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Title: Koopman Operator Based Fault Diagnostic Methods for Mechanical Systems

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Authors (names are for example only): Alexandru Nichifor
Yongzhi Qu

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SEND PAPER TO: **Professor Fu-Kuo Chang**
Organizing Chairman
Aeronautics and Astronautics Department
Stanford University
Durand Building
Stanford, California 94305, U.S.A.

Tel: 650/723-3466
Fax: 650/725-3377
E-mail: fkchang@leland.stanford.edu

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ABSTRACT

Traditionally, dynamical systems can be simulated with physics-based model when the design parameters and material property are pre-known. However, when a system is deployed in field and has suffered potential degradation, a physics-based model might be infeasible to obtain. Moreover, the non-linearity and unknown coupling between the system and contacting constraints are often hard to determine accurately. The analysis of those systems becomes practically problematic. In this paper, the Koopman operator is used to learn and represent a dynamic system in a data driven manner. This paper proposes two methods of using the Koopman operator to extract and classify critical parameters of a non-linear dynamic mechanical system for fault diagnosis. The first method proposes a model to extract key features from a dynamic system and feed the features to a neural network to classify the existence of a fault. The second method uses parameters derived from the Koopman operator to create a prediction model with healthy data. This prediction model is then used to predict future system dynamics for a measured time evolution and compare that with direct measurements when future dynamics become available. Both methods are then tested via an experimental case study and the results are discussed.

INTRODUCTION

Dynamical systems have been studied for hundreds of years starting with Henri Poincaré and his work on the chaotic motion of celestial bodies [1]. As the modeling of dynamical systems describe the world around us, it has become a focus of modern engineering. Recent efforts put towards the comprehension of modern dynamical systems include the understanding of the full evolution of physical systems, future state prediction, and optimizing system efficiency.

Alexandru Nichifor, Department of Mechanical and Industrial Engineering, University of Minnesota Duluth, 1305 Ordean Court, Duluth, Minnesota 55812, U.S.A..
Yongzhi Qu, Department of Mechanical and Industrial Engineering, University of Minnesota Duluth, 1305 Ordean Court, Duluth, Minnesota 55812, U.S.A..

Understanding how a system fully evolves over time is perhaps the underlying goal in recent studies of dynamical systems. Being able to parameterize any given system would be crucial to the analysis and solutions of governing equations. Accurately predicting dynamics has also been a goal of many studies in the past. For example, in metrology, future state predictions are being used regularly to predict the weather or natural phenomena [2]. However, much like in metrology, an issue occurs when attempting to predict the trajectories of a dynamical system too far ahead in time. The accuracy of these future state predictions is exponentially decreased over increasing time periods. The demand for system efficiency has long been regulated by the knowledge we have of the system itself.

In industrial settings, any machine downtime due to unexpected maintenance or emergency machine faults can be detrimental to a company's productivity and budget. Unforeseen degradation to a machine can be anticipated by Prognostic Health Management (PHM). PHM takes into consideration system diagnostics as well prognostic methods. The diagnostic process involves detecting and isolating faults of a system in real time. Prognostic methods are an extension of diagnosis by attempting to identify the Remaining Useful Life (RUL) of a system. Prognostics use prediction methods based on historical conditions of a system to predict a future state.

Two leading challenges when it comes to achieving the goals mentioned above are nonlinearity and unknown dynamics. A significant impediment when it comes to fully understanding a system is that the majority of systems in real world are non-linear. Once a nonlinearity is introduced into a dynamical system, the system itself becomes extremely complex to analyze. This is due to the fact that when a nonlinearity is present, the system output is no longer directly proportional to the system input. Unlike linear systems, where there is an abundance of analytical methods used to fully describe the dynamics of a system [3], handling nonlinear system is considered a challenge and existing methods are limited. Another obstacle that modern dynamical systems are facing is the unknown parameters of a system one is attempting to understand or model. In many cases, there is a gap of knowledge between the dynamic system in question and known governing equations. Without this knowledge, we are left to look for patterns in these dynamical systems which in some cases lack accuracy.

