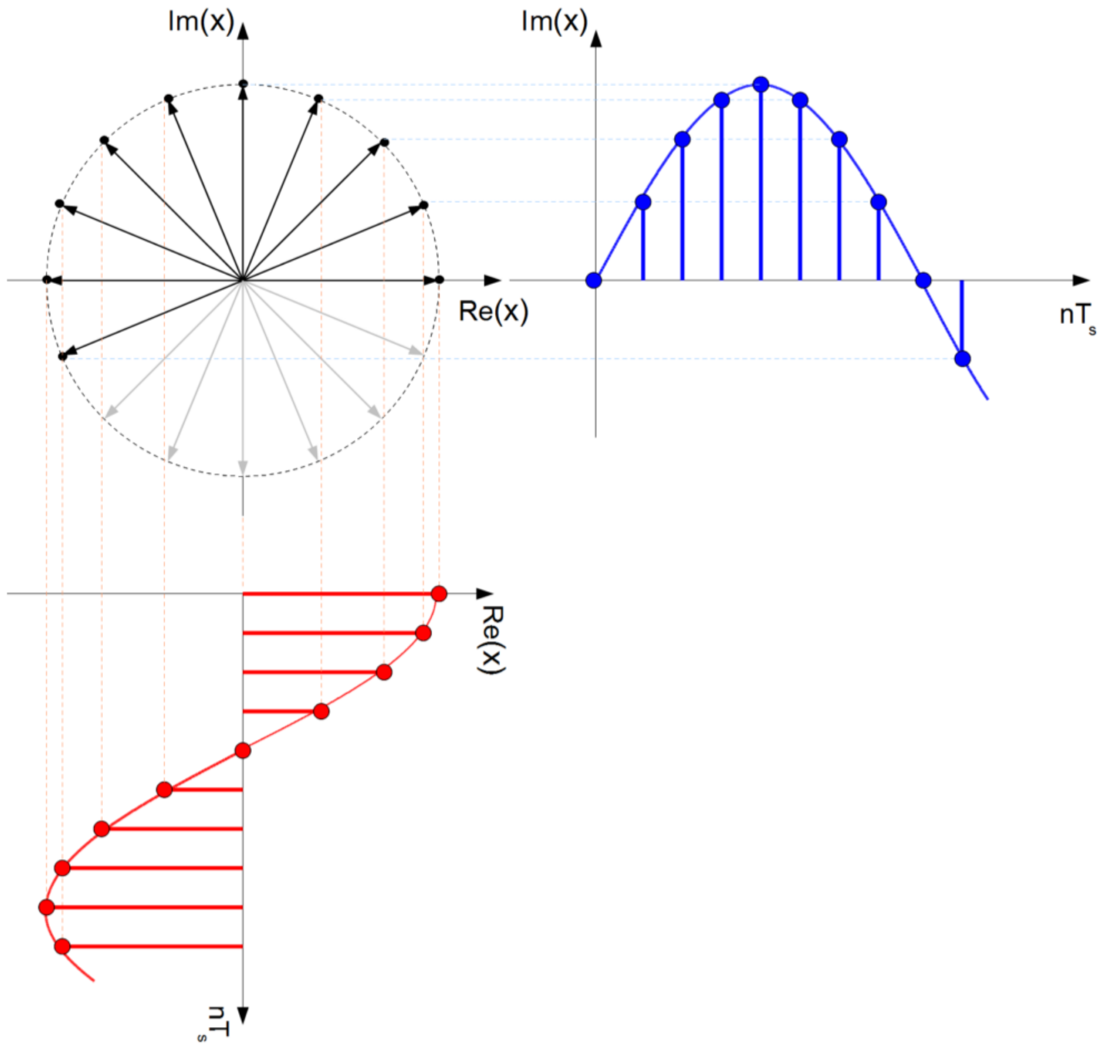


Abbildung 1: Complex hamronic sequence with period duration $T_0 = 16 \cdot T_S$

Statistical Signal Parameters

Stochastic signals must be qualified by *statistical signal parameters* within the **observation interval** $T = N \cdot T_S$.

