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Strategising in dialogues handling forward extension of enthymemes

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Abstract. A common assumption for argumentation-based dialogues is that any argument exchanged is complete, i.e. its premises entail its claim. However, in real world dialogues, agents commonly exchange enthymemes - arguments with incomplete logical structure. This paper expands a previous dialogue system which accommodates enthymemes whose premises do not directly entail the claim of the intended argument, by broadening the way participants reveal its missing elements. It also provides a rational strategy for them to generate a dialogue, without making assumptions for the argument their counterpart intends when they move an enthymeme. Such dialogue will terminate while its moves will have the correct acceptability status. Thus, we capture more realistic scenarios of how a dialogue may unfold and we provide a specific method for computational systems to address such cases, ensuring that the dialogue outcome is the appropriate one.

Keywords.
Argumentation, Enthymemes, Dialogue, Strategising

1. Introduction

Structured approaches to argumentation typically assume that arguments consist of a conclusion (claim) deductively and/or defeasibly inferred from some premises [1]. However, in real-world dialogues, humans tend to communicate arguments that are not logically complete—*enthymemes* [2]—omitting element(s) because, e.g. they assume the omitted elements can be reconstructed by their interlocutors based on information they share, or previous dialogues. Still, the recipient of an enthymeme may be incapable of correctly 'filling in' the missing information. These misunderstandings can then compromise the key desired correspondence—i.e. *soundness and completeness (**SC**)*—result for dialogical formalisms of distributed non-monotonic reasoning; that is, at any given stage of a dialogue d, d establishes the topic α iff α is the claim of a justified argument in the AF instantiated by the arguments constructed from the pooled knowledge.

Suppose that two agents Ag_1 and Ag_2 have a dialogue to decide whether information about Bob's affair should be published. The intended argument of Ag_1 is A = "If Bob is no longer a public figure ($\neg pf$) *and* information about his affair is not in the public interest ($\neg pi$), then the information should not be published ($\neg pub$). Bob is no longer a prime minister ($\neg prm$), *hence* Bob is no longer a public figure. Romantic affairs are private (arp), *hence* romantic affairs do not concern the public ($\neg ap$), *hence* information about

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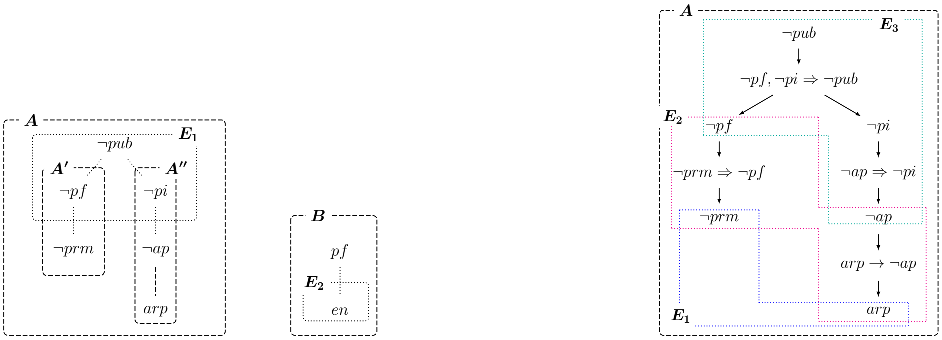

(i) (ii)

Figure 1. Graph (i) shows a formal representation of the arguments and enthymemes in our example: arguments A and B, enclosed by a dashed line, as represented in the *ASPIC*$^+$ framework where a node is a claim (or intermediate claim) and the node's children are the premises that either strictly (if they are connected with a solid line) or defeasibly (if they are connected with a dotted line) infer this claim (or intermediate claim); the sub-arguments A' and A'' of A, which backward extend A on $\neg pf$ and $\neg pi$, respectively, are enclosed by a dashed line; the enthymemes E_1 of A and E_2 of B are enclosed by a dotted line; B forward extends E_2. Graph (ii) shows A as we represent arguments in this paper, where the inference rules applied ($\Rightarrow$ and $\rightarrow$ denoting the defeasible and strict inference, respectively) are explicitly provided and a claim is connected to its premises via the inference rule applied. The enthymeme E_1 of A is an *upwards extendable enthymeme* where E_2 forward extends E_1 and E_3 forward extends E_2 as well as $E_1 \cup E_2$. Note, different colours indicate different enthymemes.

Bob's affair is not in the public interest. Therefore the information about Bob's affair should not be published." (see Fig. 1.(i)). Ag_1 may move the enthymeme E_1 = "$\neg pf$, and $\neg pi$, *hence* $\neg pub$", holding back from communicating the supports for the intermediate conclusions $\neg pf$ and $\neg pi$. Ag_2 might then query $\neg pf$ and $\neg pi$ (separately), eliciting Ag_1's arguments "$\neg prm$ *hence* $\neg pf$" and "*arp hence* $\neg ap$ *hence* $\neg pi$" (A' and A'' respectively; see Fig. 1(i)) which together '*backward extend*' E_1 to yield the complete argument A.

Now, consider that argument B = " Bob is UN envoy for the Middle East (*en*), *hence* Bob is a public figure (*pf*)." is the intended argument of Ag_2, but Ag_2 attacks E_1 with the enthymeme E_2 = *en*. It is not immediately clear why E_2 attacks E_1 (since *en* does not directly challenge any element of E_1), however it appears that Ag_2 wins the dialogue (since she *moves* E_2 against A). In other words, a mismatch may exist between the pragmatic and the logical conclusions implied by a dialogue in which enthymemes are used. As discussed in [3], to avoid misunderstanding and ensure a fully rational exchange, normative scaffolding would prompt Ag_1 to seek clarification—"what is implied by *en* such that your intended argument attacks E_1?"—to which Ag_2 might reply that *en* implies *pf*, thus *forward extending* E_2 to yield B which negates a premise in E_1.

Only a few works on formalising argumentation-based dialogues accommodate locutions that allow for requesting and providing either backward extension of enthymemes (e.g. [4,5,6,7,8]), or forward extension of enthymemes (e.g. [9]), or both (e.g. [10,11]), or enable resolution of misunderstandings that have already occurred due to use of enthymemes (e.g. [11,12]). Even fewer examine the aforementioned **SC** correspondence, specifically [7,9] (for backward and forward extension, respectively) and [12] (for misunderstandings already taken place during the dialogue). However, none of these works specify a *strategy* for determining how an agent must respond to a previous move made in the dialogue in order to ensure **SC** results are achieved. Notable exceptions are [4] and [6], but in [4] **SC** results are demonstrated based on the union of the agents' beliefs and in [6] moves are dictated by the assertion and acceptance attitude of a participant.

Our first contribution is to adapt the work described in [9] to accommodate a more flexible type of forward extension of enthymemes where the sender can reveal the enthymeme's missing elements in parts rather than all at once (similar to what can happen in real-world scenarios). Our second and main contribution is the introduction of a *rational strategy* used to generate a dialogue handling forward extension of enthymemes, and guide a participant to the kind of move they need to make to ensure that the dialogue will terminate while preserving **SC**. We demonstrate the particular steps two interlocutors must follow to finish their dialogue, reaching the "appropriate" conclusions based on the knowledge they have shared when enthymemes are involved. Our strategy does not account for the argument that an agent may assume their counterpart intended, considering only what has been revealed publicly. If we are to enable effective human-computer interaction and provide normative support for human-human dialogue, we need to account for the ubiquitous use of enthymemes in real-world dialogues and establish procedures on how to deal with them while ensuring that the right outcomes are met.

2. Preliminaries

We ground our work on [9], which uses the *ASPIC*$^+$ framework [13] to formalise enthymemes. Their definition of an *argument* leads to an alternative isomorphic structural representation of an *ASPIC*$^+$ argument (compared to the classic representation in [13]) and its characteristics. For an example, see Fig 1.(ii).

Definition 1. *Let* $\mathrm{AS} = \langle Lan, (\bar{\cdot}), \mathrm{R}, \mathsf{nom}\rangle$ *be an argumentation system, where Lan is a logical language,* $(\bar{\cdot})$ *is a function that generalises the notion of negation,* $\mathrm{R} = \mathrm{R_s} \cup \mathrm{R_{def}}$ *is a set of strict (*$\mathrm{R_s}$*) and defeasible (*$\mathrm{R_{def}}$*) inference rules, and* nom *is a naming function which assigns a name (or nominal) to each defeasible rule. Let* $\mathrm{AT} = \langle \mathrm{AS}, \mathrm{K}\rangle$ *be an argumentation theory, where the knowledge base* $\mathrm{K} \subseteq Lan$ *is a set of premises*[2]*. An* ***argument*** *is a labelled (downward directed) tree* $A = \langle \mathsf{Nd}(A), \mathsf{Ed}(A), \mathsf{lab}_A\rangle \in \mathrm{AS}$ *where:*

