Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Computer Science Groups 911-917, Academic Year 2021-2022

Mathematical Analysis
Exercise Sheet 5

23. Extrema vs. saddle points. For the following two functions $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$

$$f_1(x,y) = x^2 + y^2 \quad \text{and} \quad f_2(x,y) = x^2 - y^2,$$

a) check that both are quadratic and find their matrices;

b) check that their gradient vanishes at the origin $(0,0)$;

c) is the origin a (local) extremum point for both functions?

24. Find the local extremum points (specifying their type) of the following functions:

a) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = x^3 - 3x + y^2$.

b) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = x^3 + y^3 - 3xy$.

c) $f : (0,\infty) \times \mathbb{R} \to \mathbb{R}, \quad f(x,y) = x(y^2 + \ln^2 x)$.

d) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = x^4 + y^4 - 4(x-y)^2$.

25. Find the local extremum points (specifying their type) of the following functions:

a) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x,y,z) = z^2(1+xy) + xy$.

b) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x,y,z) = x^3 - 3x + y^2 + z^2$.

26. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined for all $(x,y) \in \mathbb{R}^2$ by

$$f(x,y) = (x^2 - y)(x^2 - 3y).$$

a) Prove that $(0,0)$ is a stationary point of f.

b) Study whether $(0,0)$ is a local minimum point of f.

c) Prove that the restriction of f to any line through $(0,0)$ attains a local minimum at $(0,0)$.