1 Introduction

–Initial Problem (graph of obs data)
–First thoughts (Hypothesis)

1.1 magnitude cut

Explanation for magnitude cut

2 Method

I simulate a synthetic stellar population using three variables to describe every given star: distance from earth (r), stellar mass (M) and age (t). In accordance to the observational data I attempt to model, distance ranges from 0 to $r_{\mathrm{max}} = 3\,\mathrm{kpc}$ and mass from $M_{\mathrm{min}} = 5\,\mathrm{M}_{\odot}$ to $M_{\mathrm{max}} = 50\,\mathrm{M}_{\odot}$. Because in first order the main sequence age (t_{ms}) of a star is a strictly monotonic increasing function of M one can define the maximum age for the oldest possible star in my sample to be $t_{\mathrm{max}} = t_{\mathrm{ms}}(M_{\mathrm{min}})$. With $M_{\mathrm{min}} = 5\,\mathrm{M}_{\odot}$ and using equation 5 of (Hurley, Pols, & Tout, 2000) this translates to $t_{\mathrm{max}} \approx 104\,\mathrm{Myr}$.

To formulate a probability density in fractional main sequence age ($\frac{dp}{d\tau}$), I need to know the probability densities for stars in space $\left(\frac{dp}{dV}\right)$, mass $\left(\frac{dp}{dm}\right)$ and age $\left(\frac{dp}{dt}\right)$. In my simulation I assume the stars are uniformly distributed in space. Because of the radial symmetry of the problem, the distribution is solely dependant on the distance from earth,

$$\frac{dp}{dV} = \frac{1}{V_{\mathrm{tot}}} = \frac{1}{\frac{4}{3} \cdot \pi \cdot r_{\mathrm{max}}^3}. \tag{1}$$

I assume the stars are distributed in mass according to the Salpeter Initial Mass Function (IMF): $\frac{dp}{dM} = A \cdot M^{-2.35}$ (Salpeter, 1955). Because the IMF follows an inverse power law, I use a logarithmic mass axis. Because of that I need to convert the IMF to a logarithmic binning using $d\log M = \frac{dM}{M \ln 10}$

$$\frac{dp}{d\log M} = \ln 10 \cdot A \cdot M^{-1.35}, \tag{2}$$

Where A is a normalization factor, that follows from:

$$1 = \int_{M_{\mathrm{min}}}^{M_{\mathrm{max}}} A \cdot M^{-2.35} dM = \left[-1.35 \cdot A \cdot M^{-1.35}\right]_{M_{\mathrm{min}}}^{M_{\mathrm{max}}}, \tag{3}$$

$$A = \frac{1.35}{M_{\mathrm{min}}^{-1.35} - M_{\mathrm{max}}^{-1.35}}. \tag{4}$$

I assume a constant star formation rate, so the probability density in age is $\frac{dp}{dt} = \frac{1}{t_{\mathrm{ms}}}$. The age axis is also defined logarithmic to make sure massive stars with small t_{ms} are correctly represented. Because of this I need to find $\frac{dp}{d\log t}$ using $d\log t = \frac{dt}{t \ln 10}$;

$$\frac{dp}{d\log t} = \frac{t\ln 10}{t_{\rm ms}}. \tag{5}$$

With this information I can formulate the overall probability density function for the stars in my sample with regard to τ

$$\frac{dp}{d\tau} = \frac{dp}{dV}dV \cdot \frac{dp}{d\log t}d\log t \cdot \frac{dp}{d\log M}d\log M \cdot \frac{1}{d\tau}. \tag{6}$$

To implement a magnitude cut I need to find the visual magnitude (V) for any given star. The first step is to use analytical formulations derived from a stellar evolution model by Hurley et al. (2000) which approximates the stellar evolution as a function of initial mass ($M_{\rm ini}$), fractional main sequence age (τ) and metalicity (Z). To do this, I need to find τ. Equation 5 of (Hurley et al., 2000) gives me the main sequence age for any given mass $t_{\rm ms}(M)$ and τ then becomes: $\tau(M) = \frac{t}{t_{\rm ms}(M)}$. I use Z=0.02 for all stars to simulate a metalicity similar to that of our sun according to Grevesse & Sauval (1998). I then use Equation 12 and 13 from the same paper to compute luminosity (L) and radius (R) for stars on the main sequence[1].

