Practice 02: Intro to ML

MADMO, 2021

girafe ai

Outline

1. ML thesaurus and notation
2. Naïve Bayes classifier

ML thesaurus

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01

ML thesaurus

Denote the ***dataset***.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

ML thesaurus

Observation (or datum, or data point) is one piece of information.

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John	22	5	4	Brown	English	5	TRUE
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In many cases the observations are supposed to be ***i.i.d.***

- ***independent***
- ***identically distributed***

ML thesaurus

Feature (or predictor) represents some special property.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

ML thesaurus

These all are features

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Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
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ML thesaurus

These all are features

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ML thesaurus

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ML thesaurus

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Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

ML thesaurus

And even the name is a ***feature***

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

ML thesaurus

The ***design matrix*** contains all the features and observations.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Features can even be multidimensional, we will discuss it later in this course.

ML thesaurus

Target represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Target can be either a ***number*** *(real, integer, etc.) – for* ***regression*** *problem*

ML thesaurus

Target represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
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*Or a **label** – for **classification** problem*

ML thesaurus

Target represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Mark can be treated as a label too (due to finite number of labels: 1 to 5). We will discuss it later.

ML thesaurus

Further we will work with the numerical target (mark)

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)
John	22	5	4	Brown	English	5
Aahna	17	4	5	Brown	Hindi	4
Emily	25	5	5	Blue	Chinese	5
Michael	27	3	4	Green	French	5
Some student	23	3	3	NA	Esperanto	2

ML thesaurus

The ***prediction*** contains values we predicted using some ***model***.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.5
Aahna	17	4	5	Brown	Hindi	4	4.5
Emily	25	5	5	Blue	Chinese	5	5
Michael	27	3	4	Green	French	5	3.5
Some student	23	3	3	NA	Esperanto	2	3

One could notice that prediction just averages of Statistics and Python marks. So our ***model*** can be represented as follows:

$$\hat{\text{mark}}_{ML} = \frac{1}{2}\text{mark}_{Statistics} + \frac{1}{2}\text{mark}_{Python}$$

ML thesaurus

The ***prediction*** contains values we predicted using some ***model***.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.5
Aahna	17	4	5	Brown	Hindi	4	4.5
Emily	25	5	5	Blue	Chinese	5	5
Michael	27	3	4	Green	French	5	3.5
Some student	23	3	3	NA	Esperanto	2	3

Different models can provide different predictions:

$$\hat{\text{mark}}_{ML} = \frac{1}{2}\text{mark}_{Statistics} + \frac{1}{2}\text{mark}_{Python}$$

ML thesaurus

The ***prediction*** contains values we predicted using some ***model***.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	1
Aahna	17	4	5	Brown	Hindi	4	5
Emily	25	5	5	Blue	Chinese	5	2
Michael	27	3	4	Green	French	5	4
Some student	23	3	3	NA	Esperanto	2	3

Different models can provide different predictions:

$$\hat{\text{mark}}_{ML} = \text{random}(\text{integer from } [1;\, 5])$$

ML thesaurus

The ***prediction*** contains values we predicted using some ***model***.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	1
Aahna	17	4	5	Brown	Hindi	4	5
Emily	25	5	5	Blue	Chinese	5	2
Michael	27	3	4	Green	French	5	4
Some student	23	3	3	NA	Esperanto	2	3

Different models can provide different predictions.

*Usually some **hypothesis** lies beneath the model choice.*

Naïve Bayes classifier

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02

Naïve Bayes classifier

Let's denote:

- Training set $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^{n}$, where
  - $\mathbf{x}_i \in \mathbb{R}^p$, $y_i \in \{C_1, \ldots, C_k\}$ for k-class classification

Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

or, in our case

$$P(y_i = C_k|\mathbf{x}_i) = \frac{P(\mathbf{x}_i|y_i = C_k)P(y_i = C_k)}{P(\mathbf{x}_i)}$$

Naïve Bayes classifier

Let's denote:

- Training set $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^{n}$, where
  - $\mathbf{x}_i \in \mathbb{R}^p$, $y_i \in \{C_1, \ldots, C_K\}$ for K-class classification

$$P(y_i = C_k|\mathbf{x}_i) = \frac{P(\mathbf{x}_i|y_i = C_k)P(y_i = C_k)}{P(\mathbf{x}_i)}$$

Naïve assumption: features are ***independent***

Naïve Bayes classifier

$$P(y_i = C_k|\mathbf{x}_i) = \frac{P(\mathbf{x}_i|y_i = C_k)P(y_i = C_k)}{P(\mathbf{x}_i)}$$

Naïve assumption: features are ***independent:***

$$P(\mathbf{x}_i|y_i = C_k) = \prod_{l=1}^{p} P(x_i^l|y_i = C_k)$$

Naïve Bayes classifier

$$P(y_i = C_k|\mathbf{x}_i) = \frac{P(\mathbf{x}_i|y_i = C_k)P(y_i = C_k)}{\cancel{P(\mathbf{x}_i)}}$$

Optimal class label:

$$C^* = \arg\max_k P(y_i = C_k|\mathbf{x_i})$$

To find maximum we even do not need the denominator

But we need it to get probabilities

Let’s Practice

Thanks for attention!

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Model validation and evaluation

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Supervised learning problem statement

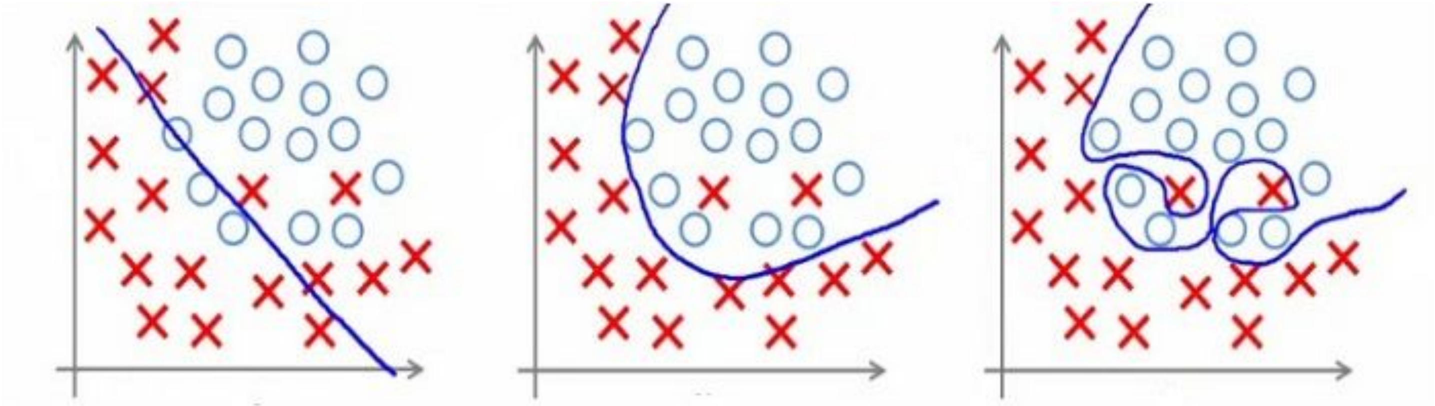
Let's denote:

- Training set $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^{n}$, where
  - $(\mathbf{x} \in \mathbb{R}^p,\ y \in \mathbb{R}\ )$ for regression
  - $\mathbf{x}_i \in \mathbb{R}^p$, $y_i \in \{+1, -1\}$ for binary classification

