Mandatory assignment STK4500 1 of 1

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Exercise 1

a)

We start by showing that $\mu_{m\ m-1}(t) = m\mu(t)$ for $m \in S \setminus \{0\}$. We have that $Z = \{Z_t\}_{t \geq 0}$ represents the number of people alive at time t, with state space $S = \{0, \ldots, N\}$. We can define Z_t as:

$$Z_t = \sum_{i=1}^{N} X_t^{(i)} \quad X_t^{(i)} = \begin{cases} 1, & \text{person i is alive at time t} \\ 0, & \text{person i is dead at time t} \end{cases}$$

Since every person in the group have same age and mortality μ, we get that:

$$\mu(t) = \lim_{h \downarrow 0} \frac{P(X_{t+h}^{(i)} = 0 | X_t^{(i)} = 1)}{h}$$

Now by following the definition we get:

$$\mu_{m\ m-1}(t) = \lim_{h \downarrow 0} \frac{P(Z_{t+h} = m - 1 | Z_t = m)}{h}$$

By definition of conditional probability we have:

$$P(Z_{t+h} = m - 1 | Z_t = m) = \frac{P(Z_{t+h} = m - 1, Z_t = m)}{P(Z_t = m)}$$

Now let's work a bit with the sets:

$$\{Z_{t+h} = m - 1, Z_t = m\} = \bigcup_{i=1}^{m} \{X_{t+h}^{(i)} = 0, X_{t+h}^{(j)} = 1, j = 1, \ldots, m, j \neq i\}$$

$$= \bigcup_{i=1}^{m} \{X_{t+h}^{(i)} = 0, X_t^{(i)} = 1, X_{t+h}^{(j)} = 1, j = 1, \ldots, m, j \neq i\}$$

$$= \bigcup_{i=1}^{m} \{A^{(i)}, B^{(i)}, C^{(j)}\}$$

We have that $A^{(i)} \cap B^{(i)}$ is independent of $C^{(j)} = \{X_{t+h}^{(j)} = 1, j = 1, \ldots, m, j \neq i\}$ furthermore $\alpha_{i,j} = \{A^{(i)}, B^{(i)}, C^{(j)}\}$ is a disjoint sequence of sets, thus:

$$P(\{Z_{t+h} = m - 1, Z_t = m\}) = P\left(\bigcup_{i=1}^m \alpha_{i,j}\right)$$

$$= \sum_{i=1}^m P(\alpha_{i,j})$$

$$= \sum_{i=1}^m P(X_{t+h}^{(i)} = 0, X_t^{(i)} = 1)P(X_{t+h}^{(j)} = 1, j = 1, \ldots, m, i \neq j)$$

$$= \sum_{i=1}^m P(X_{t+h}^{(i)} = 0|X_t^{(i)} = 1)P(X_t^{(i)} = 1)P(X_{t+h}^{(j)} = 1, j = 1, \ldots, m, i \neq j)$$

Now the last two sets are independent, meaning that we get:

$$P(X_t^{(i)} = 1)P(X_{t+h}^{(j)} = 1, j = 1, \ldots, m, i \neq j) = P((X_t^{(i)} = 1, X_{t+h}^{(j)} = 1, j = 1, \ldots, m, i \neq j)$$
$$= P(Z_t = m)$$

This leaves us with:

$$P(\{Z_{t+h} = m - 1, Z_t = m\}) = \sum_{i=1}^m P(X_{t+h}^{(i)} = 0|X_t^{(i)} = 1)P(Z_t = m)$$

This gives us the desired result:

$$\mu_{m\,m-1}(t) = \lim_{h\downarrow 0} \frac{P(\{Z_{t+h} = m - 1, Z_t = m\})}{P(\{Z_t = m\})}$$

$$= \sum_{i=1}^m \lim_{h\downarrow 0} P(X_{t+h}^{(i)} = 0|X_t^{(i)} = 1)P(Z_t = m)\frac{1}{P(Z_t = m)}$$

$$= \sum_{i=1}^m \mu(t)$$

$$= m\mu(t)$$

We are now asked to show that $\mu_{mn}(t) = 0$ for $|m - n| \geq 2$, let's assume that $N = 2$, so that i can manage this exercise:

$$\mu_{20}(t) = \lim_{h\downarrow 0} \frac{p_{20}(t, t + h)}{h}$$

Let's work with the probability in this case:

$$p_{20}(t, t + h) = P(Z_{t+h} = 0|Z_t = 2)$$
$$= P(X_{t+h}^{(2)} = \dagger, X_{t+h}^{(1)} = \dagger|X_t^{(2)} = *, X_t^{(1)} = *)$$
$$= P(X_{t+h}^{(2)} = \dagger|X_t^{(2)} = *, X_t^{(1)} = *) \times P(X_{t+h}^{(1)} = \dagger|X_t^{(2)} = *, X_t^{(1)} = *)$$
$$= P(X_{t+h}^{(2)} = \dagger|X_t^{(2)} = *) \times P(X_{t+h}^{(1)} = \dagger|X_t^{(1)} = *)$$

Here we used the fact that lives are independet, as well as the Markov property. Now letting $h \to 0$, we get:

$$\lim_{h\downarrow 0} P(X_{t+h}^{(2)} = \dagger|X_t^{(2)} = *) = 0$$

This leaves us with:

$$\mu_{20}(t) = 0$$

b)

We are asked to argue for why $p_{mn}(t,s) = 0$ for $n \geq m+1$, this follows from the fact that if this were not the case we would let the transition $\dagger \to *$ to be allowed. Hence this needs to be zero.

We were also asked to argue for $n \leq m$ that:

$$p_{mn}(t,s) = \binom{m}{n} p(t,s)^n [1 - p(t,s)]^{m-n}$$

This follows from the fact that we have a group of friends with same age, same mortality μ and the fact that they have independent lives. We thus get a sequence of independent Bernoulli trials where we have the same probability of success(survival) and failure(death), when we then sum up the number of success in $[t,s]$ we get a Binomial-distribution, and hence the probability is as above.

We are also asked to show that $s \mapsto p_{mn}(t,s)$ satisfy Kolmogorov's forward equation: We start by stating how the equation looks like in this situation:

$$\frac{d}{ds} p_{mn}(t,s) = -p_{mn}(t,s)\mu_n(s) + \sum_{k \neq m} p_{mk}(t,s)\mu_{kn}(s)$$

$$= p_{mn}(t,s)\mu_{nn}(s) + p_{m\,n+1}(t,s)\mu_{n+1\,n}(s)$$

We now recall that the row sum of the transition rate matrix is zero as well as the fact from **a)**:

$$\mu_{n\,n-1}(s) + \mu_{nn} = 0$$
$$\Downarrow$$
$$\mu_{nn}(s) = -\mu_{n\,n-1}(s) = -n\mu(s)$$

Thus we now end up with the followin equation:

$$\frac{d}{ds} p_{mn}(t,s) = -n\mu(s)p_{mn}(t,s) + (n+1)\mu(s)p_{m\,n+1}(t,s)$$

We will now actually take the derivative of this thing:

$$\frac{d}{ds} p_{mn}(t,s) = -\binom{m}{n} np(t,s)^{n-1} p(t,s)\mu(s)(1 - p(t,s))^{m-n} + \binom{m}{n} p(t,s)^n (m-n)(1 - p(t,s))^{m-n-1} p(t,s)\mu(s)$$

$$= -n\mu(s)p_{mn}(t,s) + \binom{m}{n}(m-n)p(t,s)^{n+1}(1 - p(t,s))^{m-n-1}\mu(s)$$

$$\binom{m}{n}(m-n) = \frac{m!}{n!(m-n)!}(m-n) = \frac{m!}{n!(m-n-1)!}$$
$$\Updownarrow$$
$$\binom{m}{n}(m-n)\frac{n+1}{n+1} = \frac{m!}{(n+1)!(m-n-1)!}(n+1)$$

This leaves us with:

$$\frac{d}{ds} p_{mn}(t,s) = -n\mu(s)p_{mn}(t,s) + (n+1)\mu(s)\binom{m}{n+1}p(t,s)^{n+1}(1 - p(t,s))^{m-n-1}$$

$$= -n\mu(s)p_{mn}(t,s) + (n+1)\mu(s)p_{m\,n+1}(t,s)$$

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c)

We assume that $\int_0^\infty \mu(u)du = \infty$, this means that

$$p(0, \infty) = \lim_{s \to \infty} p(0, s) = \lim_{s \to \infty} e^{\int_0^s \mu(u)du} = 0$$

