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{Cluster-G5}

pasul E:

$$p_{i1} \stackrel{def}{=} P(comp_1|x_i) \stackrel{F.B.}{\underset{F.P.T}{=}} \frac{P(x_i|comp_1)\cdot P(comp_1)}{P(x_i|comp_1)\cdot P(comp_1) + P(x_i|comp_2)\cdot P(comp_2)} = \frac{\mathcal{N}(x_i|\mu=4, \sigma_1^2=1)}{\mathcal{N}(x_i|\mu=4, \sigma_1^2=1) + \mathcal{N}(x_i|\mu=4,\sigma_2^2=4)}$$

($P(comp_1) = 1/2$)

$$= \frac{\mathcal{N}(x_i-4|0,1)}{\mathcal{N}(x_i-4|0,1) + \frac{1}{2}\mathcal{N}\left(\frac{x_i-4}{2}\Big|0,1\right)} = \frac{e^{-\frac{(x-4)^2}{2}}}{e^{-\frac{(x-4)^2}{2}} + \frac{1}{2}e^{-\frac{(x_i-4)^2}{4}}}$$

$i=1$: $e^{-\frac{(4-4)^2}{2}} = e^0 = 1 \Rightarrow p_{11} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \Rightarrow p_{12} = \frac{1}{3}$

$i=2$: $\mathcal{N}(4.6|4,1) = \mathcal{N}(0.6|0,1) = 0.33$

$\mathcal{N}(4.6|4,2^2) = \frac{1}{2}\mathcal{N}\left(\frac{0.6}{2}\Big|0,1\right) = \frac{1}{2}\mathcal{N}(0.3|0,1) = 0.38\cdot\frac{1}{2} = 0.19$

$$\Rightarrow p_{21} = \frac{0.33}{0.33+0.19} = \frac{33}{52}$$

$$\Rightarrow p_{22} = \frac{19}{52}$$

$i=3$: $\mathcal{N}(2|4,1) = \mathcal{N}(2|0,1) = 0.05$

$\mathcal{N}(2|4,2^2) = \frac{1}{2}\mathcal{N}\left(\frac{2}{2}\Big|0,1\right) = \frac{1}{2}\mathcal{N}(1|0,1) = 0.24\cdot\frac{1}{2} = 0.12$

$$\Rightarrow p_{31} = \frac{0.05}{0.05+0.12} = \frac{5}{17}$$

$$\Rightarrow p_{32} = \frac{12}{17}$$

pasul M:

$$Q(\mu) \stackrel{def}{=} E_{Z|X,\mu^{(t)}}[\ln P(\underset{X,Z}{Y}|\mu)]$$

$Y = y_1, y_2, y_3$ $\quad y_i \stackrel{not.}{=} (x_i, z_{i1}, z_{i2})$ cu $z_{i1}, z_{i2} \in \{0,1\}$ și $z_{i1}+z_{i2}=1$

$$\ln P(\underset{x_i,z_{i1},z_{i2}}{y_i}|\mu) = \ln P(x_i, z_{i1}, z_{i2}|\mu) = \ln\Big(\underbrace{P(x_i|z_{i1},z_{i2},\mu)}_{\frac{1}{\sqrt{2\pi}}e^{-\frac{(x_i-\mu^{(t)})^2}{2\cdot\sigma}}}\cdot\underbrace{P(z_{i1},z_{i2}|\mu)}_{1/2}\Big)$$

σ: $\begin{cases}1 & \text{dacă } z_{i1}=1\\ 2^2 & \text{dacă } z_{i2}=1\end{cases}$

$e^{-\sum_{j=1}^{2} z_{ij}\cdot\frac{(x_i-\mu)^2}{2\cdot\sigma_j^2}}$ unde $\sigma_1=1$, $\sigma_2=2$

$$= -\frac{1}{2}\ln(2\pi) - \sum_{j=1}^{2} z_{ij}\frac{(x_i-\mu)^2}{2\cdot\sigma_j^2} - \ln 2$$

$$= -\frac{3}{2}\ln 2 - \frac{1}{2}\ln\pi - \sum_{j=1}^{2} z_{ij}\frac{(x_i-\mu)^2}{2\cdot\sigma_j^2}$$