Research Idea

我想要解決什麼問題?

在多目標追蹤任務中同時達到 A & B，所以要透過 Dynamic Role assignment 來均衡 task A & B 的分配

Target Position

↓

Global voronoi Partition

↓

Compute Global Voronoi cells' centroids

↓

Treat agent's centroid as target with mixing cost function

↓

use gradient descent to move

Mixing function

Dynamic role assignment

Voronoi Cell (Global / Local)

Gradient Descent / Ascent

High-order Voronoi (k-covered region)

Power Diagram (Weighted Voronoi) ✓

K-coverage problem (NP-Hard)

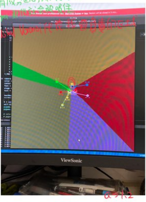
目前的效果

1. 沒有照到Target的人會考慮比較大的區域 (確保所有Target都有人照到 (所有 agent 都有照到 Target)

⇒ 沒有人照到的 Target 就会在我的 Voronoi 裡 就会想去追

⇒ 沒有做分配，所以如果移动过程中有照到其他 Target 那他就会被吸住

Global Voronoi代表我自責的區域



$a \to 1, 2$

發現還是会發生一追二跟 二追一後 沒辦法 使 b 去追 1 or 2

$b, c \to 3$

⇒ free player 要考慮其他 target?

式者是 power diagram 的 objective 再設計一下 就可以

Hierarchical Voronoi-based Multi-Target Tracking

key words:

Voronoi-Tree map

Mixing function

Voronoi diagram for directional anisotropic sensor

power diagram

$$\varphi_{res}(x) = \frac{\|x-p\|^{\lambda}}{R^{\lambda+1}} \left(R\cos\alpha - \lambda(\|x-p\| - R\cos\alpha)\right)$$

Role: leader

I. Partition the convex environment such that each target can guarantee at least one pursuer.

1.

a. Sensor with greater range should account for larger voronoi cell

b. Sensor with less information within their FoV (local Voronoi cell) should account for larger voronoi cell

Power Diagram

i. what if the sensor is heading toward the middle point in between two targets within its global voronoi cell?

2.

- Global Mode → Local Mode

→ 切換標準:

1. Global → Local ➡ Reach Global Voronoi cell centroid

2. Local → Global ➡ Another agent has higher quality within its local Voronoi cell.

Role: Coordinator

II. Deployment of extra agents

a. Assigning them to Leaders' Voronoi area

Inserting to balance the treemap

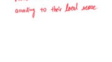
(Each branch should contribute equal quality?)

b. Perform Second Voronoi within the leader's voronoi cell

⇒ N layer Treemap (Voronoi Tree map)

L_i : i^{th} Leader　C_{ij} : j^{th} Coordinator w.r.t i^{th} Leader

Ex: 3 targets



Role on the Same branch can be interchangable according to their local score.

Scenario

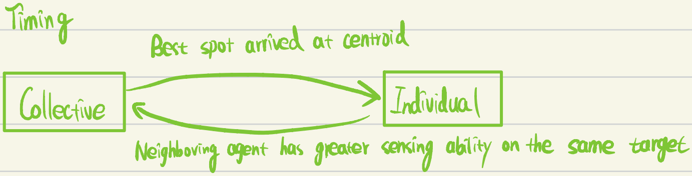
Multi-Target Tracking

- Task assignment
- Tracking

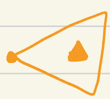
I. Task Assignment through Weighted Voronoi Diagram

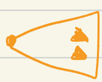
Power Diagram {
1. ~~Greater sensing range induces larger weight~~ (R not $R\cos(\alpha)$)
2. More unused sensing ability results in larger weight

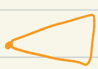
- **Mode Switching to balance Individual and Collective quality**
  - Collective mode:
    Move the best sensing spot toward the centroid of Global Voronoi Diagram
  - Individual mode:
    Move the FoV via gradient descent.
  - Switch Timing



★ How to compute unused sensing capacity?

 → Ideal configuration

 → excess information

 → insufficient of information

ver 2. Schedule

1. Implement Power Diagram with disk of sensing FoV
2. Add local orientation estimation
3. Add Task allocation feature

$$g(q) := \frac{\|q-p\|^{\lambda}}{R^{\lambda+1}}\left(R\cos\alpha - \lambda(\|q-p\| - R\cos\alpha)\right)$$

$$\frac{\partial g(q)}{q_x} = \frac{\left((q_x-p_x)^2+(q_y-p_y)^2\right)^{\frac{\lambda}{2}}}{R^{\lambda+1}}\left(R\cos\alpha - \lambda\left((q_x-p_x)^2+(q_y-p_y)^2\right)^{\frac{1}{2}} + \lambda R\cos\alpha\right)$$

$$= \frac{R\cos\alpha}{R^{\lambda+1}}\left((q_x-p_x)^2+(q_y-p_y)^2\right)^{\frac{\lambda}{2}}$$

$$- \frac{\lambda}{R^{\lambda+1}}\left((q_x-p_x)^2+(q_y-p_y)^2\right)^{\frac{\lambda-1}{2}}$$

$$+ \frac{\lambda R\cos\alpha}{R^{\lambda+1}}\left((q_x-p_x)^2+(q_y-p_y)^2\right)^{\frac{\lambda}{2}}$$

λ

$$= \frac{\cos\alpha}{R^{\lambda}} \left(\lambda \cdot (x-P_x)\left((x-P_x)^2+(y-P_y)^2\right)^{\frac{\lambda-2}{2}} \right)$$

$$+ \frac{\lambda \cos\alpha}{R^{\lambda}} \left(\lambda \cdot (x-P_x)\left((x-P_x)^2+(y-P_y)^2\right)^{\frac{\lambda-2}{2}} \right)$$

$$- \frac{\lambda}{R^{\lambda+1}} \left((\lambda-1)(x-P_x)\left((x-P_x)^2+(y-P_y)^2\right)^{\frac{\lambda-3}{2}} \right.$$

$$= \frac{\cos\alpha}{R^{\lambda}} \left(\lambda (x-P_x) \cdot \|q-P\|^{\lambda-2} \right)$$

$$+ \frac{\lambda\cos\alpha}{R^{\lambda}} \left(\lambda \cdot (x-P_x) \cdot \|q-P\|^{\lambda-2} \right)$$