This paper will introduce a Koopman operator based method for obtaining features of non-linear dynamical systems to be used in machine diagnostics. We will map a non-linear system to a linear system using the Koopman operator, solve for this operator using spectral decomposition, then use the results as parameters for system diagnostics. The rest of the paper is organized as follows: Section 2 will present the fundamentals of the Koopman operator and Dynamic Mode Decomposition. Section 3 will describe two proposed methods for fault diagnostic of dynamical systems. Sections 4 discusses the results from the proposed methods with gearbox test cases. Finally, section 5 concludes the paper.

BACKGROUND

Dynamical systems are typically non-linear systems which are, in practice, difficult to analyze. The Koopman operator is a linear operator that can help construct a linear relationship for the "temporal" dynamics of a non-linear system [4]. This is done based on the assumptions that if we "lift" the states of the system to a high dimensional observable space with all non-linear terms captured, it is possible to express the original non-linear dynamics with a linear model. Therefore, using the Koopman operator for dynamic system analysis, one can get a linear representation of the system and extract key features which further enables the prediction and control of the original non-linear system.

Traditionally, discrete time dynamics are found in the state space and are modeled as $x_{n+1} = F(x_n)$, where F is a time evolution operator that brings the system from one time to the next [4]. Figure 1 is an illustration of the "lifting" of the dynamics from the state space to the observable space. In this new space, the states x_k are represented by functions of the state, or observables, $g(x_k)$. For a discrete time based system, the dynamics can be illustrated as $g(x_{k+1}) = g \circ F(x_k) = Kg(x_k)$, where K is the Koopman operator which evolves the observables forward in time. The number of states depicted by "n" are typically much lesser than the number of observables depicted by "k". In the state space, the dynamics are non-linear and finite dimensional. Conversely in the observable space, the dynamics are now linear but augmented to higher dimensions.

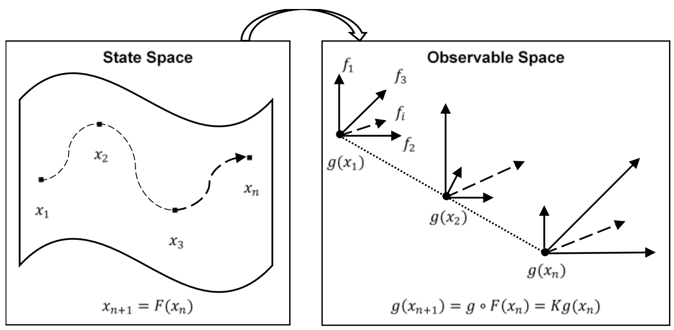

Figure 1. Koopman operator in an observable space.

Once the Koopman operator is obtained, spectral decomposition is performed to extract its eigenvalues and eigenvectors. Then, the Koopman modes and mode amplitudes are defined, which in turn can be used to reconstruct the original system states. In this paper, the modes and mode amplitudes can then be considered as features of the original state.

The purpose of dynamic mode decomposition (DMD) [5] in this paper is to extract the Koopman modes and mode amplitudes. DMD at its core is a data-driven linear regression problem. Consider a data set of vectors $\{x_1, x_2, \ldots, x_n\}$ where every vector $x_i \in \mathbb{R}^n$ and each vector is an evolution forward in time. Each vector x_i is referred to as a snapshot in time. A collection of such snapshots forms a high dimensional matrix $\boldsymbol{X} = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^{m\,x\,n}$. The linear operator A represents the relationship between the two high dimensional snapshot matrices:

$$Y \approx AX. \tag{1}$$

DMD utilizes dimensionality reduction via SVD to approximate the best-fit linear operator A. The following steps will show how the DMD algorithm extracts the operator A and extracts its modes and mode amplitudes[6]:

STEP 1: Once the data is separated into the snapshot pairs compute the SVD of X:

$$X = U\Sigma V^* \tag{2}$$

where $U \in \mathbb{C}^{n\,x\,r}$, $\Sigma \in \mathbb{R}^{r\,x\,r}$ is a diagonal matrix, $V \in \mathbb{C}^{m\,x\,r}$ and r is low rank approximation of X.

