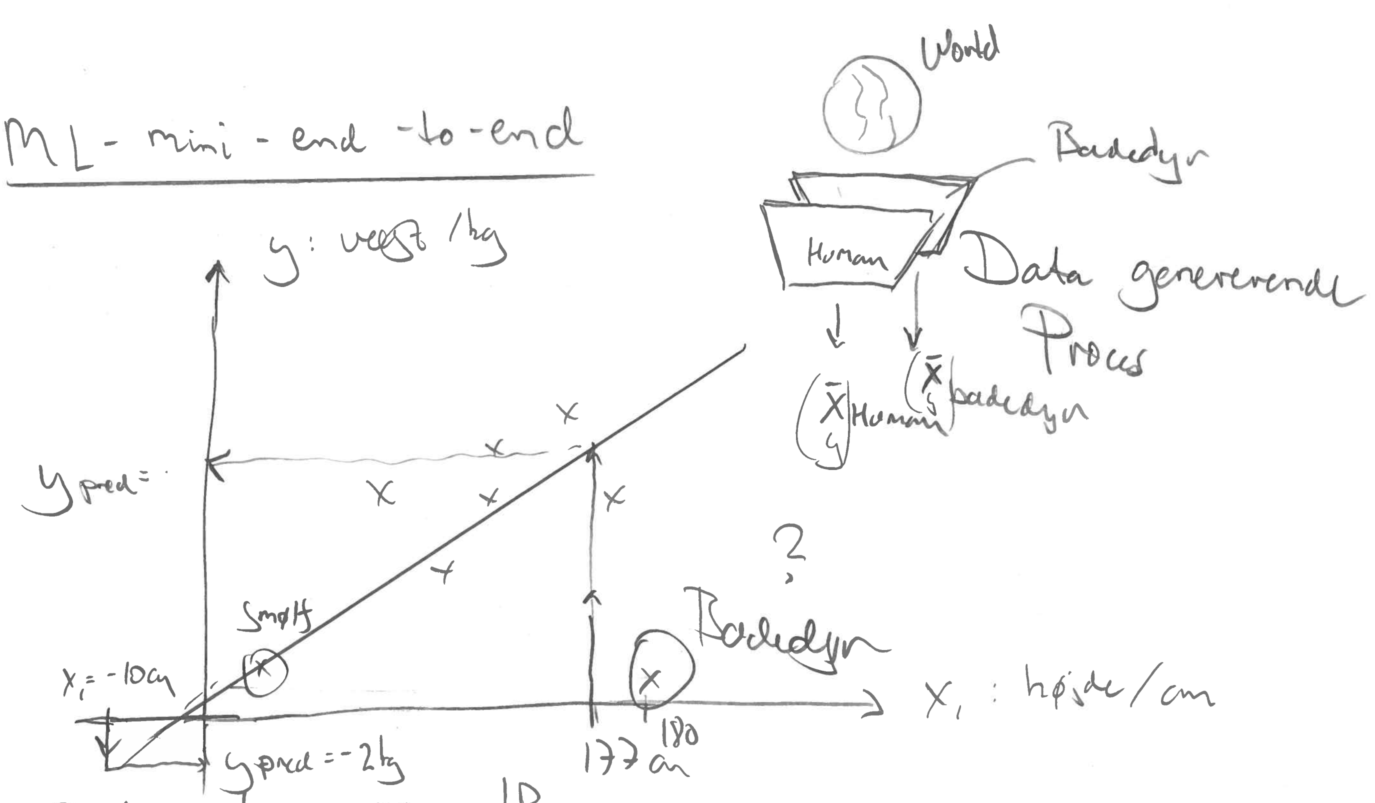
ITMAL

Noter til L01

(3/9-2020)

ML - mini - end - to - end



<u>Model</u> : Linear reg. , 1D

$$\boxed{\begin{aligned} y &= \cancel{\alpha x + \beta} \\ &= \theta_1 x_1 + \theta_0 \cdot 1 \\ &= w_1 x_1 + w_0 \cdot 1 \end{aligned}}$$