1. $\mathsf{lab}_A : \mathsf{Nd}(A) \rightarrow Lan \cup \mathrm{R}$ *is a node labelling;*
2. $\mathsf{Ed}(A) \subseteq \mathsf{Nd}(A) \times \mathsf{Nd}(A)$ *is a set of edges such that if* $(n_i, n_j) \in \mathsf{Ed}(A)$*, then either:*
   (a) $\mathsf{lab}_A(n_i) \in Lan$, $\mathsf{lab}_A(n_j) \in \mathrm{R}$ *and* $\mathsf{con}(\mathsf{lab}_A(n_j)) = \mathsf{lab}_A(n_i)$*, or*
   (b) $\mathsf{lab}_A(n_i) \in \mathrm{R}$ *and* $\mathsf{ant}(\mathsf{lab}_A(n_i)) = \{\mathsf{lab}_A(n_j) \mid (n_i, n_j) \in \mathsf{Ed}(A)\} \subseteq Lan$*,*

   and if $(n_i, n_j), (n_i, n_k) \in \mathsf{Ed}(A)$ *and* $n_j \neq n_k$ *then* $\mathsf{lab}_A(n_j) \neq \mathsf{lab}_A(n_k)$*;*
3. *for every node* $n \in \mathsf{Leaves}(A)$*, we have* $\mathsf{lab}_A(n) \in K$*;*
4. $|\mathsf{Roots}(A)| = 1$*;*
5. $\mathsf{lab}_A(\mathsf{Conc}(A)) \in Lan$*, where* $\mathsf{Conc}(A)$ *is the unique element in* $\mathsf{Roots}(A)$*;*

where $\mathsf{ant}(r) = \{p_1, \ldots, p_n\}$ *denote the set of antecedents of r and* $\mathsf{con}(r) = p$ *denote the consequent of r for* $p, p_1, \ldots, p_n \in Lan$ *and* $r \in \mathrm{R}$, $\mathsf{Leaves}(A) = \{n_i \in \mathsf{Nd}(A) \mid \nexists (n_i, n_j) \in \mathsf{Ed}(A)\}$*, and* $\mathsf{Roots}(A) = \{n_i \in \mathsf{Nd}(A) \mid \nexists (n_j, n_i) \in \mathsf{Ed}(A)\}$*. Let* $\mathrm{Args_{AT}}$ *denote the set of arguments instantiated using the elements in* AT *and* A^* *denote the class of all arguments (i.e.,* $\mathrm{Args_{AT}} \subseteq \mathrm{A}^*$*) and define* $\mathsf{Rules}(A) = \{\mathsf{lab}_A(n) \in \mathrm{R} \mid n \in \mathsf{Nd}(A)\}$*, for all* $A \in \mathrm{A}^*$.

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[2]Note we do not utilise the *ASPIC*$^+$ distinction between the disjoint sets of axiom ($\mathrm{K_a}$) and ordinary ($\mathrm{K_p}$) premises ($\mathrm{K} = \mathrm{K_a} \cup \mathrm{K_p}$), whereby only ordinary premises are fallible and so can be challenged/attacked.

The binary attack/defeat relations over arguments are defined as for *ASPIC*$^+$ arguments, i.e. an attack from argument X to argument Y may succeed as a defeat, contingent on preferences defined over the argument X and the targeted sub-argument of Y, if X's claim conflicts with an ordinary premise or the consequent or name of a defeasible rule in Y [13]. We assume that there is a unique preference order defined over arguments which is a strict total order. We later refer to the usual concepts of an argument framework $\mathrm{AF} = \langle \mathrm{Args_{AT}}, \mathrm{Dfs} \rangle$ instantiated by an argumentation theory AT (as defined in [13]), and a complete labelling L on AF (as defined in [14]).

Similarly to [9], an *enthymeme* E of an argument A is a forest (i.e. a disjoint union) of trees, whose nodes and edges are a subset of the nodes and edges of A, and the label of each node in E is the same label of the corresponding node in A (e.g. see Fig 1.(ii)).

We focus on the forward extension of enthymemes, for which the only missing information needed to reconstruct the complete argument is the information between the roots of the trees of the enthymeme and the root of the intended argument. We define an *upwards extendable enthymeme* E of an argument A as an enthymeme which includes the leaves of A, and for each root t of a tree in E, the label of t is an element of *Lan*, meaning that either it is a premise, when t is also a leaf, or the consequent of a rule used in E. We make sure that if E includes an inference rule r, then it also contains the antecedents of r. For an example of an upwards extendable enthymeme see Fig. 1.(ii)[3].

Definition 2. *An* ***enthymeme of an argument*** $A = \langle \mathsf{Nd}(A), \mathsf{Ed}(A), \mathsf{lab}_A \rangle$ *is a labelled forest of (downward directed) trees* $E = \langle \mathsf{Nd}(E), \mathsf{Ed}(E), \mathsf{lab}_E \rangle$ *(written* $E \leq A$*) such that* $\emptyset \neq \mathsf{Nd}(E) \subseteq \mathsf{Nd}(A)$*,* $\mathsf{Ed}(E) \subseteq \mathsf{Ed}(A) \cap (\mathsf{Nd}(E) \times \mathsf{Nd}(E))$ *and for every node* $n \in \mathsf{Nd}(E)$*,* $\mathsf{lab}_E(n) = \mathsf{lab}_A(n)$*. We say that* E *is an* ***upwards extendable enthymeme of*** A *iff* (i) $\mathsf{Leaves}(E) = \mathsf{Leaves}(A)$*,* (ii) $\forall n \in \mathsf{Roots}(E)$*,* $\mathsf{lab}_E(n) \in L$ *and* (iii) $\forall r \in \mathsf{Rules}(E)$ *and* $\forall p \in \mathsf{ant}(r)$*,* $\exists n' \in \mathsf{Nd}(E)$ *such that* $\mathsf{lab}_E(n') = p$*. Note if* $\mathsf{Conc}(A) \in \mathsf{Roots}(E)$*, then* $\mathsf{Roots}(E) = \{\mathsf{Conc}(A)\}$ *and* $E = A$*.*

In [9] the *upwards extension of* E *that yields* A is the enthymeme E' (unique up to isomorphism) of A which consists of all the elements between the roots of E and the claim of A, including the roots of E and the claim of A. Here, we provide a broader definition of a *forward extension* of E (capturing the concept of an upwards extension) to be an enthymeme E' which also contains the roots of E as well as elements between them and the claim of A, but not necessarily the claim[4] of A.

Definition 3. *Let* A *be an arguments where* $E, E' \leq A$ *are enthymemes. We say that* E *is a* ***forward extension*** *of* E' *(denoted as* $E' \ll_{fw} E$*) iff* (i) $\mathsf{Leaves}(E) = \mathsf{Roots}(E')$*,* (ii) $\forall n \in \mathsf{Leaves}(E)$*,* $n \notin \mathsf{Roots}(E)$*,* (iii) $\forall n \in \mathsf{Roots}(E)$*,* $\mathsf{lab}_E(n) = \mathsf{lab}_{E'}(n) \in Lan$*, and* (iv) $\forall r \in \mathsf{Rules}(E)$ *and* $\forall p \in \mathsf{ant}(r)$*,* $\exists n \in \mathsf{Nd}(E)$ *such that* $\mathsf{lab}_E(n) = p$*.*

The union $E_1 \cup E_2$, where E_1 is an upwards extendable enthymeme of A and E_2 forward extends E_1, is also an upwards extendable enthymeme of A (e.g. see Fig. 1.(ii)).

Corollary 1. *Let* E *be an upwards extendable enthymeme of an argument* A *and* $E' \leq A$ *such that* $E \ll_{fw} E'$*. Then* $E \cup E'$ *is an upwards extendable enthymeme of* A*.*

[3]Note, the enthymeme E_1 in Fig. 1.(ii) does not contain a single conclusion as arguments usually do. This case provides also an example of why an enthymeme cannot always be viewed as any other argument.

[4]In this way, we can provide the missing elements in parts until we reach the claim of A (e.g. see Fig 1.(ii)), rather than all at once as happens in [9] which further restricts the participants of the dialogue.

3. Well-formed enthymeme dialogue

The *well-formed enthymeme dialogue* d appearing in [9] (referred to as '*wfe*-dialogue' for short) is a sequence of moves, where each move is a 5-tuple that comprises the move's *sender*, *location*, *content*, *reply* and *target*. An assert move is used to posit an upwards extendable enthymeme that targets (i.e., is moved as a defeat against) an enthymeme that has been previously asserted (except for the first assert move, which has no target). *Prop* must start by asserting an *argument*, the conclusion of which we call the *topic* of the dialogue. Note that, while the target m' of a move m consists of the enthymeme E', this does not necessarily imply a valid defeat relation from the content E of m (i.e., E may not validly defeat E' according to the $ASPIC^+$ definition of defeat).

Here we propose a modified version of d, as follows. An and-so move is used to request a forward extension of a previously moved enthymeme, while a hence move replies to a previous and-so move providing a forward extension of the questioned enthymeme. We add a stop location to indicate a participant's intention to end the dialogue. When two stop moves are made consecutively a *wfe*-dialogue terminates, indicating that neither agent wishes to continue the dialogue. Since participants may be mistaken in their assumptions for the other's intended arguments, moves are made depending on the information revealed related to an enthymeme (as we will see later). An agent cannot ask for a forward extension of the first argument moved in d as its conclusion is the topic of d, and interlocutors alternate turns, but cannot repeat moves.

