Spatial interpolation and kriging

EDS 222

Tamma Carleton

Fall 2021

Announcements/check-in

- Final projects guidelines

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- **Change in office hours** for week of 11/15 to Thursday 11/18 1:30pm-2:30pm

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- **No class** 11/11; **remote class** 11/23, **no class** 11/25
- Final project presentations: 12/2 9:30-10:45am (Bren Hall 1414); 12/7 8-10:30am (Bren Hall 14**2**4)
    - You will *randomly* be assigned a slot (slots announced 11/24)

Today

Refresher: types of spatial data

Points, vector, raster/field, dynamic raster/field

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Kriging: a powerful form of interpolation

Variogram, kriging

Types of spatial data

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Examples : Elevation. Temperature. Wind direction.

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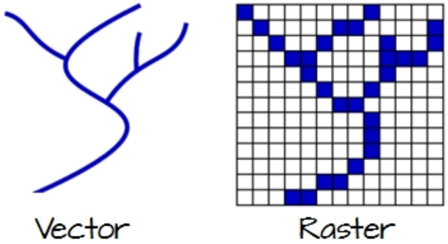

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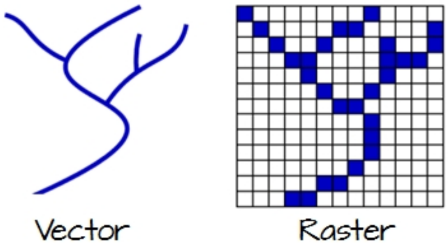

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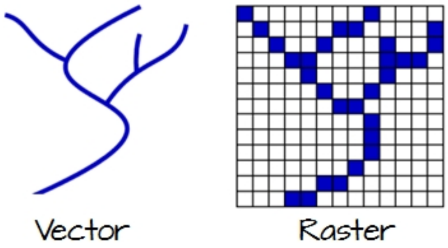

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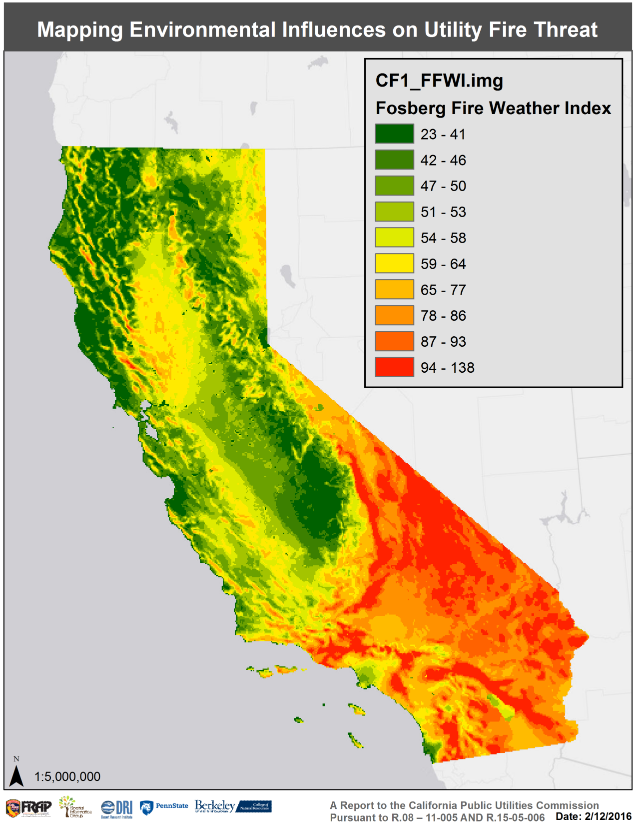
- Luckily, `R` makes it easy to switch back and forth (but we need to be careful and intentional when transforming!)

