1 Introduction

–Initial Problem (graph of obs data)
–First thoughts (Hypothesis)

2 Method

–Which Method did I use and why?
For this approach I used two programs: main.py and lumiradius.py.

I had the choice between two different approaches: A montecarlo approach and a grid approach.

In my program I use the grid approach. I span a grid in distance-mass-age. The range of these three variables can be easily adjusted, as can the binning. In accordance to the observational data I try to explain, distance goes from 0 to $r_{max} = 3kpc$. Mass will also be in the same parameters, as the observational data: from $M_{min} = 5M_{\odot}$ to $M_{max} = 50M_{\odot}$. The mass-distribution will follow the Salpeter Initial Mass Function (IMF)(see Salpeter, 1955). Because the IMF follows an inverse power law, I use a logarithmic mass-grid.
Age can be implemented in two different ways: I could either use a seperate age-axis for every star going from 0 to $t_{ms}(M)$ or I can use a single axis for all stars spanning from 0 to $t_{ms}(M_{min})$. Because the main sequence age (t_{ms}) of a star is a strictly monotonic increasing function of M $t_{ms}(M_{min}) = t_{max}$. **I use the second approach**. With $M_{min} = 5M_{\odot}$ this translates to $t_{max} = 104Myr$. This Axis will also be logarithmic to make sure massive stars with small t_{ms} are correctly represented.
The three informations I have about any given star, are its mass, its age and its distance from earth. From these three informations I need to derive its fractional main sequence age (τ), its apparent magnitude (V) and ultimately the probability density for all stars.
First I will implement a stellar evolution model by Hurley, Pols, & Tout (2000) which approximates the stellar evolution as a function of initial mass (M_{ini}), fractional main sequence age (τ) and metalicity(Z).
To do this, first I need to find a function for τ. Using equation 5 (Hurley et al., 2000, page 547) I know the main sequence age (t_{ms}) and τ then becomes: $\tau = \frac{t}{t_{ms}}$. Since I only include stars on the main sequence, I can safely include the condition: $\tau < 1$ to cut down on computing time. Equation 12 and 13 from the same paper are very powerful equations to compute luminosity and radius for a star on the main sequence.
This Paper does reference Tout, Pols, Eggleton, & Han (1996) for Zero Age Mainsequence (ZAMS) Radii and Luminosities. However there is one error in this Paper I had to correct. In Equation 1 $\gamma + M^3$ has to be $\gamma \cdot M^3$.

The program does allow for freely changeable metalicities. For my purposes I use Z=0.02 for all stars to simulate a metalicity similar to that of our galactic neighborhood.

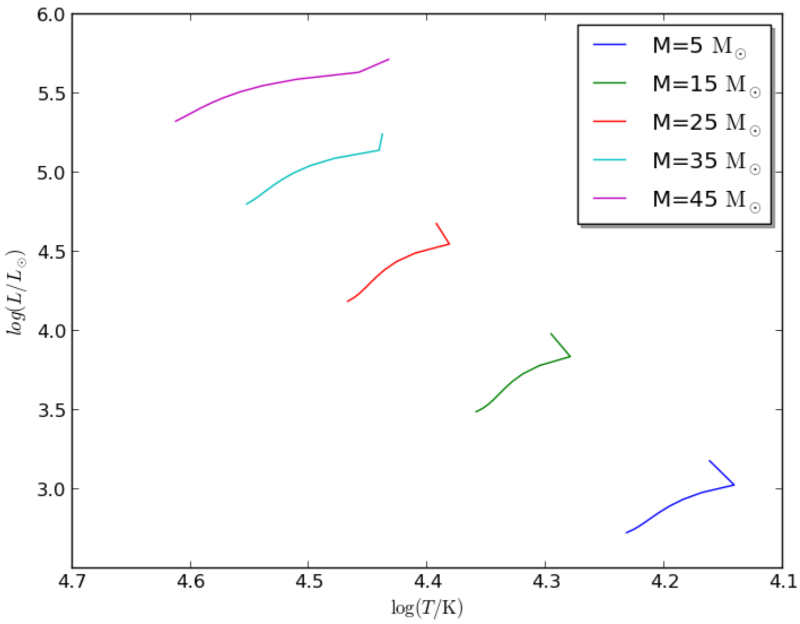

Figure 1: HRD based on lumiradius

I now run this program for every possible mass-age tuple and save the results in a matrix. This way I don't have to call the program for every distance-mass-age triple and massively cut down on computing time.
With all this I now know distance, mass, age, fractional main sequence age, luminosity and radius for any given star. I can now use these informations to compute the apparent magnitudes.

