Problem 2: Interaction between a magnetic dipole and a charged particle

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1 Introduction

The objective of this problem set is to model a static magnetic dipole, and then simulate the interaction between a charged particle and such a dipole. The purpose of this is to gain insight into the interaction between the solar wind and the earthly magnetic field.

Both the aforementioned tasks were tackled with Python. Due to the inefficiency of this programming language, the interaction between charged particles and magnetic dipoles was also modeled in c++. The code may be found on my github-page [2].

2 Theory

This project is wholly based on the expression for the magnetic field around an ideal magnetic dipole, as well as the Lorentz-force equation in the case that $\vec{E} = 0$.

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right) \tag{1}$$

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

3 Numerical procedure for dipole B-field

Consider the Earth-Sun system. The $\hat{x}$-axis is defined to be the sun-earth-axis. Furthermore, the $\hat{z}$-axis to defined to be perpendicular to the ecliptic of the Earth. The $\hat{x}$-axis changes throughout the year, but kinematics related to this change is neglected. The earth is assumed to have a tilt of 23.7°, rotated with respect to the $\hat{y}$ direction. The direction of this tilt varies throughout the year (in relation to the $\hat{x}$-axis), but I have somewhat arbitrarily chosen this orientation.

The earthly dipole is positioned in the centre of the earth, which again is placed in $(0, 0, 0)$. Its southern pole is set to point along the northern pole of the earth. The real direction of this moment varies throughout the day, however such variation is neglected.

These assumptions allow the calculation of the surrounding $\vec{B}$-field by the use of (1). This field is calculated the xy, yz and xz-planes. The B-field calculated in each case is projected down into the given plane.

4 Results for dipole B-field

Figure 1 demonstrates the aforementioned field projections. The earth is colored in as a black circle. In each plot, there are streamlines following the direction of $\vec{B}$. Additionally, every plot is colored according to the projected field strength.

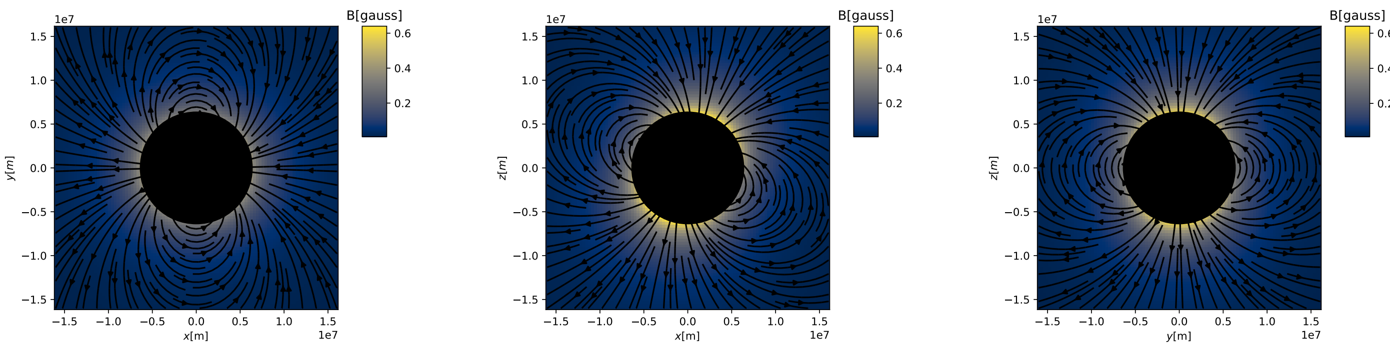

Figure 1: Plots of the magnetic fields in the xy, yz and xz-planes

5 Numerical procedure for solar wind interaction

The aforementioned description of the earthly magnetic dipole is maintained during simulation of charged particles originating from the sun. Only the magnetic interaction between solar wind particles and the earth are considered, as this is assumed to dominate other forces such as gravitational ones.

Propagation of the particles is based on an RK4-procedure. To legitimize the calculated solutions, the particle speeds have been plotted additionally, as this should be conserved during the magnetic interaction. Only protons having charge $1.60 \cdot 10^{-19}$C and mass $1.67 \cdot 10^{-27}$kg are considered.