Spatial interpolation

Spatial interpolation

In environmental data science, we are **often interested in modeling fields**



Spatial interpolation

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Spatial interpolation

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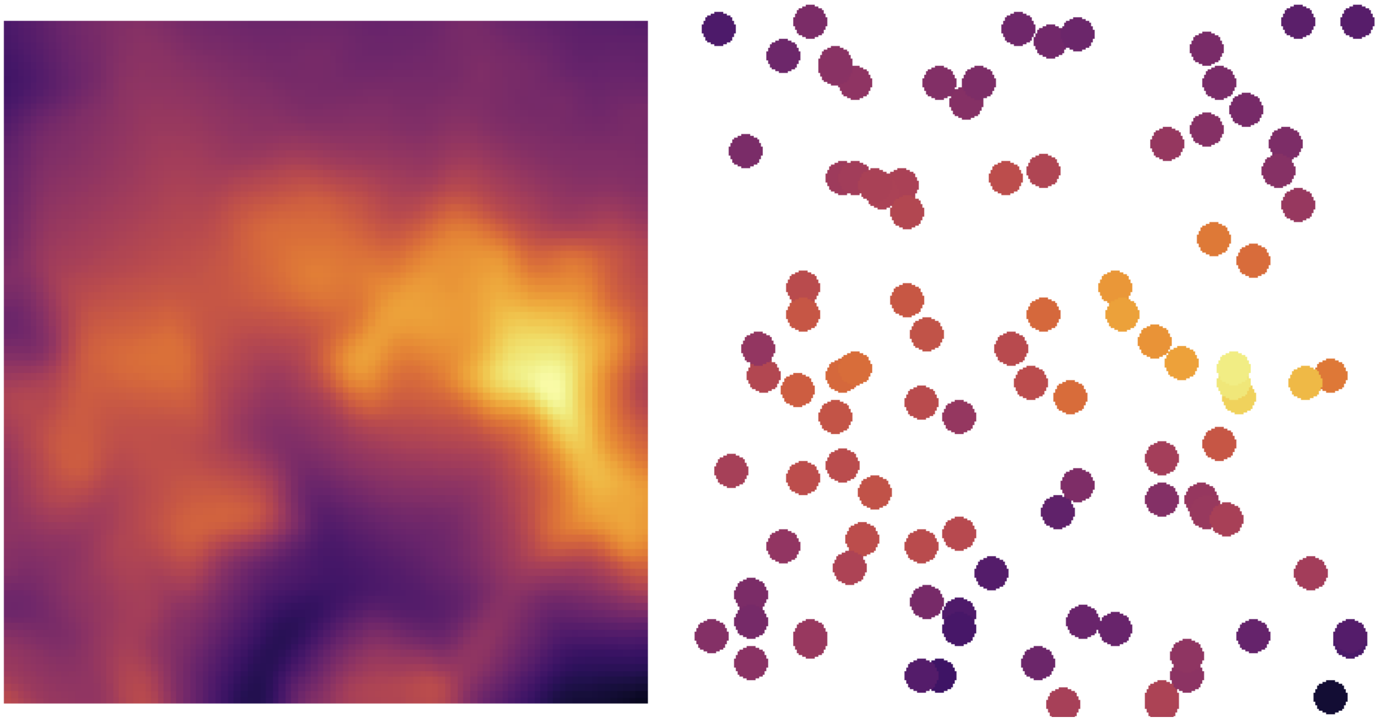
That means we only have data from a *sample*, not a census of the *population*



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For example:

- Predicting "gold grades" across South Africa using a few borehole samples (the problem of Daniel *Krige*!)
- Predicting depth to groundwater across California using monitoring wells
- Predicting air pollution across China using monitoring stations

Spatial interpolation in math

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Spatial interpolation aims to predict $Z(x_0)$ using a linear combination of the values in the sampled locations:

$$\hat{Z}(x_0) = \sum_{i=1}^{m} \lambda_i Z(x_i)$$

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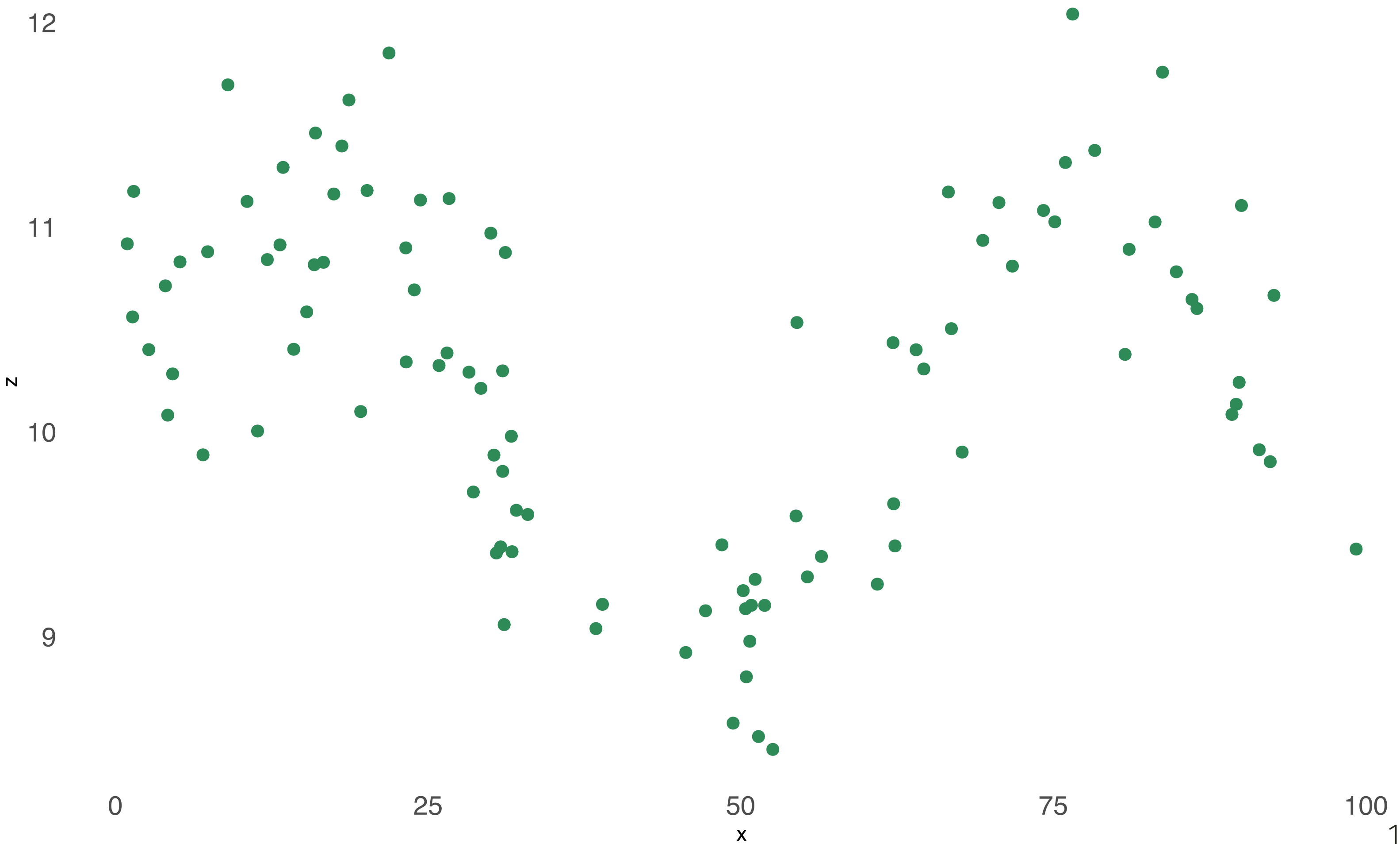
- All spatial interpolation methods assume or derive a set of λ's to compute $\hat{Z}$'s