Figure 1 shows a Hertzsprung Russel Diagram (HRD) for five different stars on their main sequence based on the evolutionary model of Hurley et al. (2000). To obtain temperature (T) I use the following equation with σ being the Stefan-Boltzmann-Constant:

$$T = \sqrt[4]{L/(\sigma 4\pi R^2)}. \tag{7}$$

With distance, mass, age, fractional main sequence age, luminosity and radius I can compute the apparent magnitudes using the following equations:

$$M_{\rm V} = V - 5\cdot\log(r) + 5 - A(r), \tag{8}$$

$$M_{\rm bol} = M_{\rm V} + BC, \tag{9}$$

$$\log(L/{\rm L}_\odot) = 0.4\cdot(4.72 - M_{\rm bol}). \tag{10}$$

Where $M_{\rm V}$ is the absolute visual magnitude, $\log(r)$ is the logarithm to base ten of the distance from earth. A(r) is the reddening, $M_{\rm bol}$ is the absolute bolometric magnitude and BC is the Bolometric Correction after a model by (Flower, 1996). Using equations 8, 9 and 10 I can compute the apparent visual magnitude V:

$$V = 5\cdot\log(r) - 5 + A(r) + 4.72 - \frac{\log(L/{\rm L}_\odot)}{0.4} - BC. \tag{11}$$

To get a rough approximation for reddening, I use Amôres & Lépine (2005, Figure 9). I linearly interpolate the three intervals between: $r = 0\,{\rm kpc}$, $A = 0$; $r = 1\,{\rm kpc}$, $A = 0.9$; $r = 2\,{\rm kpc}$, $A = 2.25$ and $r = 3\,{\rm kpc}$, $A = 3.273$.

[1] Hurley et al. (2000) cites Tout, Pols, Eggleton, & Han (1996) for Zero Age Mainsequence (ZAMS) Radii and Luminosities. In Equation 1 of the cited paper there is a typo, where: $\gamma + M^3$ has to be $\gamma \cdot M^3$.

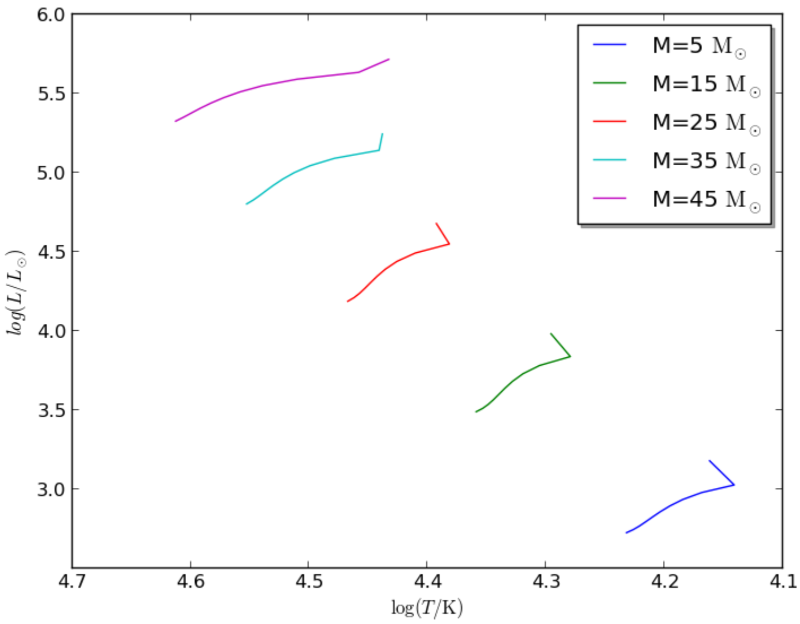

Figure 1: HRD for five different stars on their main sequence based on the evolutionary model of Hurley et al. (2000)

3 Tests

I conducted a few consistency and numerical tests.:

3.1 consistency checks

The theoretical magnitudes (V_{comp}) were computed using stellar radius, luminosity and distance. The observational magnitudes (V_{obs}) were taken from a sample of 150 stars between M=5M$_\odot$ and M=48M$_\odot$ from (Castro et al., 2014)

Figure **??** shows the difference between computed and observed magnitudes as a function of mass. There is a significant deviation towards higher V_{comp} at high masses. This can be explained using Figure 3:

My code uses an evolutionary model similar to model a). V_{obs} however is obtained using a model similar to model b). Model b) accounts for convective overshooting to a higher degree and thus the lifetime of stars is expanded. This would account for minor deviations. Because of the difference in position of TAMS in the HRD, **I STILL DON'T GET IT**. The massive deviations between $M = 25M_\odot$ and $M \approx 43M_\odot$ can be explained by taking into account inflation. Near TAMS on the main sequence massive stars will undergo an expansion of their envelopes. This gives rise to a big shift on the HRD.