STEP 2: A low rank approximation of A is defined as $\tilde{A}$:

$$\tilde{A} \triangleq U^{*}\mathrm{Y}\mathrm{V}\Sigma^{-1} \tag{3}$$

STEP 3: Compute spectral decomposition of $\tilde{A}$:

$$\tilde{A}w = \lambda w \tag{4}$$

where w is a matrix containing the eigenvectors of $\tilde{A}$ and λ is a diagonal matrix containing the eigenvalues of $\tilde{A}$.

STEP 4: Using the eigenvectors, w, DMD modes can be reconstructed to describe the high dimensional operator A.

$$\varphi = YV\Sigma^{-1}w \tag{5}$$

STEP 5: Compute the mode amplitudes:

$$b = (w\lambda)^{-1}x_1 \tag{6}$$

The eigenvalues and modes of the dynamical system shown in equation (1) are advantageous in describing the system dynamics. DMD modes are vectors that are spatial representations of the dynamics and the eigenvalues respective to their modes are growth or decay rates. Mode amplitudes define the significance that each mode has on the system.

PROPOSED METHODOLOGY

Here, an overview of two proposed methods for analysis of dynamic systems using the Koopman Operator are presented. Both methods can be employed as fault detection tools. The overall goal of both methods is to create an accurate model from healthy data, test it against experimentally or simulated faulty data, and be able to determine if a fault is present. The first method uses a neural network to build and test a model while the second build a predict model and uses statistical analysis methods to test the prediction future response.

The two methods begin by sharing the same feature extraction process. Either traditional DMD or extended DMD can be performed to solve for the Koopman operator as well it's spectral properties. The extracted Koopman eigenvalues and eigenvectors, Koopman modes and mode amplitudes are defined as key features of the original non-linear dynamical system. From there, the two methods differ and are described more thoroughly in the following.

The first method feeds the mode amplitudes obtained from DMD into a neural network model for healthy and faulty data classification. Once the mode amplitudes are extracted, they are treated as features of the dynamical system and are imported into a simple neural network. An overview of this method is presented in Figure 2.

The second method build a prediction model, which creates a dynamic function over time. This method uses the modes extracted from DMD as well as the Koopman eigenvalues to create a prediction model. The model outputs predicted future states of a dynamical system. Once the desired outputs are obtained, the model is tested for accuracy by known statistical metrics. An overview of this methods is presented in Figure 3.

