Курс МАДМО базовый

Лекция 9
Градиентный бустинг

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МФТИ, осень 2021

Outline

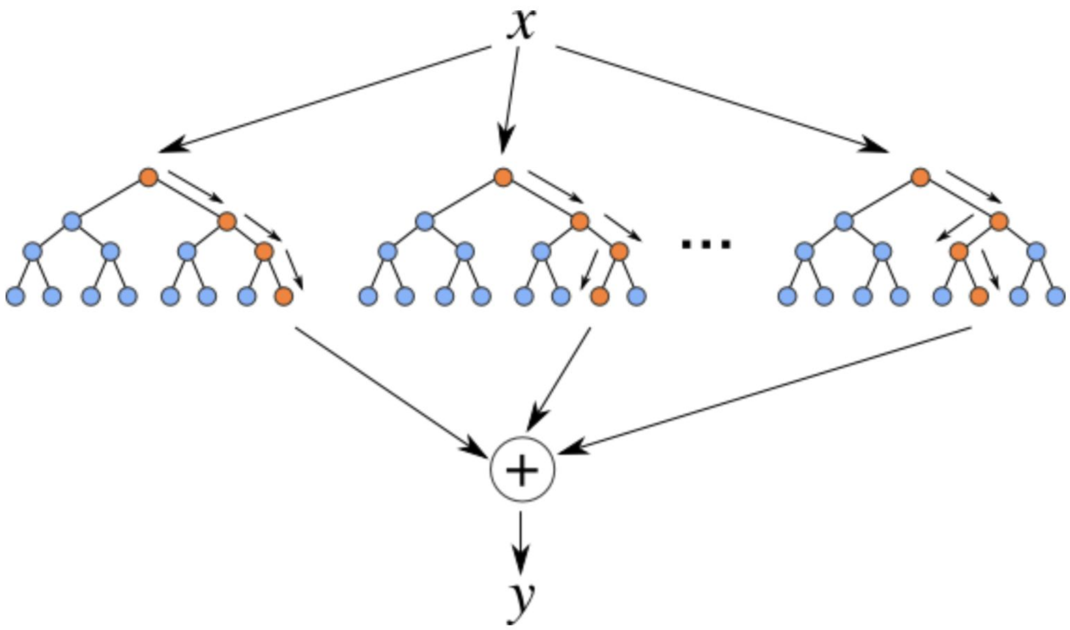
1. Bagging recap
2. Simple boosting
3. Gradient boosting theory