expected / mean	quadratic mean	variance
DC-component	average power *(w/ DC)*	average power *(w/o DC)*
$\mu_x = \frac{1}{N}\sum_{i=0}^{N-1} x[i]$	$\rho_x^2 = \frac{1}{N}\sum_{i=0}^{N-1} x[i]^2 = P_{avg}$	$\sigma_x^2 = \frac{1}{N}\sum_{i=0}^{N-1} (x[i] - \mu_x)^2 = P_{AC}$

Signal Operations

Correlation

	cross-correlation	auto-correlation
Static	$R = \frac{1}{N}\sum_{i=0}^{N-1} x[i]y[i]$	$R = \frac{1}{N}\sum_{i=0}^{N-1} x[i]x[i]$

	cross-correlation	auto-correlation
Linear	$r_{xy}[n] = \sum_{i=-\infty}^{\infty} x[i]y[i+n]$	$r_{xx}[n] = \sum_{i=-\infty}^{\infty} x[i]x[i+n] = P_{avg}$

For **linear correlation** the resulting length of r_{xy} equals

$$N_{xy} = N_x + N_y - 1$$

and the range of shifts for the computation is given by

$$-N_x + 1 \leq n \leq N_y - 1$$

For signals differing in length, zero-padding can be applied.

Convolution

The *Convolution* involves folding the time-displaces signal around the point $n = 0$

$$z[n] = \sum_{i=-\infty}^{\infty} x[i]y[-i+n] \tag{0.1}$$

A convolution equals a polynomial multiplication.

The range of shifts for the computation is given by

$$0 \leq n \leq N_x + N_y - 2$$

The Convolution described in Gleichung 0.1 is called a **linear convolution** and can be applied to two signals of different length

$$z[n] = x[n] * y[n] = y[n] * x[n]$$

```
z = conv(x, y)
```

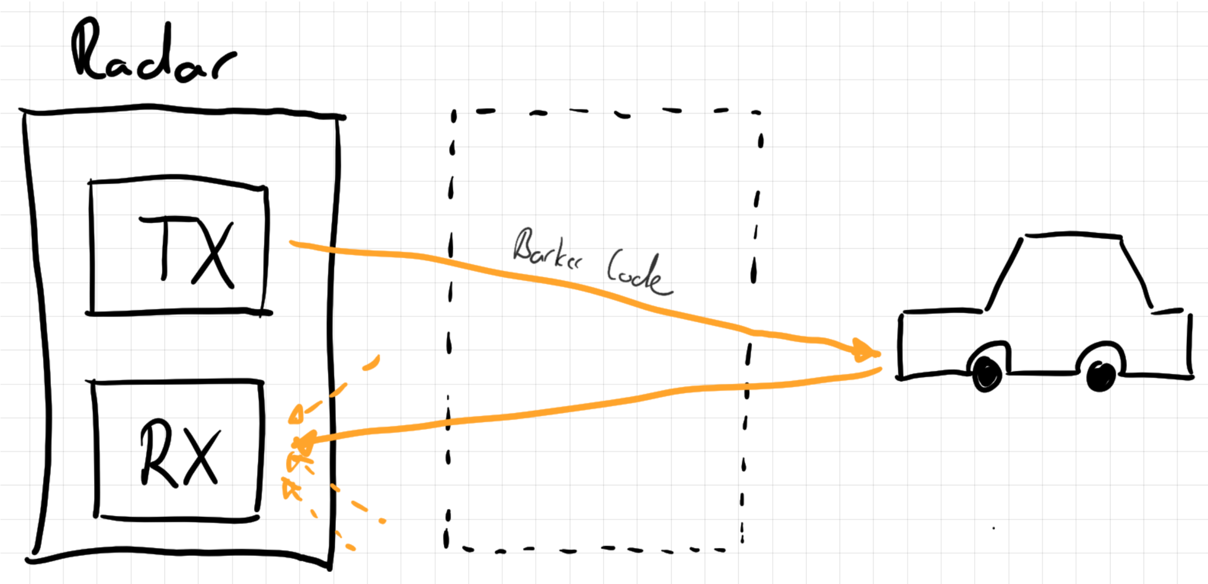
There is also the **circular convolution** which requires both signals to be of equal length N. If necessary, *zero padding* can be applied. The resulting signal then also is of length N.