The behaviour of such protons vary greatly depending on their initial conditions. According to Britannica, there is a constant solar wind containing particles having velocity around $800\frac{\text{km}}{\text{s}}$ [1]. The code simulates protons originating on the $\hat{x}$-axis having this velocity. In accordance with the 'problem plan' I have tested the simulation with various other velocities, but the results of this paper are only based on the specified value.

Firstly, the simulation considered a proton originating 15 million km ($1.5 \cdot 10^{10}$m) from the Earth, meaning $\frac{9}{10}$ of the way from the sun to the earth. For distances greater than this, the magnetic forces are negligible, so simulating the entire path would be a waste of computation. This first simulation was performed With a stepsize of 3ms, and a simulated time of $3 \cdot 10^4$s.

Secondly, the program simulates a proton being 'created' closer to the Earth, in a state where it is already embedded deep in the magnetic field of the planet. It is now created 0.1 million km ($1 \cdot 10^8$m) from the Earth. The same stepsize of 3ms is used, along with a simulated time of $5 \cdot 10^4$s.

In either simulation, the normed velocity of the proton is plotted as a function of time.

6 Results and discussion of solar wind interaction

The particle-paths are colored according to the time at which the particle was in the given location. Brighter colors are more recent. The final position of the particle is denoted by a black spot. The Earth is not plotted, but it is located at $(0, 0, 0)$.

The results of the first simulation is demonstrated in figure 2. The figure clearly demonstrates the proton being deflected upon approaching the Earth. This fits with the well known property that planetary magnetic fields deflect solar wind particles.

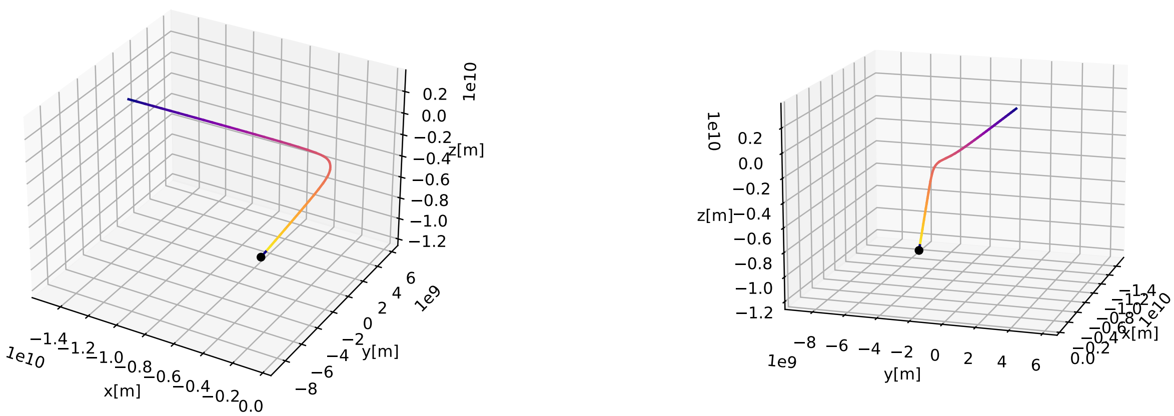

Figure 2: Plots showing two different views of a proton originating far away from the Earth being deflected upon entering its magnetic field.

The validity of this solution is strengthened by how the velocity (and therefore also the energy) is conserved with a high degree of accuracy, as demonstrated in Figure 3:

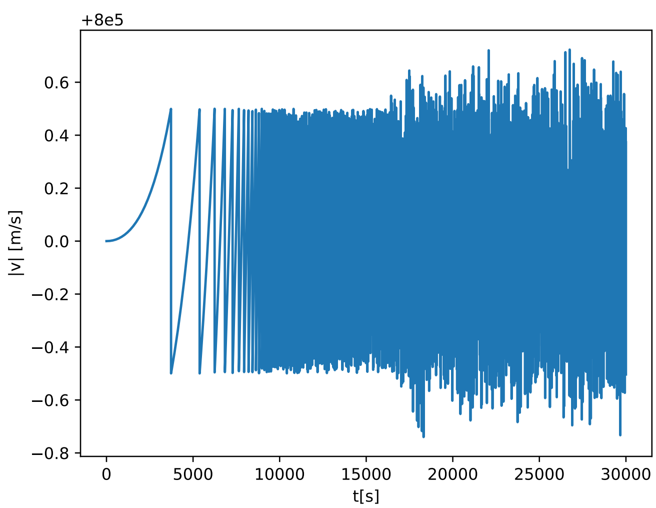

Figure 3: Energy as a function of time. The relative velocity deviations are on the order of 10^{-6}.

