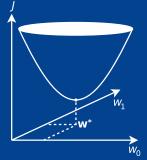


MACHINE LEARNING LESSON 4: Training I

CARSTEN EIE FRIGAARD





L04: Training I

Agenda

- Discuss your WIKI contribution...(5 min)
- Exercise from L03:

Exercise: L03/metrics.ipynb

▶ L04 Training I: Training a linear regression model

Exercise: L04/linear_regression_1.ipynb Exercise: L04/linear_regression_2.ipynb

After the class:

yet another WIKI page, deadline *L06*, 05-03-2019 (in two weeks).

- Next lesson: L05 Training II: Training and model concepts
- NOTE: on page # in [HOML] ...up to 7 editions.

Editor: Nicole Tache
Production Editor: Nicholas Adams
Copyeditor: Rachel Monaghan
Proofreader: Charles Roumeliotis

March 2017: First Edition

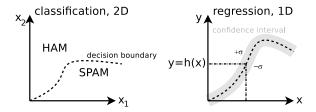
Revision History for the First Edition
2017-03-10: First Release

RESUMÉ: Classification vs. Regression

Given the following

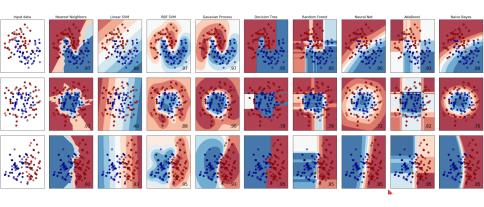
$$h: \mathbf{x} \to \mathbf{y}$$

- if y is discrete/categorical variable, then this is classification probl
- if y is real number/continuous, then this is a regression problem.



RESUMÉ: Classification

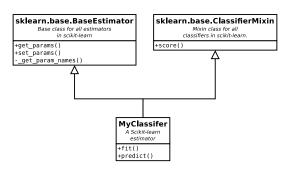
Decision Boundaries for different Models and Datasets



Souce code: L03/Extra/plot_classifier_comparison.ipynb in [GITMAL].

RESUMÉ: The Scikit-learn Fit-Predict Interface

learn





- module private: one underscore
- class-private: two underscores

via mangled names.

- ...NOTE: no virtual void fit() = 0; declaration in python!
- ...for modules, private funs can still be accessed via a hack?!
- ...src file: /opt/anaconda3/pkgs/.../sklearn/base.py

RESUMÉ: Exercise:

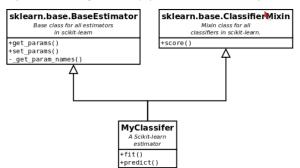
L03/dummy_classifier.ipynb

A dummy classifier for the fit-predict interface, plus intro to a Stochastic Gradient Decent method (SGD)

Qb Implement a dummy binary classifier

Follow the code found in [HOML], p84, but name you estimator $\mbox{DummyClassifier}$ instead of $\mbox{Never5Classifyer}$.

Here our Python class knowledge comes into play. The estimator class hierarchy looks like



All Scikit-learn classifiers inherit form BaseEstimator (and possible also ClassifierMixin), and they must have a fit-predict function pair (strangely not in the

Nomenclature

For a binary classifier

NAME	SYMBOL	ALIAS
true positives	TP	
true negatives	TN	
false positives	FP	type I error
false negatives	FN	type I error type II error

Nomenclature

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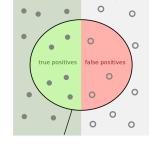
and $N = N_P + N_N$ being the total number of samples and the number of positive and negative samples respectively.

