

Mathematical Analysis
Exercise Sheet 10

Compute

47. $\int_0^\infty e^{-x^2} dx$

48. $\int_0^\infty \frac{\sin x}{x} dx$

49. $\int_0^\infty e^{-x} \frac{\sin x}{x} dx$

50. The **Euler integrals** of first and second kind (also called **beta** and **gamma** functions) are special functions¹ given by

$$B(a, b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx, \quad a, b > 0 \quad \text{and} \quad \Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx, \quad a > 0.$$

- a) Check the convergence of $B(a, b)$ (for fixed a, b);
- b) Check the convergence of $\Gamma(a)$ (for fixed a);
- c) Prove that $\Gamma(a+1) = a\Gamma(a)$ for $a > 0$ and $\Gamma(n+1) = n!$ for $n \in \mathbb{N}$;
- d) Prove that $\Gamma(\frac{1}{2}) = \int_0^\infty e^{-x^2} dx$.

¹in this case functions defined by means of parametric improper integrals