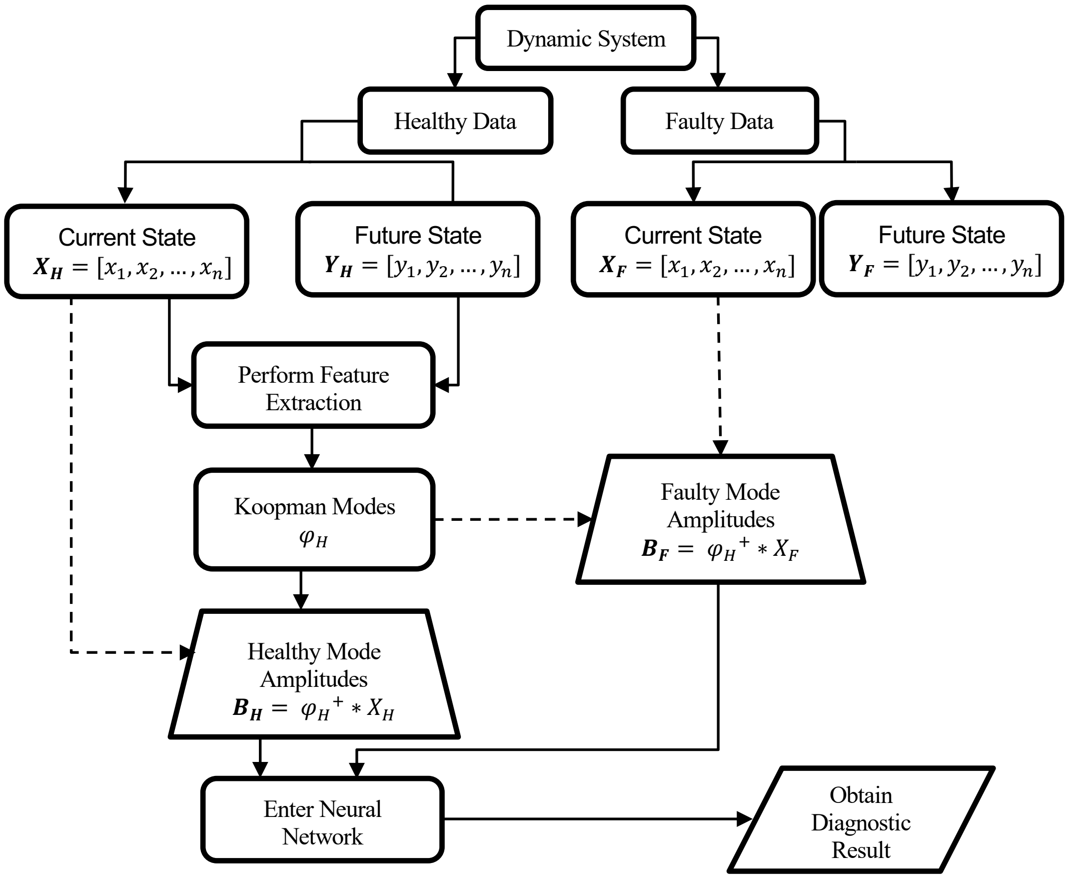

Figure 2. Classification methods using Koopman mode amplitudes

The proposed prediction model approach is based on Koopman eigenvalues and Koopman modes. This method of fault diagnosis begins identically the previous method and the feature extraction methodology. This model can be used to recreate the original dynamics to some time-step forward. The equation for this model is written as:

$$(y_\tau) = \varphi \lambda^\tau \varphi^+ x_k \tag{7}$$

where x_k is the original snapshot data, y_τ is the predicted future state data, φ and λ are the Koopman eigenmodes and eigenvalues respectively and finally τ indicates the time-step chosen for prediction. Any snapshot, y_k, can be inserted into the model above to produce a prediction τ steps away from the original data. As seen in Figure 3, when the current state of the faulty data set is imported in Equation (7) the result is the predicted future state of the faulty data set. The output, y_τ, from the prediction model and the original data can be compared using any valid known statistical methods to test the accuracy of the model. This prediction model acts as a dynamic function over time. It describes how the system will evolve over time. Much like how time is the independent variable in the derivative model, τ, is the only independent variable in this prediction model. By altering τ in the above model, the prediction can be expanded over a desired number of time steps. The goal of this method is not only to detect a possible fault in the system, but also to predict the possible occurrence of a fault before it occurs.