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true negatives

false negatives

[https://en.wikipedia.org/wiki/Precision_and_recall]

Precision, recall and accuracy, F_1 -score, and confusion matrix

```
precision, p = \frac{TP}{TP+FP}

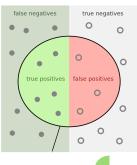
recall (or sensitivity), r = \frac{TP}{TP+FN}

accuracy, a = \frac{TP+TN}{TP+TN+FP+FN}

F_1-score, F_1 = \frac{2pr}{p+r}
```

Precision, recall and accuracy, F_1 -score, and confusion matrix

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F_1\text{-score}, \qquad F_1 = \frac{2pr}{D+F}
```





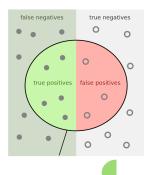


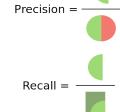
Precision, recall and accuracy, F_1 -score, and confusion matrix

precision,
$$p = \frac{TP}{TP+FP}$$
recall (or sensitivity),
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accuracy,
$$a = \frac{TP+TN}{TP+TN+FP+FN}$$

$$F_1\text{-score}, \qquad F_1 = \frac{2pr}{p+r}$$

Сс	onfusion Matrix, 🛚 🕽		
		actual	
		true	false
	predicted true	TP	FP
	predicted false	FN	TN





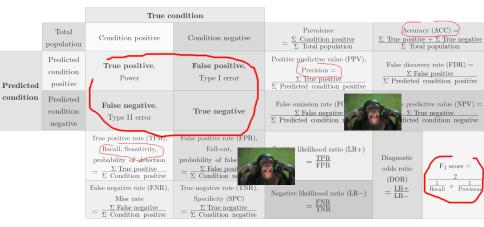
 $NOTE_0$: you can *compare* precision... F_1 -score, but not necessarily the cost, J.

NOTE₁: beware of matrix transpose and interpretation of 'TP/TN'!

Nomenclature for the Confusion Matrix

		True condition				
	Total population	Condition positive	Condition negative	$\begin{aligned} & & & \text{Prevalence} \\ & = \frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}} \end{aligned}$	Σ True positiv	$cev (ACC) = \frac{1}{e^2 + \Sigma True negative}$ al population
Predicted condition	Predicted condition positive	True positive, Power	False positive, Type I error	Positive predictive value (PPV), $\begin{array}{c} \text{Precision} = \\ \Sigma \text{ True positive} \\ \Sigma \text{ Predicted condition positive} \end{array}$	$\begin{aligned} & \text{False discovery rate (FDR)} = \\ & & \Sigma & \text{False positive} \\ & \Sigma & \text{Predicted condition positive} \end{aligned}$ $& \text{Negative predictive value (NPV)} = \\ & & \Sigma & \text{True negative} \\ & \Sigma & \text{Predicted condition negative} \end{aligned}$	
	Predicted condition negative	False negative, Type II error	True negative	$\frac{\text{False omission rate (FOR)} = }{\sum \text{False negative}}$ \(\Sigma\) Predicted condition negative		
		$ \begin{aligned} & \text{True positive rate (TPR),} \\ & \text{Recall, Sensitivity,} \\ & \text{probability of detection} \\ & = \frac{\sum \text{True positive}}{\sum \text{Condition positive}} \\ & \text{False negative rate (FNR),} \\ & \text{Miss rate} \\ & = \frac{\sum \text{False negative}}{\sum \text{Condition positive}} \end{aligned} $	False positive rate (FPR), Fall-out, Fall-out, probability of false alarm = $\frac{\Sigma}{\Sigma}$ False positive $\frac{\Sigma}{\Sigma}$ Condition negative True negative rate (TNR), Specificity (SPC) = $\frac{\Sigma}{\Sigma}$ True negative $\frac{\Sigma}{\Sigma}$ Condition negative grative $\frac{\Sigma}{\Sigma}$ Condition negative	$\begin{array}{c} \text{Positive likelihood ratio (LR+)} \\ = \frac{\text{TPR}}{\text{FPR}} \\ \\ \text{Negative likelihood ratio (LR-)} \\ = \frac{\text{FNR}}{\text{TNR}} \end{array}$	Diagnostic odds ratio (DOR) $= \frac{LR+}{LR-}$	$F_1 \text{ score} = \frac{2}{\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}}$

Nomenclature for the Confusion Matrix



Mr. Itmal



prevalence, positive predictive value, etc. not important to know at all!

Accuracy Paradox...

