K13 Overview

K13 is a two state model, meaning that we have $S = \{*, \dagger\}$, giving us:

$$\begin{aligned} P(t,s) &= \begin{bmatrix} p_{**}(t,s) & p_{*\dagger}(t,s)\\ 0 & 1 \end{bmatrix} \;\;\text{and}\;\; \Lambda(s) = \begin{bmatrix} \mu_{**}(s) & \mu_{*\dagger}(s)\\ 0 & 0 \end{bmatrix} \end{aligned}$$

Kolmogorov's backward equation: $$\begin{aligned} \partial_{t}p_{ij}(t,s) &= \mu_{i}(t)p_{ij}(t,s) - \sum_{k \neq i}\mu_{ik}(t)p_{kj}(t,s) \\ &\Downarrow \\ \partial_{t}p_{**}(t,s) &= \mu_{*}(t)p_{**}(t,s) \end{aligned}$$

From the fundamental theorem of calculus we have: $$\begin{aligned} -\int_{t}^{s}\mu_{*}(u)du &= -[M_{*}(s) - M_{*}(t)] \\ &= -M_{*}(s) +M_{*}(t) \\ &\Downarrow \\ \partial_{t}\left(- \int_{t}^{s}\mu_{*}(u)du \right) &= \partial_{t}(M_{*}(t)) = \mu_{*}(t) \end{aligned}$$

Using this we get: $$ p_{**}(t,s) = \exp\left(-\int_{t}^{s}\mu_{*}(u)du \right) $$

The row sum of the intensity matrix Λ sums to zero, furthermore we use the convention that $\mu_{i}(t) = - \mu_{ii}(t)$, this yields: $$ p_{**}(t,s) = \exp\left(-\int_{t}^{s}\mu_{*\dagger}(u)du \right) $$

Translating this to K13, where one also accounts for the calculation year Y, one gets: $$\begin{aligned} p_{**}^{x}(t,s) = p_{**}(x+t, x + s) &= \exp\left(-\int_{t}^{s}\mu_{Kol}(x+u, Y+u)du \right) \end{aligned}$$