$$z[n] = x[n] \circledast_N y[n] = y[n] \circledast_N x[n]$$

The circular convolution corresponds to matrix multiplication In order to compute $x[n] \circledast_N y[n]$, the NN-matrix constructed from circular shifting y must be multiplied with vector x.

```
z = convmtx(x,y)
```

Anwendung: Radar



Um bei einem Radar nur auf das gewünschte Signal zu reagieren, also auf das eigene, wird vom Radar ein Barker-Code ausgesendet. Über Korrelation kann so die Laufzeit eindeutig zugeordnet werden.

> **ℹ Barker-Code**
>
> Es können auch andere Codes ausgesendet werden, die verwendeten Signale müssen jedoch sehr gute Autokorrelationseigenschaften aufweisen.

Analog-to-Digital & Digital-to-Analog Conversion

Steps of A/D- and D/A-Conversion

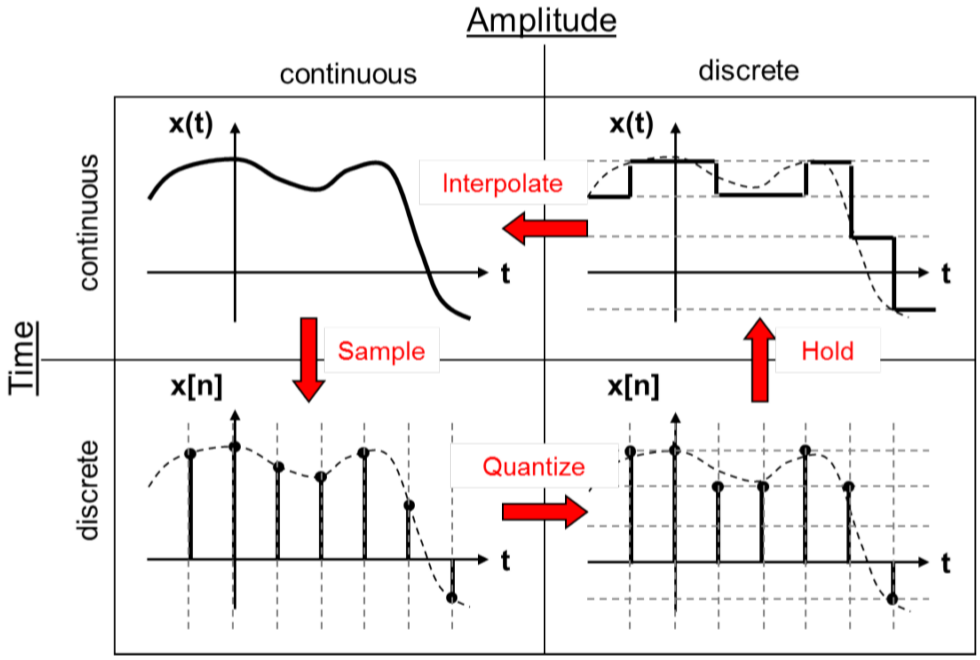


Abbildung 2: Signal classification in a/d- and d/a-conversion

A/D

Sample: Signal values are recorded at sampling rate f_S. This yields a train of pulses.

Quantize: The discrete signal values are mapped to a given number of quantization levels.

Code: The quantified values can be stored in a coded way. DSPs most often store the quantified values.