```
class ParadoxClassifier(BaseEstimator):
def fit(self, X, y=None):
    pass
def predict(self, X):
    assert X.ndim==2
return np.ones(X.shape[0],dtype=bool)
```

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Test via the breast cancer Wisconsin dataset...

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            assert X.ndim==2
5
            return np.ones(X.shape[0],dtype=bool)
    Test via the breast cancer Wisconsin dataset...
    print('The Accuracy Paradox: a naive classifer')
    X, y_true = load_breast_cancer(return_X_y=True)
3
    X_train, X_test, y_train, y_test
      = train_test_split(X, y_true, test_size=0.2, shuffle=True)
    clf = ParadoxClassifier()
    clf.fit(X_train, y_test)
8
    v_pred = clf.predict(X_test)
9
    a = accuracy_score(y_pred, y_test)
    print(' acc=', a,', N=', y_pred.shape[0])
12
```

NOTE₀: for MNIST, a dum classify as '5' $\sim a = 10\%$ NOTE₁: for MNIST, a dum classify not-as '5' $\sim a = 90\%$

Accuracy Paradox...

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                                      prints: acc=0.6228070175438597,
    clf.fit(X_train, y_test)
8
                                              N = 114
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Linear Regression: In one dimension

The well know linear equation

$$y(x) = \alpha x + \beta$$

or changing some of the symbol names, so that $h(\mathbf{x}; \mathbf{w})$ means the **predicted** value from x for a parameter set \mathbf{w} , via the hypothesis function

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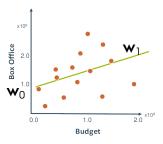
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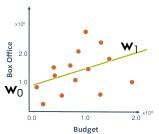
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Question: how do we find the \mathbf{w}_n 's?

Linear Regression: Hypotheis Function in N-dimensions

For 1-D:

$$h(x^{(i)}; w) = w_0 + w_1 x^{(i)}$$

Linear Regression: Hypotheis Function in N-dimensions

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The same for N-D:

$$h(\mathbf{x}^{(i)}; \mathbf{w}) = \mathbf{w}^{\top} \begin{bmatrix} 1 \\ \mathbf{x}^{(i)} \end{bmatrix}$$
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```

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, by convention in the following...

yielding the vector form of the hypothesis function

$$h(\mathbf{x}^{(i)}; \mathbf{w}) = \mathbf{w}^{\top} \mathbf{x}^{(i)}$$

Linear Regression: Loss or Objective Function

Individual loss, via a square difference

$$L^{(i)} = (h(\mathbf{x}^{(i)}; \mathbf{w}) - y^{(i)})^{2}$$
$$= (\mathbf{w}^{\top} \mathbf{x}^{(i)} - y^{(i)})^{2}$$

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= $(\mathbf{w}^{\mathsf{T}}\mathbf{x}^{(i)} - y^{(i)})^2$

and to minimize all the $L^{(i)}$ losses (or indirectly also the MSE or RMSE) is to minimize the sum of all the individual costs, via the total cost function J

$$\begin{aligned} \mathsf{MSE}(\mathbf{X}, \mathbf{y}; \mathbf{w}) &= \frac{1}{n} \sum_{i=1}^{n} L^{(i)} \\ &= \frac{1}{n} \sum_{i=1}^{n} \left(\mathbf{w}^{\top} \mathbf{x}^{(i)} - y^{(i)} \right)^{2} \\ &= \frac{1}{n} ||\mathbf{X} \mathbf{w} - \mathbf{y}||_{2}^{2} \end{aligned}$$

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$$= \frac{1}{n} ||\mathbf{X} \mathbf{w} - \mathbf{y}||_{2}^{2}$$

Ignoring constant factors, this yields our linear regression cost function

$$J = \frac{1}{2}||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$$

$$\propto \mathsf{MSE}$$

Minimizing the Linear Regression: The argmin concept

Our linear regression cost function was

$$J(\mathbf{X}, \mathbf{y}; \mathbf{w}) = \frac{1}{2} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$$

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$$\mathbf{w}^* = \operatorname{argmin}_{\mathbf{w}} J(\mathbf{X}, \mathbf{y}; \mathbf{w})$$

= $\operatorname{argmin}_{\mathbf{w}} \frac{1}{2} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$