Bagging recap

01

Random Forest

Bagging + RSM = Random Forest

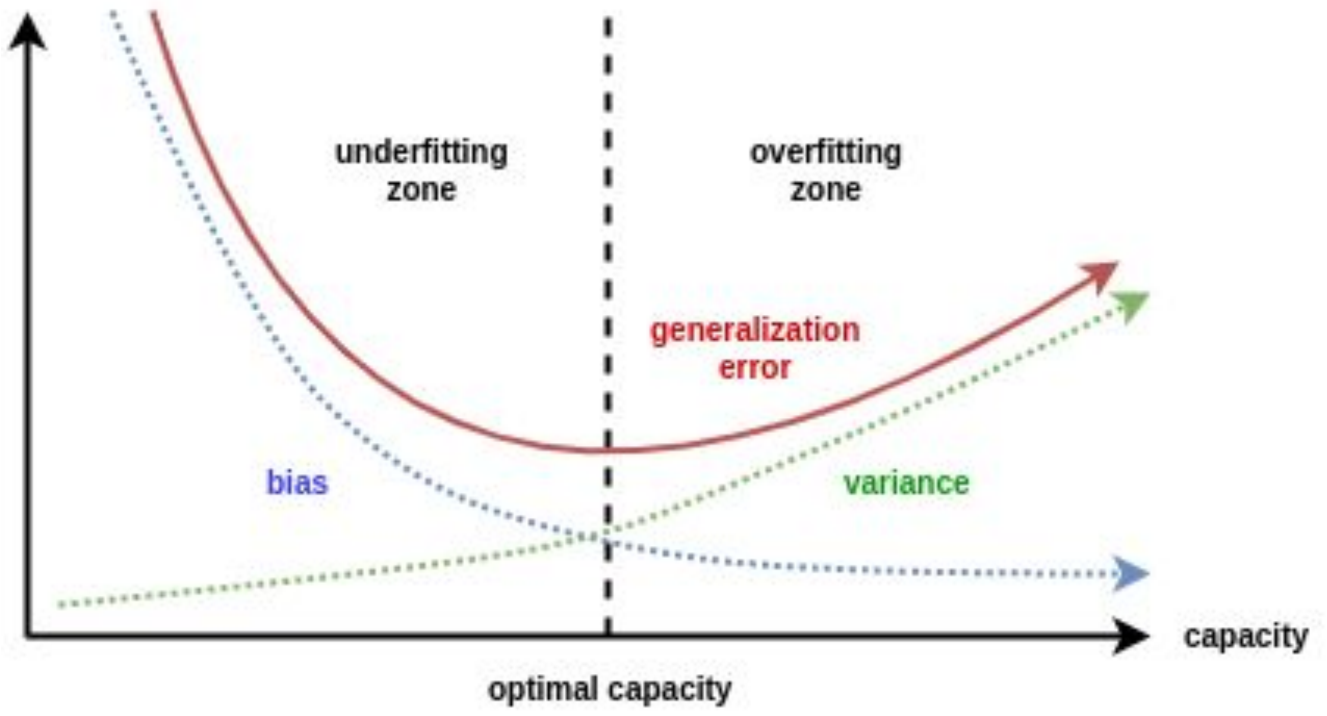


Random Forest

- One of the greatest “universal” models.
- There are some modifications: Extremely Randomized Trees, Isolation Forest, etc.
- Allows to use train data for validation: OOB

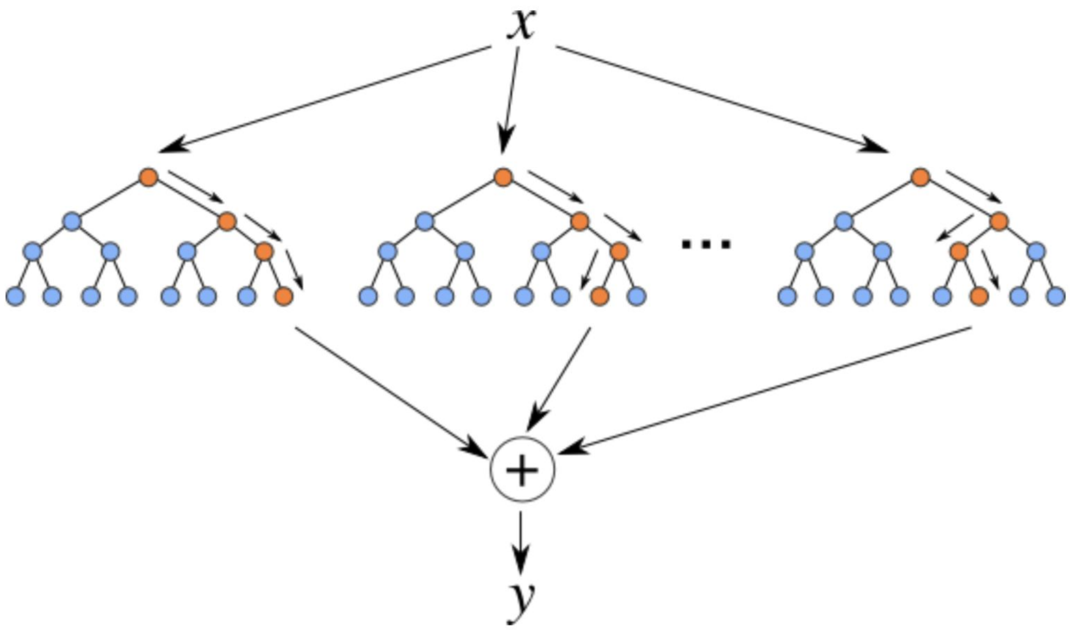
$$\mathrm{OOB} = \sum_{i=1}^{\ell} L\left(y_i, \frac{1}{\sum_{n=1}^{N}[x_i \notin X_n]} \sum_{n=1}^{N} [x_i \notin X_n] b_n(x_i)\right)$$

Bias-variance tradeoff



Random Forest

Is Random Forest decreasing bias or variance by building the trees ensemble?