D/A

Decode: The coded samples are converted back into a suitable representation for the digital-to-analog conversion method used.

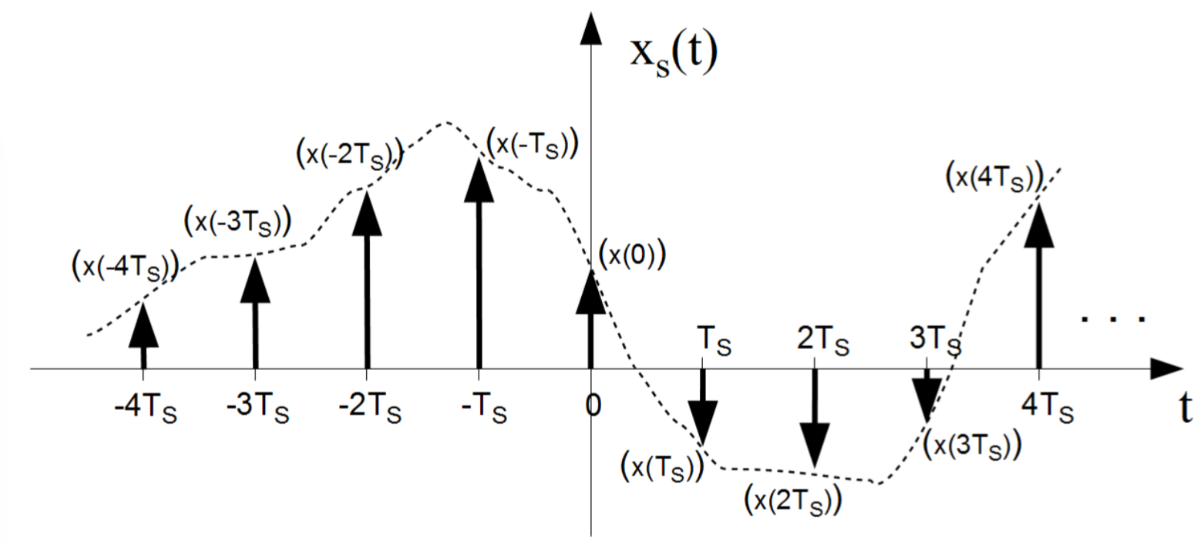
Hold: A momentary discrete signal value is constant over the sample period T_S.

Interpolate: The continuous staircase signal form is smoothed by a low-pass-filter.

Sampling and Aliasing

Sampling a time-continuous signal $x(t)$ corresponds to a multiplication with a Dirac impulse series. The resulting signal $x_S(t)$ can be regarded as a train of weighted Dirac impulses.

$$x_S(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_S)$$



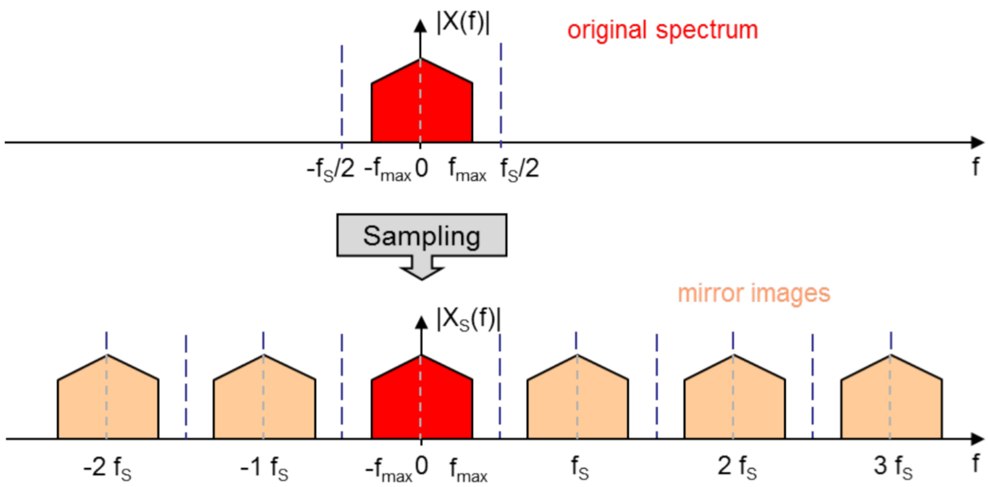
Through the application of the Fourier property $x(t)e^{j2\pi f_0 t} \circ\!\!-\!\!\bullet X(f - f_0)$ we obtain the frequency spectrum of the sampled signal as

