1 Introduction

–Initial Problem (graph of obs data)
–First thoughts (Hypothesis)

2 Method

–Which Method did I use and why?
For this approach I used two programs: main.py and lumiradius.py.

I had the choice between two different approaches: A montecarlo approach and a grid approach.

In my program I use the grid approach. I span a grid in distance-mass-age. The range of these three variables can be easily adjusted, as can the binning. In accordance to the observational data I try to explain, distance goes from 0 to $r_{max} = 3kpc$. Mass will also be in the same parameters, as the observational data: from $M_{min} = 5M_{\odot}$ to $M_{max} = 50M_{\odot}$. The mass-distribution will follow the Salpeter Initial Mass Function (IMF)(see Salpeter, 1955). Because the IMF follows an inverse power law, I use a logarithmic mass-grid.
Age can be implemented in two different ways: I could either use a seperate age-axis for every star going from 0 to $t_{ms}(M)$ or I can use a single axis for all stars spanning from 0 to $t_{ms}(M_{min})$. Because the main sequence age (t_{ms}) of a star is a strictly monotonic increasing function of M $t_{ms}(M_{min}) = t_{max}$. **I use the second approach**. With $M_{min} = 5M_{\odot}$ this translates to $t_{max} = 104Myr$. This Axis will also be logarithmic to make sure massive stars with small t_{ms} are correctly represented.
The three informations I have about any given star, are its mass, its age and its distance from earth. From these three informations I need to derive its fractional main sequence age (τ), its apparent magnitude (V) and ultimately the probability density for all stars.
First I will implement a stellar evolution model by Hurley, Pols, & Tout (2000) which approximates the stellar evolution as a function of initial mass (M_{ini}), fractional main sequence age (τ) and metalicity(Z).
To do this, first I need to find a function for τ. Using equation 5 (Hurley et al., 2000, page 547) I know the main sequence age (t_{ms}) and τ then becomes: $\tau = \frac{t}{t_{ms}}$. Since I only include stars on the main sequence, I can safely include the condition: $\tau < 1$ to cut down on computing time. Equation 12 and 13 from the same paper are very powerful equations to compute luminosity and radius for a star on the main sequence.
This Paper does reference Tout, Pols, Eggleton, & Han (1996) for Zero Age Mainsequence (ZAMS) Radii and Luminosities. However there is one error in this Paper I had to correct. In Equation 1 $\gamma + M^3$ has to be $\gamma \cdot M^3$.
insert HRD illustrating lumiradius

The program does allow for freely changeable metalicities. For my purposes I use Z=0.02 for all stars to simulate a metalicity similar to that of our galactic neighborhood.

I now run this program for every possible mass-age tuple and save the results in a matrix. This way I don't have to call the program for every distance-mass-age triple and massively cut down on computing time.
With all this I now know distance, mass, age, fractional main sequence age, luminosity and radius for any given star. I can now use these informations to compute the apparent magnitudes.

$$M_V = V - 5 \cdot \log_{10}(distance) + 5 - Red \tag{1}$$

$$M_{bol} = M_V + BC \tag{2}$$

$$\frac{L}{L_\odot} = 0.4 \cdot (4.72 - M_{bol}) \tag{3}$$

Where M_V is the absolute visual magnitude, Red is the reddening as a function of distance, M_{bol} is the absolute bolometric magnitude and BC is the Bolometric Correction. Using equations 1, 2 and 3 I can now compute the apparent visual magnitude V:

$$V = 5 \cdot \log_{10}(distance) - 5 + Red + 4.72 - \frac{L}{L_\odot \cdot 0.4} - BC \tag{4}$$

To get a rough approximation for reddening, I use Amôres & Lépine (2005, Figure 9) and interpolate between these values 1:0.9, 2:2.25 and 3:3.273
The next thing I need to know are the probability densities for stars in space $\left(\frac{dp}{dV}\right)$, mass $\left(\frac{dp}{dm}\right)$ and age $\left(\frac{dp}{dt}\right)$. In my simulation I assume a homogenous distribution of stars. This makes finding a probability density for space very easy. Because of the radial symmetry of my problem the distribution becomes solely dependant on the distance from earth:

$$\frac{dp}{dV} = \frac{1}{V_{tot}} = \frac{1}{\frac{4}{3} \cdot \pi \cdot maxdistance^3} \tag{5}$$