We assume that two participants, *Prop* and *Op*, share the same underlying argumentation system, ensuring they can understand each other, except that the defeasible rules they are aware of as well as their knowledge bases may differ, i.e. $\mathrm{AT}_{Ag} = \langle \mathrm{AS}_{Ag}, \mathrm{K}_{Ag} \rangle$ where $\mathrm{AS}_{Ag} = \langle Lan, (\bar{\cdot}), R_{Ag}, \mathsf{nom} \rangle$ for $Ag \in \{Prop, Op\}$ and $\mathrm{R}_{Ag} = \mathrm{R}_s \cup \mathrm{R}_{def}^{Ag}$. We also assume that the number of arguments they can instantiate is finite. In Table 1, we give

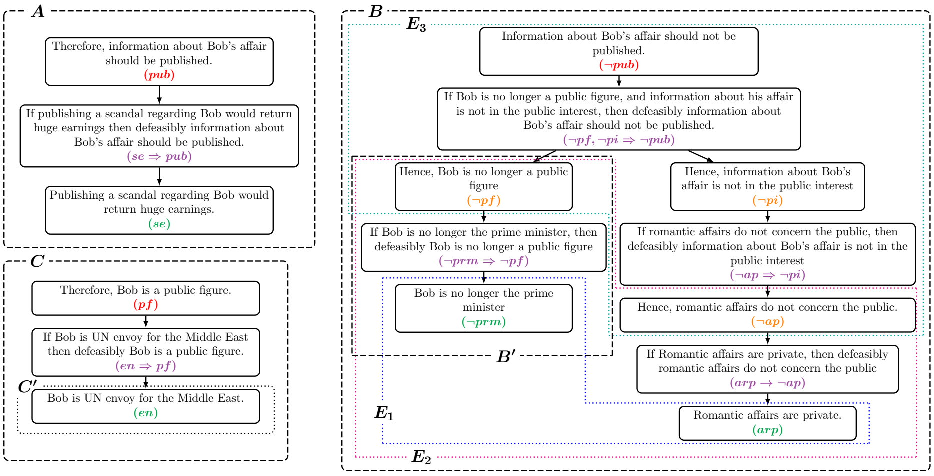


Figure 2. Internal structure of enthymemes and intended arguments of moves in d (enclosed by a dotted/dashed line, respectively, where different colours indicate different enthymemes) described in Table 1 where B' is a sub-argument of B. The natural language translation of each premise (green), inference rule (purple), intermediate conclusion (orange) and claim of intended argument (red) is shown.

Step	***wfe*-dialogue** $d = [m_0, \dots, m_4]$	**IntArg**(m_i)	**ResEnth**(m_i, d_i)
1	$m_0 = (Prop, \mathsf{assert}, A, \emptyset, \emptyset)$	A	A
2	$m_1 = (Op, \mathsf{assert}, E_1, \emptyset, m_0)$	B	E_1
3	$m_2 = (Prop, \mathsf{and\text{-}so}, \emptyset, m_1, \emptyset)$	–	–
4	$m_3 = (Op, \mathsf{hence}, E_2, m_2, \emptyset)$	B	$E_1 \cup E_2$
5	$m_4 = (Prop, \mathsf{assert}, C', \emptyset, m_1)$	C	C'

Table 1. A *wfe*-dialogue d. See Fig. 2 for internal structure of enthymemes (see Def. 2), intended arguments of moves (see Def. 4) and resulting enthymemes (see Def. 6) during d.

an example of a *wfe*-dialogue d (see in Fig. 2 the internal structure of enthymemes and intended arguments of moves made in d together with a natural language translation).

A *dialogue framework* of a *wfe*-dialogue d in [9] is a 4-tuple $\mathrm{DF} = \langle \mathrm{M}, \mathrm{T}, \mathrm{Rep}, \mathrm{Sup} \rangle$ where M is the set of moves made in d, T is a binary defeat relationship between moves Rep is a binary reply relationship and Sup is a binary support relationship. When an agent provides a forward extension E' with a move m_i, they do so as a response to a move m which questions the content E of a move m_j. Thus, m_i *supports* m_j. Here, we exclude from M the stop moves made in d as they do not have any content, reply or target.

Definition 4. *A **well-formed enthymeme dialogue** (wfe-dialogue) $d = [m_0, \dots, m_\ell]$ between Prop and Op is a sequence of **moves** $d = [m_0, m_1, \dots, m_\ell]$ such that $m_i = \langle \mathtt{s}(m_i), \mathtt{l}(m_i), \mathtt{c}(m_i), \mathtt{re}(m_i), \mathtt{t}(m_i) \rangle$, where $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{and\text{-}so}, \mathsf{hence}, \mathsf{stop}\}$, $m_i \neq m_j$ for all $i \neq j$, and for all $i \leq \ell$:*

1. $\mathtt{s}(m_i) = Prop$ *if i is even, otherwise* $\mathtt{s}(m_i) = Op$;
2. *If $i = 0$, then* $\mathtt{l}(m_i) = \mathsf{assert}$, $\mathtt{c}(m_i) \in \mathrm{A}^*$, *and* $\mathtt{re}(m_i) = \mathtt{t}(m_i) = \emptyset$;
3. *If $0 < i$ and* $\mathtt{l}(m_i) = \mathsf{assert}$, *then* $\mathtt{c}(m_i)$ *is an upwards extendable enthymeme of an* $A \in \mathrm{A}^*$, $\mathtt{t}(m_i) = m_j$ *such that* $\mathtt{l}(m_j) = \mathsf{assert}$, *and* $\mathtt{re}(m_i) = \emptyset$ *where* $0 \leq j < i$;
4. *If* $\mathtt{l}(m_i) = \mathsf{and\text{-}so}$, *then* $\mathtt{re}(m_i) = m_j$, $\mathtt{l}(m_j) \in \{\mathsf{assert}, \mathsf{hence}\}$, *and* $\mathtt{t}(m_i) = \emptyset$ *where* $0 < j < i$;
5. *If* $\mathtt{l}(m_i) = \mathsf{hence}$, *then* $\mathtt{t}(m_i) = \emptyset$ *and there exist* m_j, m_k *such that* $\mathtt{re}(m_i) = m_j$, $\mathtt{l}(m_j) = \mathsf{and\text{-}so}$, $\mathtt{re}(m_j) = m_k$ *and* $\mathtt{c}(m_k) \ll_{fw} \mathtt{c}(m_i)$, *where* $0 < k < j < i$;
6. *If* $\mathtt{l}(m_i) = \mathsf{stop}$, *then* $\mathtt{c}(m_i) = \mathtt{re}(m_i) = \mathtt{t}(m_i) = \emptyset$.

If $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{hence}\}$, $\mathsf{IntArg}(m_i) \in \mathrm{A}^*$ *is the **intended argument** of m_i.*
*The **topic** of d is the label of the conclusion of the argument moved in m_0. If $\exists m_i, m_j$ ($0 \leq i < j \leq \ell$) such that $j = i+1$ and* $\mathtt{l}(m_i) = \mathtt{l}(m_j) = \mathsf{stop}$, *then $j = \ell$. We say that d is **terminated** iff* $\mathtt{l}(m_\ell) = \mathtt{l}(m_{\ell-1}) = \mathsf{stop}$.
We denote the set of all moves as M^*, the set of all wfe-dialogues as D^*, and when we refer to a wfe-dialogue $d_i = [m_0, \dots, m_i]$ and a move m_{i+1} such that $d_{i+1} = [m_0, \dots, m_i, m_{i+1}]$, then we imply that d_{i+1} extends *d_i by m_{i+1}.*

Definition 5. *The **dialogue framework** of a wfe-dialogue $d = [m_0, \dots, m_\ell]$ is a tuple $\mathrm{DF} = \langle \mathrm{M}, \mathrm{T}, \mathrm{Rep}, \mathrm{Sup} \rangle$ where:*

- $\mathrm{M} = \{m_i \mid i \leq \ell$ *and* $\mathtt{l}(m_i) \neq \mathsf{stop}\}$ *is the set of moves of d;*
- $\mathrm{T} \subseteq \mathrm{M} \times \mathrm{M}$ *is a binary* defeat *relation such that* $\mathtt{t}(m_i) = m_j$;
- $\mathrm{Rep} \subseteq \mathrm{M} \times \mathrm{M}$ *is a binary* reply *relation such that* $\mathtt{re}(m_i) = m_j$;
- $\mathrm{Sup} \subseteq \mathrm{M} \times \mathrm{M}$ *is a binary* support *relation such that* $(m_i, m_j) \in \mathrm{Sup}$ *iff* $\exists m \in \mathrm{M}$ *such that* $\mathtt{re}(m_i) = m$ *and* $\mathtt{re}(m) = m_j$.

We let $\sim_{\text{Sup}} \subseteq \text{M} \times \text{M}$ *be an equivalence relation on* M *defined as the reflexive, symmetric, transitive closure of* Sup*, where* $m_i \sim_{\text{Sup}} m_j$ *denotes that either* $m_i = m_j$ *or* $(m_i, m_j) \in \text{Sup}^+$ *or* $(m_j, m_i) \in \text{Sup}^+$*, where* Sup^+ *is the transitive closure of* Sup*.*

The *resulting enthymeme* of an assert move m in a *wfe*-dialogue d is an enthymeme $\mathsf{ResEnth}(m,d)$ which is the union of the contents of the supporting moves of m in d (see Table 1). Intuitively, every time an agent forward extends an enthymeme, a missing part of the intended argument of m is shown. Thus, a new enthymeme is constructed which is the union of the questioned enthymeme and its forward extension (recall also Corollary 1).

Definition 6. *Let* $d = [m_0, \ldots, m_\ell]$ *be a wfe-dialogue and* $\text{DF} = \langle \text{M}, \text{T}, \text{Rep}, \text{Sup} \rangle$ *be the dialogue framework of* d*. Let* m_i*, where* $0 \leq i \leq \ell$*, be a move such that* $\mathtt{l}(m_i) = \mathsf{assert}$*. The* ***resulting enthymeme of*** m_i ***in*** d *is* $\mathsf{ResEnth}(m_i, d) = \{\mathsf{c}(m_j) \mid m_j \sim_{\text{Sup}} m_i \text{ and } 0 \leq j \leq i\}$*.*

A *complete labelling* L_{DF} *on a dialogue framework* $\text{DF} = \langle \text{M}, \text{T}, \text{Rep}, \text{Sup} \rangle$ *of a wfe-dialogue d* is defined as follows: 1) the label of a move $m \in \text{M}$ is IN iff (a) for every move m' that targets or replies to m, m' is labelled OUT; and (b) if m supports a move m'' then m'' is labelled IN (intuitively, since m's enthymeme forward extends m'''s enthymeme, then any challenge on m'' is a challenge on m, and so must be IN in order for m to be IN); 2) $m \in \text{M}$ is labelled OUT iff there is a move m' that targets or replies to m such that m' is labelled IN; or there is a move m'' that m supports (i.e. m's enthymeme forward extends m'''s enthymeme) and m'' is labelled OUT.