} Generelt NN

<u>DATA</u>

1D: $\bar{X} = [x_1]$ 2D: $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 3D: $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Model Parametre

$\bar{\Theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$ $\bar{\Theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$ $\bar{\Theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

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ML - mini e-to-e

Hypotesis Funktion:

$$\boxed{h(\bar{x};\bar{\theta}) = \bar{y}_{pred}} \quad (\hat{y})$$

input DATA — model params. — bias led

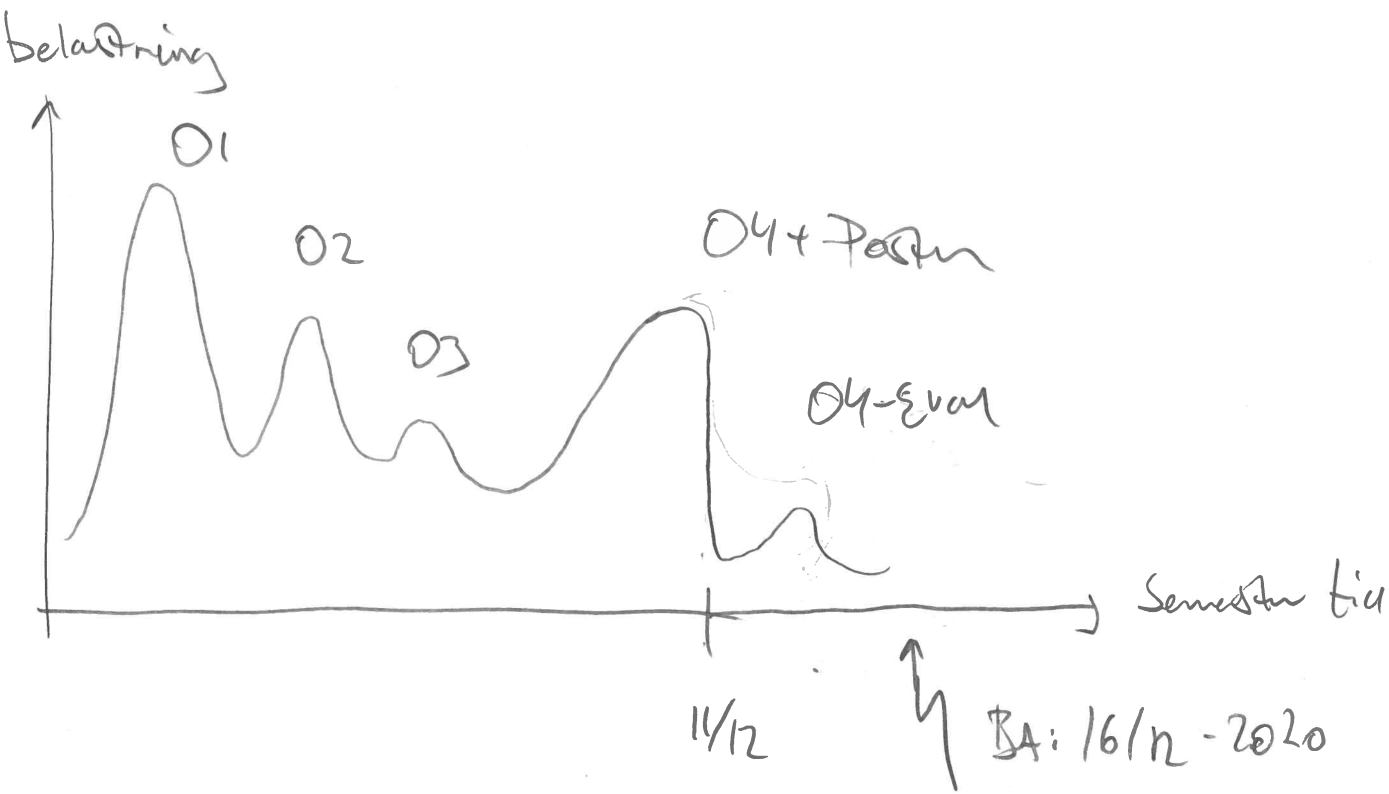
$$= \underbrace{\theta_1 x_1 + \theta_0 \cdot 1}_{\text{for } h \,=\, \text{linear reg.}}$$