Interpolation in one dimension

Consider one-dimensional space where values y depend on location x



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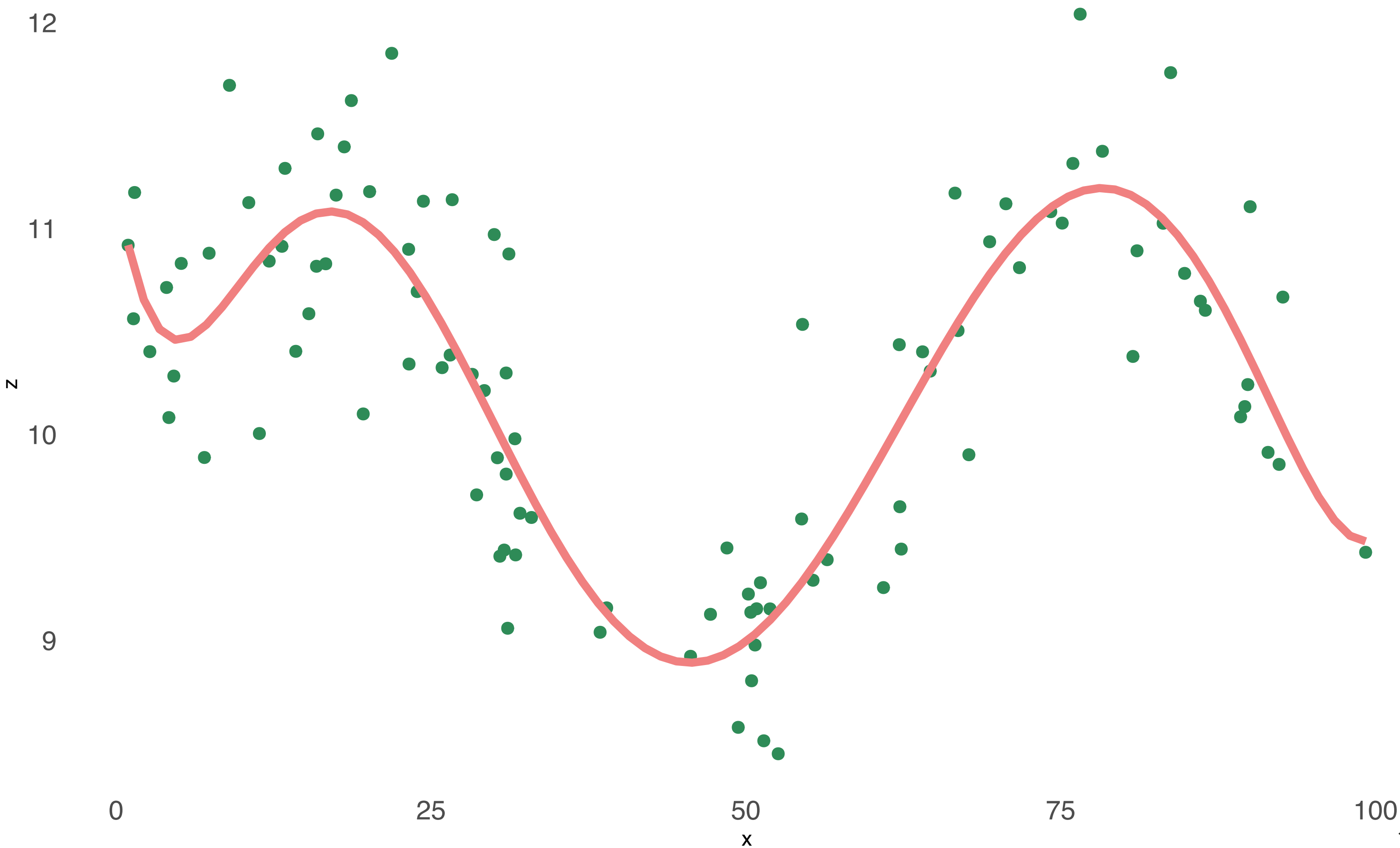

Interpolation in two dimensions

Often we have data for an outcome z observed in 2-D space: $z(x, y)$

[Draw it!]