The deviation to negative values is mostly caused by extinction. It is most prevalent in lower absolute ages. Stars of lower ages are primarily found in regions of active star formation. This would mean, that they are found in regions

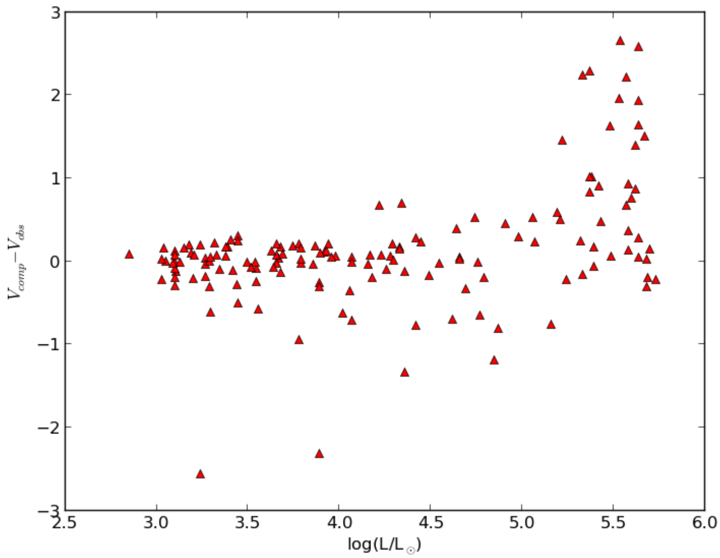

Figure 2: Difference between computed and observed magnitudes as a function of $\log(L/L_{\odot})$

of high gas density. This gas will cause extinction and thus increase V_{obs}. My model for extinction does not take into account these density fluctuations.

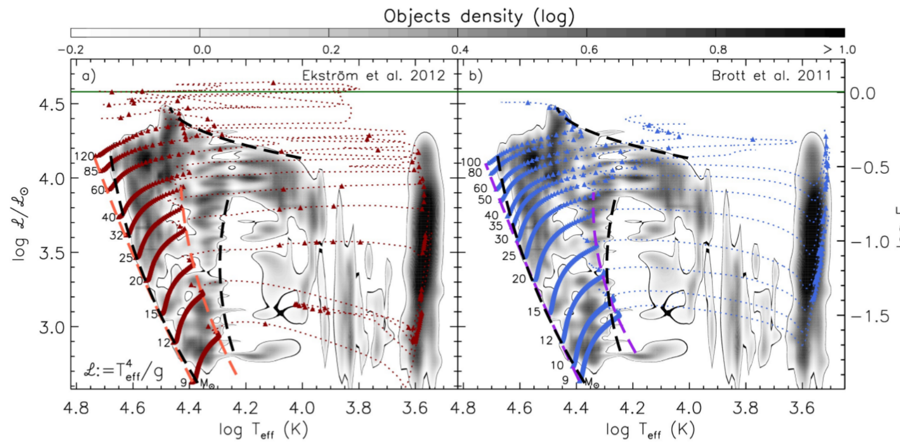

Figure 3: Grey scale representation of the probability density distribution of the location of 575 Galactic stars in the sHRD. Overlayed are stellar evolution tracks for non-rotating stars with solar composition a)Ekström et al. (2012) and b)Brott et al. (2011). The ZAMS and TAMS positions of the models are connected through orange and purple dashed lines. Red and blue triangles are placed on the tracks separated by 0.1 Myr (Castro et al., 2014)

