Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Computer Science Groups 911-917, Academic Year 2021-2022

Mathematical Analysis
Exercise Sheet 1

Before we start: Do you know how to sketch the graph of the following functions $ax^2 + bx + c$, $e^{\pm x}$, $\ln x$, $\sqrt{x}$, $|x|$?

Sketch the graph of

1. $f : [0, \infty) \to \mathbb{R}$, $f(x) = x^2 e^{-x}$.

2. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{x^2}{1 + x^2}$.

3. $f[0, \infty) \to \mathbb{R}$, $f(x) = \dfrac{x^3}{1 + x^2}$.

Homework

HW 4. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{x^4}{1 + x^2}$. Sketch the graph of f and find a polynomial function $P(x)$ (of minimal degree) such that

$$|f(x) - P(x)| \to 0 \quad \text{as} \quad x \to \infty.$$

(Counterexample) The above property, that f is asymptotically polynomial at ∞, does not hold for all smooth functions. Give a counterexample, i.e., give an example of a function for which $|f(x) - P(x)| \to 0$ as $x \to \infty$ does not hold for any polynomial P.

HW 5. Sketch the graph of $f : [0, \infty) \to \mathbb{R}$, $f(x) = e^{-x} \sin(x)$. [This function describes damped oscillations.]