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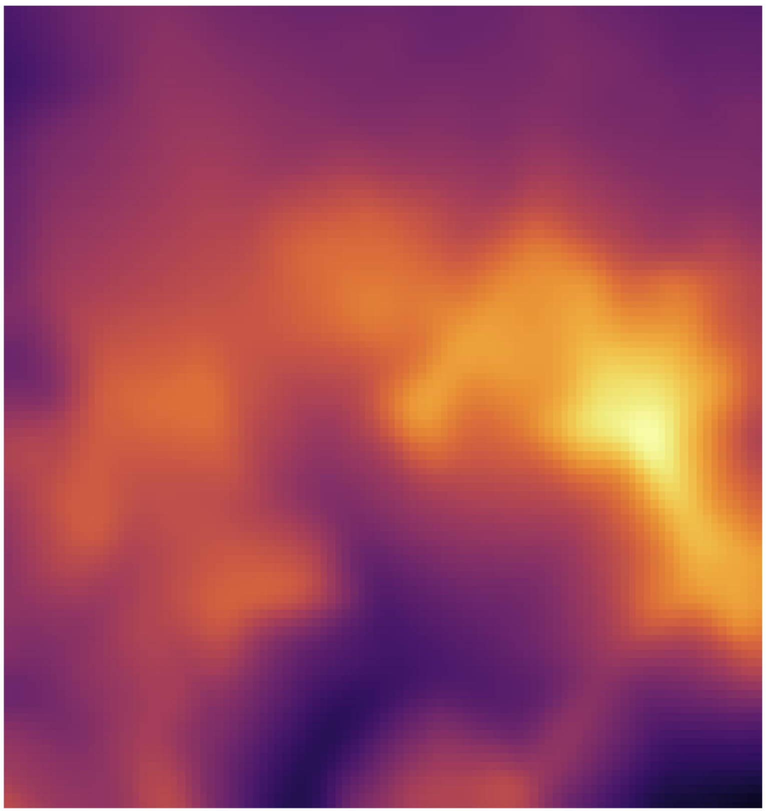
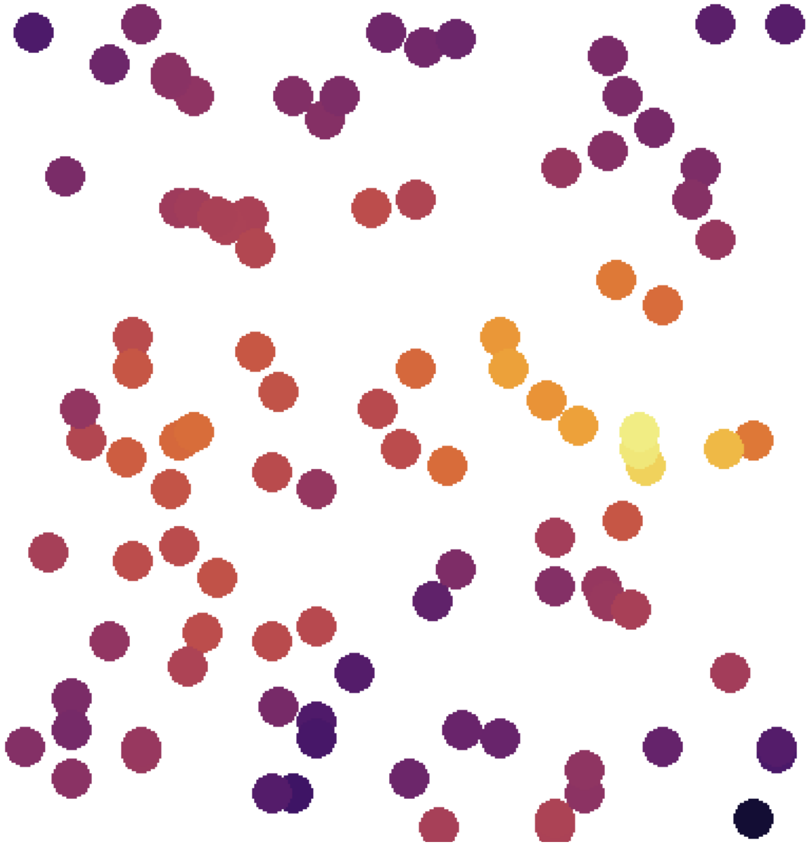
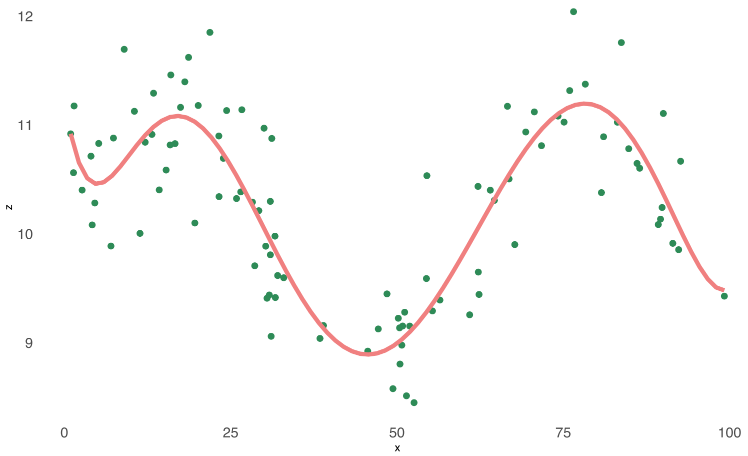