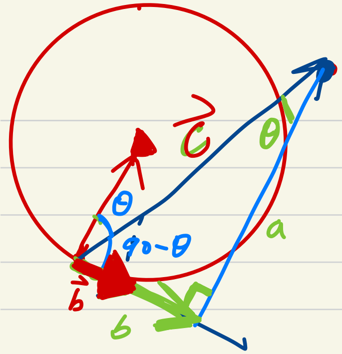
$$- \frac{\lambda}{R^{\lambda+1}} \left((\lambda-1)(x-P_x) \cdot \|q-P\|^{\lambda-3} \right.$$

$$= \frac{\cos\alpha}{R^{\lambda}} \cdot (x-P_x) \cdot \|q-P\|^{\lambda-2} (\lambda^2+\lambda) - \frac{\lambda}{R^{\lambda+1}} \left((\lambda-1)(x-P_x) \cdot \|q-P\|^{\lambda-3} \right)$$

Positional Control

$$\min \int_V \|P_i - q\|^2 \, dp \quad s.t. \ \underline{\underline{\|P_i - C_v\| = R\cos\alpha}}$$ 别管

$$\vec{CP} - R \cdot \cos\alpha \cdot \frac{\vec{CP}}{\|\vec{CP}\|} = \vec{CP}\left(1 - \frac{R\cos\alpha}{\|\vec{CP}\|}\right)$$



$$\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|^2}\cdot\vec{b}$$

- [x] Tracker Implementation
- [ ] Interceptor Implementation
- [ ] Treemap Sorting
- [ ] Mode Switching (Tracker & Interceptor)

Distributive Role selection algorithm

For Sensor Node $S_i, i \in \{0, \ldots\ldots, n\}$

if Targets exist in S_i's LocalVoronoiCell
  $S_i \leftarrow$ Tracker
else
  $S_i \leftarrow$ Interceptor

Distributive Tracker Control Law

T ← neighboring Tracker
I ← Suburdinated Interceptor

1. ComputeGlobalVoronoi(T)
2. ComputeLocalVoronoi(T)
3. Move to Centroid & Allign with Geo Centroid.
4. Compute branch score(I)

Distributive Interceptor Control Law

T ← Landlord Tracker
I ← neighboring Interceptor with Same landlord

1. ComputeSubGlobalVoronoi(I,T)
2. ComputeLocalVoronoi(I,T)
3. Move to Centroid

I. Control Law

$\lambda C_1 + (1-\lambda) C_2$

$\boxed{\text{* How to design weight } \lambda \text{ ?}}$

C_1 : Move Sweet Spot to Global Voronoi Centroid

C_2 : Move UAV to Sub Global Voronoi Centroid

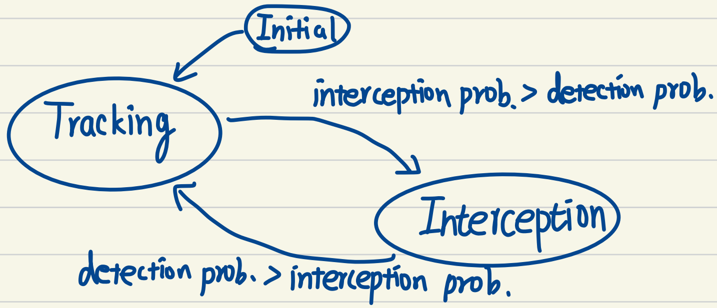
3 ways to design the controller

a. linear Combination using probability

b. Sliding mode control

c. RISE control ???

II. Switching Diagram.



Cost function :

(Joint detection probability)·(Joint Interception probability)

Multi-target to single target

*有可能出現大家都進到 Tracking Mode

- Detection Quality

$\int_Q q_{res} \cdot q_{per}$

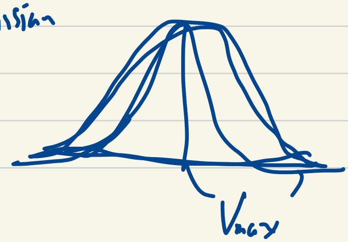
q_{res}: beta function 

q_{per}: normalized perspective

- Interception Quality

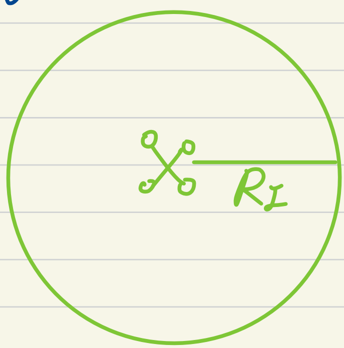
$\int_Q q_{dist}$

q_{dist}: Gaussian



III Sensing Model

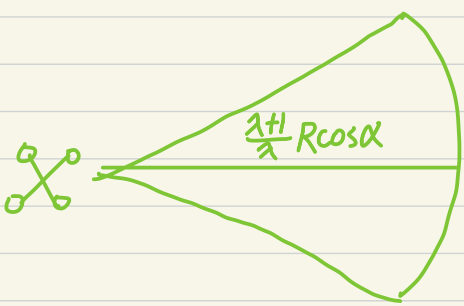
a. Close-range model for interception



R_I : Interception Range

$(R_I \propto V_{imax})$

b. Long-range model for tracking



λ : Beta function Param.