Definition 7. *Let* $\text{DF} = \langle \text{M}, \text{T}, \text{Rep}, \text{Sup} \rangle$ *be the dialogue framework of a wfe-dialogue* d*. We define a* ***complete labelling*** *on* DF *to be a (total) function* $\mathsf{L} : \text{M} \to \{\text{IN}, \text{OUT}, \text{UNDEC}\}$ *such that for every* $m_i \in \text{M}$*:*

1. $\mathsf{L}(m_i) = \text{IN}$ *iff for all* $m_j \in \text{M}$*:*
   (a) *if* $(m_j, m_i) \in \text{T} \cup \text{Rep}$ *then* $\mathsf{L}(m_j) = \text{OUT}$*, and*
   (b) *if* $(m_i, m_j) \in \text{Sup}$ *then* $\mathsf{L}(m_j) = \text{IN}$*;*
2. $\mathsf{L}(m_i) = \text{OUT}$ *iff there is some* $m_j, m_k \in \text{M}$ *such that:*
   (a) *if* $(m_j, m_i) \in \text{T} \cup \text{Rep}$ *then* $\mathsf{L}(m_j) = \text{IN}$*, or*
   (b) $(m_i, m_j) \in \text{Sup}$*, then* $\mathsf{L}(m_j) = \text{OUT}$*.*

As in [9], an *argumentation theory* AT_d *instantiated by a wfe-dialogue d* is a tuple $\text{AT}_d = \langle \text{AS}_d, \text{K}_d \rangle$, where *Lan*, $(\bar{\cdot})$, nom, and R_s of AS_d are those shared by the participants of d (as assumed in Definition 4), the *defeasible rules* of AS_d is the set of all the defeasible rules revealed during d and the *premises* in AT_d (i.e. K_d) is the set of the labels of the leaves of the enthymemes that have been moved during d with an assert move. The latter is because the content E of an assert move is an upwards extendable enthymeme and so its leaves are the premises of the argument A from which E was constructed (see Definition 2), unlike hence moves which contain forward extensions of enthymemes.

An *argumentation theory of an agent Ag instantiated by a wfe-dialogue d* is a tuple $\text{AT}_d^{Ag} = \langle \text{AS}_d^{Ag}, \text{K}_d^{Ag} \rangle$ such that the defeasible rules of AS_d^{Ag} is the set of defeasible rules revealed during d together with the defeasible rules that Ag knows, and K_d^{Ag} is the set of premises revealed during d together with the premises that Ag knows (all other elements remain the same as above).

Definition 8. *Let $d = [m_0, \ldots, m_\ell]$ be a wfe-dialogue between Prop and Op, where* $\mathrm{AS}_{Ag} = \langle Lan, (\bar{\cdot}), R_{Ag}, \mathsf{nom} \rangle$ *is the argumentation system for* $Ag \in \{Prop, Op\}$ *and* $R_{Ag} = \mathrm{R_s} \cup \mathrm{R}^{Ag}_{def}$*. Then the following hold:*

- *The* **set of defeasible rules** *and* **premises in** d *are defined as* $\mathsf{DefDRules}(d) = \{\mathsf{lab}_{\mathsf{c}(m_i)}(n) \in \mathrm{R}_{def} \mid \mathsf{c}(m_i)$ *is an enthymeme and* $n \in \mathsf{Nd}(\mathsf{c}(m_i)), 0 \leq i \leq \ell\}$ *and* $\mathsf{DPrem}(d) = \{\mathsf{lab}_{\mathsf{c}(m_i)}(n) \in Lan \mid \mathtt{l}(m_i) = \mathsf{assert} \wedge n \in \mathsf{Leaves}(\mathsf{c}(m_i)), 0 \leq i \leq \ell\}$, *respectively.*
- *The* **argumentation theory instantiated by dialogue** d *is* $AT_d = \langle AS_d, K_d \rangle$, *where* $AS_d = \langle Lan, (\bar{\cdot}), \mathrm{R_s} \cup \mathsf{DefDRules}(d), \mathsf{nom} \rangle$ *and* $K_d = \mathsf{DPrem}(d)$, *and the* **argumentation theory of** Ag **instantiated by** d *is* $AT^{Ag}_d = \langle AS^{Ag}_d, K^{Ag}_d \rangle$, *where* $AS_d = \langle Lan, (\bar{\cdot}), \mathrm{R_s} \cup \mathsf{DefDRules}(d) \cup \mathrm{R}^{Ag}_{def}, \mathsf{nom} \rangle$ *and* $K_d = \mathsf{DPrem}(d) \cup K_{Ag}$.

4. Rational strategy

We define a *rational strategy (**RS**)* that two agents can employ to generate a *wfe*-dialogue d, specifying the move each participant needs to make based on previous moves made in d to ensure that it will terminate while preserving **SC**. We have chosen the characterisation 'rational ' because our goal is to achieve a rational outcome for d (based on the information exchanged during d). We also believe that our strategy is a sensible one to use when an agent cannot or does not want to make assumptions regarding what argument their interlocutor may intend when moving an enthymeme. This is because, if they are mistaken, false results may be achieved.

An **RS** for a participant Ag of a *wfe*-dialogue $d_i = [m_0, \ldots, m_i]$ given d_i, is a move m_{i+1} such that $d_{i+1} = [m_0, \ldots, m_i, m_{i+1}]$ is also a *wfe*-dialogue, i.e. it preserves the conditions set in Definition 4. When behaving reasonably, a rational expectation is that Ag moves an enthymeme E that is part of an intended argument A which is instantiated based on their private knowledge and the information exchanged thus far in the dialogue.

Before moving an enthymeme, Ag needs to make sure that no misunderstandings will occur during the dialogue if they are to achieve a rational outcome. Thus, if the last move in d_i is a move m_i whose content E_i is part of a resulting enthymeme E_j of a move m_j that does not defeat the resulting enthymeme E_k of its target m_k, then Ag needs to ask the sender Ag' of m_i, using an $\mathsf{and\text{-}so}$ move, for a forward extension of E_i. Then, Ag' must immediately make a hence move m_k to respond to Ag, considering the same intended argument they had in mind when E_j was moved. Notice that m_j might be the same as m_i.

After Ag' has revealed their intended argument verifying a defeat relationship between the resulting enthymemes E_j and E_k of m_j and m_k, respectively, they need to make certain that: if there is an argument A, instantiated based on the knowledge exchanged thus far in the dialogue, which defeats the resulting enthymeme E_j of a move m_j formed in d, then A is the intended argument of Ag's next move m_{i+1} and the target of m_{i+1} is m_j. If such an A does not exist, but Ag can instantiate an argument B that defeats the resulting enthymeme C of a previous move m_k in d then Ag can move an enthymeme with m_{i+1} targeting m_k, otherwise they can use the stop move to indicate their intent to finish the dialogue. Recall, by Definition 6, resulting enthymemes are defined only for assert moves, whereas by Definition 4 only assert moves have targets which are other assert moves.

Definition 9. *Let $d_i = [m_0, \ldots, m_i]$ be a wfe-dialogue between Prop and Op for $0 \leq i$ and $d_{-1} = \emptyset$. A **rational strategy** for a participant $Ag \in \{Prop, Op\}$ in d_i (denoted as $\mathsf{strg}_{Ag}(d_i)$) is a move m_{i+1} such that $d_{i+1} = [m_0, \ldots, m_i, m_{i+1}]$ is a wfe-dialogue and:*

1. *for every $-1 \leq i$, $\mathsf{strg}_{Ag}(d_i) = m_{i+1}$ such that if $\mathtt{c}(m_{i+1}) \in \mathrm{E}^*$, then $\mathtt{c}(m_i) \leq \mathsf{IntArg}(m_i) \in \mathrm{Args}_{\mathrm{AT}^{Ag}_{d_i}}$ (for $i = -1$, $\mathrm{Args}_{\mathrm{AT}^{Ag}}$);*
2. *if $0 \leq i$, then:*
   (a) *if $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{hence}\}$, $m_j \sim_{\mathrm{Sup}} m_i$ and $\mathsf{ResEnth}(m_j, d_i)$ does not defeat $\mathsf{ResEnth}(\mathtt{t}(m_j), d_i)$, then $\mathtt{l}(m_{i+1}) = \mathsf{and\text{-}so}$ and $\mathtt{re}(m_{i+1}) = m_i$;*
   (b) *else if (a) is not the case and $\mathtt{l}(m_i) = \mathsf{and\text{-}so}$, then $\mathsf{IntArg}(m_{i+1}) = \mathsf{IntArg}(\mathtt{re}(m_i))$, $\mathtt{l}(m_{i+1}) = \mathsf{hence}$ and $\mathtt{re}(m_{i+1}) = m_i$;*
   (c) *else if (a)-(b) do not hold and there exists $A \in \mathrm{Args}_{\mathrm{AT}_{d_i}}$ such that A defeats $\mathsf{ResEnth}(m_j, d_i) \in \mathrm{A}^*$, then $\mathsf{IntArg}(m_{i+1}) = A$ and $\mathtt{t}(m_{i+1}) = m_j$;*
   (d) *else if (a)-(c) do not hold and there is a move m_j in d_i where $\mathtt{l}(m_j) = \mathsf{assert}$ and $\mathsf{ResEnth}(m_j, d_i) \in \mathrm{A}^*$, then either: (i) $\mathtt{l}(m_{i+1}) = \mathsf{assert}$, $\mathtt{t}(m_{i+1}) = m_j$ and $\mathsf{IntArg}(m_{i+1})$ defeats $\mathsf{ResEnth}(m_j, d_i)$, or (ii) $\mathtt{l}(m_{i+1}) = \mathsf{stop}$.*