Now, lets look at the sets again:

$$\{Z_s = 0\} = \bigcap_{i=0}^N \{X_s^{(i)} = \dagger\}$$

$$\Downarrow$$

$$P(\{Z_s = 0\}) = P\left(\bigcap_{i=0}^N \{X_s^{(i)} = \dagger\}\right) = \prod_{i=1}^N P(\{X_s^{(i)} = \dagger\}) = (1 - p(0, s))^N$$

Now, if we apply the limit on the sets:

$$P(\lim_{s \to \infty} Z_s = 0) = \left(1 - \lim_{s \to \infty} p(0, s)\right)^N = (1 - p(0, \infty))^N = 1$$

Exercise 2

In this exercise we are considering a socalled Tontine of Friends in Continous time. We are dealing with a regular continous markov chain X consisting of a total of N friends. We focus on one of the participants called the chosen one referred to as $C1$.

Our state space S looks like the following:

$$S = \{0, 1, \ldots, N-1\} \times \{*, \dagger\}$$

We also use the convention that $(m, *)$ means: m participants in the group except $C1$ are alive and the chosen one, $C1$ is alive. $(m, \dagger)$ means that m participants in the gruop except $C1$ are alive, while $C1$ is dead.

a) Transition Probabilities

We are asked to argue for the following:

$$\mu_{(m,*)(m-1,\dagger)}(t) = 0$$
$$\mu_{(m,*)(m-1,*)}(t) = \mu_{(m,\dagger)(m-1,\dagger)}(t) = m\mu(t)$$

$\mu_{(m,*)(m-1,\dagger)}(t) = 0$ means that the group consisting of m persons goes to $m-1$ i.e one has died, and also at the same time $C1$ have gone from $*$ to $\dagger$. This is zero as it represents instantanoius jumps of size two immideately.

$\mu_{(m,*)(m-1,*)}(t) = \mu_{(m,\dagger)(m-1,\dagger)}(t) = m\mu(t)$: $\mu_{(m,*)(m-1,*)}(t)$: the group goes from $m \to m-1$, while $* \to *$. From **Exercise 1** we have that the lives are assumed to be stochastically independent, same age and have the same force of mortality $\mu(t)$. Now: if the group goes from $m \to m-1$ and $C1$ remains in $*$ we have that there are $\binom{m}{m-1}$ ways to go from m to $m-1$, and given the fact that all have same force of mortality we get: $\mu_{(m,*)(m-1,*)}(t) = \binom{m}{m-1}\mu(t) = m\mu(t)$. Exact same argument applies for $\mu_{(m,\dagger)(m-1,\dagger)}(t)$.

$\mu_{(m,*)(m,\dagger)}(t)$: the group remians of m people, while $C1$ has gone from $* \to \dagger$, given the fact that all were assumed to have the same mortality rate we get: $\mu_{(m,*)(m,\dagger)}(t) = \mu(t)$

We also have that $p_{(m,j)(n,j)}(t,s) = 0$ for $m, n \in \{0, \ldots, N-1\}$ $n \geq m+1, j \in \{*, \dagger\}$, if this probability was non-zero we would allowd for the transition $\dagger \to *$, which is not reasonable, hence this probability is zero.

We now let $n \leq m$, we then have:

$$p_{(m,*)(n,*)}(t,s) = \binom{m}{n} p(t,s)^{n+1} [1 - p(t,s)]^{m-n}$$

We here have that all individuals have the same survival-probability $p(t,s)$

$$p_{(m,*)(n,*)}(t,s) = P[\text{Group goes from m to n in [t,s], C1 remains in * for [t,s]}]$$

$$\overset{indep.lives}{=} P[\text{Group goes from m to n in [t,s]}]P[\text{C1 remains in * for [t,s]}]$$

$$= \binom{m}{n} p(t,s)^n [1 - p(t,s)]^{m-n} \times p(t,s)$$

$$= \binom{m}{n} p(t,s)^{n+1} [1 - p(t,s)]^{m-n}$$

We also have:

$$p_{(m,*)(n,*)}(t,s) = P[\text{Gruop goes from m to n in [t,s], C1 goes from alive to dead in [t,s]}]$$

$$\overset{indep.lives}{=} P[\text{Gruop goes from m to n in [t,s]}]P[\text{C1 goes from alive to dead in [t,s]}]$$

$$= \binom{m}{n} p(t,s)^n [1 - p(t,s)]^{m-n} \times [1 - p(t,s)]$$

$$= \binom{m}{n} p(t,s)^n [1 - p(t,s)]^{m-n+1}$$

b)