$$M_V = V - 5 \cdot \log_{10}(distance) + 5 - Red \tag{1}$$

$$M_{bol} = M_V + BC \tag{2}$$

$$\frac{L}{L_\odot} = 0.4 \cdot (4.72 - M_{bol}) \tag{3}$$

Where M_V is the absolute visual magnitude, Red is the reddening as a function of distance, M_{bol} is the absolute bolometric magnitude and BC is the Bolometric Correction. Using equations 1, 2 and 3 I can now compute the apparent visual magnitude V:

$$V = 5 \cdot \log_{10}(distance) - 5 + Red + 4.72 - \frac{L}{L_\odot \cdot 0.4} - BC \tag{4}$$

To get a rough approximation for reddening, I use Amôres & Lépine (2005, Figure 9) and interpolate between these values 1:0.9, 2:2.25 and 3:3.273
The next thing I need to know are the probability densities for stars in space $\left(\frac{dp}{dV}\right)$, mass $\left(\frac{dp}{dm}\right)$ and age $\left(\frac{dp}{dt}\right)$. In my simulation I assume a homogenous

distribution of stars. This makes finding a probability density for space very easy. Because of the radial symmetry of my problem the distribution becomes solely dependant on the distance from earth:

$$\frac{dp}{dV} = \frac{1}{V_{tot}} = \frac{1}{\frac{4}{3} \cdot \pi \cdot maxdistance^3} \tag{5}$$

I assume a constant star formation rate, so the probability density in age would be similar to the density in distance. I do however use a logarithmic binning in age. Thus I can not simply use $\frac{dp}{dt}$ but instead need to find $\frac{dp}{d\log t}$ using the following: $\frac{dp}{dt} = \frac{1}{t_{ms}} \quad \wedge \quad d\log t = \frac{dt}{t \ln 10}$

$$\frac{dp}{d\log t} = \frac{t \ln 10}{t_{ms}} \tag{6}$$

I assume, that the stars are distributed in mass following the Salpeter IMF: $\frac{dp}{dM} = A \cdot M^{-2.35}$(Salpeter, 1955). Similar to the density function in age, I need to convert it for my logarithmic binning using $dlogM = \frac{dM}{M \ln 10}$

$$\frac{dp}{d\log M} = \ln 10 \cdot A \cdot M^{-1.35} \tag{7}$$

Where A is a normalization factor, that needs to be computed.

$$1 = \int_{M_{min}}^{M_{max}} A \cdot M^{-2.35} dM = \left[-1.35 \cdot A \cdot M^{-1.35}\right]_{M_{min}}^{M_{max}} \tag{8}$$

$$A = \frac{1.35}{M_{min}^{-1.35} - M_{max}^{-1.35}} \tag{9}$$

With this information I can now formulate an overarching probability density function with regard to τ

$$\frac{dp}{d\tau} = \frac{dp}{dV} dV \cdot \frac{dp}{d\log t} d\log t \cdot \frac{dp}{d\log M} d\log M \cdot \frac{1}{d\tau} \tag{10}$$

I now split τ in 20 bins. For every possible star in my sample I will check $\tau \leq 1$ and $V \leq 9$ This way I count every star on the main sequence, that falls below my magnitude cut.

The binning can be freely adjusted. To optimize runtime of my program I conducted a few tests to adjust the binnings in all three dimensions.

3 Tests

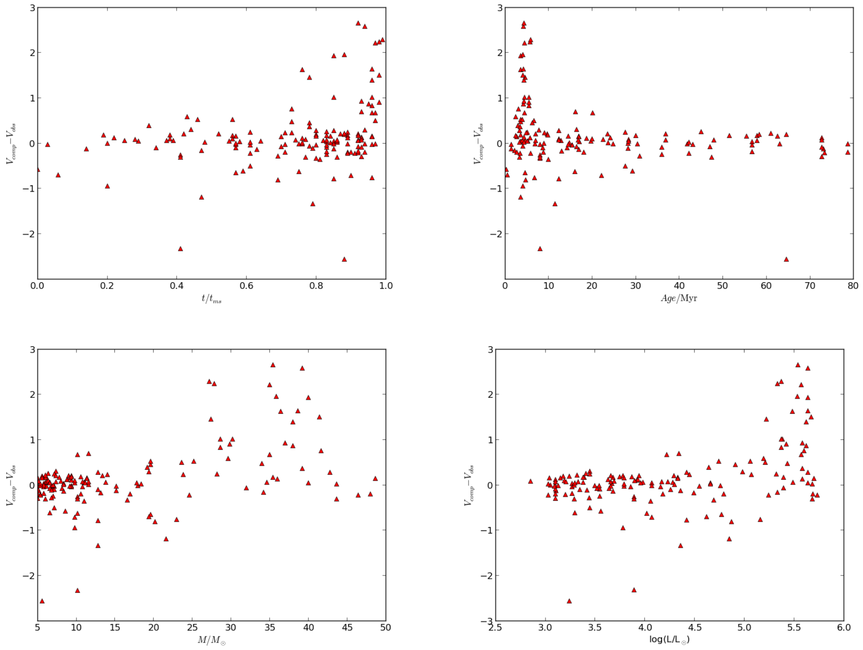

Figure 2: Difference between computed and observed magnitudes as a function of fractional main sequence age (top left), total age (top right), Mass (bottom left) and $log_{10}(r/\mathrm{kpc})$ (bottom right). The theoretical magnitudes (V_{comp}) were computed using stellar radius, luminosity and distance.