$$X_S(f) = \frac{1}{T_S} \sum_{k=-\infty}^{\infty} X(f - kf_S)$$

! Observation

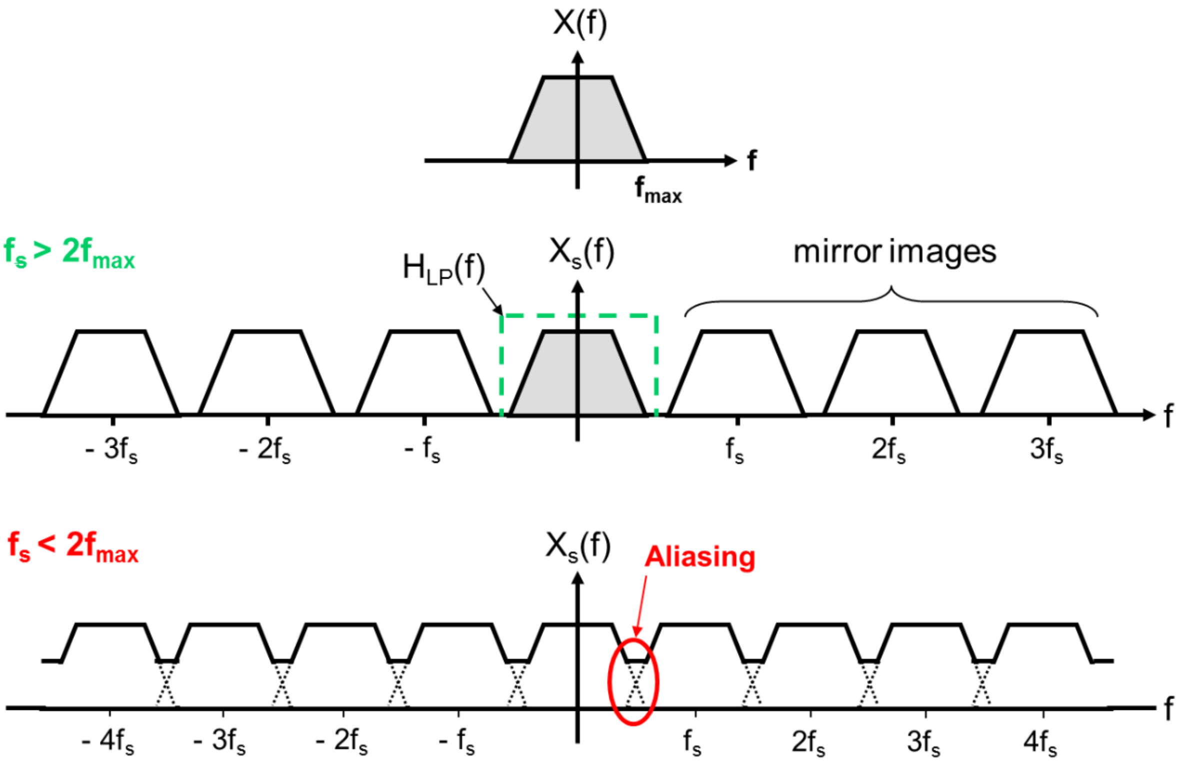
The frequency of the analog signal $x(t)$ consists of the original spectrum $X(f)$ superimposed (*überlagert*) by mirror images of the spectrum

$$f_k = k \cdot \frac{f_S}{N}$$



Aliasing

> **Sampling Theorem**
>
> An analog signal $x(t)$ with $X(f) = 0$ for $|f| > |f_{max}|$ is uniquely defined by its sample values $x[n] = x(nT_S)$, if for the sampling frequency $F_S = \frac{1}{T_S}$ holds:
>
> $$f_s > 2 \cdot f_{max}$$