~~(0) Find DATA)~~ ITMAL

ML:
1) vælg $h(\cdot)$
2) træn/fit; $\bar{\theta} \Rightarrow$ optimize, best $\bar{\theta}^* \Rightarrow$ minimer fejl $(L(\cdot))$
3) predict på nye data $\Rightarrow$ generalisering

X) Problemer i) data outliers; ii) dimensionalitet; iii)/...

ITMAL Noter : LØ1



MMLS - Recap

Matrix

$$\bar{\bar{X}} = \begin{array}{c|c} & n \\ \hline m & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

Vector

$$\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_d \end{bmatrix}$$

$$\bar{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_d \end{bmatrix}$$

$(m \times n)$ matrix, 1-indexed

Size-constraints:

$$\bar{\bar{X}} + \bar{\bar{Y}} \stackrel{?}{=}$$

$$\bar{\bar{X}} \cdot \bar{\bar{Y}} \stackrel{?}{=} \longrightarrow \begin{array}{c|c} & n \\ \hline n & \begin{bmatrix} & \cdot & \end{bmatrix} \end{array} \begin{array}{c|c} & p \\ \hline n & \begin{bmatrix} & \cdot & \end{bmatrix} \end{array} = \begin{array}{c|c} & p \\ \hline n & \begin{bmatrix} & & \end{bmatrix} \end{array}$$

E.g.

$$\begin{array}{c|c} & 4 \\ \hline 3 & \begin{bmatrix} & & \end{bmatrix} \end{array} \begin{array}{c|c} & 2 \\ \hline 4 & \begin{bmatrix} & \end{bmatrix} \end{array} = \begin{array}{c|c} & 2 \\ \hline 3 & \begin{bmatrix} & \end{bmatrix} \end{array}$$

Python Matrix

Numpy

$$\underline{X}_{demo} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 10 & 20 & 30 & 42 \end{bmatrix}$$

Build-in

python - list-of-list ≃ matrix

z = [[42]]

X.shape = (2, 4)

$X[\emptyset, 1] \stackrel{?}{=} 2.\emptyset$

$X[\emptyset, -1] \stackrel{?}{=} 4.\emptyset$

X[:, 1] = [2, 20]   row / column ?

Numpy

↓

row ≃ column

ML Design Matrix

Sample 1

[HOML]

for $d=4$, p. 40

$$\bar{X}^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_d^{(1)} \end{bmatrix} \stackrel{\downarrow}{=} \begin{bmatrix} -118 \\ 33{,}9 \\ \cancel{1446}\ 1416 \\ 38000 \end{bmatrix}$$

