



LESSON 2: Classification

Cost function, Supervised classification, Performance metrics

CARSTEN EIE FRIGAARD

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$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_d^{(2)} \\ \vdots & & & \vdots \\ x_1^{(n)} & x_2^{(n)} & \cdots & x_d^{(n)} \end{bmatrix} = \begin{bmatrix} (\mathbf{x}^{(1)})^T \\ (\mathbf{x}^{(2)})^T \\ \vdots \\ (\mathbf{x}^{(n)})^T \end{bmatrix}$$



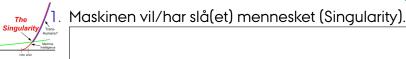
Agenda

Cost Function, Supervised Classification, Performance Metrics

- 1. Admin (afleversformat, grupper, etc.)
- 2. Spørge-minutter, ca. 10-15 min.
 - NOTE: optages IKKE
- Forelæsning
 - Resumé
 - Linær algebra og cost funktionen, J
 - Opgave: L02/cost_function.ipynb
 - Supervised binær klassifikation
 - fundamental ML supervised lærings-proces,
 - Opgave: L02/dummy_classifier.ipynb
 - Scores/Performace metrics
 - Opgave: L02/performance_metrics.ipynb
- 4. Opgaveregning på Discord..

Konceptet Machine Learning [ML]

Hvad er dine forventninger til ML-løsninger og ML i fremtiden?



- 2. Enormt potetiale i ML + ny teknologi, jubii.
- 3. Der er potential + noget varm-lufts boble.
- 4. Jeg er moderat skeptisk overfor ny teknologi (jeg er dog ikke så skeptisk som CEF).
- 5. Jeg råber 'There-is-No-Silver-Bullet'; det er salgs gas.

RESUMÉ: The toolset for ML

A list of our toolbox

- Python: our prefeered language for ML,
- Anaconda: a particular distibution of python, that we will use,
- Jupyter notebooks: interactive coding and visualization for python (alt: Spider, PyCharm),
- NumPy, SciPy, Pandas, Matplotlib, Seaborn: numerical computation and data visualization libraries for python,
- Scikit-learn: machine learning tools.

RESUMÉ: Jupyter Crash Couse

Jupyter need-to-know:

- Ctrl+Enter: executes cell,
- Shift+Tab: help for function under cusor,
- Shift+Tab repeated: extended help,
- Tab: 'tab'-completion??

Jupyter magic commands:

- %matplotlib inline: pull in the matplotlib,
- %reset -f: reset all vars (or -sf),
- %run filename.ipynb; execute code from another notebook or python file,
- %load filename.py: copy contents of the file and paste into the cell,
- ! dir: executes a shell command.

RESUMÉ: Jupyter Crash Course

Jupyter shortcuts:

 To modes: command mode (blue) and edit-mode (green),

In []: a=1

ESC: goto command mode (from edit mode),

Keyboard shortcuts

The Jupyter Notebook has two different keyboard input modes. **Edit mode** allows you to type code/text into a cell and is indicated by a green cell border. **Command mode** binds the keyboard to notebook level actions and is indicated by a grey cell border with a blue left margin.

Command Mode (press Esc to enable)

E: find and replace

Ctrl-Shift-P: open the command palette

Enter: enter edit mode

Shift-Enter: run cell, select below

Ctrl-Enter: run selected cells

Alt-Enter: run cell, insert below

Shift-J: extend selected cells below

A: insert cell above
B: insert cell below
X: cut selected cells
C: copy selected cells

Shift-V: paste cells above

RESUMÉ: Python Libraries Crash Course

A lot of modules/libraries are available for python, here we will use:

- numpy: numerical data representation module, for say vectors, matrices etc,
- matplotlib: Matplotlib is a Python 2D plotting library which produces publication quality figures.

Other libraries, typically used in ML, are:

- pandas: python data analysis library, a module for loading/saving and handling large data set,
- scipy: python library used for scientific computing and technical computing.

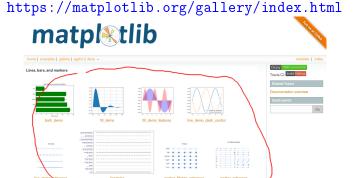
but we try to stick to numpy in this course, ...and note that numpy .matrix is depricated!

