Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Computer Science Groups 911-917, Academic Year 2021-2022

Mathematical Analysis
Exercise Sheet 3

13. Let $x = (1, 2, -1)$ and $y = (2, 0, 1) \in \mathbb{R}^3$.
a) Compute $x + y$, $x - y$, $\|x\|$, $\|y\|$, $x \cdot y$.
b) Find $z \in \mathbb{R}^3$ such that $z \perp x$, $z \perp y$ and $\|z\| = 1$.

14. Let $x = (0, 2)$, $y = (2, 1) \in \mathbb{R}^2$. Find the intersection of the segment $[x, y]$ with the sphere[1] centered at the origin and of radius 2

$$S := \left\{z \in \mathbb{R}^2 : \|z\| = 2\right\}.$$

15. Prove that the following properties hold for any $x, y \in \mathbb{R}^d$
a) $\|x + y\|^2 - \|x - y\|^2 = 4x \cdot y$
b) $\|x + y\|^2 + \|x - y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$ (parallelogram identity)

16. Find the first order partial derivatives of the following functions:
a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = \cos x \cos y - \sin x \sin y$.
b) $f : (0, +\infty) \times (0, +\infty) \to \mathbb{R}$, $f(x, y) = x^y$.
c) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x, y, z) = (x + y + z)[(2^x)^y]^z$.
d) $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^* \to \mathbb{R}$, $f(x, y, z) = xe^y/z$.

Homework

HW 17. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and let $g : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$g(x, y) = f(x^2 + y^2),\ \forall\, (x, y) \in \mathbb{R}^2.$$

Prove that for any $(x, y) \in \mathbb{R}^2$ we have

$$y\frac{\partial g}{\partial x}(x, y) - x\frac{\partial g}{\partial y}(x, y) = 0.$$

HW 18. Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ be partially differentiable functions. Prove that

$$\nabla(fg)(c) = f(c)\nabla g(c) + g(c)\nabla f(c),\ \forall\, c \in \mathbb{R}^n.$$

[1] Here, the term *sphere* is used in the generic sense of boundary of a ball in arbitrary dimension. In dimension one a *sphere* consists only of the endpoints of an interval (degenerate case), in dimension two the *sphere* is a circle while in dimension three the standard notion of a 3D sphere is recovered.