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Data Dependence in Matrix Multiplication

```
int a[n,n], b[n,n], c[n,n]

for i=1 to n
    for j=1 to n
      S0 c[i,j]=0
        for k=1 to n
          S c[i,j]=c[i,j]+a[i,k]*b[k,j]
           end for
      end for
  end for
```

Statement S_0 is at loop level 2, S at level 3

Maximum common loop index of S_0 and S is 2

Execution index sets

- $[S_0] = [1:100]^2$
- $[S] = [1:100]^3$

Control sets

- $[S_0, S_0] = \{((i,j),(i',j') \in [S_0] \times [S_0] :\ (i,j) < (i',j')\}$
- $[S, S_0] = \{((i,j,k),(i',j') \in [S] \times [S_0] :\ (i,j) < (i',j')\}$
- $[S_0, S] = \{((i,j),(i',j',k') \in [S_0] \times [S] :\ (i,j) \leq (i',j')\}$
- $[S, S] = \{((i,j,k),(i',j',k') \in [S] \times [S] :\ (i,j,k) < (i',j',k')\}$

Input/output sets

- $DEF(S_0) = \{\mathsf{c[i,j]}\}$
- $USE(S_0) = \{\mathsf{i,j}\}$
- $DEF(S) = \{\mathsf{c[i,j]}\}$
- $USE(S) = \{\mathsf{c[i,j]}, \mathsf{a[i,k]}, \mathsf{b[k,j]}, \mathsf{i}, \mathsf{j}, \mathsf{k}\}$

Check for dependences

1. $S_0(i,j)\ \delta\ S_0(i',j')$

   - no pair of iteration vectors $((i,j),(i',j')) \in [S_0, S_0]$ satisfies DEP-2
   - no common variable: in every instance of S_0 a different element $\mathsf{c}[\mathsf{i},\mathsf{j}]$ is accessed

2. $S(i,j,k)\ \delta\ S_0(i',j')$

   - no pair $((i,j,k),(i',j')) \in [S, S_0]$ satisfies DEP-2
   - no common variable: $S_0(i',j')$ cannot be executed after $S(i,j,k)$ with the same values of i, j, and thus access the same $\mathsf{c}[\mathsf{i},\mathsf{j}]$)

3. $S_0(i,j)\ \delta\ S(i',j',k')$

   (a) with $(i,j) < (i',j')$, no pair of iteration vectors $((i,j),(i',j',k')) \in [S_0, S]$ satisfies DEP-2 (no common variable)

   (b) with $(i,j) = (i',j')$ the same $\mathsf{c}[\mathsf{i},\mathsf{j}]$ is accessed in $S_0(i,j)$ and $S(i',j',k')$ and DEP-2 is satisfied

   - DEP-3 holds for $k' = 1$
   - dependences: $S_0(i,j)\ \delta_{\infty,true}\ S(i,j,1)$ and $S_0(i,j)\ \delta_{\infty,out}\ S(i,j,1)$
   - loop independent dependence: $S_0\ \delta^{(=,=)}_{\infty,true,out}\ S$

4. $S(i,j,k)\ \delta\ S(i',j',k')$

   - DEP-2 holds for $(i,j,k) <_3 (i',j',k')$ with $v = \mathsf{c}[\mathsf{i},\mathsf{j}]$
   - DEP-3 holds for $k = k' - 1$
   - loop carried dependence at level 3: $S\ \delta^{(=,=,<)}_{3,true,anti,out}\ S$

References

[1] H. Zima, B. Chapman. *Supercompilers for Parallel and Vector Computers.* ACM Press Frontier Series, Addison-Wesley, 1990.