R : Desired Sensing Range

α : Angle of View

IV Voronoi Diagram

a. Global Voronoi Diagram

- For Tracker only
- intends to evenly assign the target to tracker
- used to derive tracker controller

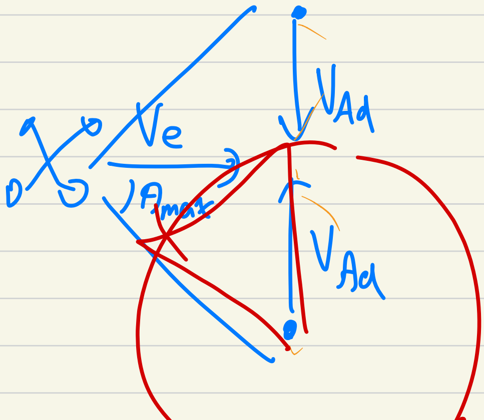
b. Sub Global Voronoi Diagram

- For both Tracker and Interceptor
- intends to distribute interceptors considering trackers position
- used to derive interceptor controller

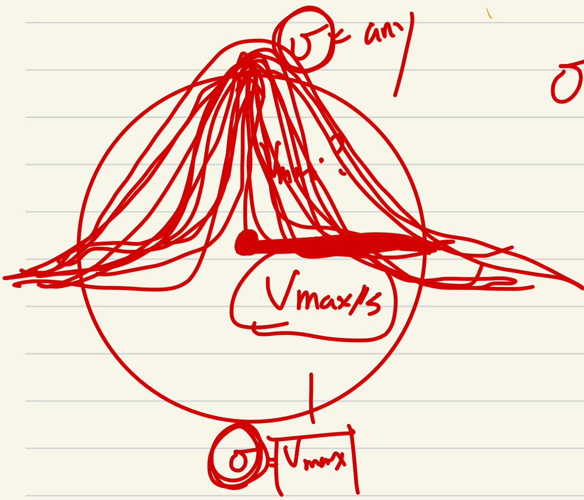
c. Local Voronoi Diagram

- derived from Tracker's FoV

V Evaluation Matrices



考虑 Sensor Range & Interception Range



σ depends on which interception method we chose

$\frac{1}{10m} = 0.1$

Research Schedule

- [ ] ~~map current sensing quality to detection probability~~
- [ ] ~~map distance to interception probability~~
- [ ] ~~implement sensor mode switching with detection & interception probability~~
- [ ] bridging the gap between two control laws
- [ ] Implement the code in a distributive manner.

Q1. How to determine the timing of behavior switching

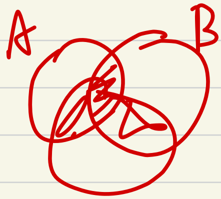
1. Comparison of individual's interception score and tracking score

=> independent of its neighbor

=> achieve highest quality of the cost function is (int + tracking)

2. Comparison of neighbors's tracking score.

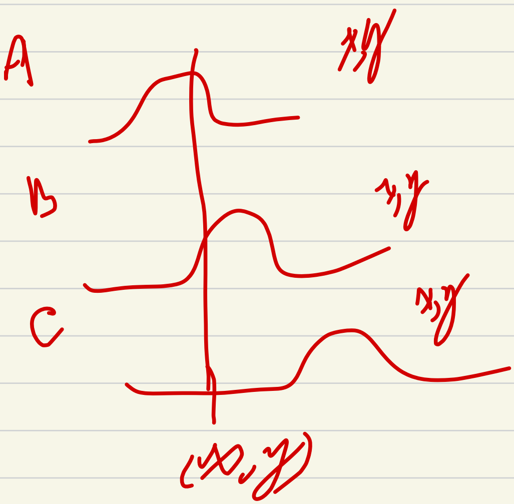
=> if the amount of useful tracking information



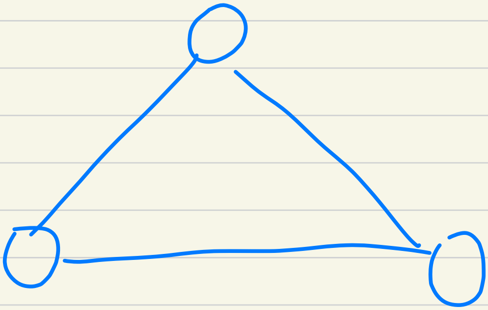
A: 0.4
B: 0.6
C: 0.5

Σ ? $P(A)+P(B)-P(A\cap B)$

$1-0.24=0.76$



Incentive to switch: U(current modal) - U(switched modal)



Use Auction Based Algorithm to bid for the switch if the incentive is positive.

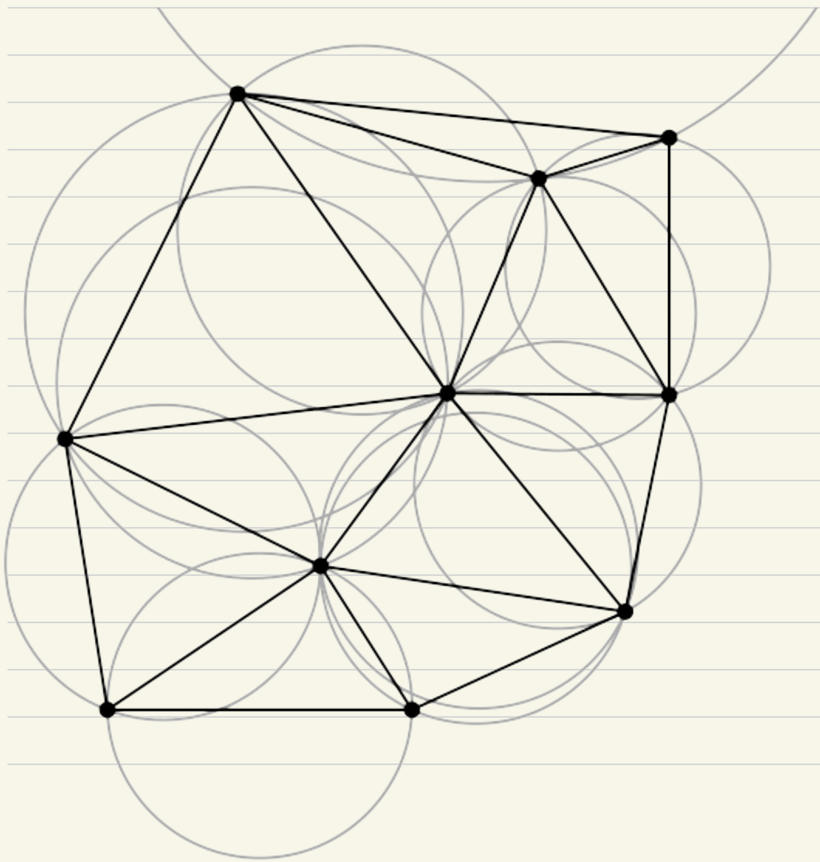
⇒ problem: Only one agent will switch at a time step?

