Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Computer Science Groups 911-917, Academic Year 2021-2022

# Mathematical Analysis
## Exercise Sheet 4

**19. The Schwarz Inequality.** Let $x \cdot y = x_1 y_1 + \ldots + x_d y_d$ the Euclidean scalar product in $\mathbb{R}^d$ and $\|x\| = \sqrt{x \cdot x}$ the associated norm. Prove that

$$|x \cdot y| \leq \|x\| \|y\| \quad \text{for any} \quad x, y \in \mathbb{R}^d.$$

**20. All linear maps are of the form $a \cdot x$.** Let $T : \mathbb{R}^d \to \mathbb{R}$ be a linear map, i.e.,

$$T(x+y) = T(x) + T(y) \quad \text{and} \quad T(\alpha x) = \alpha T(x) \quad \text{for all } x, y \in \mathbb{R}^d,\ \alpha \in \mathbb{R}$$

Prove that there exists $a_T \in \mathbb{R}^d$ such that

$$T(x) = a_T \cdot x \quad \text{for all } x \in \mathbb{R}^d.$$

**21. The Gradient and Hessian of a linear map.** Let $T : \mathbb{R}^d \to \mathbb{R}$, $T(x) = a \cdot x$, be a linear map with $a = (a_1, \ldots, a_d) \in \mathbb{R}^d$. Compute its gradient and Hesse matrix.

**22. The Gradient and Hessian of a quadratic map.** Let $Q : R^d \to \mathbb{R}$, $Q(x) = \sum_{i,j=1}^d a_{ij} x_i x_j$ with $a_{ij} \in \mathbb{R}$ and $a_{ij} = a_{ji}$ be a quadratic map. Compute its gradient and Hesse matrix of $Q$.