Babeş-Bolyai University, Faculty of Mathematics and Computer Science Computer Science Groups 911-917, Academic Year 2021-2022

Mathematical Analysis Exercise Sheet 10

Compute

47.
$$\int_{0}^{\infty} e^{-x^2} dx$$

$$48. \int_0^\infty \frac{\sin x}{x} \mathrm{d}x$$

$$49. \int_0^\infty e^{-x} \, \frac{\sin x}{x} \mathrm{d}x$$

50. The **Euler integrals** of first and second kind (also called **beta** and **gamma** functions) are special functions¹ given by

$$B(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx$$
, $a,b>0$ and $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$, $a>0$.

- a) Check the convergence of B(a, b) (for fixed a, b);
- b) Check the convergence of $\Gamma(a)$ (for fixed a);
- c) Prove that $\Gamma(a+1) = a\Gamma(a)$ for a > 0 and $\Gamma(n+1) = n!$ for $n \in \mathbb{N}$;
- d) Prove that $\Gamma(\frac{1}{2}) = \int_0^\infty e^{-x^2} dx$.

¹in this case functions defined by means of parametric improper integrals