RESUMÉ: Matplotlib Crash Course

Visualizations can be created in multiple ways:

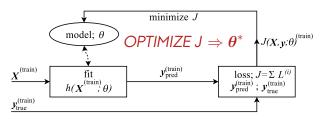
- ► matplotlib
- pandas: (via matplotlib),
- seaborn: statistically-focused plotting methods.

And we will stick to matplotlib, don't re-invent the wheel; find demos here



RESUMÉ

Data-flow model for supervised learning



X^(train): trænings data input,

loose notation: $\mathbf{X}^{(\text{train})} = \mathbf{X}^{(i)}$ for $\forall i \in \text{train set}$

 θ : model parametre,

h: hypothesis function; types of ML algos,

y(train): training data output,

 $\mathbf{y}_{\text{pred}}^{(\text{train})}$: predicted (train) data output,

 $L^{(i)}$: individual loss (distance),

J: loss/cost/error/objective function (summeret)

Exercise: L02/cost_function.ipynb

The Design Matrix

Say, we have d features for a given sample point. This d-sized feature column vector for a data-sample i is then given by

$$\mathbf{x}^{(i)} = \begin{bmatrix} x_1^{(i)} & x_2^{(i)} & \cdots & x_d^{(i)} \end{bmatrix}^T$$

The full data matrix \mathbf{X} and target column vector \mathbf{y} are then constructed out of n samples of these feature vectors

$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_d^{(2)} \\ \vdots & & & \vdots \\ x_1^{(n)} & x_2^{(n)} & \cdots & x_d^{(n)} \end{bmatrix} = \begin{bmatrix} (\mathbf{x}^{(1)})^T \\ (\mathbf{x}^{(2)})^T \\ \vdots \\ (\mathbf{x}^{(n)})^T \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n)} \end{bmatrix}$$

(and **X** and **y** are sometimes concantenated into a single matrix!)

Exercise: L02/cost_function.ipynb

Distance/norms

The \mathcal{L}_2 Euclidian norm for a vector of size n is defined as

$$\mathcal{L}_2: ||\mathbf{x}||_2 = \left(\sum_{i=1}^n |x_i|^2\right)^{1/2}$$

and thus via linear algebra and vector inner-dot product $\mathcal{L}_2^2: ||\mathbf{x}||_2^2 = \mathbf{x}^{\top}\mathbf{x}$

The distance between two vectors is given by
$$d(\mathbf{x}, \mathbf{y}) = ||\mathbf{x} - \mathbf{y}||_2$$

$$= \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{1/2}$$

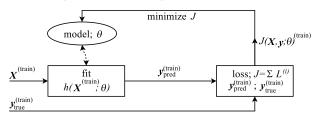
$$= \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{1/2}$$

The general \mathcal{L}_p norm is given by

$$\mathcal{L}_{p}: \ ||\mathbf{x}||_{p} = \left(\sum_{i} |x_{i}|^{p}\right)^{1/p}; \ \text{norm:} \left\{ \begin{array}{l} \mathcal{L}_{p}(\mathbf{x}) = \mathbf{0}, \ \Rightarrow \mathbf{x} = \mathbf{0} \\ \mathcal{L}_{p}(\mathbf{x} + \mathbf{y}) \leq \mathcal{L}_{p}(\mathbf{x}) + \mathcal{L}_{p}(\mathbf{y}), \\ \text{(triangle inequality)} \\ \mathcal{L}_{p}(\alpha \mathbf{x}) = |\alpha| \mathcal{L}_{p}(\mathbf{x}) \end{array} \right.$$

Exercise: L02/cost_function.ipynb

Data-flow model for supervised learning



Express J in terms of vectors and matrices using the \mathcal{L}_2

$$J(\mathbf{X}, \mathbf{y}_{true}; \boldsymbol{\theta}) = \frac{1}{n} \sum_{i=1}^{n} L^{(i)}$$

$$= \frac{1}{n} \sum_{i=1}^{n} d(h(\mathbf{X}^{(i)}) - \mathbf{y}_{true}^{(i)})^{2}$$

$$= \frac{1}{n} ||h(\mathbf{X}) - \mathbf{y}_{true}||_{2}^{2} \sum_{\text{Equation 2-1. Root Mean Square Error (RMSE)}} \frac{1}{n} ||\mathbf{y}_{pred} - \mathbf{y}_{true}||_{2}^{2}$$