Figure 2 shows the difference between computed and observed magnitudes as a function of fractional main sequence age (top left), total age (top right), Mass (bottom left) and $log_{10}(r/\mathrm{kpc})$ (bottom right).
The top graphs show, that there is a high offset to higher V_{comp} at high τ and low absolute ages. This implies, that this disagreement shows up in high mass stars near TAMS. This can be explained using (Castro et al., 2014, figure 1). My code uses an evolutionary model similar to the one used at the lefthand side. V_{obs} however is obtained using a model similar to the righthand side. The right model accounts for convective overshooting to a higher degree and thus the lifetime of a star is expanded. This agrees with the graph at the bottom left. The very high deviations are only seen in stars of Mass between $M = 25M_{\odot}$ and $M \approx 43M_{\odot}$. The convective overshooting, which extends the life of stars in the righthand side can explain for minor deviations. The massive differences between $M = 25M_{\odot}$ and $M \approx 43M_{\odot}$ can be explained by taking into account inflation however. Near TAMS on the main sequence massive stars will undergo an expansion of their envelopes. The right model takes this into account, whereas the left one does not. The luminosity limit for stars in the milky way for such an effect to occur is at about $log(L/L_{\odot}) = 5.3$

The deviation to negative values is most prevalent in lower absolute ages. This supports my hypothesis, that this is caused by extinction. Stars of lower ages are primarily found in regions of active star formation. This would mean, that they are found in regions of high gas density fluctuation. This gas will cause extinction and thus increase V_{obs}.

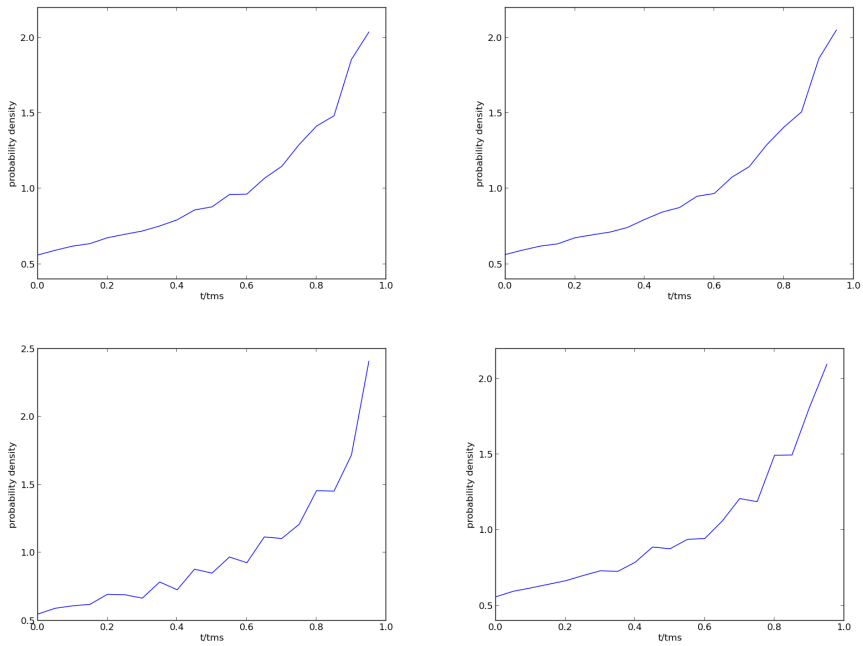

Figure 3: Probability density function for all stars in my sample with a magnitude cut at V=9 with binnings of 100-100-100 (top left) 10-100-100 (top right) 100-50-100 (bottom left) and 100-100-50 (bottom right) in distance-mass-aghe.

Figure 3 shows the probability density function for all stars in the sample with a magnitude cut at V=9 with different distance-mass-age binnings. The top left graph shows the reference graph with a binning of 100 for all variables. The top right one has the binning in distance reduced by a factor of ten. Both graphs are very similar. This means, that distance will be very robust to low binnings.

The bottom now shows the same graph with the binning in mass (left) and age (right) halved. Here the discrepancies are very obvious. Because of this I use a binning of 100-1000-1000 for my final data.

