1 Candidate inflationary regions from toy models

Here we present a simple example of supervised learning problem. The aim of problem is trying to check whether some members of a family of effective field theories can host slow-roll inflation, and how to find the field space region accommodating for inflation.

The physical question. Firstly, let us specify the family of effective field theories that we will examine. We consider four-dimensional effective field theories described by the action

$$S_{\rm EFT} = \int {\rm d}^4 x\,\sqrt{-g}\left(\frac{1}{2}M_{\rm P}^2 R - \frac{1}{2}M_{\rm P}^2\, G_{ij}(\varphi)\partial\varphi^i\cdot\partial\varphi^j - V(\varphi)\right)\,, \tag{1.1}$$

We assume that the effective description is endowed with only two scalar fields, which we denote as a and s, collected as $\varphi^i = (a, s)$. Furthermore, we assume that the field space metric is $G_{ij} = \frac{1}{2s^2}\delta_{ij}$. Indeed, since the field space metric does not depend on the scalar field a, the kinetic terms of the scalar fields are invariant under any shift $a \to a + c$, for some $c \in \mathbb{R}$. For this reason, we shall refer to a as '*axion*', and call its partner, s, '*saxion*'.

The scalar potential is assumed to acquire the following form

$$V(a,s) = \frac{(e-am)^2}{2s^p} + \frac{1}{2}m^2 s^q\,, \tag{1.2}$$

for some integral parameters $e, m \in \mathbb{Z}$, and $p, q \in \mathbb{Z}_{>0}$. The scalar potential breaks the axion shift symmetry to just a discrete subgroup thereof: in fact, the scalar potential (1.2) is invariant under shifts $a \to a+n$, with $n \in \mathbb{Z}$, provided that the parameter e gets also shifted as $e \to e+nm$. Clearly, if the parameter e is frozen, the axion shift symmetry is spontaneously broken.

The family of effective field theories (1.1), described by the scalar potentials (1.2), mimic the effective field theories stemming from the compactification of Type IIB string theory over an orientifolded Calabi-Yau three-fold: in fact, within these theories, the axion a and saxion s can be seen as describing a single, dynamical complex stricture modulus $t = a + is$, and e and m can be regarded as Ramond-Ramond background fluxes. The field space metric introduced above, as well as the structure of the scalar potential (**??**) resemble those emerging from effective field theories defined in a field space region close to a complex structure boundary located at infinite field distance.

We would like to investigate whether, in the effective field theories so defined, slow-roll inflation is strictly forbidden. In order to realize slow-roll inflation, it is crucial that the first slow-roll parameter ε remains very small, $\varepsilon \ll 1$, along the inflationary path. Along geodesic paths, the first slow-roll parameter ε can be recast as

$$\varepsilon = \frac{\gamma^2}{2} \tag{1.3}$$

where we have introduced the *de Sitter coefficient*:

$$\gamma = \frac{\|\nabla V\|}{V}\,, \qquad \text{with} \qquad \|\nabla V\|^2 := G^{ij}\partial_i V \partial_j V\,, \tag{1.4}$$

Therefore, as a preliminary, exploratory analysis, we can check which regions, in the two-dimensional space spanner by the axion and the saxion, display a small first-slow roll parameter (1.3), for these regions cannot accommodate inflationary paths.

Such regions can be machine-learned, simply by knowing the values that the first-slow parameter acquires at some points in the field space. In the following, we will firstly show how to craft a database that can be fit the analysis, and then we will show how to machine-learn the allowed inflationary regions.

Creation of a database. The scalar potentials in (1.2) come in families: each choice of parameters (p, q), (e, m) specifies a single scalar potential. For simplicity, let us momentarily fix the parameters $e = m = 1$; then, we will study possible inflationary regions for some values of p, q.