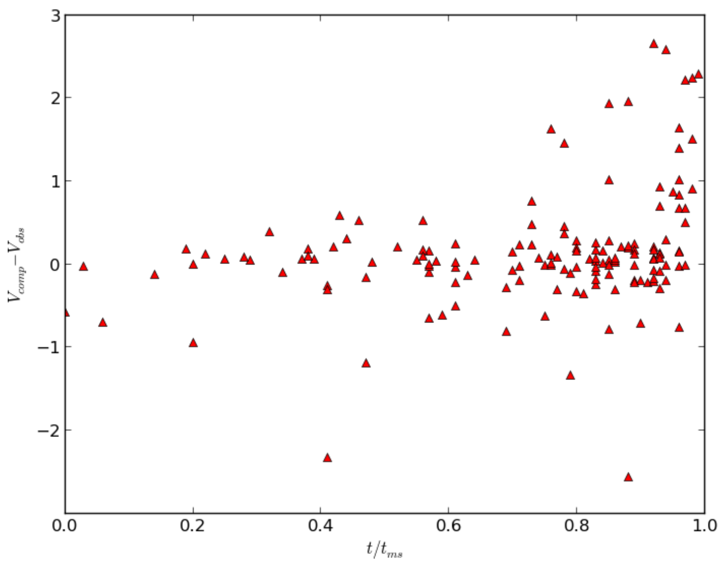

Figure 4: Difference between computed and observed magnitudes as a function of age

3.2 numerical tests

Figure 5 shows the probability density function for all stars in my sample with a magnitude cut at V=9 in four graphs. Graph 1 shows both the function with a

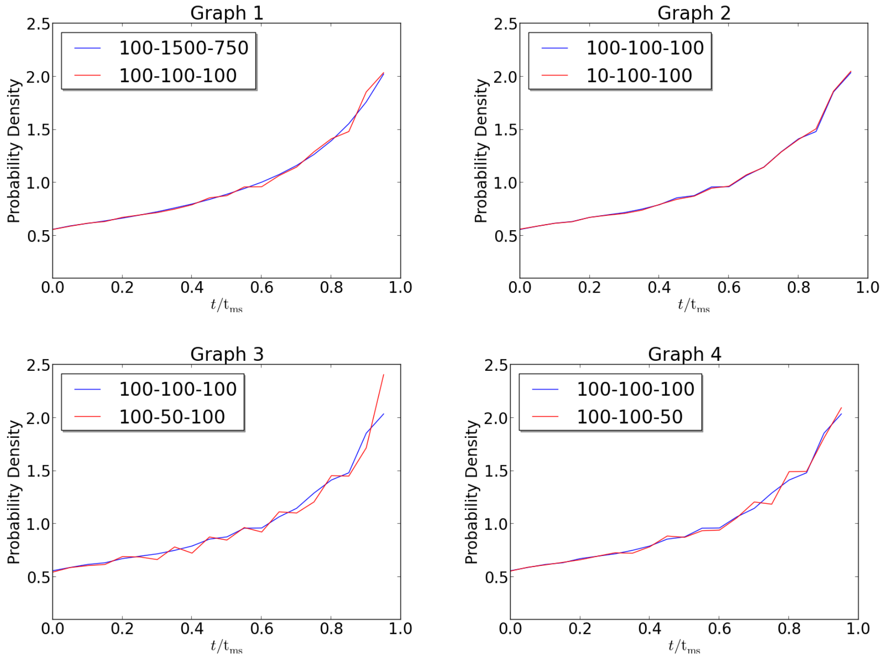

Figure 5: Probability density function for all stars in my sample with a magnitude cut at V=9 with different binnings. The legend shows the binning of each line. The first value being distance, the second being mass and the third being age.

resolution of 100-100-100 and the final results without noise for reference. One can see from this graph that with this resolution there is noise that needs to be eliminated. I don't know what variables play the biggest part in creating this noise. To understand this, in Graphs 2, 3 and 4 I change the resolution of a select variable and compare them to the curve of 100-100-100 resolution.

In Graph 2 one can see, that even tho I cut the resolution of distance by one order of magnitude, the curves only deviate very little. This tells me, that distance will be very robust to a low resolution. Graphs 3 and 4, even tho I only cut the resolution in half, both show a significant increase in noise, Graph 3 showing the greatest increase. Using these results, I determine a binning of 100-1500-750 to yield an appropriate resolution to maximize accuracy and minimize computing time.

