K13 Documentation

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Overview

This is a two-state model, meaning that we have $S = \{*, \dagger\}$ and:

$$P(t,s) = \begin{bmatrix} p_{**}(t,s) & p_{*\dagger}(t,s) \\ 0 & 1 \end{bmatrix} \text{ and } \Lambda(s) = \begin{bmatrix} \mu_{**}(s) & \mu_{*\dagger}(s) \\ 0 & 0 \end{bmatrix}$$

Kolmogorv's backward equation is given by:

$$\begin{aligned} \partial_t p_{ij}(t,s) &= \mu_i(t) p_{ij}(t,s) - \sum_{k \neq i} \mu_{ik}(t) p_{kj}(t,s) \\ &\Downarrow \\ \partial_t p_{**}(t,s) &= \mu_*(t) p_{**}(t,s) \end{aligned}$$

From The Fundamental Theorem of Calculus:

$$\begin{aligned} -\int_t^s \mu_*(u)du &= -[M_*(s) - M_*(t)] \\ &= -M_*(s) + M_*(t) \\ &\Downarrow \\ \partial_t \left(-\int_t^s \mu_*(u)du \right) &= \partial_t [M_*(t)] = \mu_*(t) \end{aligned}$$

We have that the row sum of the intensity matrix Λ sums to zero, and we use the convention that $\mu_i(t) = -\mu_{ii}(t)$, this gives us:

$$\begin{aligned} p_{**}(t,s) &= \exp\left(-\int_t^s \mu_*(u)du \right) \\ &= \exp\left(-\int_t^s \mu_{*\dagger}(u)du \right) \end{aligned}$$

The only thing we need to account for in K2013, is the dependency of calculation year Y, this gives us:

$$p^x_{**}(t,s) = p_{**}(x+t, x+s) = \exp\left(-\int_t^s \mu_{Kol}(x+u, Y+u)du \right)$$