Portofolio (Part I)

1. Plot $f:[0,\infty)\to\mathbb{R}$, $f(x)=xe^{x}$.
Find and add to the plot the two Taylor polynomials of degree $=2$ that approximate f locally at $x_0=0$ and x^* the maximum point of f.

2. Let $f: D\subseteq\mathbb{R}\to\mathbb{R}$ be Lipschitz continuous
(i.e. $\exists L>0$ s.t. $|f(x)-f(y)|\le L|x-y|$ $\forall x,y\in D$)
Prove that
a) f Lip. cont. $\Rightarrow$ f cont (on D)
b) $f:[0,\infty)\to\mathbb{R}$, $f(x)=\sqrt{x}$ is not Lip. cont. on $[0,\infty)$,
but f is Lip. cont. on $[a,\infty)$ for any $a>0$.

3. Let $f:\mathbb{R}^2\to\mathbb{R}$, $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=0.\end{cases}$

Study the continuity and partial differentiability of f at $(x,y)=(0,0)$.

4. Find the box of fixed volume $(=1)$ and minimal surface, i.e., solve

$$f(x,y,z)=2xy+2yz+2zx\longrightarrow\text{min!}$$

subject to $g(x,y,z)=xyz-1=0$.

What about the 2D case (rectangle of area 1 and minimal perimeter)?

Portofolio (Part II)

5. Find the antiderivative of $\frac{1}{(x^2+4)^2}$
(i.e. compute $\int \frac{dx}{(x^2+4)^2}$)

6. Compute $\iint_D \frac{xy}{x^2+1}\, dx\, dy$, where $D=[1,2]\times[0,1]$

7. Let D be delimited by $y=x^2$, $x=2$, $y=0$,
$\iint_D (x+2y) = ?$

8. $D=\{(x,y,z)\in\mathbb{R}^3 :\ 1\le x^2+y^2+z^2\le 4,\ z\ge 0\}$
Compute $\iiint_D \frac{dx\,dy\,dz}{x^2+y^2+z^2}$

9. Using differentiation w.r.t. a parameter, compute
$\int_0^1 \frac{x^5-1}{\ln x}\, dx$

10. a) Prove that $\Gamma(\frac{1}{2}) = 2\int_0^\infty e^{-x^2} dx$
b) Compute $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$