Examining the methods for measuring Market Risk

Andreas Theodoulou

University of Bristol
School of Mathematics

09 May 2018

Contents

Motivation

- Risks of Financial Institutions risks [4]
- Risk Management process
- Financial Risk and Market Risk
- Assessment: Value at Risk (VaR) and Expected Shortfall (ES) (Origins) [1]
- Importance [5]
- Project's Purpose

Risk Measures

Definition

Let S_t is the final price at which a security is traded at time t, t refers to a specific market day. Then the **returns** (log-returns) are defined by [3]

$$R_{t+1} = ln(S_{t+1}) - ln(S_t) \tag{1}$$

Definition

Let $R_{PF,t+1}$ denote the random variable of portfolio returns at time t. Then the $\mathbf{q - \%VaR}$ is defined by [3]

$$(R_{PF,t+1} < -VaR^q_{t+1}) = q \tag{2}$$

Definition

Let $R_{PF,t+1}$ be defined as above. The $\mathbf{q - \%ES}$ is defined by,[3]

$$ES^q_{t+1} = E_t[R_{PF,t+1} \mid R_{PF,t+1} < -VaR^q_{t+1}] \tag{3}$$

Risk Measures

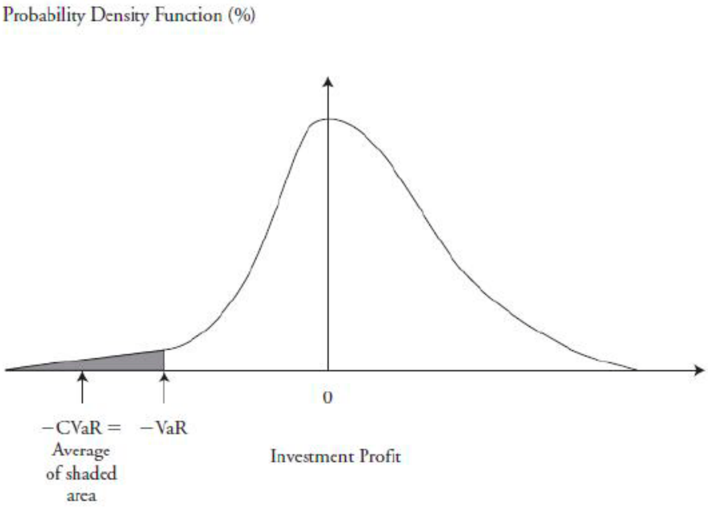


Figure: VaR and ES/CVaR on a probability density function of returns [6]

Financial Assets Characteristics

- Volatility Clustering [3]
  - high autocorrelation of daily volatility
- Leptokurtic Distributions [3]
  - Higher peak at 0
  - Fatter tails
- Leverage effect [3]
  - asymmetry of upward and downward movements.
- Uncorrelated daily returns and mean around 0. [3]

Figure: Volatility Clustering effect. [3]

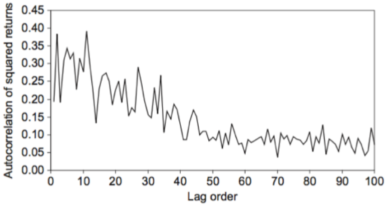


Figure: Uncorrelated Returns [3]

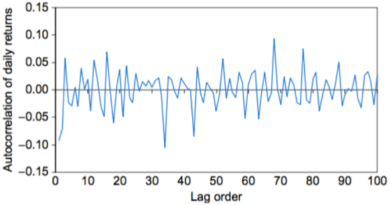
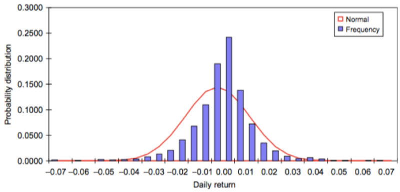


Figure: Leptokurtic Distribution. [3]



Methods

1. **Historical Simulation** [3]
   Let $\{R_{PF,t+1-\tau}\}_{\tau=1}^{m}$ denote the time series of returns, τ the times when $\widehat{VaR}$ losses where greater than VaR.

$$\widehat{VaR_{t+1}^{q}} = -Percentile(\{R_{PF,t+1-\tau}\}_{\tau=1}^{m}, 100q) \tag{4}$$

$$\widehat{ES_{t+1}^{q}} = \frac{1}{\tau}\sum_{i=1}^{\tau} r_i \tag{5}$$

2. **Parametric method** [3]

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1} \qquad \text{with } z_{t+1} \sim D(0,1) \tag{6}$$

D(0,1) will be tested for a normal distribution and t-distribution. $\mu_{t+1} = 0$.

- EWMA

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)R_t^2, \qquad \text{where } \lambda = 0.94 \qquad [3] \tag{7}$$

- GARCH

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta\sigma_t^2, \qquad \text{with } \alpha + \beta < 1 \tag{8}$$

- GJR-GARCH

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \alpha\theta\mathbb{1}_t R_t^2 + \beta\sigma_t^2 \qquad \text{for } \mathbb{1}_t = \begin{cases} 1, & \text{if } R_t < 0. \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Backtesting