I assume a constant star formation rate, so the probability density in age would be similar to the density in distance. I do however use a logarithmic binning in age. Thus I can not simply use $\frac{dp}{dt}$ but instead need to find $\frac{dp}{d\log t}$ using the following: $\frac{dp}{dt} = \frac{1}{t_{ms}} \quad \wedge \quad d\log t = \frac{dt}{t \ln 10}$

$$\frac{dp}{d\log t} = \frac{t \ln 10}{t_{ms}} \tag{6}$$

I assume, that the stars are distributed in mass following the Salpeter IMF: $\frac{dp}{dM} = A \cdot M^{-2.35}$(Salpeter, 1955). Similar to the density function in age, I need to convert it for my logarithmic binning using $dlogM = \frac{dM}{M \ln 10}$

$$\frac{dp}{d\log M} = \ln 10 \cdot A \cdot M^{-1.35} \tag{7}$$

Where A is a normalization factor, that needs to be computed.

$$1 = \int_{M_{min}}^{M_{max}} A \cdot M^{-2.35} dM = \left[-1.35 \cdot A \cdot M^{-1.35}\right]_{M_{min}}^{M_{max}} \tag{8}$$

$$A = \frac{1.35}{M_{min}^{-1.35} - M_{max}^{-1.35}} \tag{9}$$

With this information I can now formulate an overarching probability density function with regard to τ

$$\frac{dp}{d\tau} = \frac{dp}{dV} dV \cdot \frac{dp}{d\log t} d\log t \cdot \frac{dp}{d\log M} d\log M \cdot \frac{1}{d\tau} \tag{10}$$

I now split τ in 20 bins. For every possible star in my sample I will check $\tau \leq 1$ and $V \leq 9$ This way I count every star on the main sequence, that falls below my magnitude cut.

The binning can be freely adjusted. To optimize runtime of my program I conducted a few tests to adjust the binnings in all three dimensions.

3 testing stuffs

I

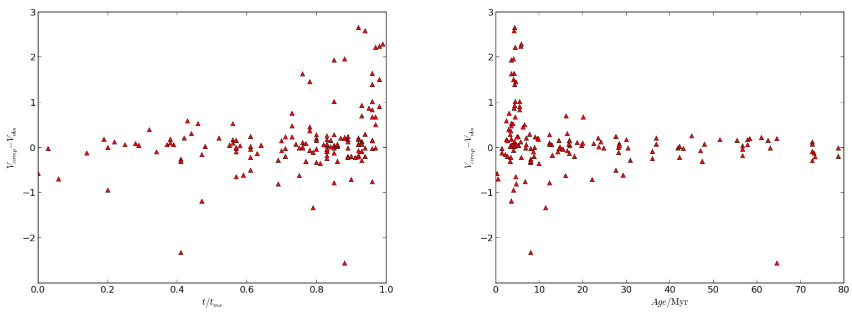

Abbildung 1: Difference between computed and observed magnitudes as a function of fractional main sequence age (left) and total age (right). The theoretical magnitudes were computed using stellar radius, luminosity and distance.

Figure 1 shows the difference between computed and observed magnitudes as a function of fractional main sequence age (left) and total age (right). It shows, that for some stars there is an offset to higher V_{comp} at high τ and low absolute ages. This implies, that this disagreement shows up in high-mass stars near TAMS. This can be explained using (Castro et al., 2014, figure 1). My code uses an evolutionary model similar to the one used at the lefthand side. V_{obs} however is obtained using a model similar to the righthand side. This subsequently means, that TAMS will shift to lower T_{eff} and higher $L/L_{\odot}$.

