Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Computer Science Groups 911-917, Academic Year 2021-2022

Mathematical Analysis
Exercise Sheet 11

51. Study the convergence of and (if convergent) compute the improper integrals

a) $\displaystyle\int_0^\infty \frac{\operatorname{arctg} x}{1+x^2}\mathrm{d}x;$

b) $\displaystyle\int_2^\infty \frac{x-1}{x^2+x+1}\mathrm{d}x;$

c) $\displaystyle\int_0^1 (\ln x)^2\,\mathrm{d}x.$

52. a) Study the convergence of and compute $\displaystyle\int_0^\infty x^2 e^{-x}\mathrm{d}x.$

b) Prove that $\displaystyle\int_0^\infty P(x)e^{-x}\mathrm{d}x = P(0)+P'(0)+\cdots+P^{(n)}(0)$ where P is a polynomial function of degree $n\in\mathbb{N}$.

53. Compute the following integrals by embedding them in parametrized families and differentiating with respect to the parameter[1]

a) $\displaystyle\int_0^1 \frac{x^5-1}{\ln x}\mathrm{d}x$; [Hint: consider $I(a)=\int_0^1 \frac{x^a-1}{\ln x}\mathrm{d}x$]

b) $\displaystyle\int_0^\infty \frac{\operatorname{arctg} 2x}{x(1+x^2)}\mathrm{d}x.$ [Here $I(y)=\int_0^\infty \frac{\operatorname{arctg} xy}{x(1+x^2)}\mathrm{d}x$]

[1] Assume that all improper (parametric) integrals are convergent.