Consensus Auction-based Switching Algorithm

1. Calculate the incentive of switching mode
   $I_i = U_i(\text{switched mode}) - U_i(\text{current mode})$, $I_i = \begin{cases} I_i, & \text{if } I_i \geq 0 \\ -1, & \text{if } I_i < 0 \end{cases}$

2. Place the incentive as the bid for auction with neighbor

3. Resolve the conflict if both the adjacent neighbors want to change mode



1. Each agent compute its incentive of changing mode

2. Each agent place the incentive as the bid for switching

3. Each agent recieves the incentive from its neighbors as bids

4. Each agent assign the admission to switching role to the one with highest bid

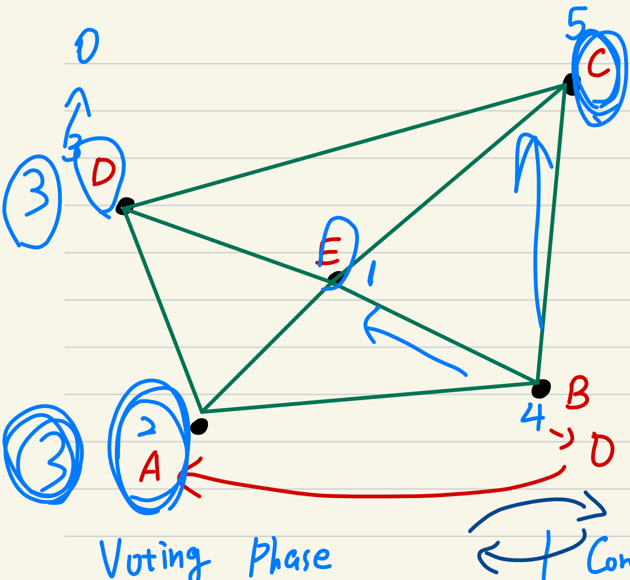
5. Each agent update its bid list including the bids from all other agents and they should switch or not

6. Each agent exchanges their bid list

7. If the agent has the right to bid and so is its neighbor then conflicts need to be resolved.
   => Deprive the right from the lower one and give it to the

8. Repeat 2 to 7 until the bid list converge or step exceeds maximum

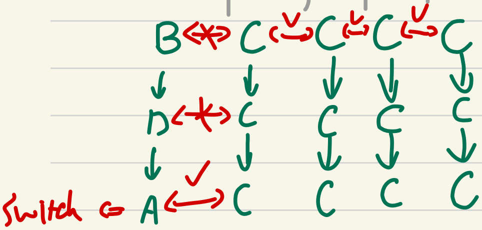
9. switch based on the bid list



Voting Phase

Vote list

	A	B	C	D	E
A	2	2	X	2	2
B	4	4	4	X	4
C	X	5	5	5	5
D	3	X	3	3	3
E	1	1	1	1	1



Consensus Phase (Take B for example)

B knows that it was voted from A's vote list. B also knows that C was voted from B, E, C. ⇒ conflict

1. B checks if C has higher bid.
   Yes: B will set its bid to 0 + random small constant
2. B will send the new bid to A
3. A will choose the second highest (D)

---

4. D will resolve the conflict by setting its bid to 0 + random small constant
5. A will eventually choose itself to switch which induces no conflict.