Indeed, let us choose some pairs $(p_{(\mathsf{I})}, q_{(\mathsf{I})})$, with $\mathsf{I} = 1, \ldots, \mathsf{N}$. Now, let us focus on a single element of these pairs, $(p_{(\mathsf{I})}, q_{(\mathsf{I})})$. For fixed $(p_{(\mathsf{I})}, q_{(\mathsf{I})})$, we scan large subsets of the field space. For instance, we may construct a 'grid' in the subset of the field space, with $a \in [a_{\min}, a_{\max}]$, $s \in [s_{\min}, s_{\max}]$; alternatively, we may pick random points in the given field space subset. Then, at each of these field space points we compute the first-slow parameter (1.3) via (1.4); to each of these points, we assign the label '1', or 'True' if at that point $\varepsilon < c$, for some x; otherwise, we assign the label '0', or 'False' to that point.

In Figure 1 is plotted a concrete example of a set of databases so constructed: we chose $\mathsf{N} = 9$, and we have conventionally set $c = \frac{1}{2}$.

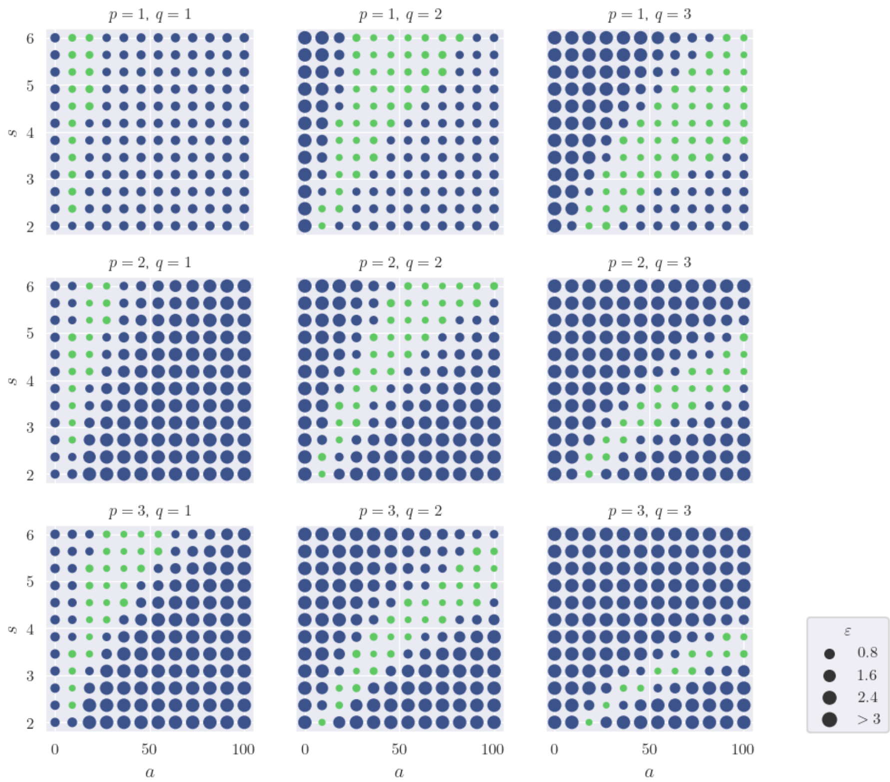

Figure 1: The first slow-roll parameter $\varepsilon = \frac{\gamma^2}{2}$ across the field space for some values of p, q. We have fixed $e = m = 1$. The size of the dots denotes the magnitude of ε at the given field space point; in particular, the green dots are those for which $\varepsilon < \frac{1}{2}$.

Machine learning the effective theories hosting valleys. As we have formulated it, the problem is a binary classification problem: for regions where field space points are labeled as

'1', slow-roll inflation is not excluded a priori; otherwise, in the other regions, where points are labeled as '0', slow-roll inflation is necessarily excluded. Out of the set of the isolated points in the databases above, one can learn the regions associated to each of the labels using a simple k-nearest neighbors algorithm. Indeed, the k-nearest neighbors algorithm allows one to learn the regions, in the field space, that are likely associated to the label '1', or '0'. In Figure 2 is the application of the k-nearest neighbors algorithm to the data sets plotted in Figure 1.

