Name:

Group:

Mathematical Analysis (R)[1]

1. Let $x = (1,1,0)\,, y = (0,1,1) \in \mathbb{R}^3$. Compute the distance $d(x,y) = \ldots\ldots\ldots$ and $x \cdot y = \ldots\ldots\ldots$

2. Let $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x_1, x_2, x_3) = x_1 x_2 x_3 + x_3^2 x_2$. The partial derivatives of f are

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3. Let $f : [a,b] \to \mathbb{R}$ be a Riemann integrable function, $\Delta = \{a = x_0, x_1, \ldots, x_{n-1}, x_n = b\}$ a division and $\xi = \{\xi_1, \xi_2, \ldots \xi_{n-1}, \xi_n\}$ with $\xi_i \in [x_{i-1}, x_i]$ a system of intermediate points. The Riemann sum associated to f, Δ, ξ is

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4. The improper integral $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^2+1} dx$ (a) converges or (b) diverges? (mark the correct answer)

5. Let $D = [1,2] \times [0,1]$. Compute $\displaystyle\iint_D xy\, dxdy = \ldots\ldots\ldots\ldots\ldots\ldots$

6. According to the Theorem of Fermat for functions of several variables, if $f : B_r(x^*) \subset \mathbb{R}^n \to \mathbb{R}$ Fréchet differentiable in x^* and x^* is a local minimum (maximum) for f, then $\ldots\ldots\ldots = 0_{\mathbb{R}^n}$

7. Give an example of a quadratic function $Q : \mathbb{R}^2 \to \mathbb{R}$.

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8. The series $\displaystyle\sum_{n=0}^{\infty} q^n$, $|q| < 1$ (a) converges or (b) diverges? (mark the correct answer)

   [Prove your claim, for +1 bonus point.]

9. Fill in the next term of the Taylor expansion $f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \ldots\ldots$

10. Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ for which $\nabla f(x_1^*, x_2^*) = (0,0)$, at $(x_1^*, x_2^*) \in \mathbb{R}^2$, but (x_1^*, x_2^*) is neither a local minimum nor a local maximum (draw the graph of f if you prefer).

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Give detailed solutions to the following exercies on the next pages.

11. Compute $\displaystyle\iint_D \frac{1}{\sqrt{x^2+y^2}} dxdy$, where $D = \left\{(x,y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 4, x \ge 0, y \ge 0\right\}$.

12. Using Lagrange's mean value Theorem and the chain rule, prove that if $f : \mathbb{R}^n \to \mathbb{R}$ differentiable and $a = (a_1, \ldots, a_n)\,, b = (b_1, \ldots, b_n) \in \mathbb{R}^n$, then there exists $c \in [a,b] \subset \mathbb{R}^n$ on the segment connecting a and b, such that $f(b) - f(a) = \nabla f(c) \cdot (b - a)$.

13. a) Prove that the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$ converges for any $0 \le x < 1$.

    b) Argue that $\displaystyle\frac{1}{1+x^2} = 1 - x^2 + x^4 - \ldots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ is a Taylor expansion.

[1] 17.02.2020