Flow chart for each agent

Receive Data

Receive data from agents within communication range

↓

Store in buffer

Exchanged data

- id
- position
- bid
- vote list

Buffer: a size one container which updates in realtime

Initialization

Compute Reward

↓

Place reward as bid

↓

Publish data

↓

Fetch from buffer

Voting phase

Construct bid list

↓

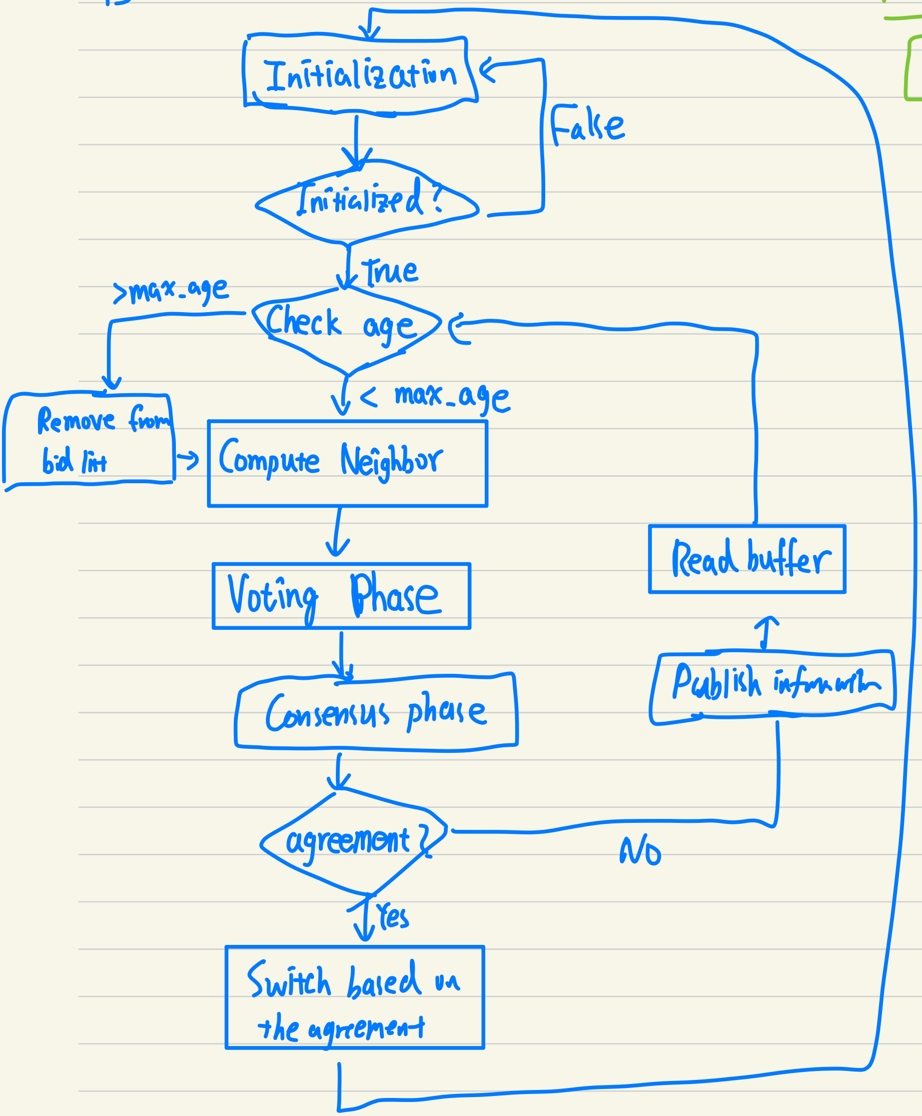
Vote

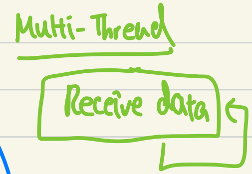
↓

Construct vote list

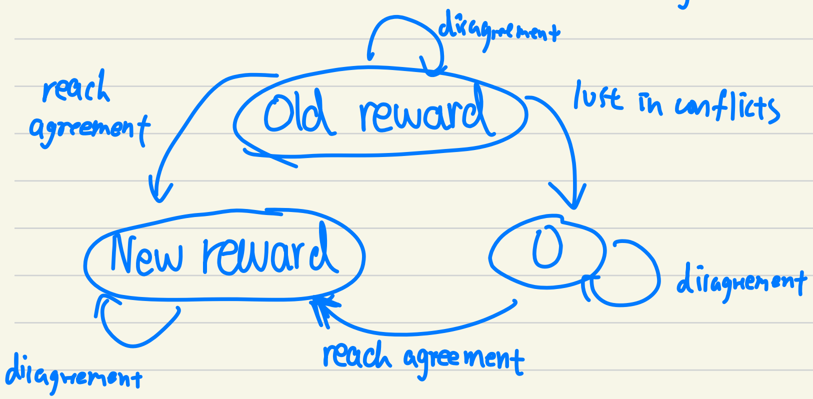
Consensus phase

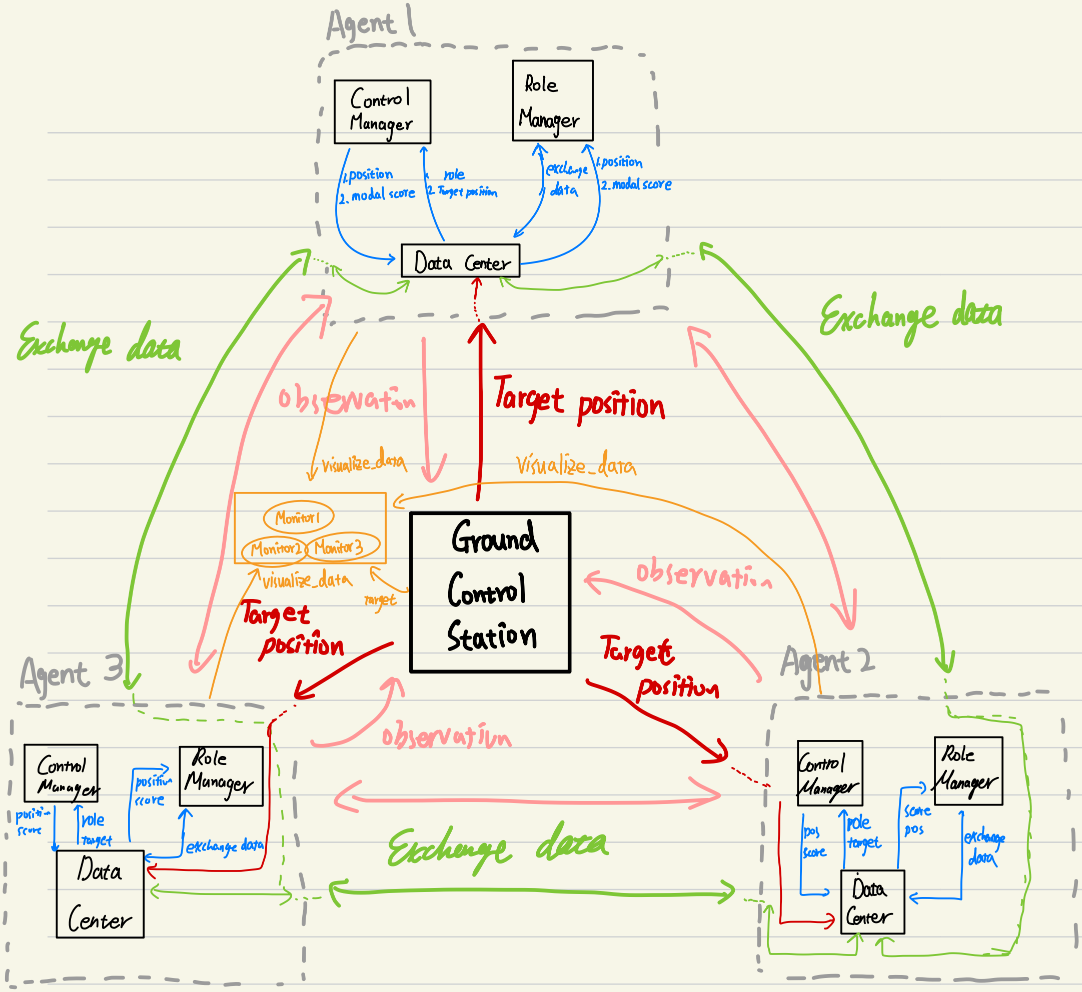
DCBSA Flow chart





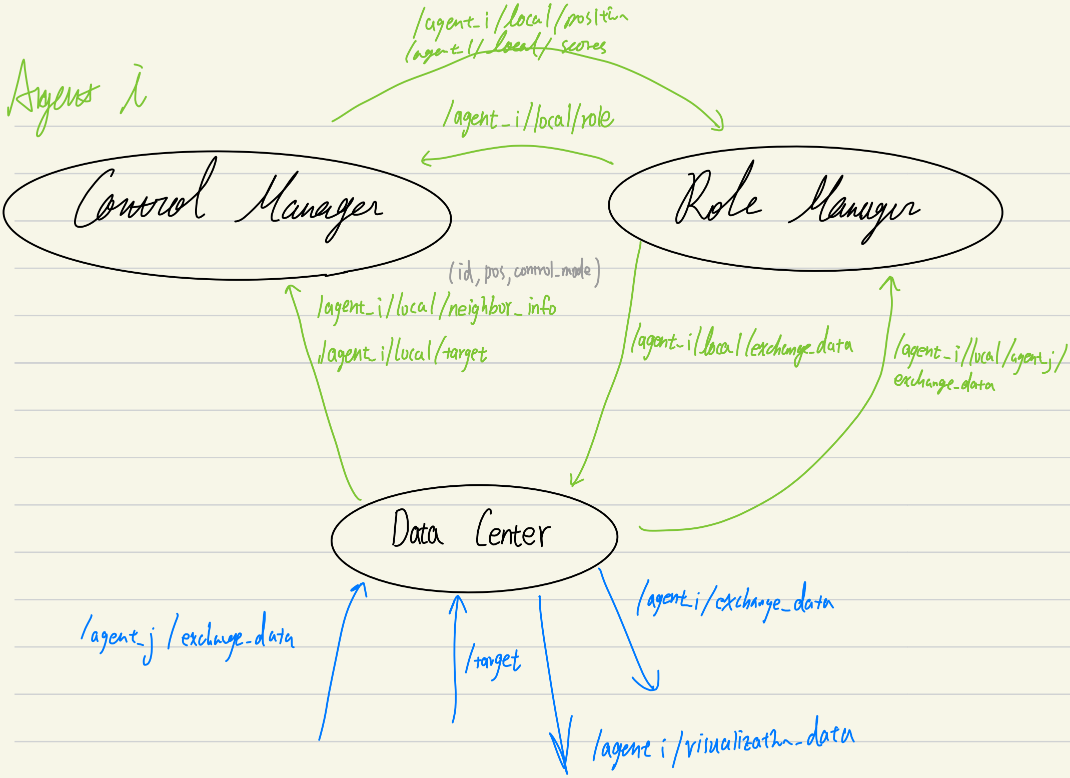
Finite State Machine of the agent's bid





Exchange data : id, position, bid , vote list , score, role

observation exchange ⇒ future work


Agent i
/agent_i/local/position
/agent_i/local/scores
/agent_i/local/role
Control Manager
Role Manager
(id, pos, control_mode)
/agent_i/local/neighbor_info
/agent_i/local/target
/agent_i/local/exchange_data
/agent_i/local/agent_j/exchange_data
Data Center
/agent_j/exchange_data
/target
/agent_i/exchange_data