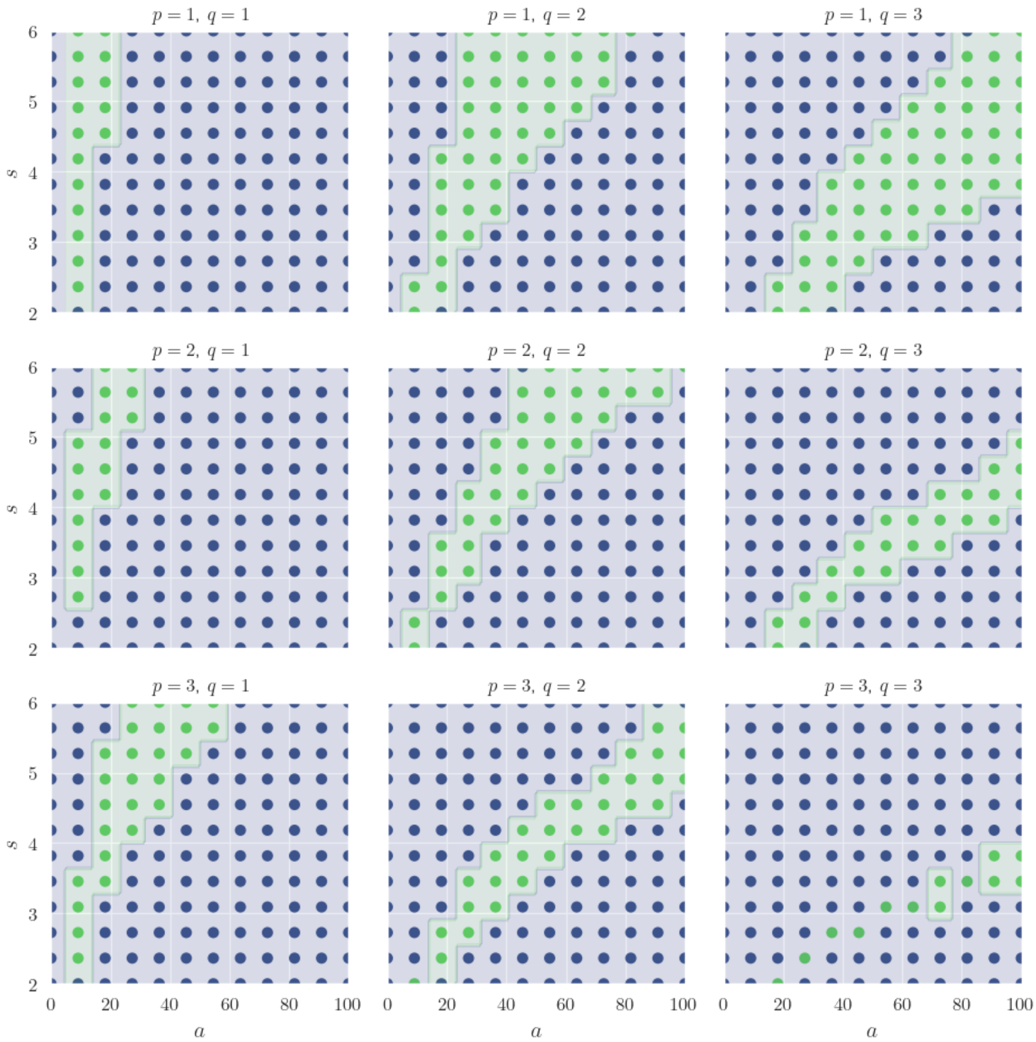

Figure 2: Here, given the dataset depicted in Figure 1, the regions were $\varepsilon < \frac{1}{2}$ or $\varepsilon \geq \frac{1}{2}$ have been learned via a k-nearest neighbor algorithm.

In Figure 3 we have performed an analogous analysis, scanning however the parameter space (e, m).