$$= \frac{1}{n} ||\mathbf{y}_{pred} - \mathbf{y}_{true}||_{2}^{2}$$

$$= \frac{1}{n} ||\mathbf{y}_{pred} - \mathbf{y}_{true}||_{2}^{2}$$

arriving at a J proportional to the MSE or \mathcal{L}_2 metric

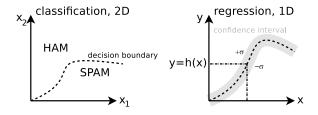
cost function:
$$J(\mathbf{X}, \mathbf{y}_{true}; \boldsymbol{\theta}) \propto \frac{1}{2} ||\mathbf{y}_{pred} - \mathbf{y}_{true}||_2^2 \propto \textit{MSE}$$

Classification vs. Regression

Given the following hypothesis function

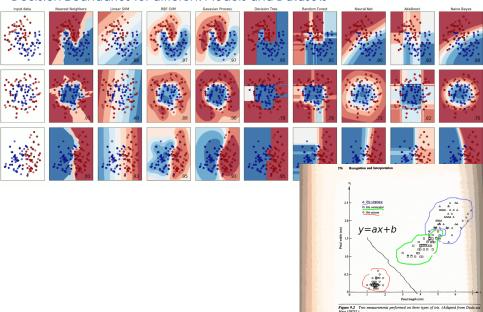
$$h(\mathbf{x}) \rightarrow y$$

- if y is discrete/categorical variable, then this is classification problem.
- if y is real number/continuous, then this is a regression problem.



Classification

Decision Boundaries for different Models and Datasets

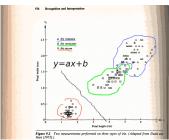


'Demo' datasæt

MNIST, Iris og Moon

Iris:

Sepal/petal længde/bredde, Mr. Fisher, 1936, "Anderson's Iris data set" sklearn.datasets.load_iris(...)







MNIST:

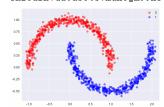
Håndskrevne tal,
preprocesseret, centrerede,
sklearn.datasets.fetch_openml('mnist_784'...



Moon:

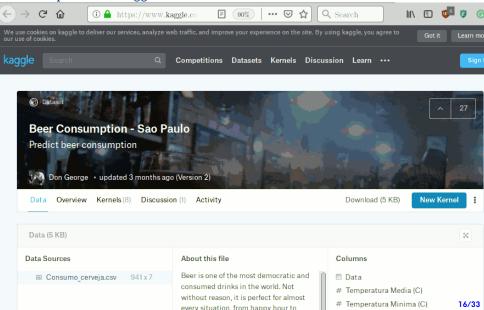
'XOR' lign.,

non-linear decision boundary, sklearn.datasets.make_moons(...)

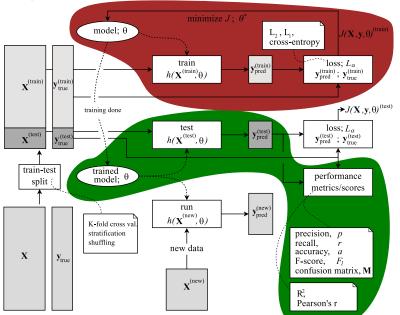


'Dit' datasæt

Fro https://www.kaggle.com...



ML Supervised Learning, Train/Test

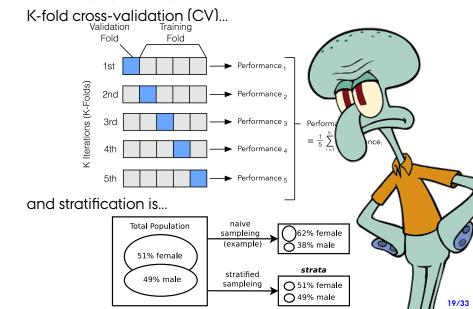


Fundamental supervised learning-proces

- i) Forbered data:
 - manuel preprocessering + visualisering (støj, outliers..)
 - ▶ label y_{true} data!!!
 - normalization, skalering
 - shuffle,
 - (stratification, K-fold cross-validation).
- ii) **Split** data i train/test.
 - analogi: skriftlig eksamenssæt på ASE: test-træningssæt (eksamen) udleveres ikke til træning inden!
- iii) Træn på trænings-data (fit)
 - ► ML træning via J,
- iv) Evaluér på test-data (predict)
 - performance metrics/scores

Forbered data: cross-validation, stratification

Bemærk: mere preprocess og k-fold cross-validation i LO3..