Simple boosting

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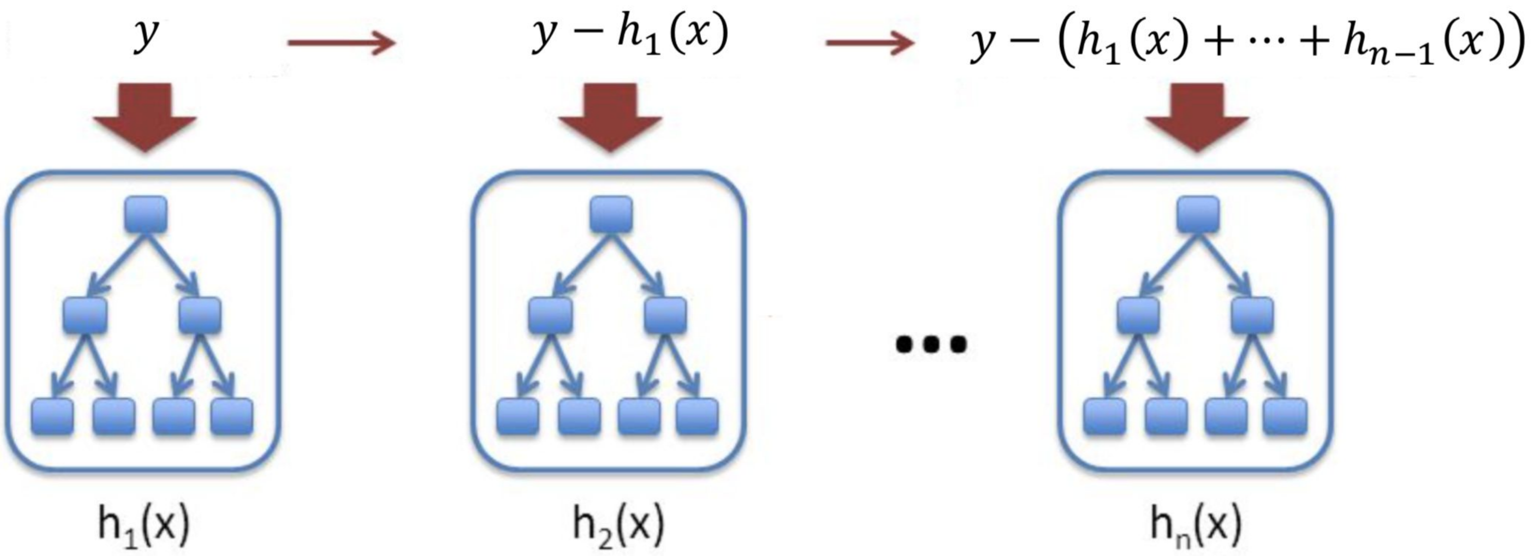
ai

02

Boosting

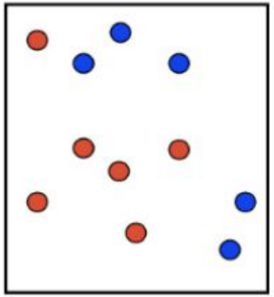
$$a_n(x) = h_1(x) + \cdots + h_n(x)$$

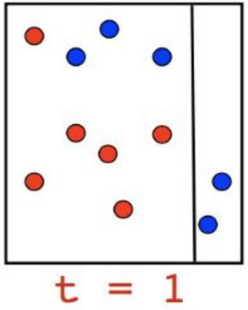


Boosting: intuition

Binary classification

Use decision stumps.





Boosting: intuition

Binary classification

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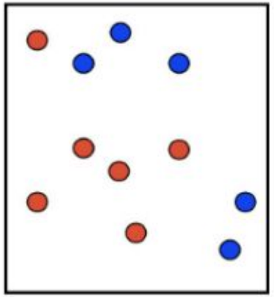