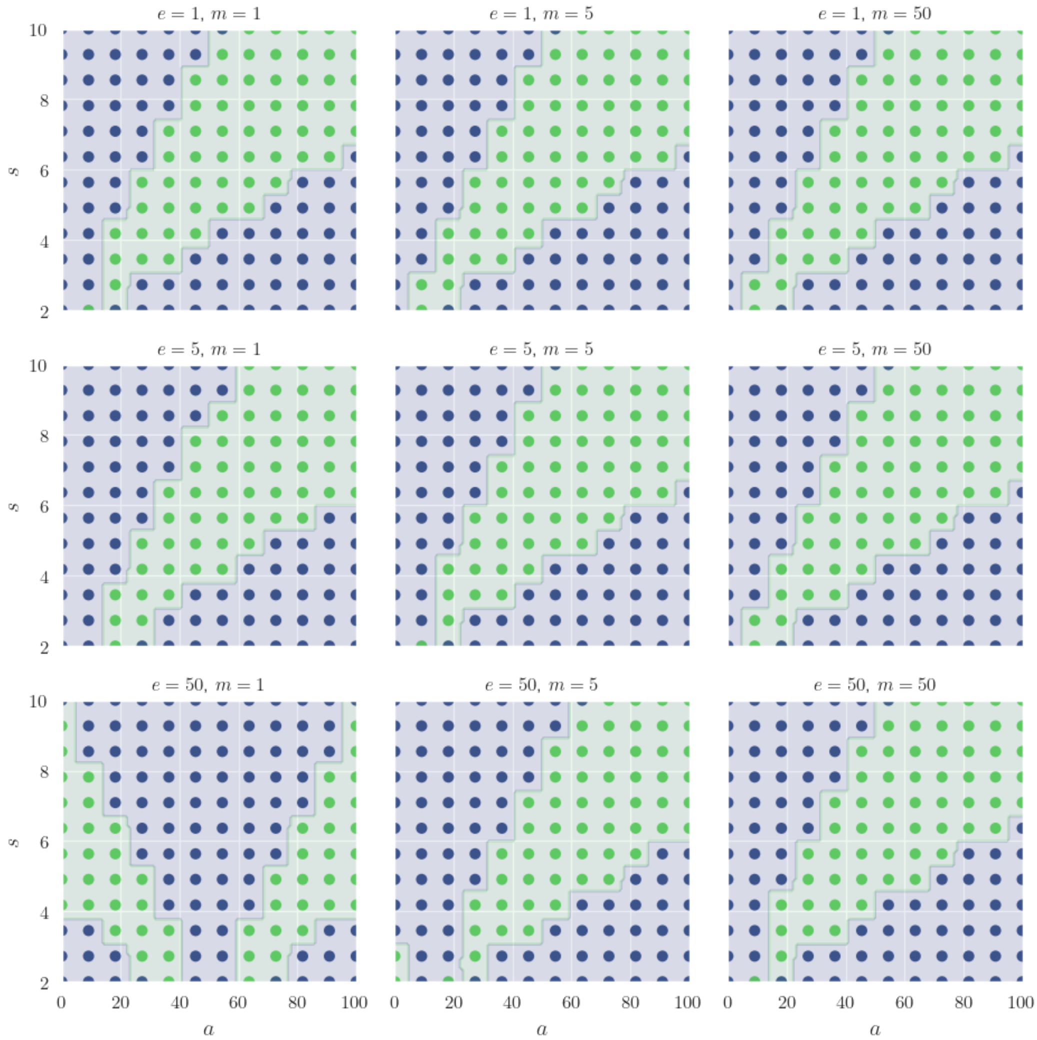

Figure 3: The first slow-roll parameter $\varepsilon = \frac{\gamma^2}{2}$ across the field space for some values of e, m. We have fixed $p = 1$, $q = 2$. The size of the dots denotes the magnitude of ε at the given field space point; in particular, the green dots are those for which $\varepsilon < \frac{1}{2}$, and the blue ones those for which $\varepsilon \geq \frac{1}{2}$. The associated regions are learned via a k-nearest neighbor algorithm.