Model $f(\mathbf{x})$ predicts some value for every object

Loss function $Q(\mathbf{x}, y, f)$ that should be minimized

Overfitting vs. underfitting



Under-fitting

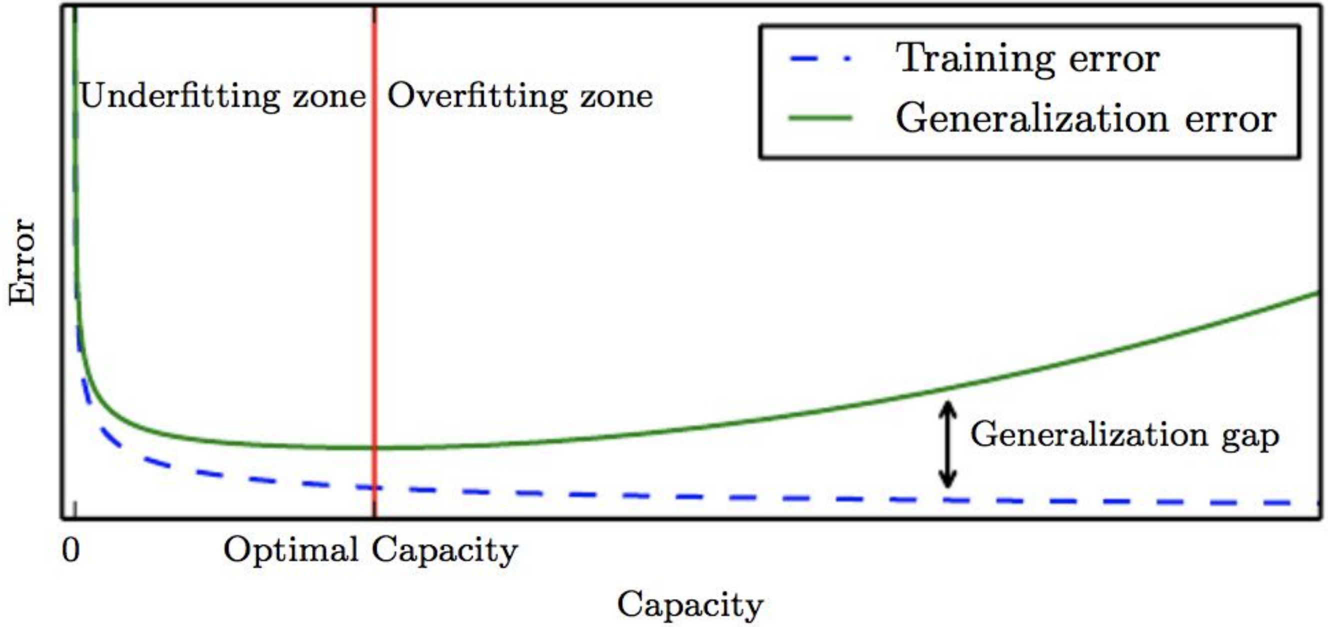
(too simple to explain the variance)

Appropriate-fitting

Over-fitting

(forcefitting -- too good to be true)

Overfitting vs. underfitting



Overfitting vs. underfitting

- We can control overfitting / underfitting by altering model's capacity (ability to fit a wide variety of functions):
- select appropriate hypothesis space
- learning algorithm's effective capacity may be less than the representational capacity of the model family

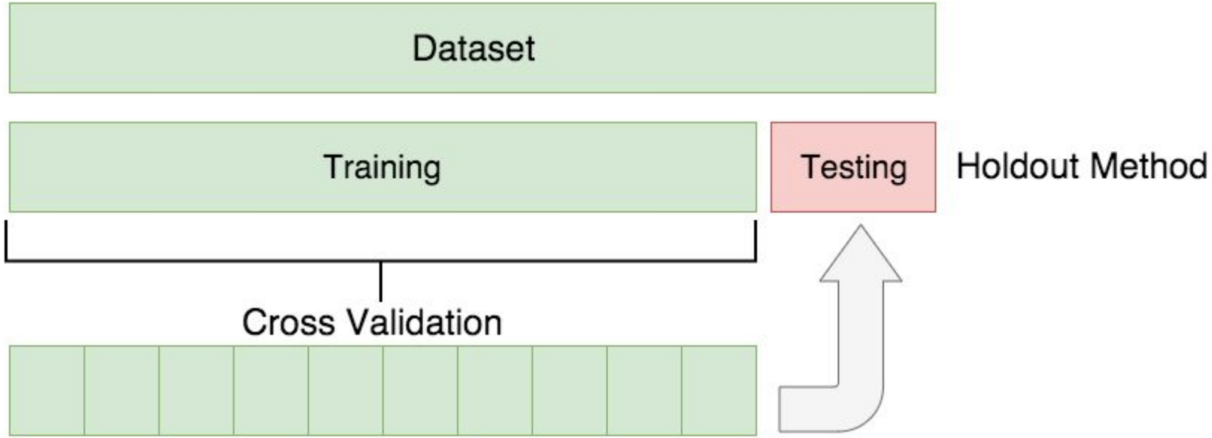
Evaluating the quality

Evaluating the quality

Is it good enough?