The result of the second simulation is shown in figure 4. The proton apparently oscillates back and forth between the northern and southern pole of the Earth. The fact that charged particles are guided towards the poles fits nicely with the theory of aurora-borealis, which is a polar phenomena. In addition to this oscillations, the particles precess clockwise around the northern pole. One may suspect this toroidal path corresponds to the Van-Allen belts of the planet. Close examination reveals an additional helical structure of these paths, as is common for charged particles propagating through magnetic fields.

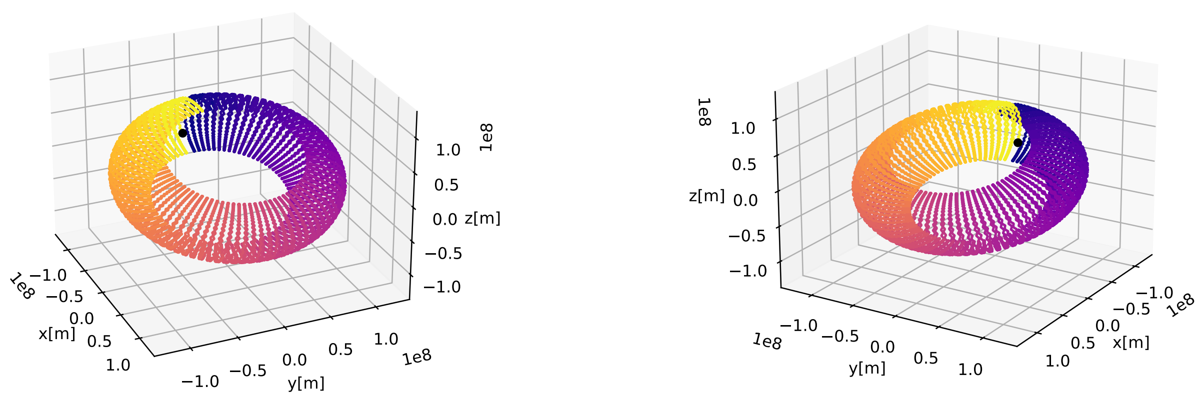

Figure 4: Path of a fast proton created close to the Earth

Once again, this plot is legimitized by its approximate conservation of speed (and hence energy) as plotted in Figure 5.

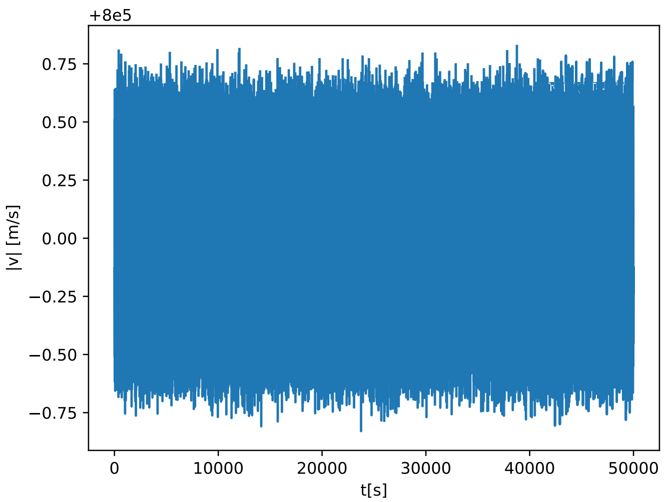

Figure 5: Particle speed as a function of time for the proton in the toroidal path. The relative variations are of order 10^{-6}.

It is not realistic that protons would be created this close to the Earth, but higher energy protons could approach the Earth, and lose energy upon collision with the atmosphere, and thus end up in this situation.

7 Conclusion

These simulations seem to numerically unveil several well known phenomena of the earthly magnetic fields.

References

[1] Britannica. *Solar wind*. URL: `https://www.britannica.com/science/solar-wind`.

[2] Andreas Stapnes. URL: `https://github.com/AndreasStapnes/DipoleModeling`.