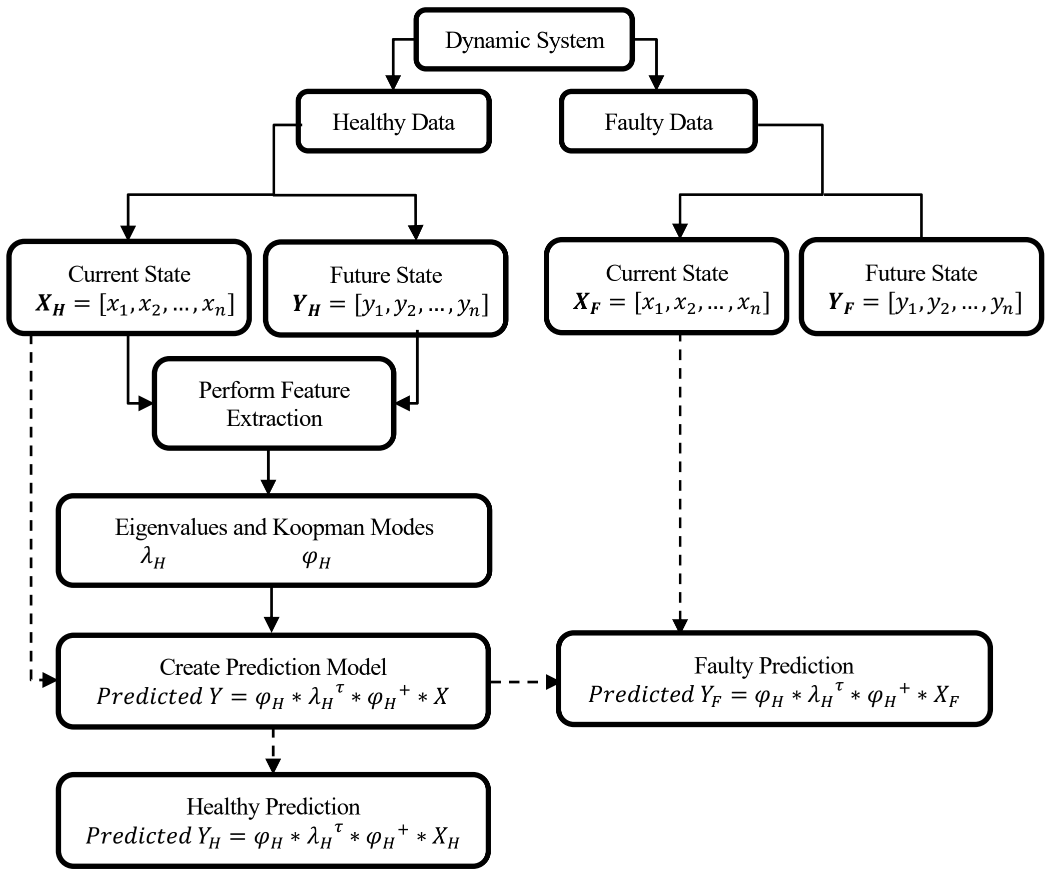

Figure 3. Illustration of prediction model based on Koopman operator

RESULTS AND DISSCUSSION

The proposed methods are tested with an experimental gearbox test rig as shown in Figure 4. Two sets of experiences are conducted. In the first sets of experiments, a single level of fault were simulated. In the second set of experiments, multiple level of faults were simulated. Triaxial vibration signals were collected and used for the methods validation.

