Gamma Function

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1 Definition

This report is concerning the non-simple function, the Gamma function, $\Gamma(x)$. This is a generalization of the factorial operator ($n!$) for integers to non-integers as well. In fact, the Gamma function satisfies $\Gamma(n) = (n-1)!$ for any positive integer.

For any complex number, z, with a real part larger than zero, $\mathrm{Re}(z) > 0$, we can define the Gamma function as [1]

$$\Gamma(z) = \int_0^\infty x^z \exp(-x) \mathrm{d}x \,. \tag{1}$$

This is known as the *Euler integral of the second kind*. This function also satisfies the equation [1]

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)}, \quad z \notin \mathbb{Z}, \tag{2}$$

such that we are able to define the Gamma function for negative real parts as well. Equation 2 can obviously be rewritten to

$$\Gamma(z) = \frac{\pi}{\sin(\pi z)\Gamma(1-z)}, \quad z \notin \mathbb{Z}, \tag{3}$$

which means that any complex number, z, with a negative real part can be calculated using the integral in equation 1 with $(1-z)$ as the argument (which must then have a positive real part).

2 Calculation

I have calculated the Gamma function using equations 1 and 3 in C# and the result is shown in figure 1. Tabulated values from the Gamma function implemented in the C language are also shown in the figure. We can clearly see why equations 2 and 3 do not hold for integers, since the function diverges at negative integers.

References

[1] *Gamma function*, available at `http://en.wikipedia.org/wiki/Gamma_function`.

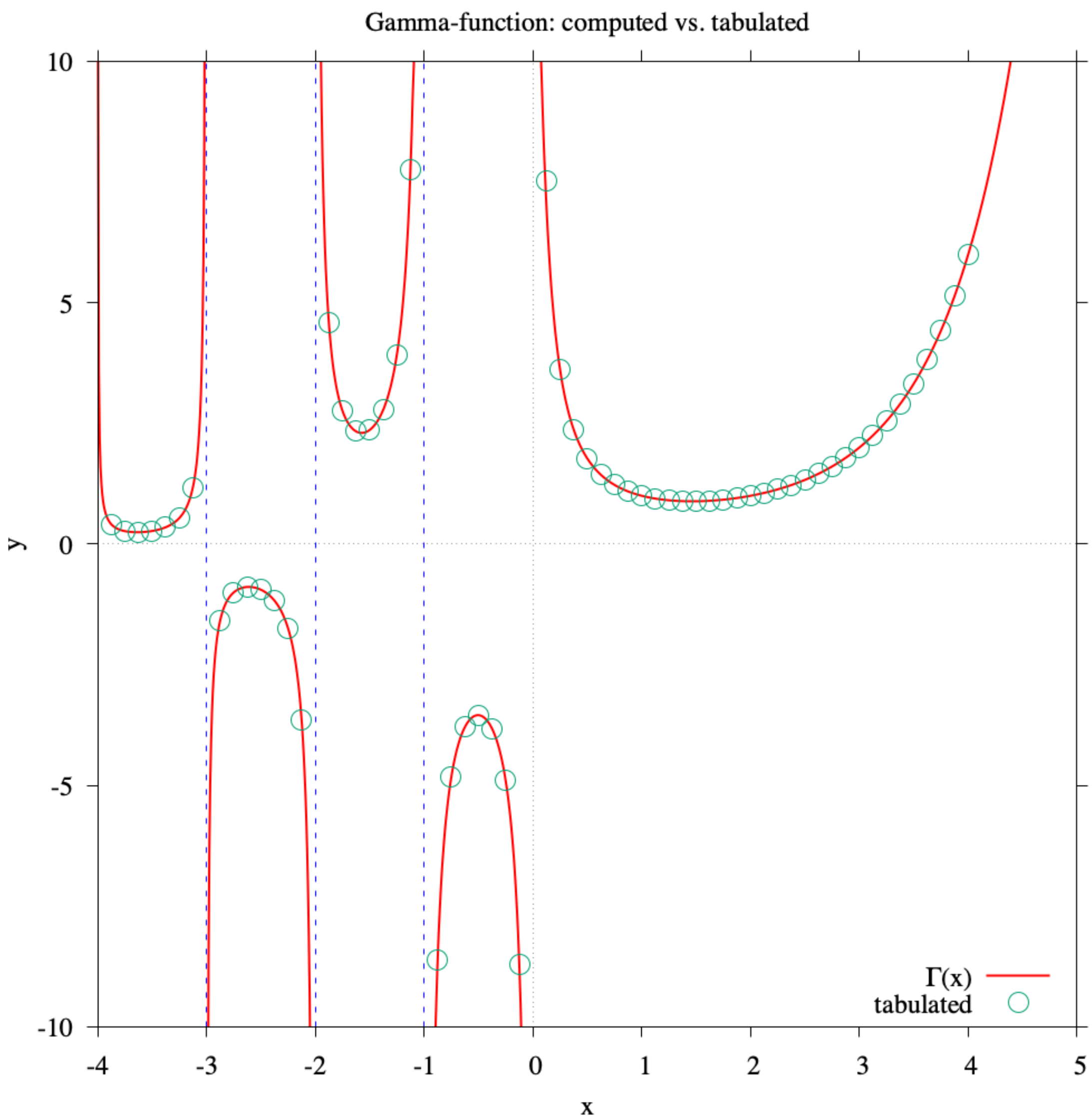

Figure 1: *The Gamma function calculated with C# along with tabulated values from the implemented Gamma function in the C language.*