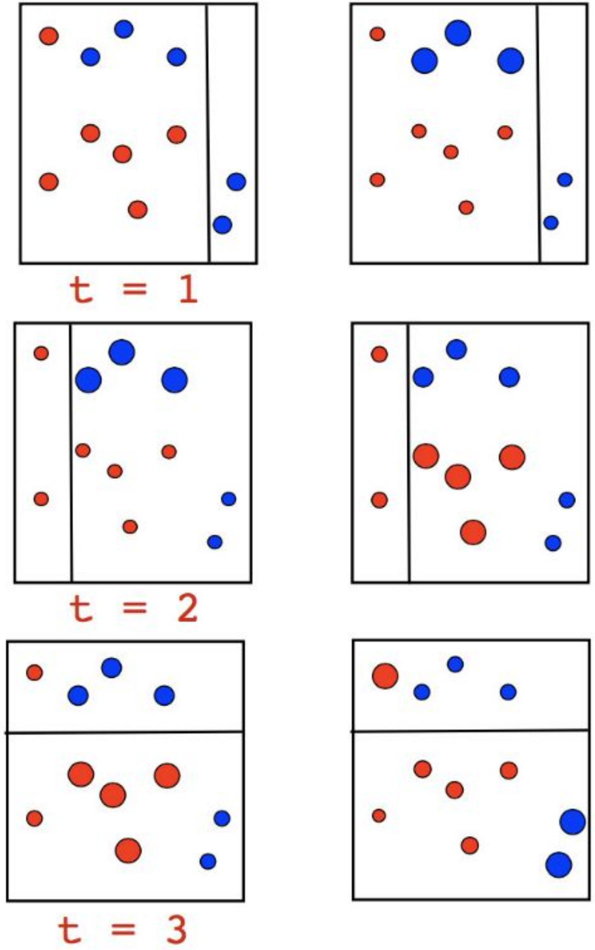

Boosting: intuition

Binary classification

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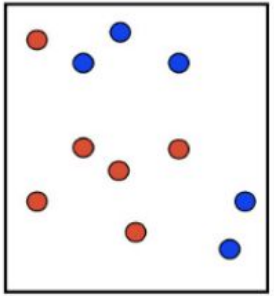
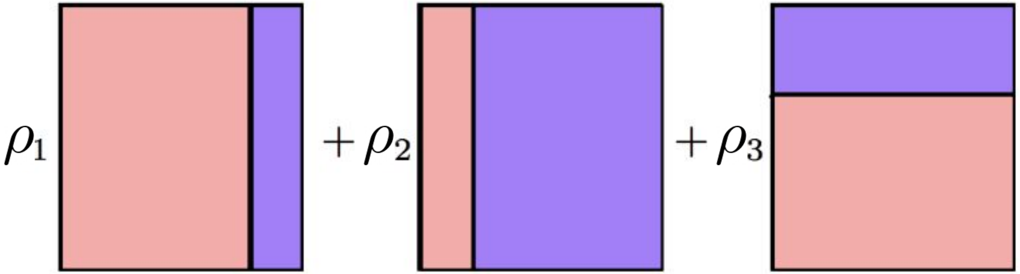
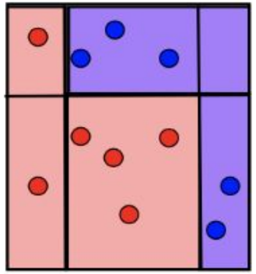
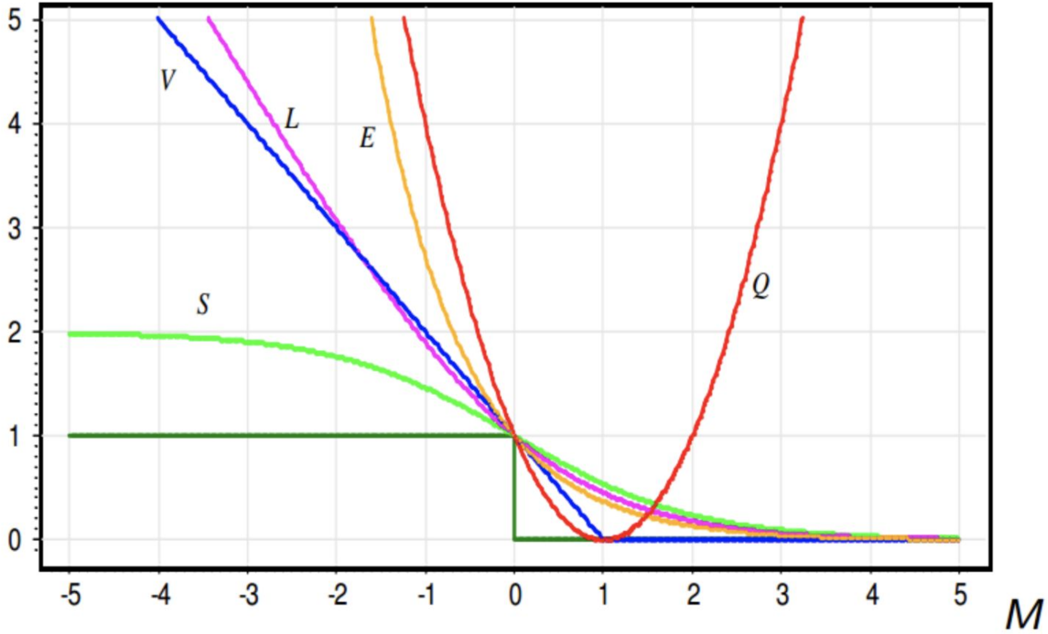




$$\hat{f}_T(x) = \sum_{t=1}^{T} \rho_t h_t(x) =$$

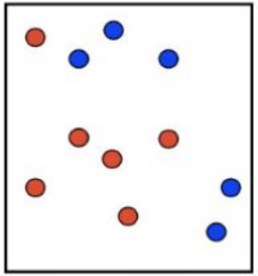


Recap: loss functions for classification



$$Q(M) = (1 - M)^2$$
$$V(M) = (1 - M)_+$$
$$S(M) = 2(1 + e^M)^{-1}$$
$$L(M) = \log_2(1 + e^{-M})$$
$$E(M) = e^{-M}$$

Boosting: AdaBoost



$$\hat{f}_T(x) = \sum_{t=1}^{T} \rho_t h_t(x)$$

$$L(y_i, \hat{f}_T(x_i)) = \exp\left(-y_i \hat{f}_T(x_i)\right) = \exp\left(-y_i \sum_{t=1}^{T} \rho_t h_t(x_i)\right)$$

$$= \boxed{\exp\left(-y_i \sum_{t=1}^{T-1} \rho_t h_t(x_i)\right)} \cdot \exp(-y_i \rho_T h_T(x_i))$$

const on step T

$$= w_i \cdot \exp(-y_i \rho_T h_T(x_i))$$

Gradient boosting theory

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Gradient boosting: theory

Denote dataset $\{(x_i, y_i)\}_{i=1,\dots,n}$, loss function $L(y, f)$.