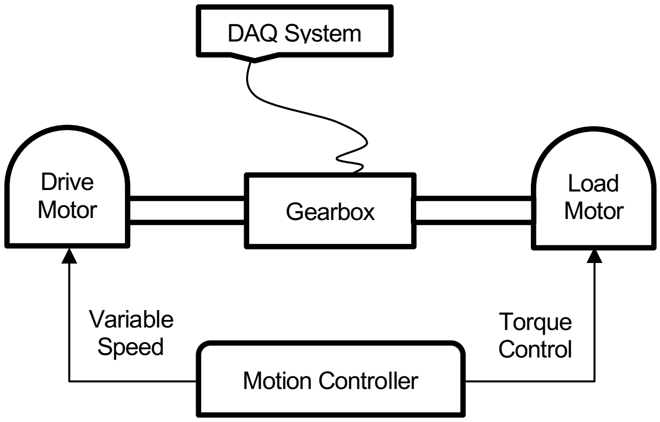

Figure 4. Symbolic graph for the experimental test rig

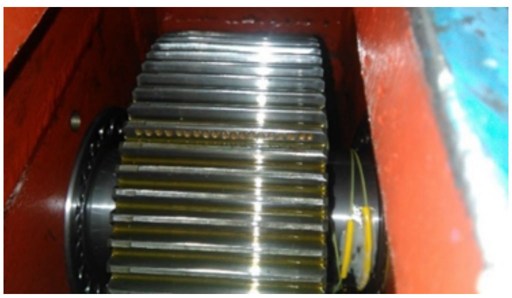

Figure 5. Single gear tooth pitted fault diagnostics

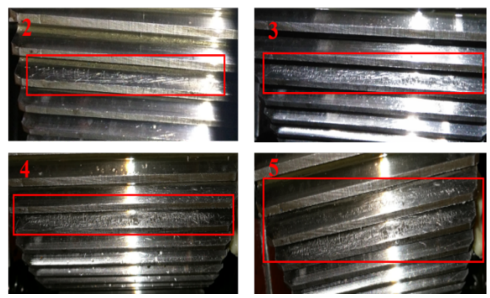

Figure 6. Multiple level fault diagnostics

TABLE I. CLASSFICATION ACCURACY FOR SINGLE LEVEL FAULTS

Trial	1	2	3	4	5	6	7	8	9	10	Average
Augmented	99.6	100	100	100	100	100	99.6	100	100	100	**99.9%**
Non-Augmented	71.5	71.9	74.0	75.0	71.9	73.1	72.7	71.0	71.4	72.2	**72.4%**

Table I shows the classification accuracy with the first proposed method for single level fault detection. The results suggest that when the data is augmented, the mode amplitude model is 99.9% accurate in determining whether or not a fault is present. However, when non-augmenting the data sets, the accuracy of the model decreases to 72.4%.

A further testing on the first method with multiple level fault data shows that the accuracy of fault diagnostic is about 99.3%, as shown in TABLE II.

TABLE II. CLASSFICATION ACCURACY FOR MULTIPLE-LEVEL FAULTS

Trial	1	2	3	4	5	6	7	8	9	10	Average
Augmented	99.4	99.3	99.3	99.3	99.3	99.4	99.3	99.3	99.4	99.4	**99.3%**
Non-Augmented	40	20	20	20	40	20	40	40	40	20	**30.0%**

The second proposed method is tested with single level fault data. A Koopman operator based prediction model was first learned with healthy data. Then, prediction was made and compared with the actual data. A summary of the RMSE percent error of the single step prediction model using healthy and faulty data is shown in Table III below. The percent error was compared for both non-augmented and augmented data. Since the prediction model itself uses the healthy Koopman eigenvalues and Koopman modes, testing the healthy state percent error was expected to have a lower value than testing the faulty state percent error. It can be seen that with both augmented data and non-augmented data, the predicted data from the health model have a much larger error when faults are presented. Therefore, based on the deviation between predicted signal and measured the signal, fault can be detected.

TABLE III. PREDICTION ACCURACY OF THE LEARNED DYNAMIC MODEL FOR HEALTHT AND FAULTY DATA

	Non-augmented	Augmented
Predicted Healthy State RMSE (%)	1.79	0.89
Predicted Faulty State RMSE (%)	2.91	1.54

CONCLUSIONS

To conclude, two methods based on Koopman operator for fault diagnosis and prediction were proposed and validated via case studies. The performance of these methods is promising for the prognostic health management (PHM) field. Both methods have the potential to be used for generic industrial applications. The first method has potential to identify faults from non-linear dynamic systems while the second method could be used for prediction and control applications. Both methods showed promising results when augmenting the original data sets from a state space to an observable space. However, when the state variables were directly used in the Koopman operator model, the accuracy of the proposed methods decreased significantly. The results from this paper indicate that augment the original data to a high dimensional space can boost diagnostic performance. Future work regarding the proposed methods is to investigate further on the robustness for multilevel fault diagnosis.

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