Multiclass/Multinomial Classification

And Introduction to Multilabel Classification

- Many classifiers are binary (HAM/SPAM)
- What to do for say a three category, like CAT/DOG/TURTLE problem?
- Divide into three CAT/NON-CAT, etc, binary classifiers and solve!
- Aka.: one-vs-rest/one-vs-all (OvA), one-against-all (OAA).
- Or the one-vs-one (OvO) method.
- NOTE: Multilabel classification is yet again differentity can categorize item into more classes, say both and DOG!
- ...and Multioutput/multilabel multiclass classificate

The Scikit-learn Fit-Predict Interface



Supervised Classification in practice

The API has one predominant object: the estimator.



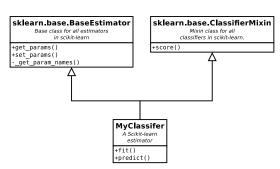
An estimator is an object that fits a model based on some training data and is capable of inferring some properties on new data. It can be, for instance, a classifier or a regressor.

All estimators implement the fit method: estimator.fit(X,y) All built-in estimators also have a set_params method, which sets data-independent parameters (overriding previous parameter values passed to __init__.

All estimators in the main scikit-learn codebase should inherit from sklearn.base.BaseEstimator.

The Scikit-learn Fit-Predict Interface





Python module and class function and member encapsulation:

- module private: one underscore
- class-private: two underscores

via mangled names.

 \dots NOTE: no virtual void fit() = 0; declaration in python!

...for modules, private funs can still be accessed via a hack?!
...src file: /opt/anaconda3/pkgs/.../sklearn/base.py

The Scikit-learn Fit-Predict Interface



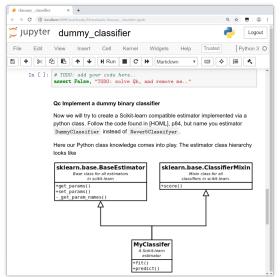
Demo..

Implementing an estimater via a python class as simple as

```
class ParadoxClassifier(BaseEstimator, ClassifierMixin):
    def fit(self, X, y=None):
        pass
    def predict(self, X):
        assert X.ndim==2
    return np.ones(X.shape[0],dtype=bool)
```

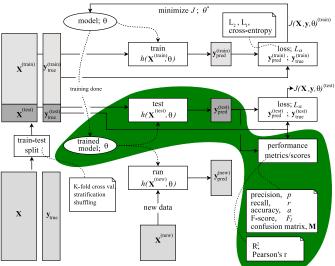
Exercise: L02/dummy_classifier.ipynb

A dummy classifier for the fit-predict interface, plus intro to a Stochastic Gradient Decent method (SGD) and introduction to the accuracy-paradox.



Evaluér på test-data: Perfomance metrics

Kort intro til konceptet performance metrics..



 $NOTE_0$: Performance metric = score.

NOTE₁: 'Performance measure' begreb bruges ikke, kun score eller perf. metric. NOTE₂: Loss er ML algo'ens 'performance mål', score er vores evalueringsmål.

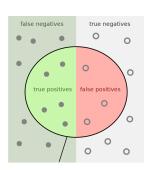
Nomenclature

For a binary classifier

NAME	SYMBOL	ALIAS
true positives	TP	
true negatives	TN	
false positives	FP	type I error
false negatives	FN	type II error

and $N = N_P + N_N$ being the total number of samples and the number of positive and negative samples respectively.