Interpolation methods

Polynomial regression



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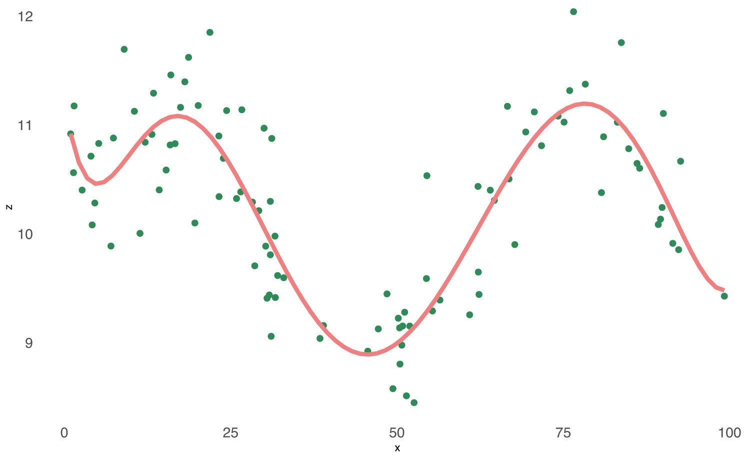

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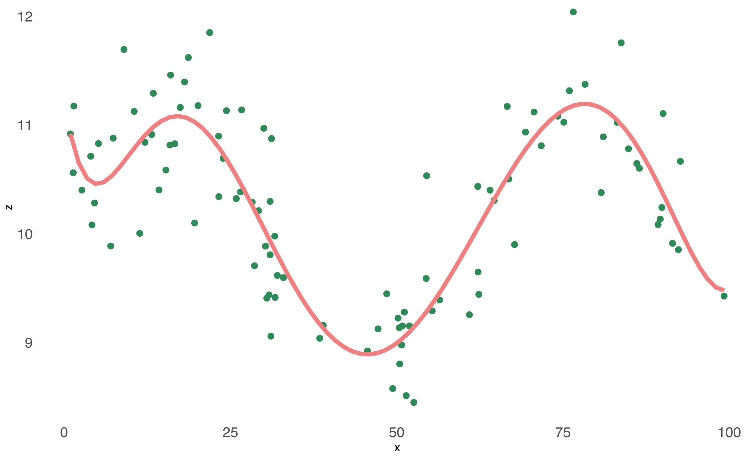

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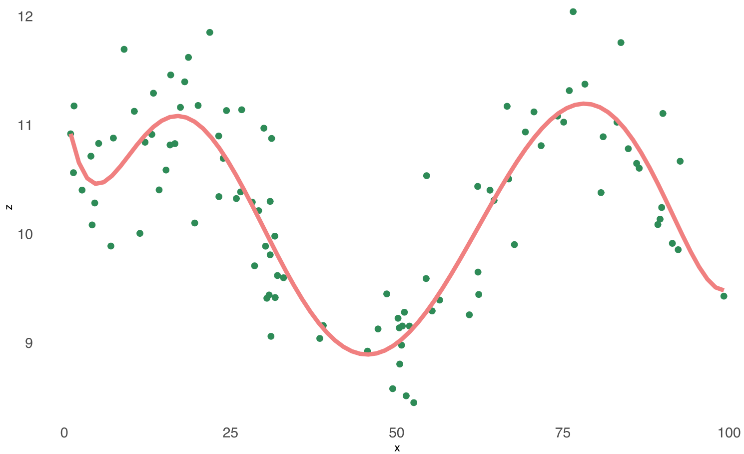

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Exact: Predicts a value identical to the measured value.