Example 1. *Considering the wfe-dialogue in Table 1, m_0 is an **RS** for Prop in d_{-1} since $\mathtt{c}(m_0) = \mathsf{IntArg}(m_0)$. Assuming that agents can indeed instantiate their intended arguments and $A \prec B$ and $B' \prec C$, then every move in d follows Definition 9.1 in d_{i-1}, $0 \leq i \leq 4$. Moreover, m_1 is also an **RS** for Op in d_0 (since $\mathsf{ResEnth}(m_0, d_0) = A \in \mathrm{A}^*$, $\mathtt{t}(m_1) = m_0$, and $\mathsf{IntArg}(m_1) = B$ defeats A, following Definition 9.2(d)), however m_4 is not an **RS** for Prop in d_3 as $\mathtt{l}(m_4) = \mathsf{assert}$ instead of $\mathsf{and\text{-}so}$ (thus not following Definition 9.2(a)). We also have that m_2 is an **RS** for Op in d_1 (as E_1 does not defeat A, $\mathtt{l}(m_1) = \mathsf{assert}$, $\mathtt{l}(m_2) = \mathsf{and\text{-}so}$ and $\mathtt{re}(m_2) = m_1$ and so following Definition 9.2(a)). Finally, m_3 is an **RS** for Prop in d_2 (as $\mathtt{l}(m_2) = \mathsf{and\text{-}so}$, $\mathtt{l}(m_3) = \mathsf{hence}$, $\mathtt{re}(m_3) = m_2$ and $\mathsf{IntArg}(m_1) = \mathsf{IntArg}(m_3) = B$ and as such it follows Definition 9.2(b)).*

5. Properties of a *wfe*-dialogue generated using rational strategies

We argue that when agents use **RS**s to conduct a *wfe*-dialogue then this will terminate and **SC** will be preserved.

Suppose that *Prop* and *Op* start a *wfe*-dialogue *d* where every move is an **RS** for the sender of the move. This means that *Prop* needs to start d_{-1} with a move m_0 whose content is the intended argument of m_0, as the topic of a *wfe*-dialogue is the claim of the content of m_0, by Definition 4. This also preserves Definition 9.1. The simplest scenario is for *Op* to continue by making a stop move (by Definition 9.2(d)), where *Prop* terminates the dialogue by repeating stop. Thus, there must be at least two moves in *d* before it terminates. If *Op* chooses to move an enthymeme against *Prop*'s argument (instead of stop) then the participants will keep making moves following Definition 9 until they have no other moves to make except for using consecutive stop moves (as an **RS** dictates). Thus, every time they make a move, both participants prioritise questioning an enthymeme that does not defeat a previous resulting enthymeme, over responding to a question, over moving any argument they can instantiate using solely information revealed during the dialogue to defeat another argument, over moving an enthymeme against the resulting enthymeme of a move or using stop.

Using only **RS**s, participants first ensure that the intended argument of an asserted enthymeme of their counterpart is revealed (by keep asking and responding to requests for forward extensions eventually the claim of the original intended argument of the challenged move will be reached) and then they move an enthymeme against it. This also means that intended arguments of moves will be the correct ones and by Definition 9.2(d) a defeat relationship will indeed exist between them. Therefore, if there is a targeting relationship between two moves in the DF then there is also a defeat relationship between their intended arguments in the AF instantiated by d.

Definitions 9.2(c) imposes that if a participant can move an argument A, instantiated using elements revealed during the dialogue, that defeats the resulting enthymeme B of a move m_i in the dialogue, then they do so by having A as the intended argument of their move m_j that targets m_i. Therefore, every such argument A will be the intended argument of a move m' targeting m in the dialogue. The *wfe*-dialogue in Table 2 has been instantiated using only **RS**s, preserving all the properties discussed.

Proposition 1. *Let d_{-1} be the empty wfe-dialogue between Prop and Op. Let* $\mathrm{DF}_{d_x} = \langle \mathrm{M}_x, \mathrm{T}_x, \mathrm{Rep}_x, \mathrm{Sup}_x \rangle$ *and* $\mathrm{AF}_{d_x} = \left\langle \mathrm{Args}_{\mathrm{AT}_{d_x}}, \mathrm{Dfs}_x \right\rangle$ *be the dialogue framework and the argument framework instantiated by the argumentation theory* AT_{d_x} *of a* $d_x \in \mathrm{D}^*$*, respectively. Then,* $\forall d_\ell \in \mathrm{D}^*$ *between Prop and Op it holds that if* $\forall m_i$*, where* $0 \leq i \leq \ell$*,* $m_i = \mathsf{strg}_{Ag}(d_{i-1})$ *is an **RS** for Ag in* d_{i-1}*, then* $\exists k$ *such that* $2 \leq k \leq \ell$ *and:*

I. $d_k \in \mathrm{D}^*$ *is terminated;*
II. *if* $\mathtt{l}(m_i) = \mathsf{assert}$*, then* $\mathsf{ResEnth}(m_i, d_k) = \mathsf{IntArg}(m_i)$*;*
III. *If there are moves* $m_j, m_h \in \mathrm{M}$ *such that* $(m_j, m_h) \in \mathrm{T}$ *then it holds that* $(\mathsf{IntArg}(m_j), \mathsf{IntArg}(m_h)) \in \mathrm{Dfs}$ *where* $m_j, m_h \in \mathrm{M}_k$*;*
IV. $\forall A, B \in \mathrm{Args}_{\mathrm{AT}_{d_k}}$ *such that* $\mathsf{ResEnth}(m_i, d_k) = A$ *and B defeats A,* $\exists m_j, m_h$ *where* $0 < i \leq h < j \leq k$*,* $\mathsf{IntArg}(m_j) = B$*,* $m_i \sim_{\mathrm{Sup}} m_h$ *and* $\mathtt{t}(m_j) = m_h$*.*

Proof. According to Definition 4, $d_{-1} = \emptyset$. By Definition 4.1 we have that $\mathtt{s}(m_i) = Prop$ if i is even, otherwise $\mathtt{s}(m_i) = Op$ for every m_i ($0 \leq i < \ell$). Additionally, by Definition 9, we have that for every m_i ($0 \leq i$) such that $m_i = \mathsf{strg}_{Ag}(d_{i-1})$, d_i will be a *wfe*-dialogue. Since *Prop* and *Op* will be using **RS**s to generate a *wfe*-dialogue d_ℓ ($0 \leq \ell$), *Prop* and *Op* do not have an **RS** when a *wfe*-dialogue has an odd and an even number of moves, respectively. Based on the above we have the following:

1. if $i = 0$, then by Definitions 4.2, and 9.1, $d_0 = [m_0]$ where $m_0 = \mathsf{strg}_{Prop}(d_{-1})$ is an **RS** for *Prop* in d_{-1} such that $m_0 = \langle Prop, \mathsf{assert}, \mathsf{IntArg}(m_0), \emptyset, \emptyset \rangle$ and $\mathsf{IntArg}(m_0) \in \mathrm{Args}_{\mathrm{AT}^{Ag}}$. By Definition 6, $\mathsf{ResEnth}(m_0, d_0) = \mathsf{IntArg}(m_0)$;
2. if $i = 1$, then by Definitions 9.1 and 9.2(d) we have that $d_1 = [m_0, m_1]$ where $m_1 = \mathsf{strg}_{Op}(d_0)$ is an **RS** for *Op* in d_0 such that $\mathtt{l}(m_1) \in \{\mathsf{assert}, \mathsf{stop}\}$ and:
   (a) if $\mathtt{l}(m_1) = \mathsf{assert}$, then by Definition 4.3 $m_1 = \langle Op, \mathsf{assert}, E_1, \emptyset, m_0 \rangle$ such that E_1 is an upwards extendable enthymeme of $\mathsf{IntArg}(m_1)$, and $\mathsf{IntArg}(m_1)$ defeats $\mathtt{c}(m_0) = \mathsf{ResEnth}(m_0, d_0) = \mathsf{IntArg}(m_0)$. Moreover, by Definition 6, $\mathsf{ResEnth}(m_1, d_1) = E_1$. We have two cases:
      i. either $E_1 = \mathsf{IntArg}(m_1)$ (remember by Definition 2 an argument is an enthymeme of itself) and so $E_1 = \mathsf{ResEnth}(m_1, d_1)$ defeats $\mathtt{c}(m_0) = \mathsf{ResEnth}(m_0, d_1)$. In this case, we can proceed to the next move m_2 which

is an **RS** for *Prop* in d_1 and so, by Definition 9, 2(d) holds (recall we have assumed a unique preference order which is a strict total order);