4 Results

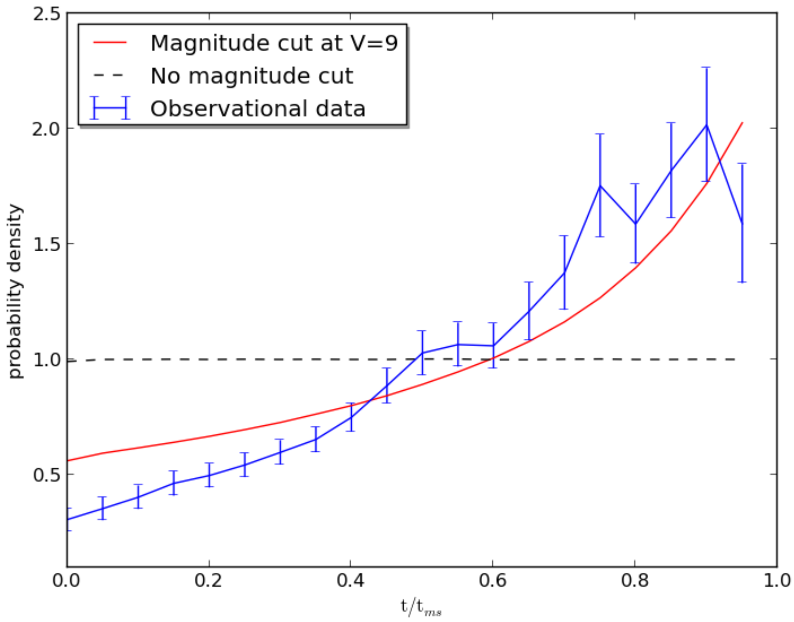

Figure 6: Probability density function of the sample, with magnitude cut at V=9 (red line), probability density function without magnitude cut (dashed line) and observational data with error bars (blue line). All computed data is obtained using a binning of 100-1500-750 in distance-mass-age. The observational data is taken from Fossati et al., 2015 (in prep.).

Figure 6 shows the probability density function with magnitude cut at V=9, probability density function without magnitude cut and observational data with error bars.
The probability density function without a magnitude cut shows, that the stars are uniformly distributed in τ. This was expected, because my simluation assumes a constant star formation rate.
Introducing a magnitude cut at V=9 reproduces a similar trend to the one we see in the observational data. This is caused by the increasing magnitude of stars There are a lot of things, that can still be implemented:

- Right now the stars are distributed homogenously in a sphere of a 3kpc radius. To make it more accurate one could model our galactic neighborhood. Namely take into account the thickness of the galactic disk, which is smaller, than 3kpc.
- The program does not take into account binary stars. This possibly has a major effect, because most massive stars are in binary systems **cite Sana et al. 2012**

- From observations we know, that we live in a part of the galaxy with little star formation. The density of young open starclusters increases the farther you move away from the solar system. As seen in Figure 7.
- My model for reddening is only approximated. It does not take into account density fluctuations in the interstellar medium. The effects of which can be seen in figures **reference to diffmag**.

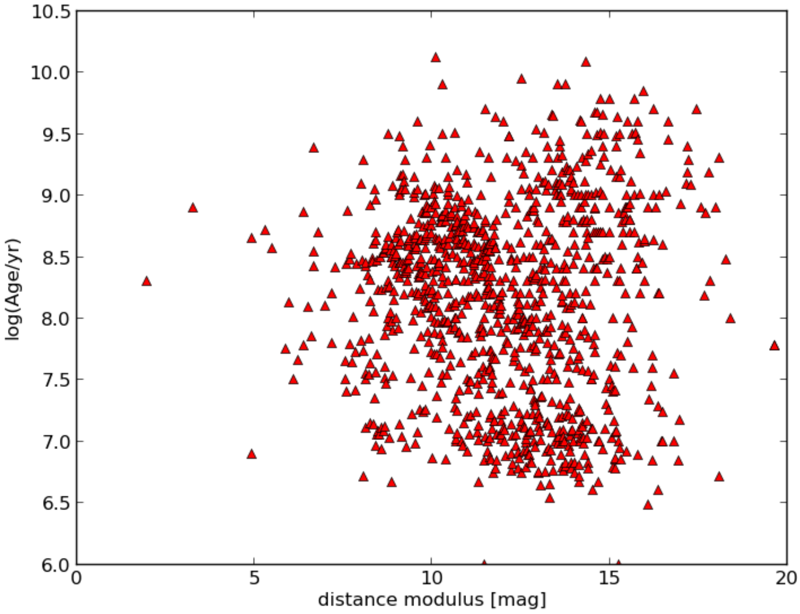

Figure 7: Open starclusters in our galaxy with distance modulus vs log(Age/yr)

5 Conclusions

References

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