We have that the contract starts at $t = 0$, and that all participants pay a single premium π_0 at this time. These premiums are invested into a fund, which we will regard as a stocastich process $S = \{S(t), t \geq 0\}$, which looks like

$$S(t) = S(0)e^{\rho t}$$

Furthermore the fund will not start paying out until we reach $t = T_0$, where after $t \geq T_0$ will have the value: $S(t) = S(0)e^{\rho T_0}$ with a constant interest rate $r > 0$, we are asked to derive $a_{(m,*)}(s)$, when we do not consider premiums.

$$a_{(m,*)}(t) = \begin{cases} 0 & , t \in [0, T_0) = 0 \\ \frac{\rho t S(0) e^{\rho T_0}}{m+1} & , t \geq T_0 \end{cases}$$

c)

In this exercise we are asked to calcualte the reserves, we first start off by remembering the formula applicable to our situation, and then translate it into our setting:

$$V_i^+(t,A) = \frac{1}{v(t)} \left[\sum_{j \in S} \int_t^\infty v(s) p_{ij}^x(t,s) da_j(s) \right]$$

Now translating this into our setting:

$$V^+_{(m,*)}(t) = \sum_{n=0}^{m} \int_t^{\infty} \frac{v(s)}{v(t)} p_{(m,*)(n,*)}(t,s) da_{(n,*)}(s)$$

$$= \sum_{n=0}^{m} \int_t^{\infty} \frac{v(s)}{v(t)} p_{(m,*)(n,*)}(t,s) \mathbb{1}_{[t \geq T_0]} \frac{\rho S(0) e^{\rho T_0}}{n+1} ds$$

$$= \rho S(0) e^{\rho T_0} \sum_{n=0}^{m} \int_{t \vee T_0}^{\infty} \binom{m}{n} \frac{p(t,s)^{n+1}[1-p(t,s)]^{m-n}}{n+1} e^{-r(s-t)} ds$$

d)

By Fubini we have:

$$V^+_{(m,*)} = \rho S(0) e^{\rho T_0} \int_{t \vee T_0}^{\infty} \sum_{n=0}^{m} \binom{m}{n} \frac{p(t,s)^{n+1}(1-p(t,s))^{m-n}}{n+1} e^{-r(s-t)} ds$$

Let $X \sim Bin(m,p)$, then:

$$E\left[\frac{1}{1+X}\right] = \sum_{n=0}^{m} \binom{m}{n} \frac{p^n(1-p)^{m-n}}{n+1}$$

We also have:

$$E\left[\frac{1}{1+X}\right] = \frac{1-(1-p)^{m+1}}{(m+1)p}$$

This means that we can rewrite $V^+_{(m,*)}(t)$:

$$V^+_{(m,*)}(t) = \rho S(0) e^{\rho T_0} \int_{t \vee T_0}^{\infty} pE\left[\frac{1}{1+X}\right] e^{-r(s-t)} ds$$

$$= \frac{\rho S(0) e^{\rho T_0}}{m+1} \int_{t \vee T_0}^{\infty} (1-(1-p(t,s))^{m+1}) e^{-r(s-t)} ds$$

e)

We start with the single premium π_0: According to the equivalence principle, we have that:

$$\pi_0 = V^+_{(N-1,*)}(0) = \frac{\rho S(0) e^{\rho T_0}}{N} \int_{T_0}^{\infty} (1-(1-p(0,s))^N) e^{-rs} ds$$

We also want the yearly premiums, from lecture notes we have: $V^+_*(t,A) = V^+_*(t,A_*) + V^+_*(t,\tilde{A}_*)$ Where we in the first expression do not consider premiums, and in the other we only care about premiums, i.e $a_{(m,*)}(t)$ remains, while:

$$\tilde{a}_{(m,*)}(t) = \begin{cases} -\pi t & , t \in [0,T_0) = 0 \\ -\pi T_0 & , t \geq T_0 \end{cases}$$

with $\dot{\tilde{a}}_{(m,*)}(t) = -\pi$ for $t \in (0,T_0)$ Now, we get the following:

$$V^+_{(m,*)}(t,\tilde{A}) = \frac{1}{v(t)} \int_t^{T_0} v(s) p_{(m,*)(n,*)}(t,s)(-\pi ds)$$

$$= -\pi \int_t^{T_0} \sum_{n=0}^{m} \binom{m}{n} p(t,s)^{n+1}(1-(p(t,s))^{m-n}) e^{-r(s-t)} ds$$

$$= -\pi \int_t^{T_0} p(t,s) e^{-r(s-t)} ds$$

Now translating the above idea, we get:

$$V^+_{(m,*)}(t, A) = V^+_{(m,*)}(t, A_*) + V^+_{(m,*)}(t, \tilde{A}_*)$$

And by the equivalence principle we will set

$$V^+_{(N-1,*)}(0, A_*) - V^+_{(N-1,*)}(0, \tilde{A}_*) = 0$$

and solve for π, leading to:

$$0 = \frac{\rho S(0)e^{\rho T_0}}{N} \int_{T_0}^{\infty} (1 - (1 - p(0, s))^N) e^{-rs} ds - \pi \int_t^{T_0} p(t, s) e^{-r(s-t)} ds$$

$$\Downarrow$$

$$\pi = \frac{\frac{\rho S(0)e^{\rho T_0}}{N} \int_{T_0}^{\infty} (1 - (1 - p(0, s))^{m+1}) e^{-rs} ds}{\int_0^{T_0} p(0, s) e^{-rs} ds}$$

f) Thiele's equation

$$\frac{d}{dt} V_i^+(t) = r(t) V_i^+(t) - \dot{a}_i(t) - \sum_{j \neq i} \mu_{ij}^x(t)(a_{ij}(t) + V_j^+(t) - V_i^+(t))$$

$$\Downarrow$$

$$\frac{d}{dt} V^+_{(m,*)}(t) = r(t) V^+_{(m,*)}(t) - \frac{\rho S(0)e^{\rho T_0}}{m+1} - \left(\mu^x_{(m,*)(m,\dagger)}(t)(-V^+_{(m,*)}(t)) + \mu_{(m,*)(m-1,*)}[V^+_{(m-1,*)}(t) - V^+_{(m,*)}(t)] \right)$$

$$= r(t) V^+_{(m,*)}(t) - \frac{\rho S(0)e^{\rho T_0}}{m+1} + \mu^x(t) V^+_{(m,*)}(t) - m\mu^x(t)[V^+_{(m-1,*)}(t) - V^+_{(m,*)}(t)]$$

$$= [r(t) + (m+1)\mu^x(t)] V^+_{(m,*)}(t) - \frac{\rho S(0)e^{\rho T_0}}{m+1} - m\mu^x(t) V^+_{(m-1,*)}(t)$$

With final condition: $V^+_{(m,*)}(T) = 0$

g) Numerical example:

We start off by the contractual information:

```r
r    <- 0.03    #interest-rate
rho  <- 0.07
S0   <- 100000
N    <- 10      #number of person
x    <- 30      #age of insured
T0   <- 40      #withdrawal
T    <- 90      #length of contract
G    <- 0       #Gender = male
Y    <- 2022    #Year
```

We were also asked to use the mortalities proposed by Finanstilsynet:

```r
#The weights given by finanstilsynet
w <- function(G,u){
  #men: G = 0
  #woman: G = 1
  #x: age in calender year t,
  if (G==0){
    return (min(2.671548-0.172480*u + 0.001485*u**2, 0))
  }
  if (G==1){
    return(min(1.287968-0.101090*u+ 0.000814*u**2,0))
  }
}


mu_kol_2013 <- function(G,u){
  #men
  if (G==0){
    return((0.241752+0.004536*10**(0.051*u))/1000)
  }
  #woman
  if (G==1){
    return((0.085411+0.003114*10**(0.051*u))/1000)
  }
}

mu_kol <- function(G,u,x,Y){
  #G: gender
  #x: age of insured
  #Y: calender year Y>= 2013
  return(mu_kol_2013(G,x+u)*(1+w(G,x+u)/100)**(Y+u-2013))
}

#survival prob for person aged x in year Y, t,s length of contract.
p <- function(G,x,Y,t,s){
  exp(-as.numeric(integrate(Vectorize(mu_kol),t,s, G=G, x=x, Y=Y)[1]))
}

temp_integrand <- function(s){
  (1-(1-p(G,x,Y,0,s))^{N})*exp(-r*s)
}
```

```
integral <- as.numeric(integrate(Vectorize(temp_integrand),T0,800)[1])

single_premium <- (rho*S0*exp(rho*T0)/N)*integral

single_premium
```