Band-Pass Sampling

$x(t)$ can be perfectly reconstructed if an integer $N \geq 0$ exists, such that $X(f) = 0$ holds for all frequencies f outside

$$-\frac{N+1}{2} f_S \leq f \leq -\frac{N}{2} f_S \qquad \text{and} \qquad \frac{N}{2} f_S \leq f \leq \frac{N+1}{2} f_S$$

For a given band-pass signal with given limits f_{min} and f_{max} it can be checked if the sampling frequency f_S can be used ($N \geq 1$)

$$\frac{2 \cdot f_{min}}{N} \geq f_S \geq \frac{2 \cdot f_{max}}{N+1}$$

For sampling with $N =$ **even** we get the mirror spectrums

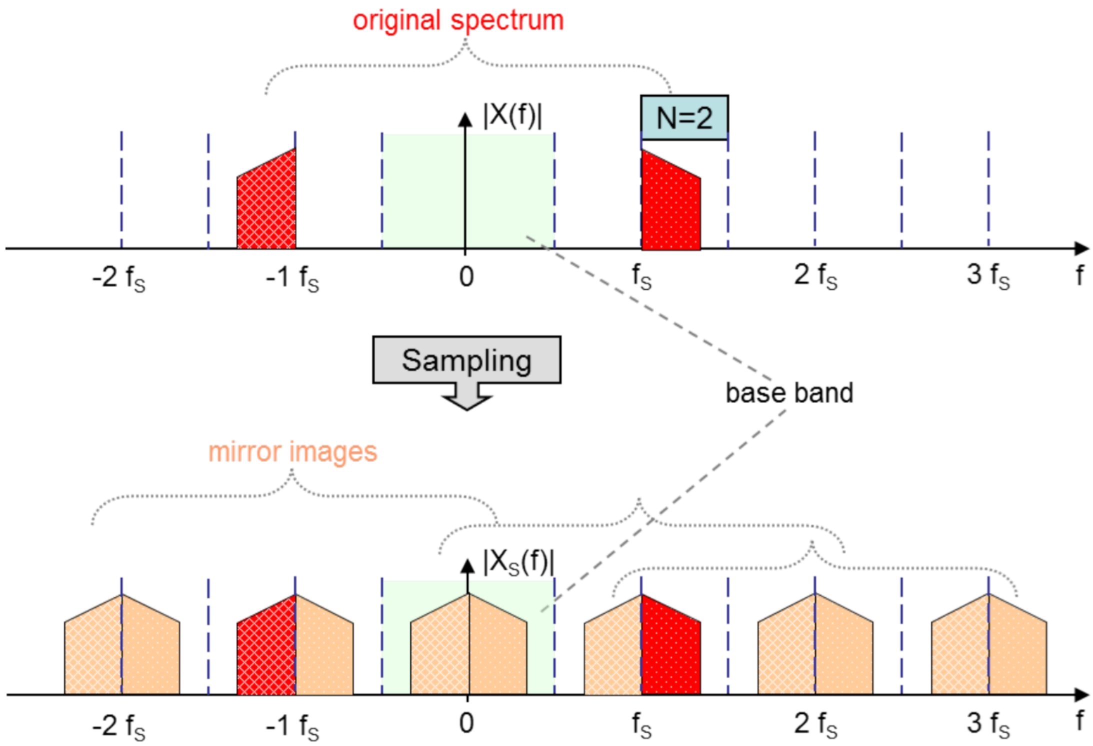


Abbildung 3: Band-pass sampling for even N

For sampling with $N =$ **odd** we get the mirror spectrums

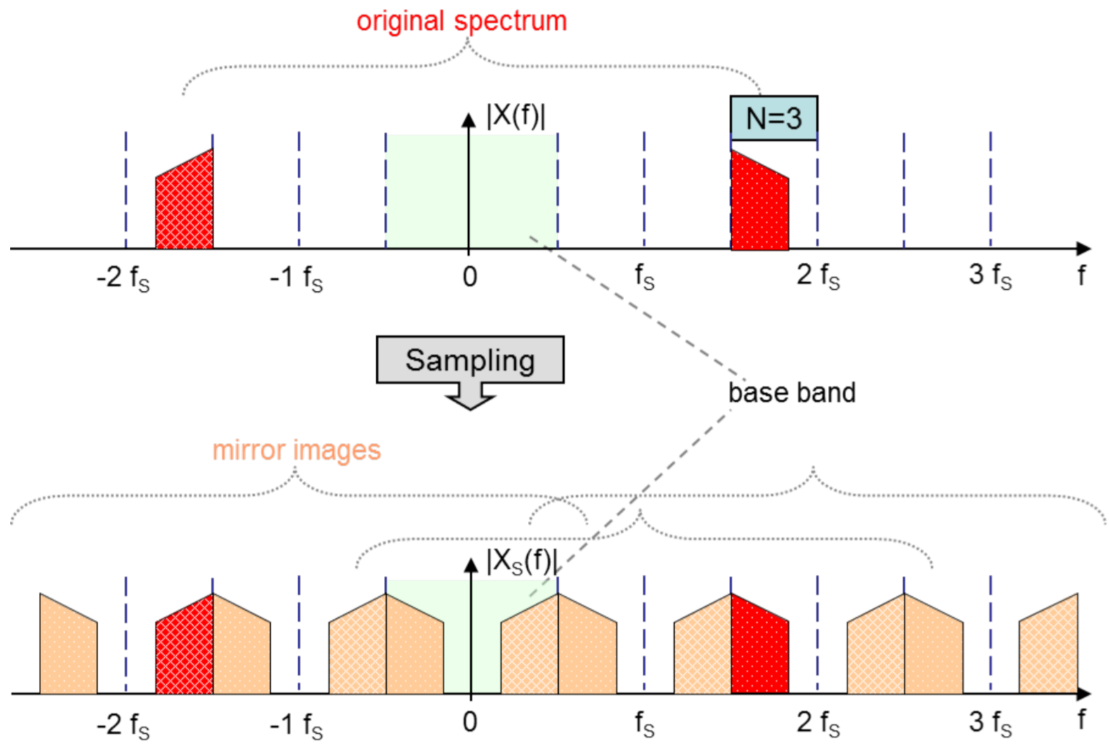


Abbildung 4: Band-pass sampling for odd N

Spectrum Correction

Note that for N odd, the original spectrum appears "inverted" in the base band. The original structure of the spectrum can be re-obtained by changing the sign of every second sample of the time-domain sequence, i.e.

