Exercises chapter 6

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Exercise 6.2

We have the following contractual information:

```
T <- 10     #length of contract
x <- 60     #age of insured
D <- 20000  #disability pension while disabeld
B <- 50000  #death benefit
r <- 0.05   #intensity rate
```

Where our state space is $S = \{*, \diamond, \dagger\}$, we have the following policy-functions:

$$a_{\diamond}(t) = \begin{cases} 0, & t < 0 \\ Dt, & t \in [0, T) \\ DT, & t \geq T \end{cases} \quad a_{*\dagger}(t) = \begin{cases} B, & t \in [0, T) \\ 0, & else \end{cases}$$

Since we for now do not care about the premiums in the policy-functions, we get

$$\dot{a}_*(t) = 0$$

and

$$\dot{a}_{\diamond}(t) = D\mathbb{1}_{[0,T)}(t)$$

We can set up thieles differential equations:

$$\frac{d}{dt}V_*^+(t) = r(t)V_*(t) - \mu_{*\diamond}^x(t)[V_{\diamond}^+(t) - V_*^+(t)] - \mu_{*\dagger}^x(t)[B\mathbb{1}_{[0,T)}(t) - V_*^+(t)]$$
$$\frac{d}{dt}V_{\diamond}^+(t) = r(t)V_{\diamond}(t) - D\mathbb{1}_{[0,T)}(t) - \mu_{*\diamond}^x(t)[V_*^+(t) - V_{\diamond}^+(t)] + \mu_{\diamond\dagger}^x(t)V_{\diamond}^+(t)$$

With finial conditions: $V_*^+(T) = V_{\diamond}^+(T) = 0$

In order to solve this numerically, we partition up $[0, T]$ in times $t_i = t_0 + ih$, with $0 = t_0 < \cdots < t_n = T$.

We also iterate backwards, giving us:

$$\frac{V_*^+(t_i) - V_*^+(t_{i-1})}{h} = rV_*(t_i) - \mu_{*\diamond}^x(t_i)[V_{\diamond}^+(t_i) - V_*^+(t_i)] - \mu_{*\dagger}^x(t_i)[B - V_*^+(t_i)]$$

We then solve for $V_*(t_{i-1})$, giving us:

$$V_*^+(t_{i-1}) = V_*^+(t_i) - h\left[rV_*^+(t_i) - \mu_{*\dagger}^x(t_i)[V_{\diamond}^+(t_i) - V_*^+(t_i)] - \mu_{*\dagger}^x(t)[B - V_*(t_i)]\right]$$

We do the same for $V_{\diamond}^+(t_{i-1})$:

$$V_{\diamond}^+(t_{i-1}) = V_{\diamond}^+(t_i) - h\left[rV_{\diamond}^+(t_i) - D\mathbb{1}_{[0,T)}(t) - \mu_{\diamond *}^x(t_i)[V_*^+(t_i) - V_{\diamond}^+(t_i)] + \mu_{\diamond\dagger}^x(t_i)V_{\diamond}^+(t_i)\right]$$

```r
#states 0:alive, 1:disabled, 2:dead

mu01 <- function(u){
  a1 <- 4*10^{-4}
  b1 <- 3.4674*10^{-6}
  c1 <- 0.138755
  return(a1 + b1*exp(c1*u))
}

mu02 <- function(u){
  a2 <- 5*10^{-4}
  b2 <- 7.5858*10^{-5}
  c2 <- 0.087498
  return(a2 + b2*exp(c2*u))
}

mu12 <- function(u){
  return(mu02(u))
}

mu10 <- function(u){
  return(0.1*mu01(u))
}
```

```r
#a.e. derivative of a_{\diamond}(t)
dis_dot <- function(t){
  if(t >= 0 && t<T){ return(D) }
  if(t>=T){ return(0) }
}

#PV of policy V0 and V1
h <- 1/12
N <- T/h

PV_act <- PV_dis <- rep(0,N+1)
PV_act[N+1] <- PV_dis[N+1] <- 0 #per definition of the policy

for(i in N:1){
  PV_act[i] <- PV_act[i+1] - h*(r*PV_act[i+1]
                                - mu01(x+(i+1)*h)*(PV_dis[i+1]-PV_act[i+1])
                                - mu02(x+(i+1)*h)*(B-PV_act[i+1])
                                )

  PV_dis[i] <- PV_dis[i+1] - h*(r*PV_dis[i+1]- dis_dot((i+1)*h)
                                - mu10(x+(i+1)*h)*(PV_act[i+1]-PV_dis[i+1])
                                + mu12(x+(i+1)*h)*PV_dis[i+1]
                                )
}
```

And from this we can find the one-time premium $\pi_0 = V_*^+(0)$:

```
pi0 <- PV_act[1]
pi0
```

```
## [1] 20608.17
```

We were asked to find the yearly premium π, we then use the method, where we create an artificial policy, where one pays a premium of NOK 1 during $[0, T)$, i.e this is the only contribution of interest, thus:

$$
\begin{aligned}
a_*^{Prem=1}(t) &= \begin{cases} 0, & t < 0 \\ -t, & t \in [0, T) \\ -T, & t \geq T \end{cases} \implies \dot{a}_*^{Prem=1}(t) = -1\mathbb{1}_{[0,T)}(t) \\
a_\diamond^{Prem=1}(t) &= 0, \ \forall t \implies \dot{a}_\diamond^{Prem=1} 0
\end{aligned}
$$