

Mathematical Analysis Exercise Sheet 12

54. Prove that the following series are divergent: a) $\sum_{n \geq 1} \operatorname{arctg} n$ and b) $\sum_{n \geq 1} \sin n$.

55. Discuss (w.r.t. $a > 0$) the convergence of the generalized harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^a}$ using

Theorem (Cauchy's condensation test) Let $(a_n)_{n \in \mathbb{N}^*}$ be a decreasing sequence with positive terms, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \sum_{n=1}^{\infty} 2^n a_{2^n}$$

are both convergent or both divergent.

56. Analyze by using only elementary inequalities (i.e., without using convergence tests) the convergence of

$$\text{a) } \sum_{n=0 \text{ or } 1}^{\infty} q^n, \quad |q| < 1, \quad \text{b) } \sum_{n=1}^{\infty} \frac{1}{n}, \quad \text{c) } \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

HW 57. Study the convergence of the following series

$$\begin{array}{ll} \text{a) } \sum_{n \geq 1} \frac{e^n}{n + 3^n}; & \text{b) } \sum_{n \geq 1} \frac{1}{n^2 - \ln n + \sin n}; \\ \text{c) } \sum_{n \geq 1} \frac{\sqrt{n+1}}{1 + 2 + \dots + n}; & \text{d) } \sum_{n \geq 1} \frac{2^n \cdot n!}{n^n}; \\ \text{e) } \sum_{n \geq 1} \frac{5^{n/2}}{n 2^n}; & \text{f) } \sum_{n \geq 1} (\arctan n)^n; \\ \text{g) } \sum_{n \geq 1} \frac{n^2}{2^{n^2}}; & \text{h) } \sum_{n \geq 1} \frac{(n+1)^n}{n^{n+2}}; \\ \text{i) } \sum_{n \geq 1} \ln \left(1 + \frac{1}{n} \right); & \text{j) } \sum_{n \geq 2} \frac{1}{n \ln n}. \end{array}$$

HW 58. Find the sum of the following series:

$$\begin{array}{ll} \text{a) } \sum_{n=1}^{\infty} (-\pi/4)^n; & \text{b) } \sum_{n=1}^{\infty} 3^{1-2n}; \\ \text{c) } \sum_{n=1}^{\infty} \binom{n+2}{3}^{-1}; & \text{d) } \sum_{n=1}^{\infty} \frac{1}{1^2 + 2^2 + \dots + n^2}; \\ \text{e) } \sum_{n=1}^{\infty} (\sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}); & \text{f) } \sum_{n=2}^{\infty} \ln \left(1 - \frac{1}{n^2} \right); \\ \text{g) } \sum_{n=0}^{\infty} \operatorname{arctg} \frac{1}{n^2 + n + 1}; & \text{h) } \sum_{n=0}^{\infty} \frac{n+1}{2^n}. \end{array}$$