ii. or $E_1 \neq \mathsf{IntArg}(m_1)$ and so $E_1 = \mathsf{ResEnth}(m_1, d_1)$ does not defeat $\mathsf{c}(m_0) = \mathsf{ResEnth}(m_0, d_1)$ as $\mathsf{Conc}(\mathsf{IntArg}(m_1)) \notin \mathsf{Roots}(E_1)$ (as explained in Section 2, a defeat from an argument X to an argument Y depends on X's claim conflicting with an ordinary premise or the consequent or name of a defeasible rule in Y). Since m_2 is an **RS** for *Prop* in d_1, by Definition 9, 2(a) holds and so, by Definition 4.4, $m_2 = \langle Prop, \mathsf{and\text{-}so}, \emptyset, m_1, \emptyset\rangle$ and $d_2 = [m_0, m_1, m_2]$, where by Definition 4 $\mathsf{IntArg}(m_2) = \emptyset$. Since m_3 is an **RS** for *Op* in d_2, by Definition 9, 3(b) holds and so, by Definition 4.5,$m_3 = \langle Op, \mathsf{hence}, E_3, m_2, \emptyset\rangle$ such that $E_1 \ll_{fw} E_3$, $\mathsf{IntArg}(m_3) = \mathsf{IntArg}(m_1)$, and $d_3 = [m_0, m_1, m_2, m_3]$. In this case, by Definitions 5 and 6, $(m_3, m_1) \in$ Sup, $\mathsf{ResEnth}(m_1, d_3) = E_1 \cup E_3$ and by Corollary 1, $\mathsf{ResEnth}(m_1, d_3)$ is an upwards extendable enthymeme of $\mathsf{IntArg}(m_1)$. Now, if $\mathsf{Conc}(\mathsf{IntArg}(m_1)) \in \mathsf{Roots}(\mathsf{ResEnth}(m_1, d_3))$ then $\mathsf{ResEnth}(m_1, d_3) = \mathsf{IntArg}(m_1)$ and so $\mathsf{ResEnth}(m_1, d_3)$ defeats $\mathsf{c}(m_0) = \mathsf{ResEnth}(m_0, d_3)$. In this case, as in 2.(a).i above, we can proceed to the next move m_4 which is an **RS** for *Prop* in d_3 following Definition 9.2(c) or 9.2(d). Otherwise, the process explained earlier is repeated until $\mathsf{ResEnth}(m_1, d_j) = E_1 \cup E_3 \cup \ldots \cup E_j = \mathsf{IntArg}(m_1)$ which defeats $\mathsf{c}(m_0) = \mathsf{ResEnth}(m_0, d_j)$, where $1 < j$ and j is odd. Notice that this holds because, by Definitions 4 and 5, $(m_j, m_{j-2}), \ldots, (m_3, m_1) \in$ Sup where $\mathtt{l}(m_j) = \mathtt{l}(m_{j-2}) = \ldots = \mathtt{l}(m_3) = \mathsf{hence}$ and $\mathtt{l}(m_1) = \mathsf{assert}$ and so by Definition 9.2(b), $\mathsf{IntArg}(m_j) = \mathsf{IntArg}(m_{j-2}) = \mathsf{IntArg}(m_3) = \mathsf{IntArg}(m_1)$. As before, *Prop* is ready to make a move which is an **RS** for *Prop* in d_j following Definition 9.2(c) or 9.2(d).

(b) if $\mathtt{l}(m_1) = \mathsf{stop}$, by Definitions 4.6 and 9.2(d), we have that $m_1 = \mathsf{strg}_{Op}(d_0)$ such that $m_1 = \langle Op, \mathsf{stop}, \emptyset, \emptyset, \emptyset\rangle$. Following Definition 9.2, only (d) holds and so $m_2 = \mathsf{strg}_{Prop}(d_1)$ such that:

i. If m_2's locution is stop then we have that $d_2 = [m_0, m_1, m_2]$ where $m_2 = \langle Prop, \mathsf{stop}, \emptyset, \emptyset, \emptyset\rangle$. and there is no $A \in \mathsf{Args}_{\mathrm{AT}_{d_\ell}}$ such that A defeats $\mathsf{IntArg}(m_0)$. Therefore I, II, III and IV hold for $k = 2$;

ii. If m_2's locutions is assert then we have that $d_2 = [m_0, m_1, m_2]$ where by Definitions 4.3, 9.1 and 9.2(d), $m_2 = \langle Prop, \mathsf{assert}, E_2, \emptyset, m_0\rangle$ such that E_2 is an upwards extendable enthymeme of $\mathsf{IntArg}(m_2)$ and $\mathsf{IntArg}(m_2)$ defeats $\mathsf{c}(m_0)$. Additionally, by Definition 6, $\mathsf{ResEnth}(m_0, d_2) = \mathsf{c}(m_0) = \mathsf{IntArg}(m_0)$ and $\mathsf{ResEnth}(m_2, d_2) = E_2$. In this case, we follow the same process as demonstrated above in 2.(a).i and 2.(a).ii (only this time *Op* makes and-so moves and *Prop* replies to them with hence moves) until $\mathsf{ResEnth}(m_2, d_j) = E_2 \cup \ldots \cup E_j = \mathsf{IntArg}(m_2)$ which defeats $\mathsf{c}(m_0) = \mathsf{ResEnth}(m_0, d_j)$, where $1 < j$ and j is even, and *Op* is ready to make a move which is an **RS** for *Op* in d_j following Definition 9.2(c) or 9.3(d).

3. if $1 < i$ and d_i is not terminated, then it means that we continue either from case 2.(a).i or 2.(a).ii or 2.(b).ii. Notice that in all three cases two intended arguments have been revealed during the dialogue. We will assume that 2.(a).ii holds, i.e. we have that $d_j = [m_0, m_1, \ldots, m_j]$, where $1 < j$ and j is odd, $\mathsf{ResEnth}(m_1, d_j) =$

$E_1 \cup \ldots \cup E_j = \mathsf{IntArg}(m_1)$ which defeats $\mathsf{c}(m_0) = \mathsf{ResEnth}(m_0, d_j)$ and *Prop* is ready to make a move which is an **RS** for *Prop* in d_j following Definition 9.3(c) or 9.3(d). However, what we discuss holds for the other two cases as well since in 2.(a).i the only difference is that $j = 1$, and in 2.(b).ii m_1 was an **RS** for *Op* in d_0 following Definition 9.3(d). where m_1's locution was stop and m_2 was an **RS** for *Prop* in d_1, following Definition 9.3(d) where m_2's locution was assert, against their own move m_0, thus $1 < j$ and j is even and *Op* is ready to make a move which is an **RS** for *Op* in d_j following Definition 9.2(c) or 9.2(d). Since m_{j+1} is an **RS** for *Prop* in d_j, by Definition 9.3(c) if $\exists A \in \mathrm{Args}_{\mathrm{AT}_{d_j}}$ such that A defeats $\mathsf{ResEnth}(m_h, d_j)$ where $0 < h \leq j$, then $m_{j+1} = \langle Prop, \mathsf{assert}, E_{j+1}, \emptyset, m_h \rangle$ and $\mathsf{IntArg}(m_{j+1}) = A$. Similarly to the beginning of 2.(a) above, either E_{j+1} is an upwards extendable enthymeme of A or $E_{j+1} = A$ where if the former holds then we repeat the same steps as in 2.(a).ii shown earlier until the whole A is revealed. The above procedure continues alternatively for the participants of the dialogue until neither can instantiate another intended argument that defeats a previous argument revealed in the dialogue (or does not want to in case all arguments that could be instantiated using the elements of the dialogue have been moved). Since both $\mathrm{Args}_{\mathrm{AT}^{Prop}}$ and $\mathrm{Args}_{\mathrm{AT}^{Op}}$ are finite, then the set of arguments instantiated by combining the argumentation theories of the agents will also be finite. Thus, there will be a point where both *Prop* and *Op* will need to use an **RS** following Definition 9.2(d)., i.e. a stop move will appear consecutively. In other words, $d_k = [m_0, m_1, \ldots, m_{j-1}, m_j, m_{j+1}, \ldots, m_{k-1}, m_k]$ such that $m_{k-1} = \mathsf{strg}_X(d_{k-2})$ and $m_k = \mathsf{strg}_Y(d_{k-1})$ are **RS**s for X and Y in d_{k-2} and d_{k-1}, respectively, and $\mathtt{l}(m_{k-1}) = \mathtt{l}(m_k) = \mathsf{stop}$ where $X = Prop$ and $Y = Op$ if k is odd, and vice versa if k is even. Therefore, by Definition 4, d_k is terminated, i.e. I holds in all cases. Additionally, we have shown in all cases that before making an assert move, a participant *Ag* makes sure that the intended argument A of the move it targets has been revealed before they assert an enthymeme. Thus, the resulting enthymeme of every assert move in the dialogue is the intended argument of this move, so II also holds in every case. Furthermore, since all the elements of intended arguments of all moves have been revealed, then these intended arguments can be instantiated in the AF_{d_k} instantiated by AT_{d_k} (by Definition 8). Moreover, by Definition 9.2(d), an agent moves an enthymeme E with a move m_j against another enthymeme E' of a move m_h only when the intended argument of m does indeed defeat the resulting enthymeme of m' in d_{j-1}. Therefore, we can infer that the intended arguments of moves, which have a targeting relationship between them, have a corresponding defeat relationship between them, which means that III holds. Lastly, we have also shown that IV holds in all cases as by Definition 9.2(c) if an agent can instantiate an argument B using elements revealed during the dialogue which defeats a previously revealed argument A, then an agent has to make an assert move, whose intended argument is B, against the assert move whose resulting enthymeme is A before they move a stop. To conclude, our Proposition holds.

□

Step	**wfe-dialogue** $d = [m_0, \ldots, m_10]$	**IntArg**(m_i)	**ResEnth**(m_i, d_i)
1	$m_0 = (Prop, \mathsf{assert}, A, \emptyset, \emptyset)$	A	A
2	$m_1 = (Op, \mathsf{assert}, E_1, \emptyset, m_0)$	B	E_1
3	$m_2 = (Prop, \mathsf{and\text{-}so}, \emptyset, m_1, \emptyset)$	–	–
4	$m_3 = (Op, \mathsf{hence}, E_2, m_2, \emptyset)$	B	$E_1 \cup E_2$
5	$m_4 = (Prop, \mathsf{and\text{-}so}, \emptyset, m_3, \emptyset)$	–	–
6	$m_5 = (Op, \mathsf{hence}, E_3, m_3, \emptyset)$	B	$E_1 \cup E_2 \cup E_3$
7	$m_6 = (Prop, \mathsf{assert}, C', \emptyset, m_1)$	C	C'
8	$m_7 = (Op, \mathsf{and\text{-}so}, \emptyset, m_6, \emptyset)$	–	–
9	$m_8 = (Prop, \mathsf{hence}, C, m_3, \emptyset)$	C	$C \cup C'$
10	$m_9 = (Op, \mathsf{stop}, \emptyset, \emptyset, \emptyset)$	–	–
11	$m_{10} = (Prop, \mathsf{stop}, \emptyset, \emptyset, \emptyset)$	–	–

Table 2. A *wfe*-dialogue d that has been instantiated using rational strategies. Internal structure of enthymemes and intended arguments of moves are shown in Fig. 2
.

