Assignment 1
Grammars & Truth Tables

1. The grammar:

   N → F | F [e] S I
   F → I [.] I
   I → D | D I
   D → [0] | [1] | [2] | … | [9]
   S → [+] | [-]

   (a) 6.02e+23

   N

   F [e] S I

   I [.] I [e] S I

   D [.] I [e] S I

   D [.] D I [e] S D I

   D [.] D D [e] S D I

   D [.] D D [e] S D D

   D [.] D D [e] [+] D D

   6 [.] D D [e] [+] D D

   6 . D D [e] [+] D D

   6 . 0 D [e] [+] D D

   6 . 0 2 [e] [+] D D

   6 . 0 2 e [+] D D

   6 . 0 2 e + D D

   6 . 0 2 e + 2 D

   6 . 0 2 e + 2 3

   (b) 1e-6
   It is impossible to otainn this number with the given grammar. In order to be able to derive this number, the grammar must be modified as follows:

   N → F | F [e] S I
   F → I | I [.] I
   I → D | D I
   D → [0] | [1] | [2] | … | [9]
   S → [+] | [-]

The rule F → I has been added so as to represent values that do not contain the „." character. Using this grammar, the value 1e-6 can be represented as such:

N

F [e] S I

I [e] S I

D [e] S I

D [e] S D

D [e] [-] D

1 [e] [-] D

1 e [-] D

1 e - D

1 e - 6

2. Grammar for formulae in propositional logic:
   F → Ap | ¬ F | F ∧ F | F ∨ F | F → F
   Ap → [P] | [Q] | [R]

   In order to represent arguments in propositional logic, expressed using sequent notation, this grammar needs to be extended as follows:

   Starting symbol: S
   S → F O F | F
   F → Ap | ¬ F | F ∧ F | F ∨ F | F → F | F [,] F
   Ap → [P] | [Q] | [R]
   O → [:] | [⊦]

   - Added the new starting symbol S with the rule S → F O F | F such that we are able to represent coherent, complete arguments.
   - Added the rule F → F [,] F such that we can enumerate multiple premises if needed.
   - Added the symbol O with the rule O → [:] | [⊦] to allow expressing wheter anargument is knowingly valid.

3. If Alice studies logic, then Bob studies it too. – A → B
   If either Alice or Bob studies logic, then Alice definitely does. – (A ∨ B) → A
   Therefore, both Alice and Bob study logic. – A ∧ B

   Atomic propositions:
   - Alice studies logic. → A
   - Bob studies logic. → B

A	B	A ∨ B	A → B	(A ∨ B) → A	A ∧ B
T	T	T	T	T	T
T	F	T	F	F	F
F	T	T	T	T	F
F	F	F	T	T	F

The argument is invalid. From the truth table, we can observe that there are cases when both premises are true but the conclusion is false.

4. $P \rightarrow Q, Q \vee R : \neg(P \wedge Q) \rightarrow R$

P	Q	R	P ∧ Q	¬(P ∧ Q)	P → Q	Q ∨ R	¬(P ∧ Q) → R
T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	F	F

From the truth table we learn that the argument is invalid as it there are cases when both presmises are true but the conclusion is false.

A propositional logic formula that includes atomic propositions P and Q (but not R) and which, when added as a premise to this argument, makes it valid is $Q \rightarrow P$.

P	Q	R	P ∧ Q	¬(P ∧ Q)	Q → P	P → Q	Q ∨ R	¬(P ∧ Q) → R
T	T	T	T	F	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F	F
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	F	F

This way, everytime the presmises are true, the conclusion is true as well, having added a premise that is false in the case where all the other premises were true and the conclusion was false.