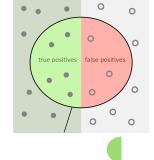
[https://en.wikipedia.org/wiki/Precision_and_recall]



Precision, recall and accuracy, F_1 -score, and confusion matrix

precision,
$$p = \frac{TP}{TP+FP}$$
recall (or sensitivity),
$$r = \frac{TP}{TP+FN}$$
accuracy,
$$a = \frac{TP+TN}{TP+TN+FP+FN}$$

$$F_1\text{-score}, \qquad F_1 = \frac{2pr}{p+r}$$



Precision =

true negatives

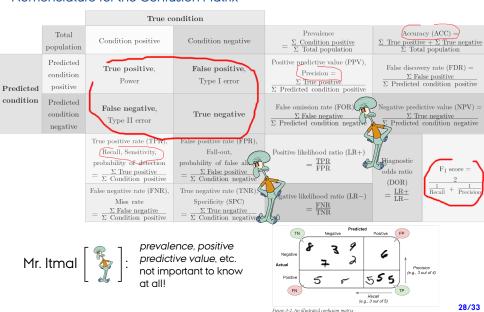
false negatives

Confusion Matrix,			
	actual		
	true	false	
prodrotod trao	TP	FP	
predicted false	FN	TNI	



NOTE₀: you can *compare* precision...*F*₁-score, but not necessarily the cost, *J*. NOTE₁: beware of matrix transpose and interpretation of *TP/TN*!

Nomenclature for the Confusion Matrix



Accuracy Paradox...

9

```
class ParadoxClassifier(BaseEstimator, ClassifierMixin):
    def fit(self, X, y=None):
        pass
    def predict(self, X):
        assert X.ndim==2
        return np.ones(X.shape[0],dtype=bool)
Test via the breast cancer Wisconsin dataset...
```

```
X_train, X_test, y_train, y_test =
     train_test_split(
       X, y_true, test_size=0.2, shuffle=True, random_state=42
   clf = ParadoxClassifier()
                                     prints: acc=0.6228070175438597.
   clf.fit(X_train, y_train)
   y_pred = clf.predict(X_test)
8
```

N = 114acc = accuracy_score(y_test, y_pred)

```
score = clf.score(X_test, y_test)
print(f' clf.score()={score} (same as accuracy_score)')
NOTE<sub>0</sub>: for MNIST, a dum classify as '5' \sim a = 10\%
```

print(f' acc={acc}, N={y_pred.shape[0]}')

NOTE₁: for MNIST, a dum classify not-as '5' $\sim a = 90\%$

More on metrics, oh-so-many!

[https://scikit-learn.org/stable/modules/classes.html#sklearn-metrics-metrics]

Classification metrics		
ee the Classification metrics section of the user guide for furt	ther details.	
metrics.accuracy_score(y_true, y_pred[,])	Accuracy classification score.	
metrics.auc(x, y[, reorder])	Compute Area Under the Curve (AUC) using the trapezoidal rule	
metrics.average_precision_score(y_true, y_score)	Compute average precision (AP) from prediction scores	· - /_)
metrics.balanced_accuracy_score(y_true, y_pred)	Compute the balanced accuracy	
metrics.brier_score_loss(y_true, y_prob[,])	Compute the Brier score.	
metrics.classification_report(y_true, y_pred)	Build a text report showing the main classification metrics	(FR
metrics.cohen_kappa_score(y1, y2[, labels,])	Cohen's kappa: a statistic that measures Inter-annotator agreement.	
metrics.confusion_matrix (y_true, y_pred[,])	Compute confusion matrix to evaluate the accuracy of a classification	
metrics.fl_score (y_true, y_pred[, labels,])	Compute the F1 score, also known as balanced F-score or F-measure	
metrics.fbeta_score(y_true, y_pred, beta[,])	Compute the F-beta score	7
metrics.hamming_loss(y_true, y_pred[,])	Compute the average Hamming loss.	Sel
metrics.hinge_loss (y_true, pred_decision[,])	Average hinge loss (non-regularized)]
metrics.jaccard_similarity_score(y_true, y_pred)	Jaccard similarity coefficient score	
metrics.log_loss (y_true, y_pred[, eps,])	Log loss, aka logistic loss or cross-entropy loss.	//
metrics.matthews_corrcoef(y_true, y_pred[,])	Compute the Matthews correlation coefficient (MCC)	
metrics.precision_recall_curve(y_true,)	Compute precision-recall pairs for different probability thresholds	
metrics.precision_recall_fscore_support()	Compute precision, recall, F-measure and support for each class	Ì

DISCUSS

Hvad er ...??



1.	I høj grad
2.	I noget grad
3.	Hverken-eller
4.	I lav grad
5.	I meget lav grad