/agent i/visualization_data

Current Problems

- Visualization in pygame
- Switching algorithm not working with controller
  - transmitted data is wrong?
  - synchronization problem?

  → stuck in dead-lock ⇒ everyone thinks it loses to others
- Correctness of controller?
  - need visualization

  → possibly correct

solved implementation problem

4/17

Why do I need to use vote mechanism and consensus algorithm while I can just let each agent determines whether they are the highest in its neighborhood?

Why do I need voting??! when finding the maximum is sufficient

1. What's the difference or improvement can I assert with the designed algorithm?
2. What should I modify to make the algorithm different?
3. How to solve the issue that in dynamic networks, switching role will not converge due to constantly changing topology?

觀察到的問題

1.

2-hop neighbor、

要改 最低的可比切?

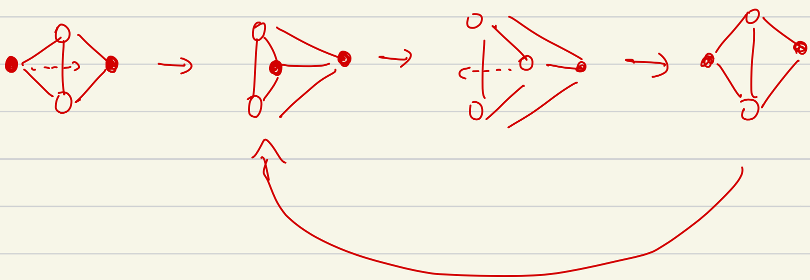
Conflict Resolution 仍然用 max 的方法 (= non-voting method)

=> consensus 改成考慮票數 => 票數的用處是什麼？如何用票數解決來回切換的問題

問題點:

1. controller 之間的切換太劇烈？ -> use backupcontroller and design 入？

2.