[1] 66209.17

```
temp_integrand_1 <- function(s){
  (1-(1-p(0,x,Y,0,s))^{N})*exp(-r*s)
}

temp_integrand_2 <- function(s){
  p(0,x,Y,0,s)*exp(-r*s)
}

integral1 <- as.numeric(integrate(Vectorize(temp_integrand_1),T0,800)[1])
integral2 <- as.numeric(integrate(Vectorize(temp_integrand_2),0,T0)[1])

above <- rho*S0*exp(rho*T0)/N*integral1
below <- integral2

yearly_premium <- above/below
yearly_premium
```

[1] 2877.21

```
#not including premiums:
PV <- function(t,x, m){
  temp_int <- function(s){
    (1-(1-p(G,x,Y,t,s))^{N})*exp(-r*(s-t))
  }
  integral <- as.numeric(integrate(Vectorize(temp_int),max(t,T0),800)[1])
  return((rho*S0*exp(rho*T0)/((m)+1))*integral)
}

#including premiums
PV_premiums <- function(t,x){
  temp_int <- function(s){
    p(G,x,Y,t,s)*exp(-r*(t-s))
  }
  integral <- as.numeric(integrate(Vectorize(temp_int),t,max(t,T0))[1])
  return(-yearly_premium*integral)
}

time <- seq(0,100,1)
PV_total <- NULL
for (i in 1:length(time)){
  PV_total[i] <- PV(i,x, N-1) + PV_premiums(i, x)
}
```

i)
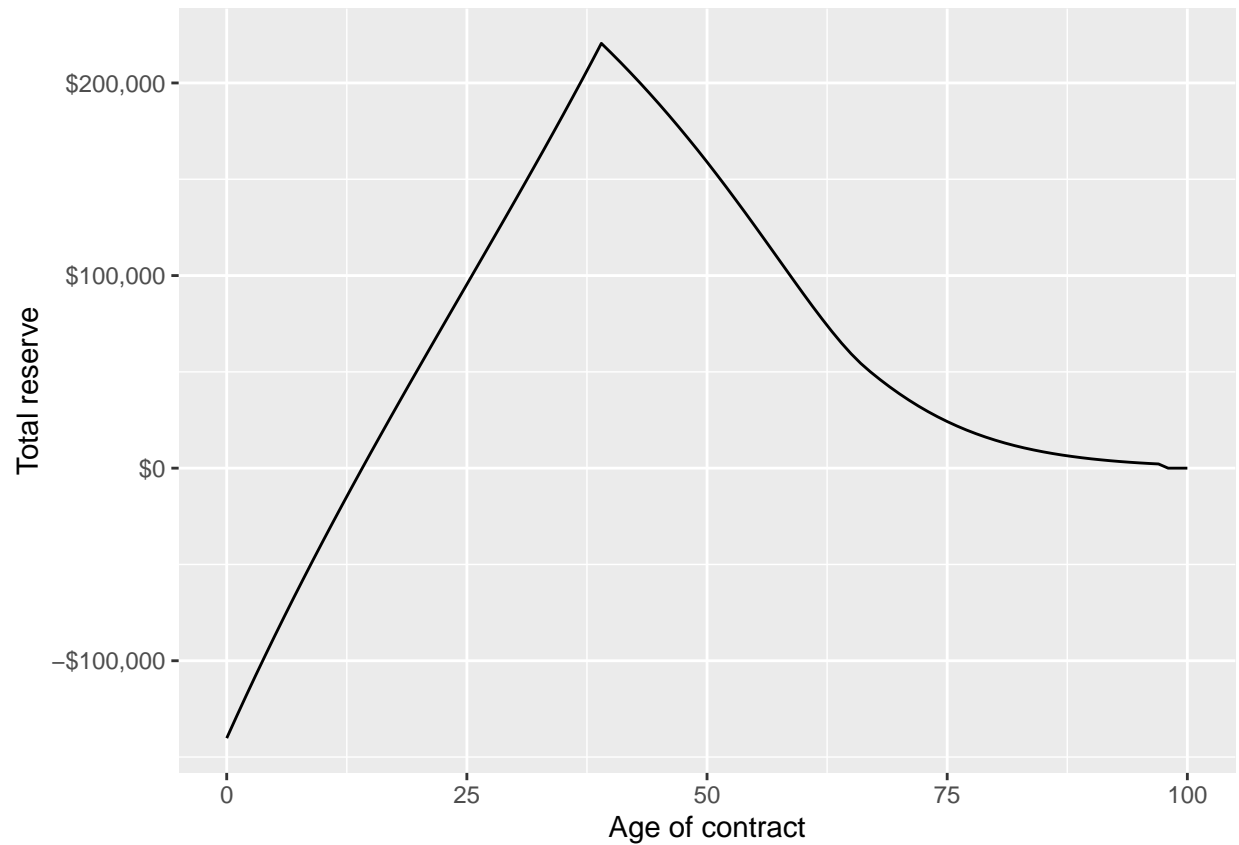
```r
#i)
m <- c(1,5,9)    #lives
t <- c(1,40,60) #times

PV_murder <- matrix(nrow = 3, ncol = 3)

for (i in 1:length(t)){
  for (j in 1:length(m))
    PV_murder[i,j] <- PV(t[i], x, m[j]-1) - PV(t[i], x, m[j])
}
PV_murder
```

```
##            [,1]     [,2]       [,3]
## [1,]   341134.4 22742.29   7580.764
## [2,]  1102847.7 73523.18  24507.727
## [3,]   488214.1 32547.60  10849.201
```