Optimal model:

$$\hat{f}(x) = \underset{f(x)}{\arg\min}\ L(y, f(x)) = \underset{f(x)}{\arg\min}\ \mathbb{E}_{x,y}[L(y, f(x))]$$

Let it be from parametric family:

$$\hat{f}(x) = f(x, \hat{\theta}),$$

$$\hat{\theta} = \underset{\theta}{\arg\min}\ \mathbb{E}_{x,y}[L(y, f(x, \theta))]$$

Gradient boosting: theory

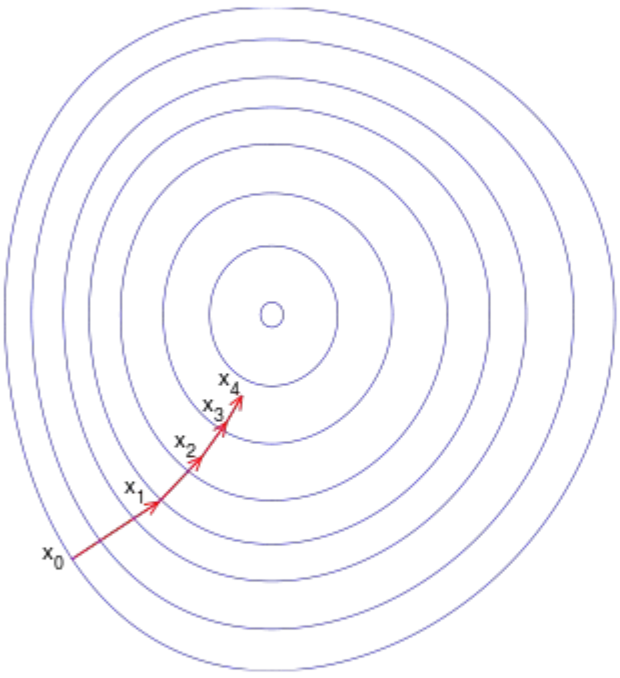
$$\hat{f}(x) = \sum_{i=0}^{t-1} \hat{f}_i(x),$$

$$(\rho_t, \theta_t) = \arg\min_{\rho,\theta} \mathbb{E}_{x,y}[L(y, \hat{f}(x) + \rho \cdot h(x, \theta))],$$

$$\hat{f}_t(x) = \rho_t \cdot h(x, \theta_t)$$

What if we could use gradient descent in space of our models?

Gradient boosting: theory



What if we could use gradient descent in space of our models?

Gradient boosting: theory

$$\hat{f}(x) = \sum_{i=0}^{t-1} \hat{f}_i(x),$$

$$r_{it} = -\left[\frac{\partial L(y_i, f(x_i))}{\partial f(x_i)}\right]_{f(x)=\hat{f}(x)}, \quad \text{for } i = 1, \ldots, n,$$

$$\theta_t = \arg\min_{\theta} \sum_{i=1}^{n} (r_{it} - h(x_i, \theta))^2,$$

$$\rho_t = \arg\min_{\rho} \sum_{i=1}^{n} L(y_i, \hat{f}(x_i) + \rho \cdot h(x_i, \theta_t))$$

Gradient boosting: theory

In linear regression case with MSE loss:

$$r_{it} = -\left[\frac{\partial L(y_i, f(x_i))}{\partial f(x_i)}\right]_{f(x)=\hat{f}(x)} = -2(\hat{y}_i - y_i) \propto \hat{y}_i - y_i$$

Gradient boosting: beautiful demo

Great demo:
http://arogozhnikov.github.io/2016/06/24/gradient_boosting_explained.html

Gradient boosting

What we need:

- Data.
- Loss function and its gradient.
- Family of algorithms (with constraints if necessary).
- Number of iterations M.
- Initial value (GBM by Friedman): constant.