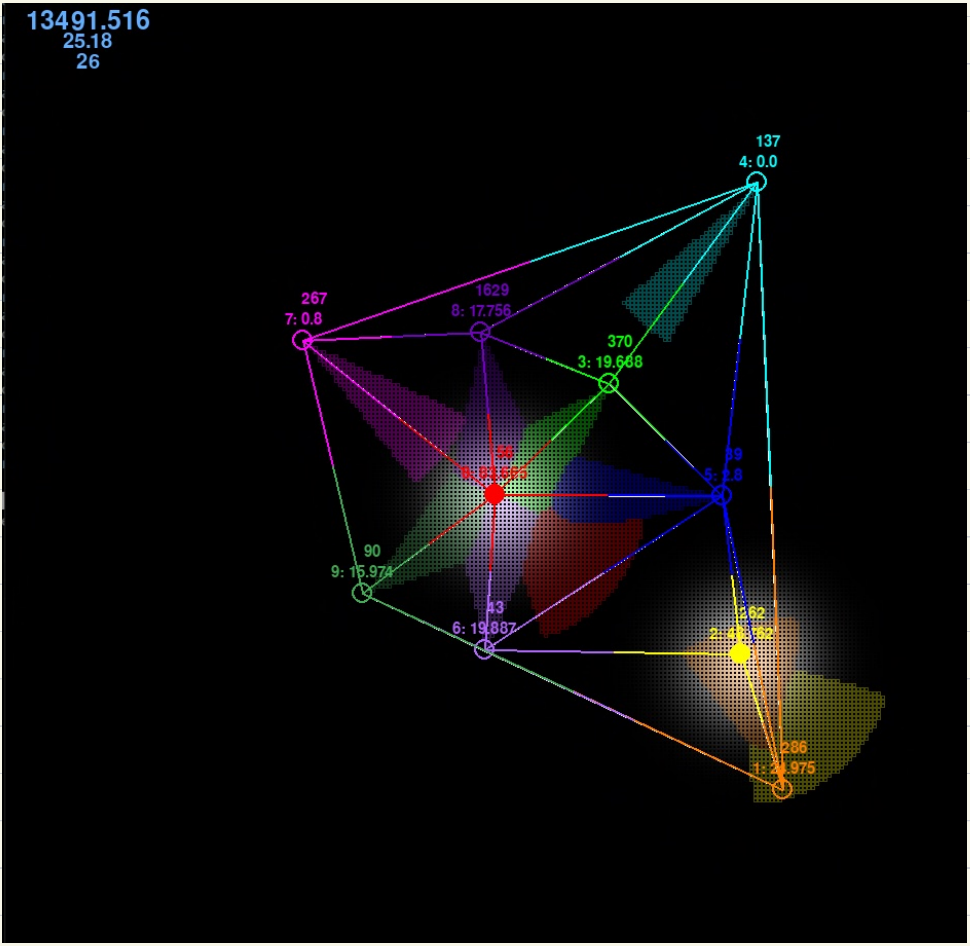


1. consider 2-hop neighbor?

2.

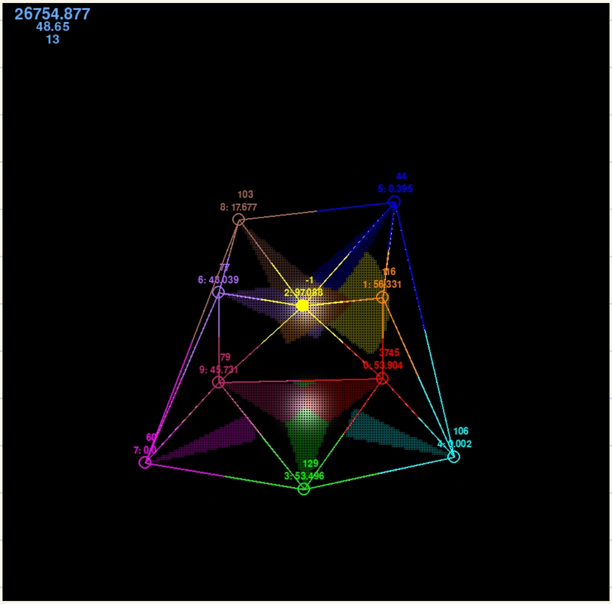
=> 會有問題是因為我只考慮切換會對其中一个 cand 有影響

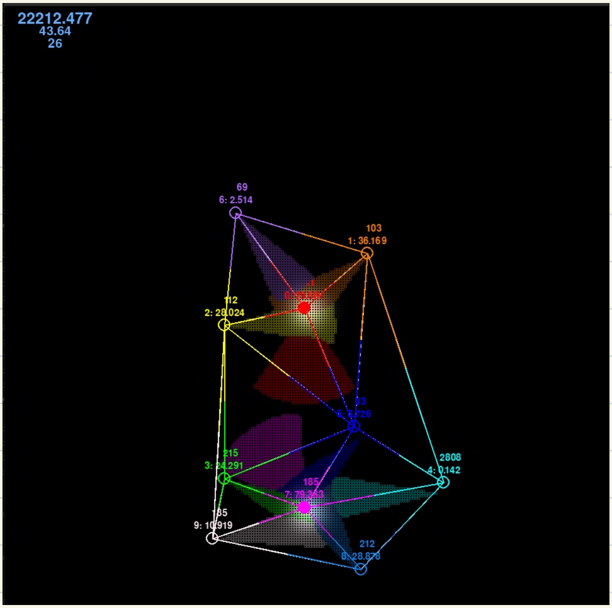
Problem 1: Cluster分配不均

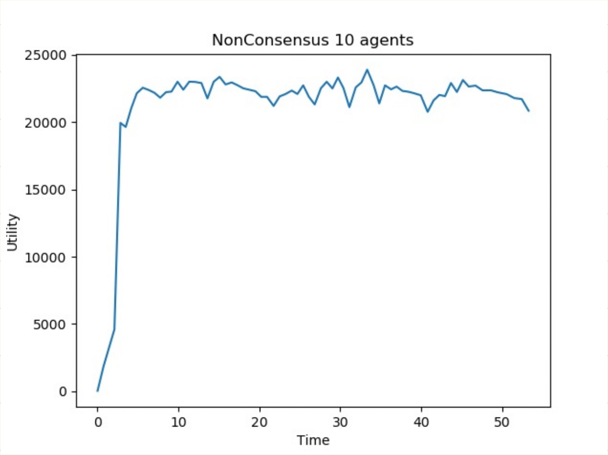


id 4, 7, 5 are useless in assisting id 0, somehow some of the tracking agent should be allocate to assist id 2.

Problem 2


26754.877
48.65
13
103
8: 17.677
44
5: 0.395
6: 43.039
2: 97.088
1: 56.331
79
9: 45.731
0: 53.904
106
4: 0.002
60
7: 0.0
129
3: 53.496


22212.477
43.64
26
69
6: 2.514
103
1: 36.169
112
2: 28.024
215
3: 24.291
2808
4: 0.142
185
212
8: 28.878


NonConsensus 10 agents
Utility
Time
25000
20000
15000
10000
5000
0
0
10
20
30
40
50

Idea1 Change the definition of reward

1. Calculate neighbors' contribution to me and send it to each neighbor, can be positive or negative
2. collect feedback from each neighbor and times a factor related to the distance (assign weights for each edge based on distance)
3. use maximum to change role

⇒ conflicts resolution also need to be redesigned

- occurs at local leader shared same neighboring node

⇒ leader with maximum reward will announce victory

⇒ when other leaders within 2-hop neighborhood received the announcement they will check if it conflicts with itself, if no then it'll change. If, yes

Idea2 Cluster-head balancing by assigning tracker to interceptor (cluster-head)