k) Comparison with annuity

In this situation we have: $\bar{S} = \{*, \dagger\}$, furthermore we do get the following policy-function:

$$\bar{a}_*(t) = \begin{cases} 0, & t \in [0, T_0) \\ Pt, & t \geq T_0 \end{cases}$$

$$\bar{V}_*^+(t) = \int_t^\infty \frac{v(s)}{v(t)} p(t,s) d\bar{a}_*(s)$$

$$= P \int_{t \vee T_0}^\infty p(t,s) e^{-r(t-s)} ds$$

From the equivalence principle we get:

$$\bar{V}_*^+(0) = V_{(N-1,*)}^+(0) = \pi_0$$

$$\Downarrow$$

$$P = \frac{\pi_0}{\int_{T_0}^\infty p(0,s) e^{-rs} ds}$$

```
temp_integrand_3 <- function(s){
  p(G,x,Y,0,s)*exp(-r*s)
}
```

```
integral_3 <- as.numeric(integrate(Vectorize(temp_integrand_3),T0,800)[1])
```

```
P <- single_premium/integral_3
P
```

```
## [1] 16234.24
```

```
PV_pension <- function(t,x){
  temp_int <- function(s){
    p(G,x,Y,t,s)*exp(-r*(t-s))
  }
  integral <- as.numeric(integrate(Vectorize(temp_int),max(t,T0), 800)[1])
  return(P*integral)
}
```

```
t1 <- seq(30, 90, 1)
Pension <- NULL
for (i in 1:length(t1)){
  Pension[i] <- PV_pension(t1[i], x)
}
Pension
```

```
##  [1] 591977.226 575789.737 560168.970 545107.605 530598.092 516638.162
##  [7] 503222.079 490351.510 478026.629 466251.758 455033.889 428338.578
## [13] 402582.289 377748.494 353821.591 330787.113 308632.461 287345.427
## [19] 266916.311 247334.764 228594.067 210686.622 193606.064 177348.363
## [25] 161908.235 147282.631 133467.879 120460.909 108259.312  96858.391
## [31]  86255.868  76447.893  67431.264  59205.113  51773.443  45152.146
## [37]  39384.122  34573.569  30927.982  27881.504  25108.354  22588.217
## [43]  20301.674  18230.244  16356.419  14663.686  13136.533  11760.451
## [49]  10521.916   9408.373   8408.204   7510.694   6705.994   5985.072
## [55]   5339.673   4762.265   4245.998   3784.651   3372.589   3004.714
## [61]   2676.422
```

```
df2 <- data.frame(t1, Pension)
colnames(df2) <- c("age_contract", "Value_pension")

df2 %>%
  ggplot(aes(x=age_contract, y = Value_pension)) +
  geom_line() +
  scale_y_continuous(labels = dollar) +
  xlab("Age of contract") +
  ylab("PV Pension")
```