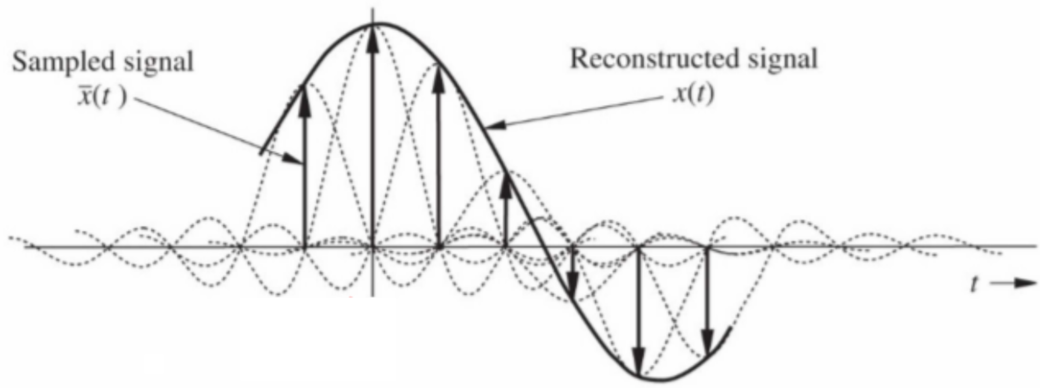
$$\tilde{x} = (-1)^n \cdot x[n]$$

Reconstruction

Ideal Reconstruction

Sampled signals w/o Aliasing can be *theoretically* reconstructed error-free. For this all mirror-spectra must be eliminated by a ideal low-pass filter. Because of the property $\mathrm{rect}\left(\frac{t}{T}\right) \circ\!\!-\!\!\bullet |T| \cdot \mathrm{si}(\pi T f)$ the ideal interpolation equals

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_S) \cdot \mathrm{sinc}(\pi f_S(t - nT_S))$$



Ideal values

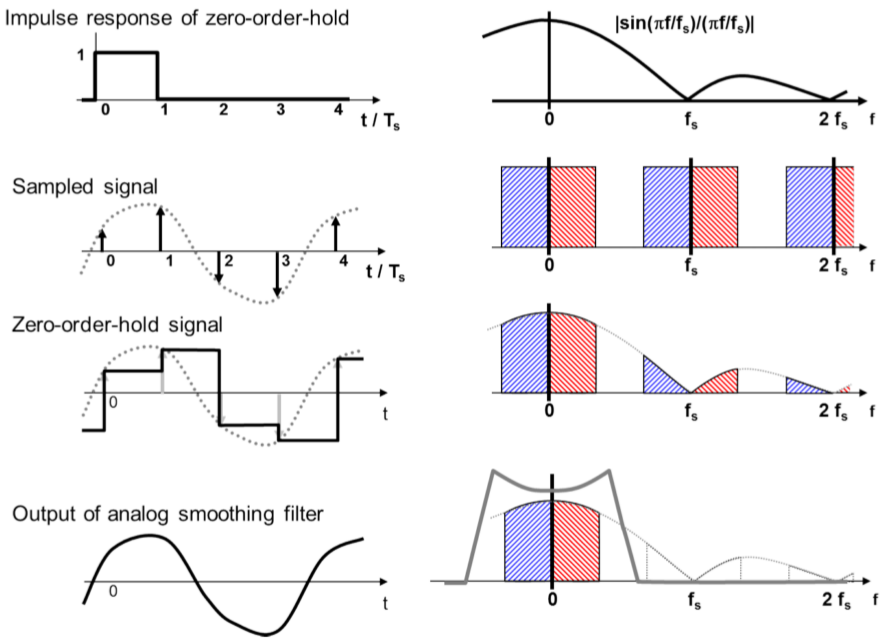
At the points $t = nT_S$ all values of except of $x(nT_S)$ equal 0. Thus at every point of $x(nT_S)$ the signal reaches the right value.
Caution! because of the infinit sum of sinc-pulses, the values between $x(nT_S)$ aren't particularly correct. Also the further to the "edge" of x you get, the more inaccurate it gets.

Practical Reconstruction

In practice Reconstruction is very often done with a simple *zero-order-holder* (ZHO). Such operation holds each sample value constant over a subsequent sample interval T_S. This results in a stair-case waveform, thus making a very poor low-pass filter. For this reason a analog low-pass filter is usually implemented.

Without analog filtering the **SNR** can be approximated as

$$\mathrm{SNR} \approx 6dB \cdot \log_2\left(\frac{f_S}{f_0}\right) - 11dB$$



Digital Signals in the Frequency Domain

Digital LTI Systems

Design of Digital Filters

Fourier Analysis of analog Signals

DFT Inside

The z-Transform