Following the above properties, we can conclude that for every *wfe*-dialogue d between two participants it holds that if the moves of d are the result of **RS**s and d is terminated then the dialectical status of the moves in the DF (determined by a complete labelling) is sound and complete in relation to the dialectical status of the arguments in the AF instantiated by the contents of the moves made in the dialogue (determined by a complete labelling). In Fig. 3 we show the dialogue and argument framework DF and AF instantiated in d given in Table 2, respectively. As observed, the status of a move m in DF whose intended argument A is not $\emptyset$ is the same as the status of A in AF.

Proposition 2. *Let $d = [m_0, \ldots, m_\ell]$ be a wfe-dialogue between participants Prop and Op where every move m_i is an **RS** for $\mathsf{s}(m_i)$ in d_{i-1} $\forall 0 \leq i \leq \ell$ and d is terminated. Let* $\mathrm{DF} = \langle \mathrm{M}, \mathrm{T}, \mathrm{Rep}, \mathrm{Sup} \rangle$ *be the dialogue framework of d, and* $\mathrm{AF} = \left\langle \mathrm{Args}_{\mathrm{AT}_d}, \mathrm{Dfs} \right\rangle$ *the argument framework instantiated by the argumentation theory* AT_d*. It follows that:*

I. For every complete labelling function L_{AF} *on* AF*, there exists a complete labelling function* L_{DF} *on* DF *such that for every* $m_i \in \mathrm{M}$*, if* $\mathsf{c}(m_i) \in \mathrm{E}^*$ *then:* $\mathsf{L}_{\mathrm{DF}}(m_i) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i))$*; and*

II. For every complete labelling function L_{DF} *on* DF*, there exists a complete labelling function* L_{AF} *on* AF *such that for every* $A \in \mathrm{Args}_{\mathrm{AT}_d}$*, if there is some* $m_i \in \mathrm{M}$ *such that* $A = \mathsf{IntArg}(m_i)$ *then:* $\mathsf{L}_{\mathrm{AF}}(A) = \mathsf{L}_{\mathrm{DF}}(m_i)$.

Proof. I. Let L_{AF} be a complete labelling on AF as usually defined such that:

$$\mathsf{L}_{\mathrm{AF}}(A) = \begin{cases} \textsc{in} & \text{if } \forall B \in \mathrm{Args}_{\mathrm{AT}_d};\ (B,A) \in \mathrm{Dfs} \text{ implies } \mathsf{L}_{\mathrm{AF}}(B) = \textsc{out}, \\ \textsc{out} & \text{if } \exists B \in \mathrm{Args}_{\mathrm{AT}_d};\ (B,A) \in \mathrm{Dfs} \text{ and } \mathsf{L}_{\mathrm{AF}}(B) = \textsc{in}, \\ \textsc{undec} & \text{otherwise.} \end{cases} \tag{1}$$

and define a new function L_{DF} on DF such that for all $m_i \in \mathrm{M}$,

$$\mathsf{L}_{\mathrm{DF}}(m_i) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) \tag{2}$$

if $\mathsf{IntArg}(m_i) \neq \emptyset$ and the usual

$$\mathsf{L}_{\mathrm{DF}}(m_i) = \begin{cases} \textsc{in} & \text{if } \forall m_j \in \mathrm{M};\ (m_j, m_i) \in \mathrm{Rep} \text{ implies } \mathsf{L}_{\mathrm{DF}}(m_j) = \textsc{out}, \\ \textsc{out} & \text{if } \exists m_j \in \mathrm{M};\ (m_j, m_i) \in \mathrm{Rep} \text{ and } \mathsf{L}_{\mathrm{DF}}(m_j) = \textsc{in}, \\ \textsc{undec} & \text{otherwise} \end{cases} \tag{3}$$

if $\mathsf{IntArg}(m_i) = \emptyset$, i.e. when $\mathtt{l}(m_i) = \mathsf{and\text{-}so}$.
We claim that L_{DF} is a complete labelling on DF, so suppose $m_i \in \mathrm{M}$:

Case 1) Suppose that $\mathsf{L}_{\mathrm{DF}}(m_i) = \textsc{in}$.

(1) Suppose to the contrary that there is some $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{Rep}$ and $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \textsc{out}$. Based on Definitions 5 and 4, there are the following cases:

(a) Let $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{hence}\}$. So, $\mathtt{l}(m_j) = \mathsf{and\text{-}so}$. Hence, based on Definition 4, $\mathsf{IntArg}(m_j) = \emptyset$. Thus, by (3) there does not exist $m_k \in \mathrm{M}$ such that $(m_k, m_j) \in \mathrm{Rep}$ and $\mathsf{L}_{\mathrm{DF}}(m_k) = \textsc{in}$. Additionally, since every move m_h in d is an **RS**, it means that by Definition 9.2(b) there is some $m_k \in \mathrm{M}$ such that $(m_k, m_j) \in \mathrm{Rep}$. Based on Definitions 4, 5, 9.1 and 9.2(b) we have that $m_i \sim_{\mathrm{Sup}} m_k$ and $\mathsf{IntArg}(m_i) = \mathsf{IntArg}(m_k)$. Based on Definition 9.1, $\mathsf{IntArg}(m_k) \neq \emptyset$. Therefore, by (2) we have that $\mathsf{L}_{\mathrm{DF}}(m_k) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_k)) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i) = \mathsf{L}_{\mathrm{DF}}(m_i) = \textsc{in}$, contrary to our assumption;

(b) Let $\mathtt{l}(m_i) = \mathsf{and\text{-}so}$. Hence, based on Definition 4, $\mathsf{IntArg}(m_i) = \emptyset$. Thus, by (3), for every $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{Rep}$ we have that $\mathsf{L}_{\mathrm{DF}}(m_j) = \textsc{out}$ contrary to our assumption.

(2) Suppose to the contrary that there is some $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{T}$ and $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \textsc{out}$. By Definitions 4and 5, $\mathtt{l}(m_i) = \mathtt{l}(m_j) = \mathsf{assert}$. Since every move m_h in d is an **RS**, it means that by Definitions 4, and 9.1, $\mathsf{IntArg}(m_i) \neq \emptyset$ and $\mathsf{IntArg}(m_j) \neq \emptyset$. According to (2), it follows that $\mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) = \textsc{in}$ and $\mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_j)) \neq \textsc{out}$. By Definition 9.1 and Proposition 1.III, we have that $(\mathsf{IntArg}(m_j), \mathsf{IntArg}(m_i)) \in \mathrm{Dfs}$ in AF. Since $\mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_j)) \neq \textsc{out}$, by 1 we have that $\mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) \neq \textsc{in}$ contrary to (2).

(3) Suppose to the contrary that there is some $m_j \in \mathrm{M}$ such that $(m_i, m_j) \in \mathrm{Sup}$ and $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \textsc{in}$. By Definitions 5 and 4, $\mathtt{l}(m_i) = \mathsf{hence}$. Following (1.a.) from before we reach a contradiction.

Conversely, suppose that $\mathsf{L}_{\mathrm{DF}}(m_i) \neq \textsc{in}$. If $\mathsf{IntArg}(m_i) = \emptyset$ then by (3), there is some $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{Rep}$ and $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \textsc{out}$, as required. Else, by (2), $\mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) \neq \textsc{in}$. According to (1), there is some $A \in \mathrm{Args}_{\mathrm{AT}_d}$ such that $(A, \mathsf{IntArg}(m_i)) \in \mathrm{Dfs}$ and $\mathsf{L}_{\mathrm{AF}}(A) \neq \textsc{out}$. By Definition 4, $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{hence}\}$. So there are the following cases:

(1) If $\mathtt{l}(m_i) = \mathsf{assert}$ then by Proposition 1.IV there exist $m_j \in \mathrm{M}$ such that $\mathtt{l}(m_j) = \mathsf{assert}$, $\mathsf{IntArg}(m_j) = \mathsf{ResEnth}(m_j, d) = A$, and $(m_j, m_i) \in \mathrm{T}$. Thus, by (2) $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \textsc{out}$. By Definition 7, it follows that for a move $m \in \mathrm{M}$ we have that $\mathsf{L}_{\mathrm{DF}}(m) \neq \textsc{in}$ iff either:

(a) $(m', m) \in \mathrm{Rep}$ and $\mathsf{L}_{\mathrm{DF}}(m') \neq \textsc{out}$;

(b) $(m', m) \in \mathrm{T}$ and $\mathsf{L}_{\mathrm{DF}}(m') \neq \mathrm{OUT}$;
(c) or $(m, m') \in \mathrm{Sup}$ and $\mathsf{L}_{\mathrm{DF}}(m') \neq \mathrm{IN}$;

So, (1.b) holds as required.

(2) If $\mathtt{l}(m_i) =$ hence by Definitions 4 and 5 there exists a unique move $m_j \in \mathrm{M}$ such that $\mathtt{l}(m_j) = \mathsf{assert}$ and $m_i \sim_{\mathrm{Sup}} m_j$. By Definitions 4, 5, 9.1 and 9.2(b), $\mathsf{IntArg}(m_i) = \mathsf{IntArg}(m_j)$. So (and following 1) $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \mathrm{IN}$. Based on Definitions 4, 5 and 6, there exists $m_k \in \mathsf{ResEnth}(m_i, d)$ such that (1.c) holds as required, i.e. $(m_i, m_k) \in \mathrm{Sup}$ and $\mathsf{L}_{\mathrm{DF}}(m_k) \neq \mathrm{IN}$.