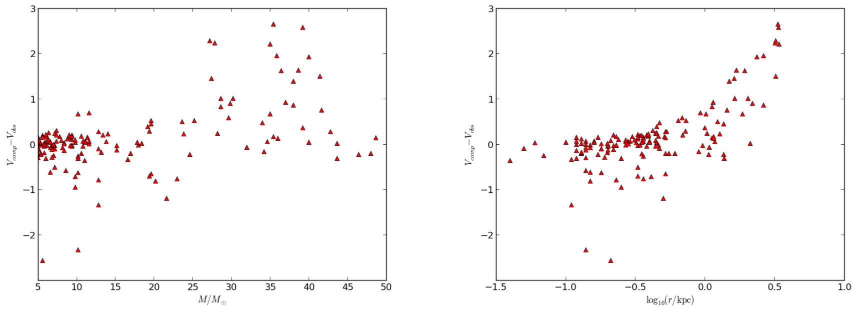

Abbildung 2: Difference between computed and observed magnitudes as a function of Mass (left) and $log_{10}(distance)$ (right). The theoretical magnitudes were computed using stellar radius, luminosity and distance.

Figure 2

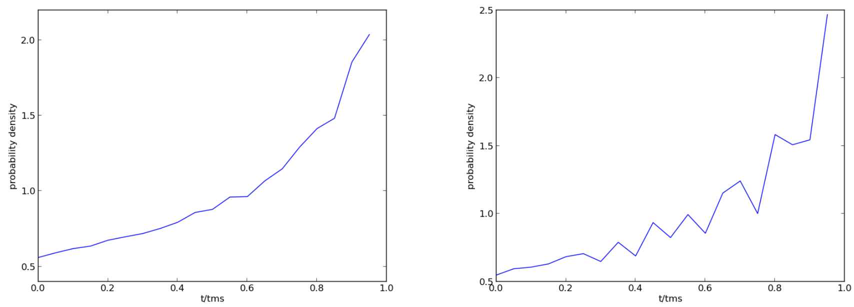

Abbildung 3: Probability density function for all stars in my sample with a magnitude cut at V=9 with binnings of 100-100-100 (left) and 50-50-50 (right) in distance-mass-age.

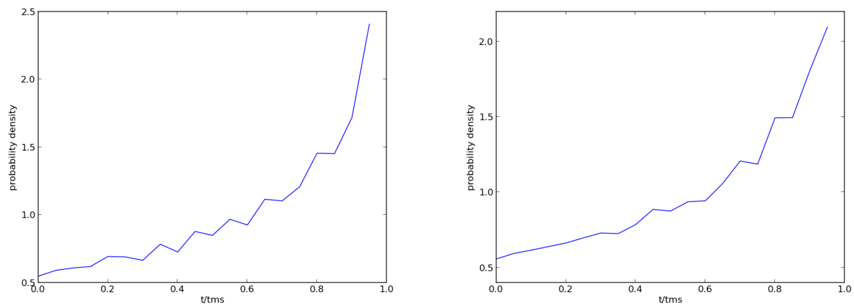

Abbildung 4: Probability density function for all stars in my sample with a magnitude cut at V=9 with binnings of 100-50-100 (left) and 100-100-50 (right) in distance-mass-age.

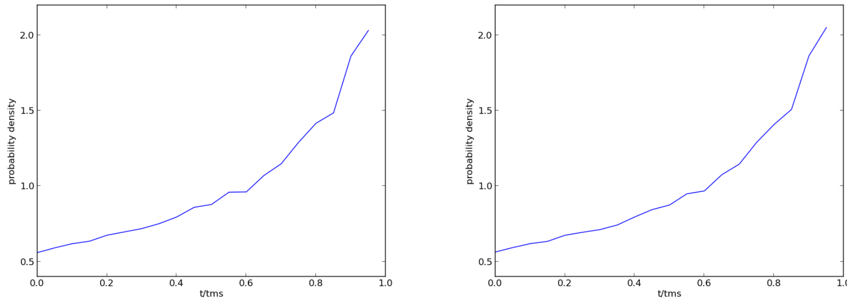

Abbildung 5: Probability density function for all stars in my sample with a magnitude cut at V=9 with binnings of 50-100-100 (left) and 10-100-100 (right) in distance-mass-age.

4 ending

Literatur

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Tout, C. A., Pols, O. R., Eggleton, P. P., & Han, Z. 1996, MNRAS, 281, 257