Image credit: Joseph Nelson [@josephofiowa]

Evaluating the quality



Evaluating the quality

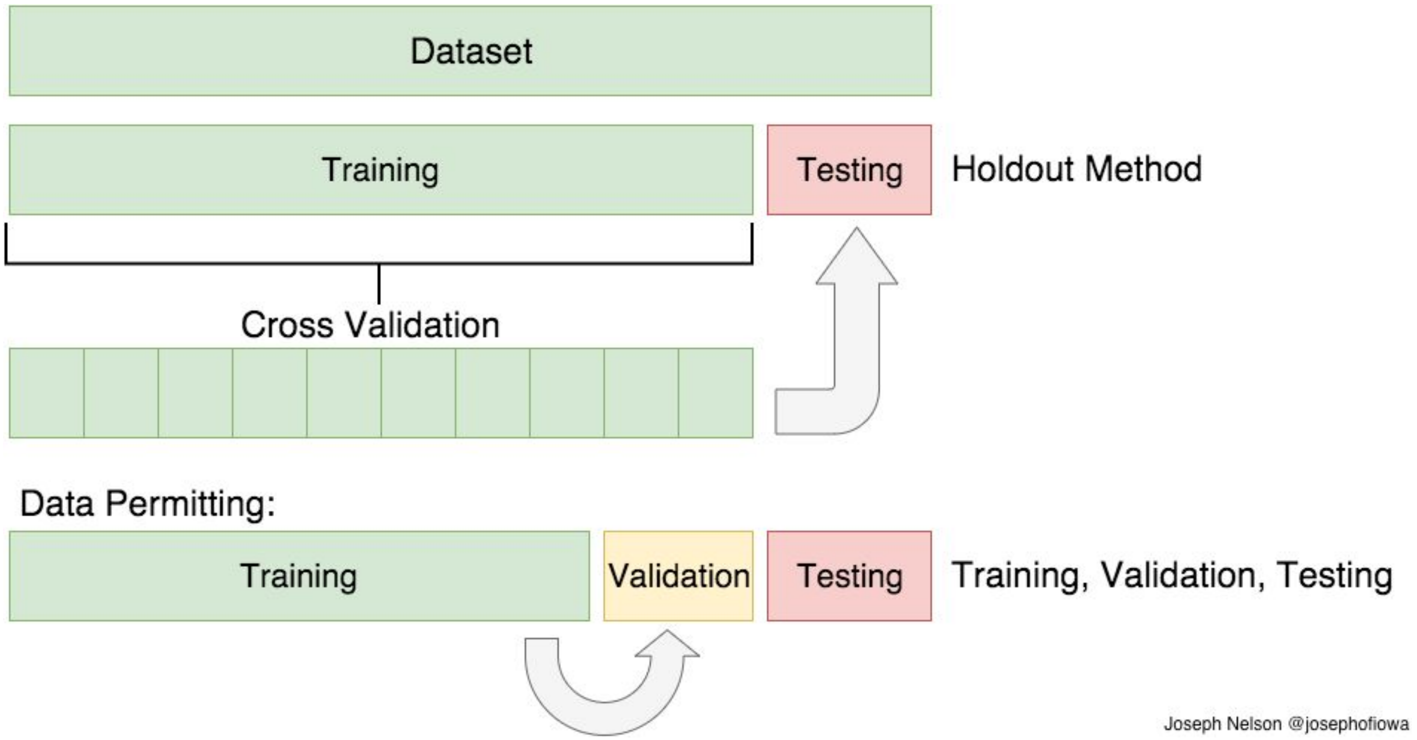


Image credit: Joseph Nelson @josephofiowa

Cross-validation