Case 2) This case is analogous to Case 1 and is proved similarly.

II. Let L_{DF} be a complete labelling on DF. According to Definition 8 and Proposition 1.II, we have that $\mathsf{IntArg}(m_i) \in \mathrm{Args}_{\mathrm{AT}_d}$ for every $m_i \in \mathrm{M}$ such that $\mathsf{IntArg}(m_i) \neq \emptyset$, and so let $\mathrm{IntArgs} = \{\mathsf{IntArg}(m_i) \mid m_i \in \mathrm{M} \text{ and } \mathsf{IntArg}(m_i) \neq \emptyset\}$, and define a function L_{AF} on AF by taking

$$\mathsf{L}_{\mathrm{AF}}(A_i) = \mathsf{L}_{\mathrm{DF}}(m_i) \tag{4}$$

for all $A_i = \mathsf{IntArg}(m_i) \in \mathrm{IntArgs}$— since every move of the dialogue is an **RS** we have that for every move m_i, m_j of the dialogue if $\mathsf{IntArg}(m_i) = \mathsf{IntArg}(m_j)$ then $\mathsf{L}_{\mathrm{DF}}(m_i) = \mathsf{L}_{\mathrm{DF}}(m_j)$ and so this function is well-defined.
For every $A \notin \mathrm{IntArgs}$ we have the usual

$$\mathsf{L}_{\mathrm{AF}}(A) = \begin{cases} \mathrm{IN} & \text{if } \forall B \in \mathrm{Args}_{\mathrm{AT}_d};\ (B,A) \in \mathrm{Dfs} \text{ implies } \mathsf{L}_{\mathrm{AF}}(B) = \mathrm{OUT}, \\ \mathrm{OUT} & \text{if } \exists B \in \mathrm{Args}_{\mathrm{AT}_d};\ (B,A) \in \mathrm{Dfs} \text{ and } \mathsf{L}_{\mathrm{AF}}(B) = \mathrm{IN}, \\ \mathrm{UNDEC} & \text{otherwise.} \end{cases} \tag{5}$$

We claim that L_{AF} is a complete labelling on AF, so suppose $A \in \mathrm{Args}_{\mathrm{AT}_d}$:

Case 1) Suppose that $\mathsf{L}_{\mathrm{AF}}(A) = \mathrm{IN}$ and suppose to the contrary that there is some $B \in \mathrm{Args}_{\mathrm{AT}_d}$ such that $(B,A) \in \mathrm{Dfs}$ and $\mathsf{L}_{\mathrm{AF}}(B) \neq \mathrm{OUT}$. It follows from (5) that $A \in \mathrm{IntArgs}$ so let $m_i \in \mathrm{M}$ be such that $A = \mathsf{IntArg}(m_i)$. By Proposition 1.IV, there are $m_j, m_k \in \mathrm{M}$ such that $\mathtt{l}(m_j) = \mathsf{assert}$, $B = \mathsf{IntArg}(m_j)$, $m_i \sim_{\mathrm{Sup}} m_k$ and $(m_j, m_k) \in \mathrm{T}$. It then follows from (4) and Definitions 4, 5, 9.1 and 9.2(b) that $\mathsf{L}_{\mathrm{DF}}(m_k) = \mathsf{L}_{\mathrm{DF}}(m_i) = \mathrm{IN}$ and $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \mathrm{OUT}$ contrary to Definition 7.
Conversely, suppose that $\mathsf{L}_{\mathrm{AF}}(A) \neq \mathrm{IN}$. If $A \notin \mathrm{IntArgs}$ then there is some $B \in \mathrm{Args}$ such that $(B,A) \in \mathrm{Dfs}$ and $\mathsf{L}_{\mathrm{AF}}(B) \neq \mathrm{OUT}$, as required. So suppose that $A = \mathsf{IntArg}(m_i)$ for some $m_i \in \mathrm{M}$.
By (4) we have that $\mathsf{L}_{\mathrm{DF}}(m_i) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) \neq \mathrm{IN}$. So there are the following cases:

(1) Suppose there is an $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{Rep}$ and that $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \mathrm{OUT}$. Since $A = \mathsf{IntArg}(m_i) \in \mathrm{IntArgs}$, we have that $A \neq \emptyset$. By Definition 4, $\mathtt{l}(m_i) \in \{\mathsf{assert}, \mathsf{hence}\}$. So, by Definitions 5 and 4, we have that $\mathtt{l}(m_j) = \mathsf{and\text{-}so}$. Additionally, by Definition 7 and since every move m_h in d is an **RS**, it means that by Definition 9.2(b) there is some $m_k \in \mathrm{M}$ such that $(m_k, m_j) \in \mathrm{Rep}$, $\mathtt{l}(m_k) = \mathsf{hence}$ and $\mathsf{L}_{\mathrm{DF}}(m_k) \neq \mathrm{IN}$. By Definitions 4, 5, 4, 5, 9.1 and 9.2(b), there exists $m_h \in \mathrm{M}$ such that $\mathtt{l}(m_h) = \mathsf{assert}$,

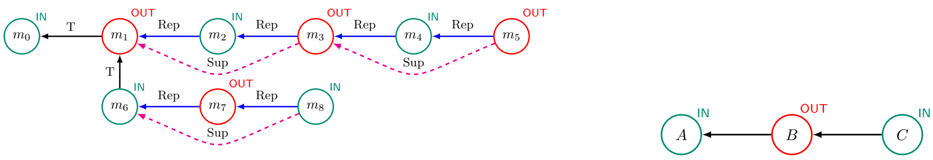
$m_h \sim_{\mathrm{Sup}} m_i \sim_{\mathrm{Sup}} m_k$ and $\mathsf{IntArg}(m_h) = \mathsf{IntArg}(m_i) = \mathsf{IntArg}(m_k)$. As a result (and also considering (4)), it follows that $\mathsf{L}_{\mathrm{DF}}(m_h) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_h))$ and $\mathsf{L}_{\mathrm{DF}}(m_i) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i))$ and $\mathsf{L}_{\mathrm{DF}}(m_k) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_k)) \neq \mathrm{IN}$, as required.

(2) Suppose there is an $m_j \in \mathrm{M}$ such that $(m_j, m_i) \in \mathrm{T}$ and $\mathsf{L}(m_j) \neq \mathrm{OUT}$. Hence, by Proposition 1.IV, there is some $B = \mathsf{IntArg}(m_j) \in \mathrm{A}_{\mathrm{AT}_d}$ such that $(B, A) \in \mathrm{Dfs}$ and, by (4), $\mathsf{L}_{\mathrm{AF}}(B) = \mathsf{L}_{\mathrm{DF}}(m_j) \neq \mathrm{OUT}$, as required.

(3) Suppose there is an $m_j \in \mathrm{M}$ such that $(m_i, m_j) \in \mathrm{Sup}$ and that $\mathsf{L}_{\mathrm{DF}}(m_j) \neq \mathrm{IN}$. By Definitions 4, 5, 9.1 and 9.2(b), $\mathsf{IntArg}(m_i) = \mathsf{IntArg}(m_j)$. Thus, and by (4), $\mathsf{L}_{\mathrm{DF}}(m_j) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_j) = \mathsf{L}_{\mathrm{AF}}(\mathsf{IntArg}(m_i)) = \mathsf{L}_{\mathrm{DF}}(m_i) \neq \mathrm{IN}$, as it is required.

Case 2) This case is analogous to Case 1 and is proved similarly.

□



(i) (ii)

Figure 3. Graph (i) shows the dialogue framework DF instantiated by the *wfe*-dialogue d from Table 2, with notations from the complete labelling L on DF. Graph (ii) shows the argument framework AF instantiated by AT_d with notations from the complete labelling L' on AF where the internal structure of arguments is shown in Fig. 2 (note we have omitted the sub-arguments of the intended arguments of moves in d as we focus on the intended arguments).

6. Conclusion and Discussion

In this paper, we address the limitations presented in [9], where a dialogue system is introduced that accounts for upwards extendable *enthymemes* (i.e. whose premises do not directly entail the claims of the arguments they support) and allows the participants to request and provide missing elements. Here we allow for a more relaxed version of a forward extension of an upwards extendable enthymeme, such that an agent can reveal the missing elements that yield the intended argument in parts instead of all at once, fitting better to real-world examples.

Additionally, although the work in [9] proves that it is possible to achieve soundness and completeness, under certain conditions, the authors do not show how this can be realised. This means that the participants of a dialogue can still be mistaken in the moves they make and **SC** results may not be accomplished. So, we also introduce a rational strategy that two agents can employ to generate a *wfe*-dialogue d accommodating forward extension of enthymemes, making sure that d will terminate while preserving **SC**. Moreover, we do so avoiding to consider arguments that an agent may assume for their interlocutor based on an enthymeme they receive from them. Instead, we build on knowledge revealed during the dialogue regarding this enthymeme. To the best of our knowledge, this is the first work that provides an explicit process for agents to start and conduct an argumentation-based dialogue accounting for forward extension of enthymemes, while also ensuring that when the dialogue finishes its outcome will be the logically correct one according to the information exchanged.

Nonetheless, our work comes with limitations which we plan to address in the future. Firstly, our proposed protocol and strategy need to be validated using real data. An implementation and an empirical assessment could help strengthening our results. Additionally, our current dialogue system concerns two parties, however it would be useful to investigate a multi-agent scenario such that more than two agents can communicate amongst them. Finally, this paper focuses on one type of enthymeme, namely upwards extendable enthymemes. Our target is to develop and implement similar strategies for other types of enthymemes that are not covered in this work.

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