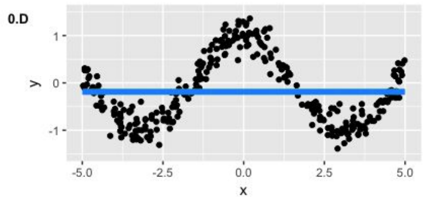
Gradient boosting: example

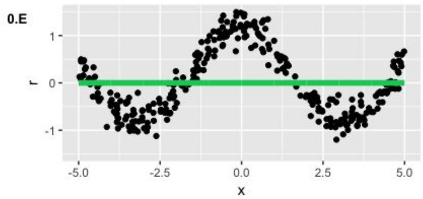
What we need:

- Data: toy dataset $y = cos(x) + \epsilon, \epsilon \sim \mathcal{N}(0, \frac{1}{5}), x \in [-5, 5]$
- Loss function: MSE
- Family of algorithms: decision trees with depth 2
- Number of iterations M = 3
- Initial value: just mean valu

Example by ODS; source: https://habr.com/ru/company/ods/blog/327250/

Gradient boosting: example



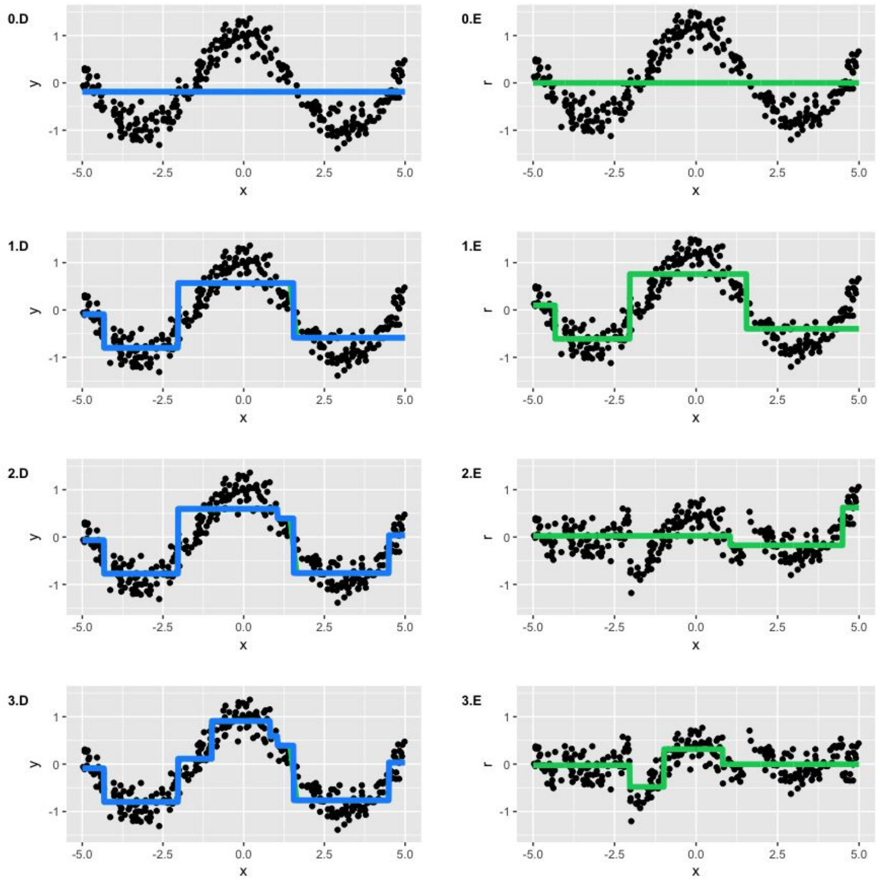


Left: full ensemble on each step.

Right: additional tree decisions.