Minimizing the Linear Regression: The argmin concept

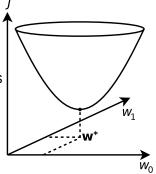
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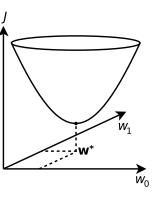
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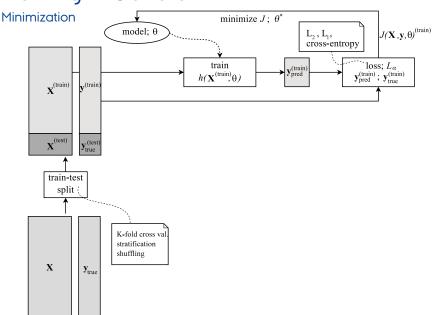
= $\operatorname{argmin}_{\mathbf{w}} \frac{1}{2} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$

and by minima, we naturally hope for

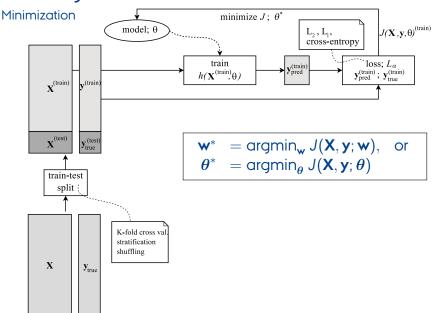
- the global minumum
- thought for non-linear models this cannot be guarantied, hitting some
 - local minimum



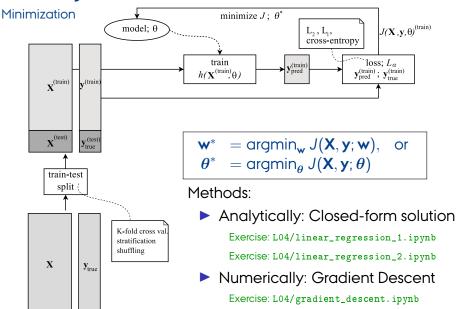
Training in General



Training in General



Training in General



Exercise: L04/linear_regression_1.ipynb

Training: The Closed-form Linear-Least-Squares Solution

To solve for \mathbf{w}^* in closed form, we find the gradient of J with respect to \mathbf{w}

$$\nabla_{\mathbf{w}} J = \left[\frac{\partial J}{\partial w_1}, \frac{\partial J}{\partial w_2}, \dots, \frac{\partial J}{\partial w_m} \right]^{\top}$$

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Taking the partial deriverty $\partial/\partial_{\mathbf{w}}$ of the J via the gradient (nabla) operator

$$\nabla_{\mathbf{w}} J(\mathbf{X}, \mathbf{y}; \mathbf{w}) = \mathbf{X}^{\top} (\mathbf{X} \mathbf{w} - \mathbf{y}) = 0$$
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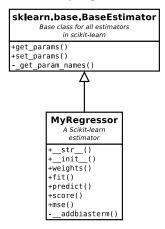
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with a small amount of matrix algegra, this gives the normal equation

$$\begin{aligned} \mathbf{w}^* &= \text{argmin}_{\mathbf{w}} \ \tfrac{1}{2} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2 \\ &= \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{y} \end{aligned}$$

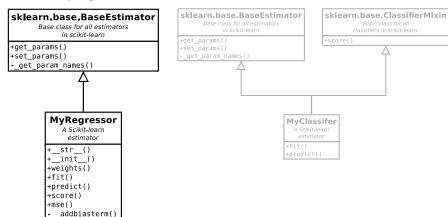
Exercise: L04/linear_regression_2.ipynb

Python class: MyRegressor



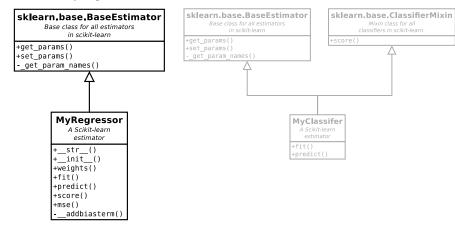
Exercise: L04/linear_regression_2.ipynb

Python class: MyRegressor



Exercise: L04/linear_regression_2.ipynb

Python class: MyRegressor



Exercise: create a linear regressor, inheriting from Base-Estimator and implement score() and mse().

NOTE: no inhering from ClassifierMixin.