Inexact: Does *not* predict a value identical to the measured value.

Polynomial regression interpolation

This is just **multiple linear regression** using spatial information as the independent variable

```
mod = lm(z~poly(x,8))
predictions = augment(mod)$.fitted
```

Interpolation methods

Nearest Neighbors (NN)

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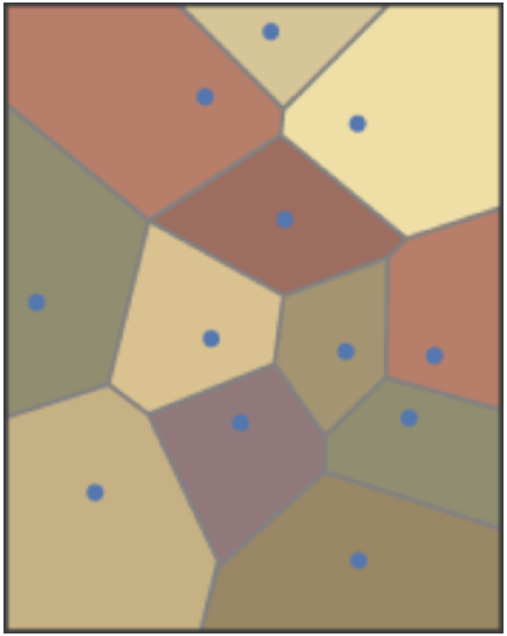

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Nearest Neighbors (NN)

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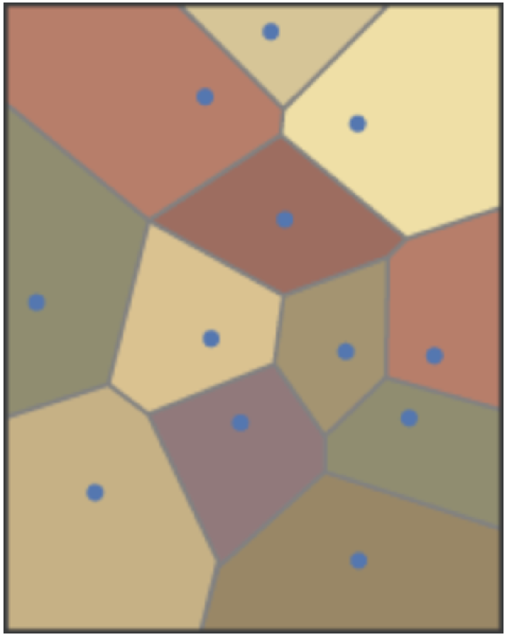

- Creates what are called "Theissen Polygons", which allocate space to the nearest sampled point

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Implementation in `R`

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library(dismo)
v <- voronoi(dta)
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Interpolation methods

Inverse distance weighting

Basic idea: weights are a decreasing function of distance from x_0 to x_i

$$\hat{Z}(x_0) = \sum_{i=1}^{m} \frac{Z(x_i) Dist(x_i, x_0)^{-p}}{\sum_{i=1}^{m} Dist(x_i, x_0)^{-p}}$$

Equivalently:

$$\lambda_i^{IDW} = \frac{1/Dist(x_i, x_0)^p}{\sum_{i=1}^{m} 1/Dist(x_i, x_0)^p}$$

where p is the "power parameter" determining how fast the weight declines as the distance between the points grows larger

Interpolation methods

Inverse distance weighting

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```
library(phylin)
idw(values, coords, grid, method = "Shepard", p = 2, R = 2, N = 15,
    distFUN = geo.dist, ... )
```

- Note the `method` argument: "Shepard" follows the math on the previous slide
- Note the `p` argument: Need to specify power parameter

Interpolation methods

There are many more!

- Piecewise linear interpolation / Delany triangulation
- Local polynomial regression
- Radial basis function (RBF)
- Kriging (of many forms)
- Many new machine-learning based methods
- Learn more in Li and Heap (2014)

Enter: Kriging

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Why?

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- Under certain assumptions it is the "best linear unbiased estimate" (sound like OLS yet??)
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We will study **kriging** and its implementation in `R` in the next lecture and lab

Slides created via the R package **xaringan**.