x_1: long / °
x_2: lat / °
x_3: inhabitants / #
x_4: median income / \$

$$y^{(1)} = 156000$$

true output : median house value / \$

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Sample 2

$\bar{x}^{(2)} = [\cdots$

$y^{(2)} =$

$\vdots$

Sample m

$\bar{x}^{(m)} = [\vdots$

$y^{(m)}$

Design Matrix

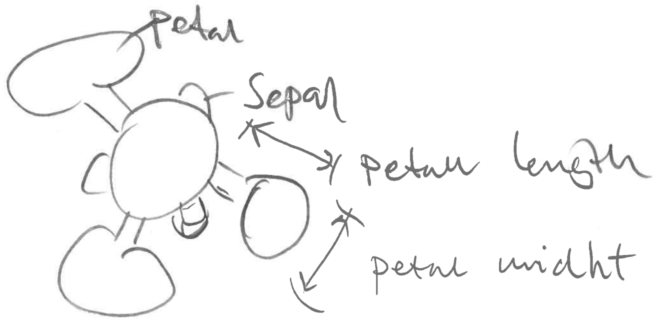
$$\bar{\bar{X}} = \begin{bmatrix} \bar{x}^{(1)T} \\ \bar{x}^{(2)T} \\ \vdots \\ x^{(m)T} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_d^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & & x_d^{(m)} \end{bmatrix} \overset{d=4}{\downarrow}{=} \begin{bmatrix} -118 & 33.9 & 1416 & 38000 \\ \vdots & & & \\ & & & \end{bmatrix}$$

d=4 (Housing Data) eg

$$\bar{y}_{true} = \begin{bmatrix} y^{(1)}_{true} \\ y^{(2)}_{tr} \\ \vdots \\ y^{(m)}_{true} \end{bmatrix} \overset{d=4}{\downarrow}{=} \begin{bmatrix} 156000 \\ \vdots \\ \\ \end{bmatrix}$$

Mogle gange augmentered form : $(\bar{\bar{X}} ; \bar{y})$

Iris Data



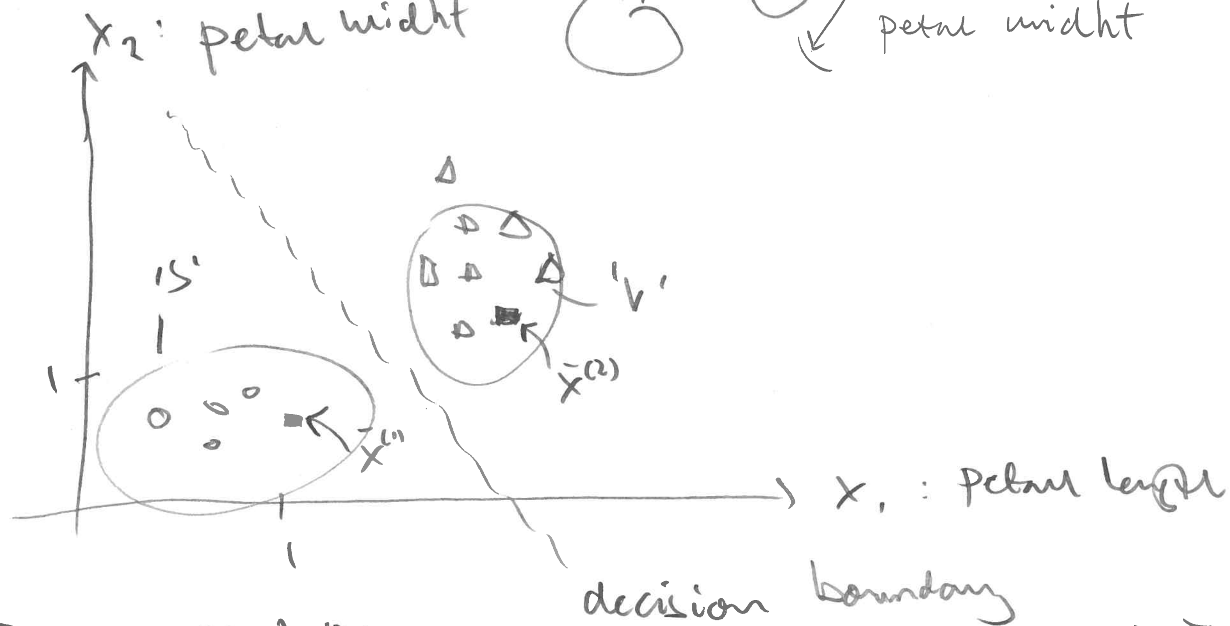
Classes:

I: Setosa 'S' o

II: Versicolor 'V' □

~~III: Verginica 'X' △~~

⸤ 2 Classes only for this demo



Design Matrix

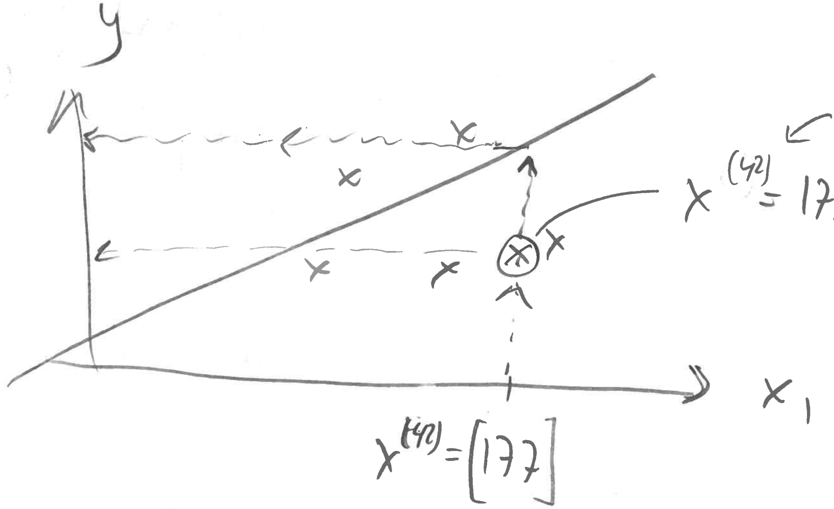
$$\bar{x}^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 1{,}2 \\ 0{,}5 \end{bmatrix} \qquad \bar{x}^{(2)} = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} 4{,}5 \\ 1{,}2 \end{bmatrix}$$

$$\left.\begin{aligned} y^{(1)}_{true} &= \text{'S'} \\ y^{(2)}_{true} &= \text{'V'} \end{aligned}\right\}$$

$$\bar{\bar{X}} = \begin{bmatrix} \bar{x}^{(1)T} \\ \bar{x}^{(2)T} \end{bmatrix} = \begin{bmatrix} 1{,}2 & 0{,}5 \\ 4{,}5 & 1{,}2 \end{bmatrix}$$

$$\bar{y}_{true} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \begin{bmatrix} \text{'S'} \\ \text{'V'} \end{bmatrix}$$

Cost Fun



lin. reg model

$$h(\bar{x}; \bar{\theta}) = y_{pred}$$

$$= \theta_1 x_1 + \theta_0 \cdot 1$$

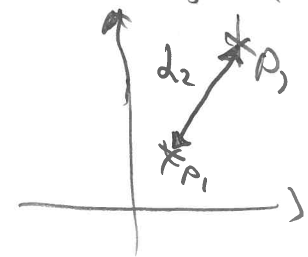
$(y = \alpha x + \beta)$

Fit: bedste $\bar{\theta}$? $\Rightarrow$ Optimize

$$\left.\begin{array}{l} h^{(i)}(\bar{x}^{(i)}, \bar{\theta}) = y^{(i)}_{pred} \\ \text{og} \quad y^{(i)}_{true} \end{array}\right\} \text{DiFF} \Rightarrow \text{Cost/error}$$

$$L^{(i)}\left(y^{(i)}_{true}, y^{(i)}_{pred}\right) = ?$$

$$\mathcal{L}_2 : \sqrt{(\bar{p}_1 - \bar{p}_2)^2}$$



Samlet tab : Cost Fun

$$J = \sum_i L^{(i)} \overset{m=2}{=} L^{(1)} + L^{(2)}$$

$$= L\left(y^{(1)}_{pred}, y^{(1)}_{true}\right) + L\left(y^{(2)}_{pred}, y^{(2)}_{true}\right)$$

$$= L\left(h(\bar{x}^{(1)}, \bar{\theta}), y^{(1)}_{true}\right) + L(\quad) \Rightarrow \boxed{\text{Solve for } \bar{\theta}^*}$$

numerisk/symbolsk