Example by ODS; source: https://habr.com/ru/company/ods/blog/327250/

Gradient boosting: example



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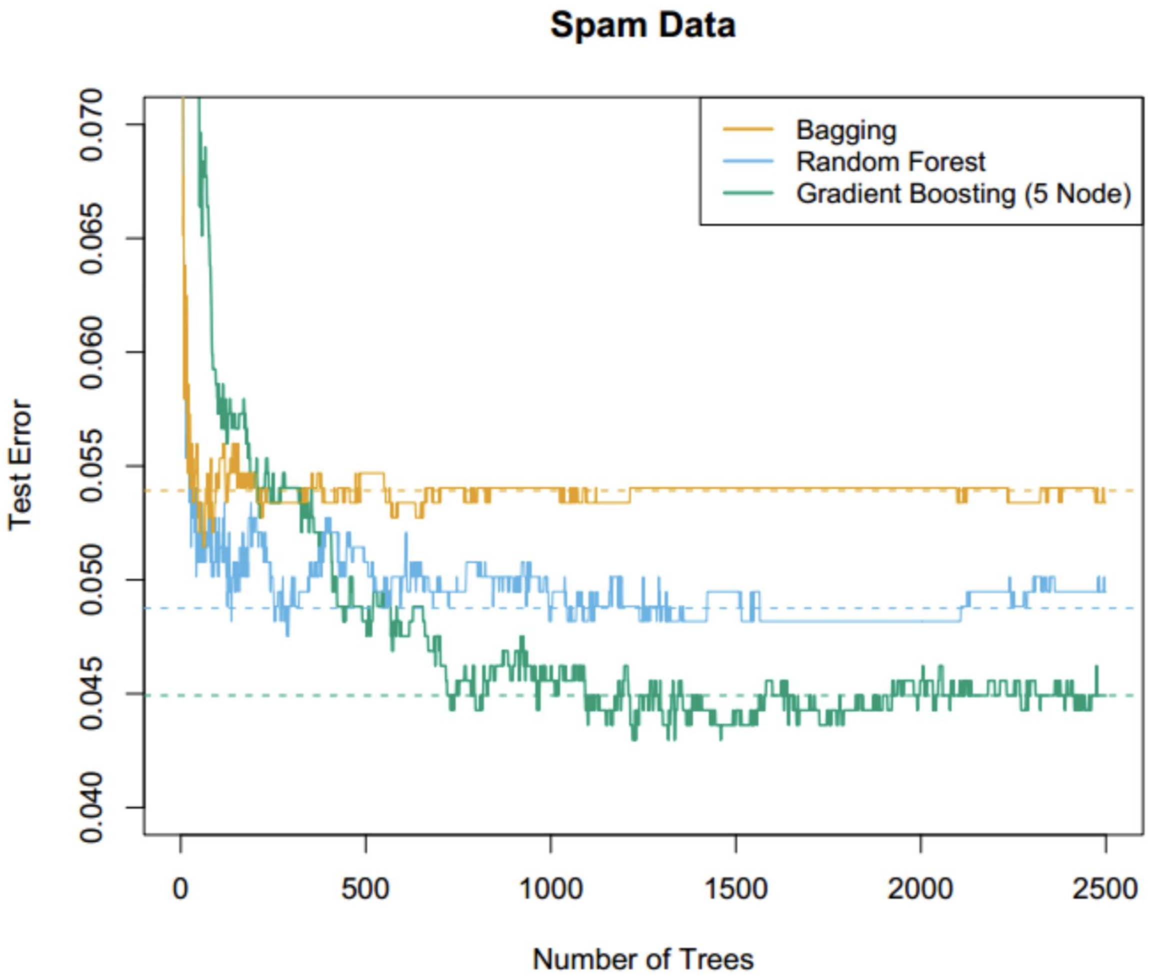
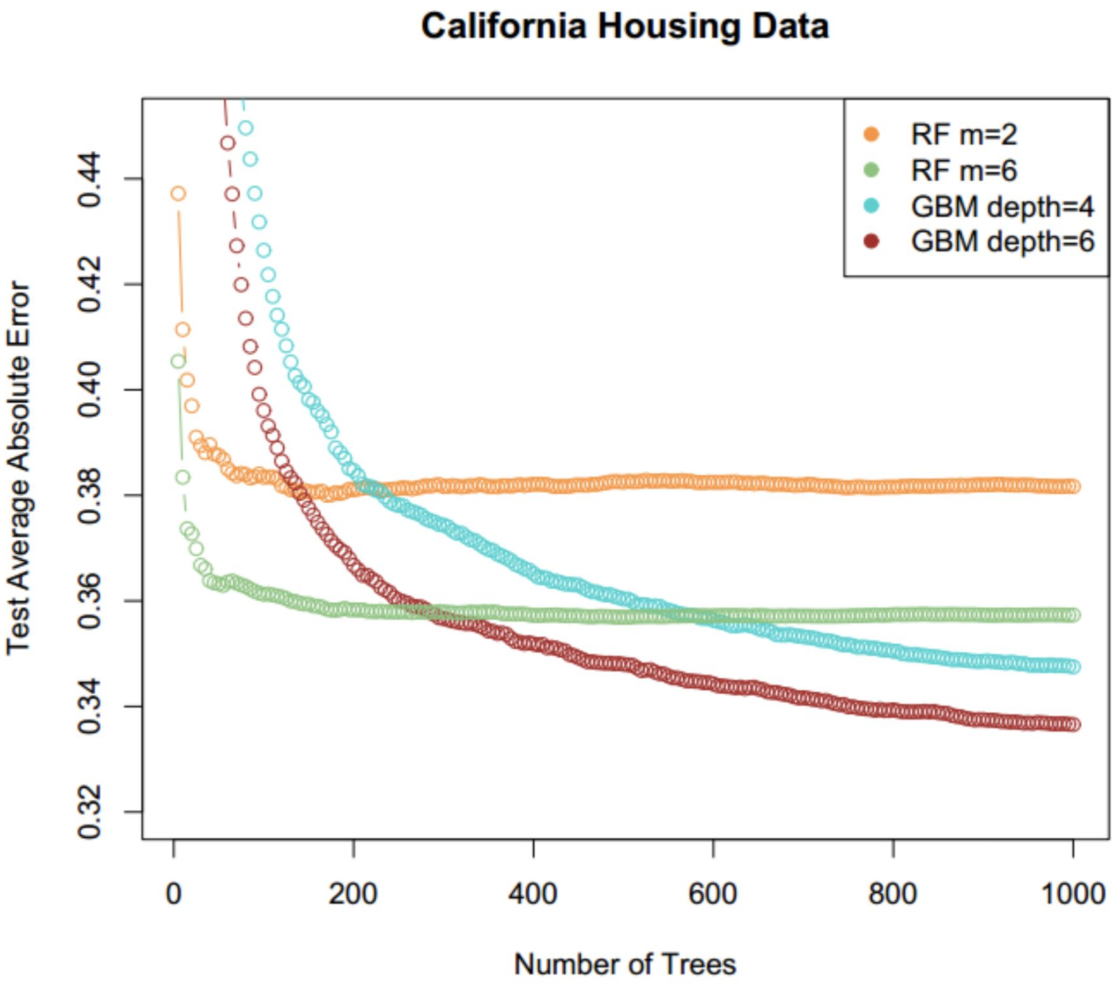
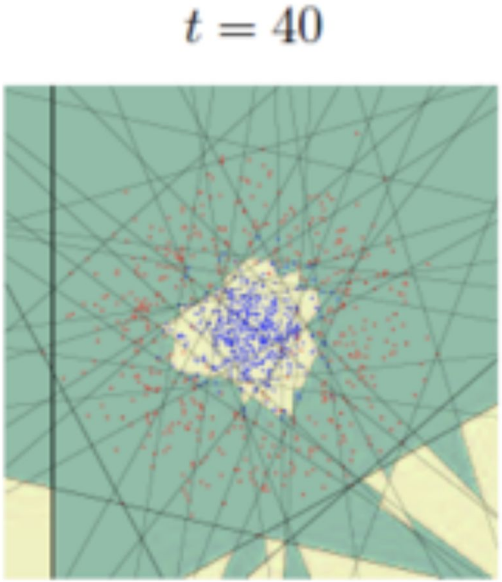
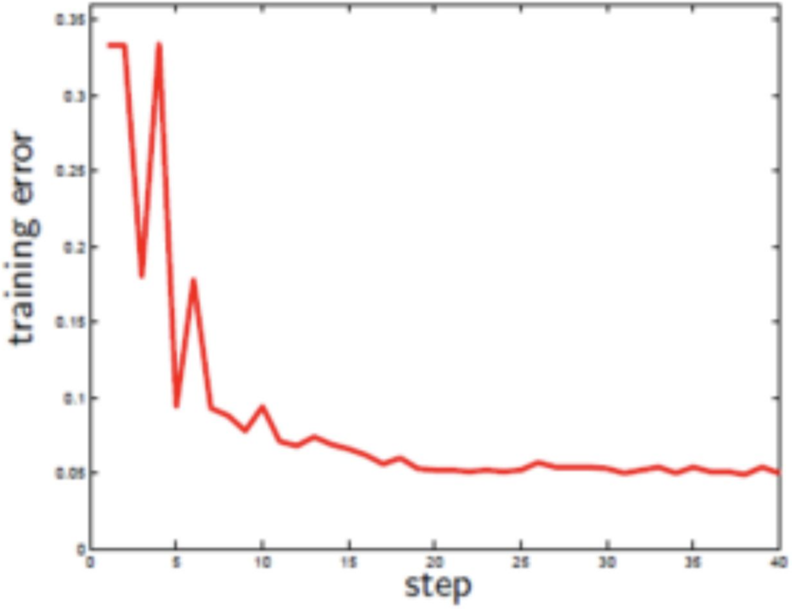


Spam Data
Bagging
Random Forest
Gradient Boosting (5 Node)
Test Error
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0
500
1000
1500
2000
2500
Number of Trees


California Housing Data
RF m=2
RF m=6
GBM depth=4
GBM depth=6
Test Average Absolute Error
0.32
0.34
0.36
0.38
0.40
0.42
0.44
0
200
400
600
800
1000
Number of Trees

Boosting with linear classification methods





Technical side: training in parallel

Which of the ensembling methods could be parallelized?

- Random Forest: parallel on the forest level (all trees are independent)
- Gradient boosting: parallel on one tree level

Revise

1. Bagging recap
2. Simple boosting
3. Gradient boosting theory

Thanks for attention!

Questions?

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