4 Results

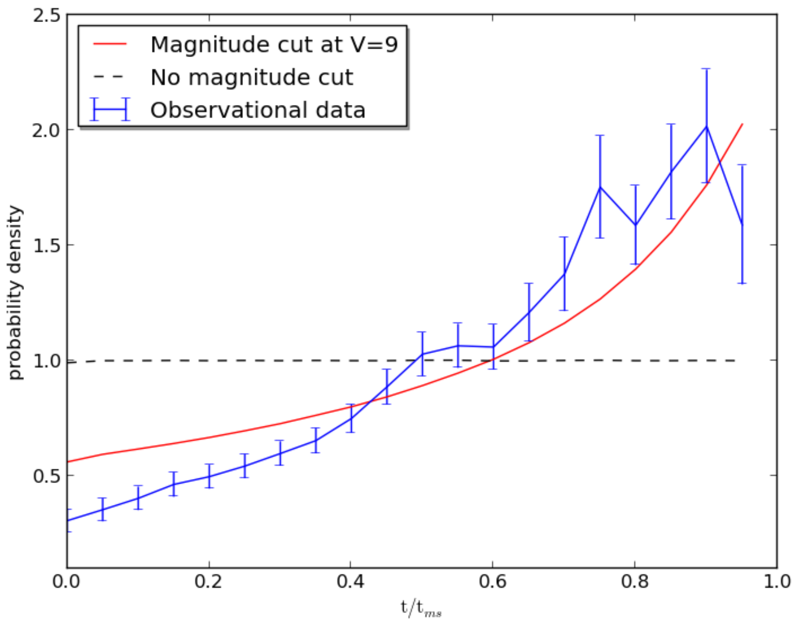

Figure 4: Probability density function with magnitude cut at V=9 (red line), probability density function without magnitude cut (black line) and observational data with error bars (blue line). All computed Data is obtained using a binning of 100-1000-1000 in distance-mass-age. The observational data is taken from **citation?**

Figure 4 shows the probability density function with magnitude cut at V=9 (red line), probability density function without magnitude cut (black line) and observational data with error bars (blue line).
The probability density function without a magnitude cut shows, that the stars are uniformly distributed in τ. This way I can confirm, that any shape the curve does have is because of the magnitude cut I implement.
Comparing the red and blue line, the blue line seems to have a higher slope, than the red line. They do however compare pretty well considering the program is only a minimalistic model. There are a lot of things, that can still be implemented:

- Right now the stars are distributed homogenously in a sphere of a 3kpc radius. To make it more accurate one could model our galactic neighborhood. Namely take into account the thickness of the galactic disk, which is smaller, than 3kpc.
- The program does not take into account binary stars.
- From observations we know, that we live in a part of the galaxy with

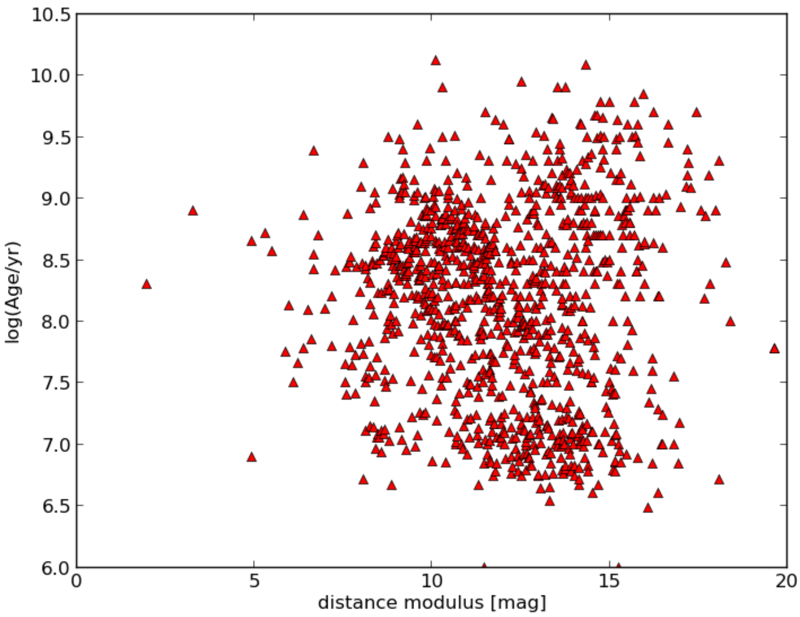

Figure 5: Shows star clusters

little star formation. The density of young open starclusters increases the farther you move away from the solar system. As seen in Figure 5.

- My model for reddening is very approximated. It does not take into account density fluctuations in the interstellar medium. The effects of which can be seen in figure 2.

5 Conclusions

References

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Hurley, J. R., Pols, O. R., & Tout, C. A. 2000, MNRAS, 315, 543

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