- **Backtesting VaR** [3]
  Let $L(\cdot)$ denote the likelihood function for an i.i.d. Bernoulli random variable, T_0 the number of days losses were greater than $\widehat{VaR}^q_{t+1}$, and T be the total VaR estimates. Also, let π be the proportion of VaR estimates failures, i.e $\hat{\pi} = T_1/T$.[2]

$$H_0 : \pi = q \tag{10}$$

$$H_1 : \pi \neq q \tag{11}$$

Test Statistic:
$$LR_{UC} = -2ln[L(q)/L(\hat{\pi})] \sim \chi^2_1 \tag{12}$$

- **Backtesting ES** [2]
  Let $P^{[q]}_{t+1}$ denote the actual cumulative distribution of returns, $F^{[q]}_{t+1}$ our estimated one and I_t be equal to 1 when $R_{PF,t+1} < \widehat{VaR}^q_{t+1}$ and 0 otherwise.[2]

$$H_0 : P^{[q]}_{t+1} = F^{[q]}_{t+1}, \qquad \text{for all t} \tag{13}$$

$$H_1 : ES^P_{t+1,q} \geq ES^F_{t+1,q} \tag{14}$$

Test Statistic:
$$Z_2 = \frac{1}{qT} \sum_{t=0}^{T} \frac{R_{PF,t+1} I_t}{ES^q_{t+1}} + 1 \tag{15}$$

Data & Methodology

Data

- 10 stock indices presented in Appendix. Each represents a portfolio.
- Estimation periods - Crisis and Post-Crisis.
- Time window - past 1000 trading days for GARCH and 250 for HS.

Methodology

- Methods - All the above as introduced.
- Confidence interval of Backtests - 95%.
- ES backtesting of t-distribution methods - assume 15 degrees of freedom.
- Volatility models coefficients - Estimated by MLE by a MATLAB function.

Ranking Statistics

- n_w: Frequency of best estimation - Methods and Distributions
- $\sum LR$, $\sum |Z|$: Average performance - Methods
- $\sum\sum LR$, $\sum\sum |Z|$: Average performance - Distributions

Data & Methodology

Hypotheses

1. Parametric method - Does each model captures the asset characteristics?
2. Performance in "difficult" forecasting periods vs "easy" forecasting periods.
3. Is a single model capable to capture best both the 95% and 99% quantile of the returns distribution?
4. Do ES estimates differ in terms of accuracy over VaR?

Figure: S&P500 daily returns. Illustration of our approach to testing Hypothesis no.2.
Source: Google

Results

Overall Performance

Figure: Frequency of best ranking statistic over all results

	Average performance	Frequency of Best estimation
EWMA	7/8	7/8
HS	1/8	1/8

Figure: Frequency of worst average performance and Total frequency of best estimation over all results

	Worst average performance	Frequency of best estimation
GJR	5/8	5/80
HS	2/8	14/80
GARCH	0/8	15/80
EWMA	1/8	46/80

Figure: Frequency of best ranking statistic over all results

	Best average performance	Frequency of best estimation
Normal	6/8	5/8
Student's	2/8	3/8

Ranking Statistics Results

Figure: 95%-VaR estimates

		Crisis				Post Crisis			
		$\sum LR$	$\sum\sum LR$	n_ω	$\sum n_\omega$	$\sum LR$	$\sum\sum LR$	n_ω	$\sum n_\omega$
Normal	HS	93.93		1		2.3		6	
	EWMA	14.99		3		3.28		4	
	GARCH	72.38	176.45	0	4	44.11	294.29	0	4
	GJR	89.08		1		246.91		0	
Student's t	EWMA	12.76		1		9.09		0	
	GARCH	26.08	139.51	2	5	33.11	375.93	0	0
	GJR	100.67		2		333.73		0	

Figure: 95%-ES estimates

		Crisis				Post-Crisis			
		$\sum\lvert Z\rvert$	$\sum\sum\lvert Z\rvert$	n_ω	$\sum n_\omega$	$\sum\lvert Z\rvert$	$\sum\sum\lvert Z\rvert$	n_ω	$\sum n_\omega$
Normal	HS	9.57		0		1.26		3	
	EWMA	1.77		2		0.45		7	
	GARCH	2.85	9.67	1	3	2.56	8.27	0	7
	GJR	5.05		0		5.26		0	
Student's t	EWMA	1.25		4		1.35		0	
	GARCH	1.76	8.18	2	7	2.65	9.98	0	0
	GJR	5.17		1		5.98		0	

- 99% VaR estimates
- 99% ES estimates

Results Discussion

- Hypothesis 1 - ability of capturing asset characteristics
- Hypothesis 2 - "difficult" vs "easy" forecasting periods
- Hypothesis 3 - 99% vs 99%
- Hypothesis 4 - ES vs VaR model performance

Conclusion

- Aim 1 - Best models for estimating VaR and ES
- Aim 2 - Illustrating the extent to which the use of the two measures is different.

Recommendations for further research

- Best distribution for financial asset returns
- Backtesting ES
- Volatility forecasting with adaptable coefficients

Thank you!

References I

- *Risk Management and Financial Institutions*, John C.Hull, 2012.
- B. S. Carlo Acerbi, *Backtesting Expected Shortfall*, MSCI Inc., 2014.
- P. F. Christoffersen, *Elements of Financial Risk Management*, 2012.
- K. Dowd, *Beyond value at risk: the new science of risk management*, John Wiley Sons, 1998.
- B. for Internaltional Settlements, *Revised framework for market risk capital requirements issued by the basel committee.*
- D. Kidd, *Value at risk and condition value at risk: A comparison*, Investment Risk and Performance Articles 2012.1, (2012).

Appendix

Table: List of the 10 Stock Indices, each representing a portfolio to be tested.

Stock Index	No. of Stocks	Region
S&P 500	500	USA
FTSE 100	100	UK
Nikkei 225	225	Japan
EuroNext 100	100	Europe
DAX	30	Germany
HSI	50	Hong Kong
ASX 200	200	Australia
S&P BSE	30	India
IBOVESPA	60	Brazil
IPC	35	Mexico