Mathematical Analysis
Exercise Sheet 6

27. Let $a = (1, 2) \in \mathbb{R}^2$. Find the point on the unit circle ($d = 2$ sphere)

$$S := \{x = (x_1, x_2) \in \mathbb{R}^2 : \|x\| = 1\}$$

which lies closest to a. Formulate and solve this as an Optimization problem.

28. Arithmetic-Geometric Mean Inequality. If $a, b, c > 0$, then

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc},$$

and equality holds if and only if $a = b = c$.

29. box of fixed volume and minimal surface.
Find the minimum of $f : \mathbb{R}^3_+ \to \mathbb{R},\ f(x, y, z) = 2xy + 2yz + 2zx$ subject to $xyz = 1$.
(Hint: use the Lagrange Multiplier Method in order to find a minimizer candidate and then use the inequality in **Ex. 28.** to prove that this candidate actually is a global minimizer.)

30. Find the local extrema of $f : \mathbb{R}^3_+ \to \mathbb{R},\ f(x, y, z) = xyz$ subject to $x + y + z = 1$.

HW 31. A large container in the shape of a rectangular solid must have a volume of $480m^3$. The bottom of the container costs $\$5/m^2$ to construct whereas the top and sides cost $\$3/m^2$ to construct. Use Lagrange multipliers to find the dimensions of the container that has minimum cost.