Schedule

✓ 1. Restore the FoV in ptz.py

2. Complete the previous proposed algorithm

3. Come up with the balancing algorithm.

Block Diagram for New Algorithm

(Reward Computation
Consensus Algorithm
Election Algorithm
Conflict Resolution)

- Reward Computation

  - Compute Contribution List

```
Contrib = { }
N = neighbors {role, score}
i = agent i
for n in N:
  current_utility = fixed n.role compute i's hood's utility
  switched_utility = switch n.role compute i's hood's utility
  Contrib [n] = switched_utility - current_utility
```

  - compute reward

```
reward = 0 , i = agent
for c, n in contribution_list, N
  reward += C[i] · 1 / (1+distance(n, i))
return reward
```

- Election Algorithm

  - winning announcement

```
if reward is max(N.reward)
  winning_list [i] = [True, reward]
else
  winning_list [i] = [False, 0]
```

- Consensus Algorithm

  - Check consensus

```
for w in winning_lists_from_neighbors:
    if self.winning_list != w:
        return unstable
return stable
```

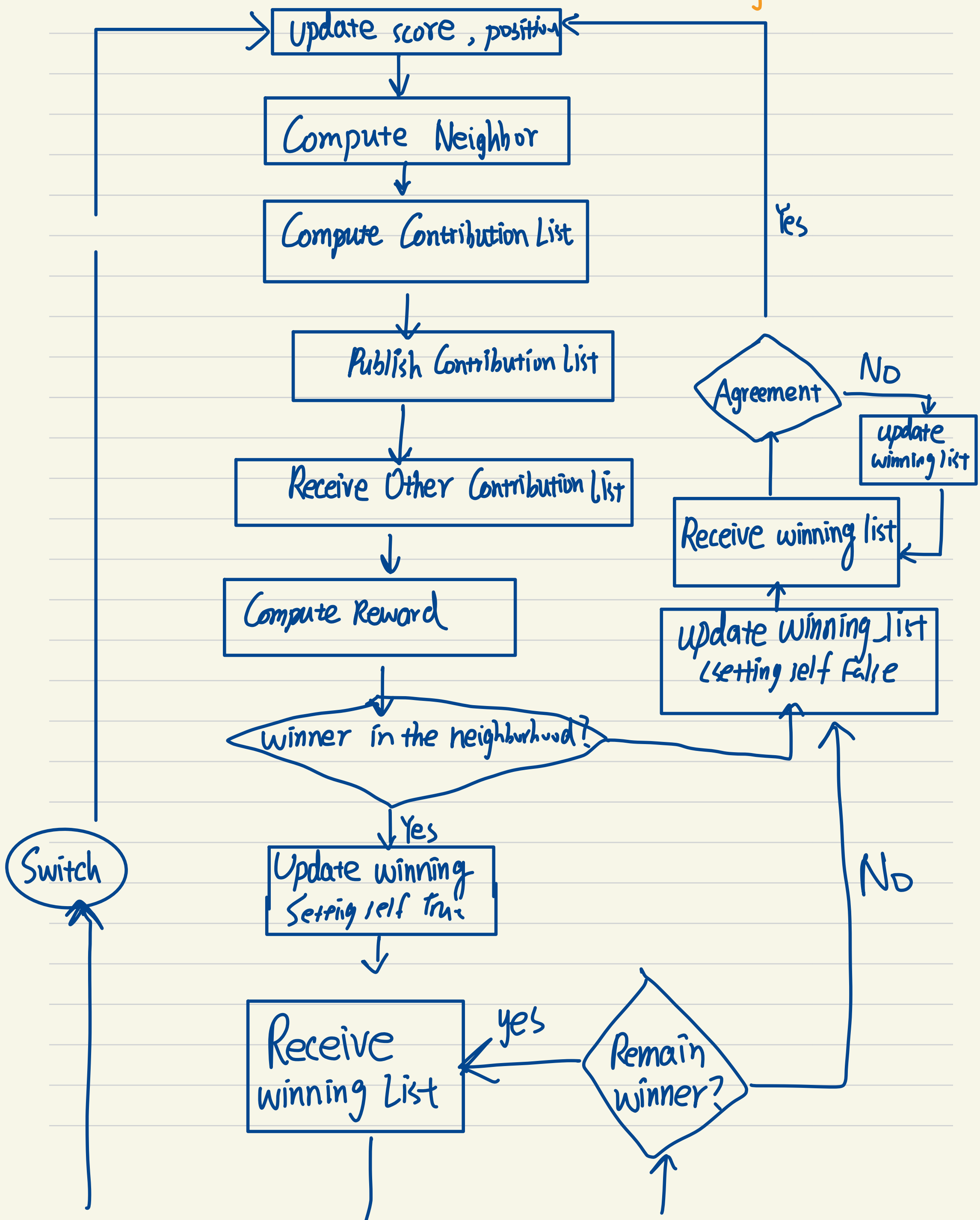
  - 2-hop coflict resolution → since the reward is also related to 2-hop neighbors

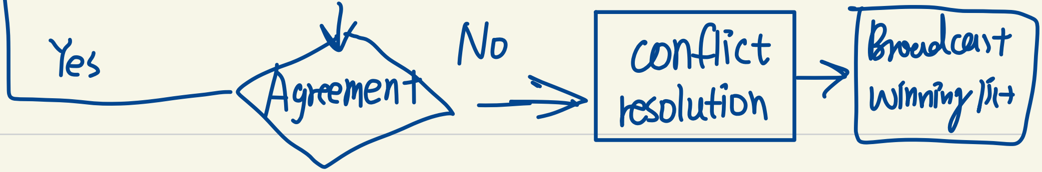
```
N = neighbors
if self.win == true:
    for n in N.neighbors:
        if winning_list[n][0] = true
            if reward > winning_list[n][1]:
                winning_list[n] = [false, 0]

        else
            winning_list[i] = [false, 0]

pub_winning_list
```

Flow chart for new Consensus Leader Election Algorithm




Yes
Agreement
No
conflict resolution
Broadcast winning list

拆解問題成兩大塊

1、控制

a. ~~cover~~ directional sensor coverage control

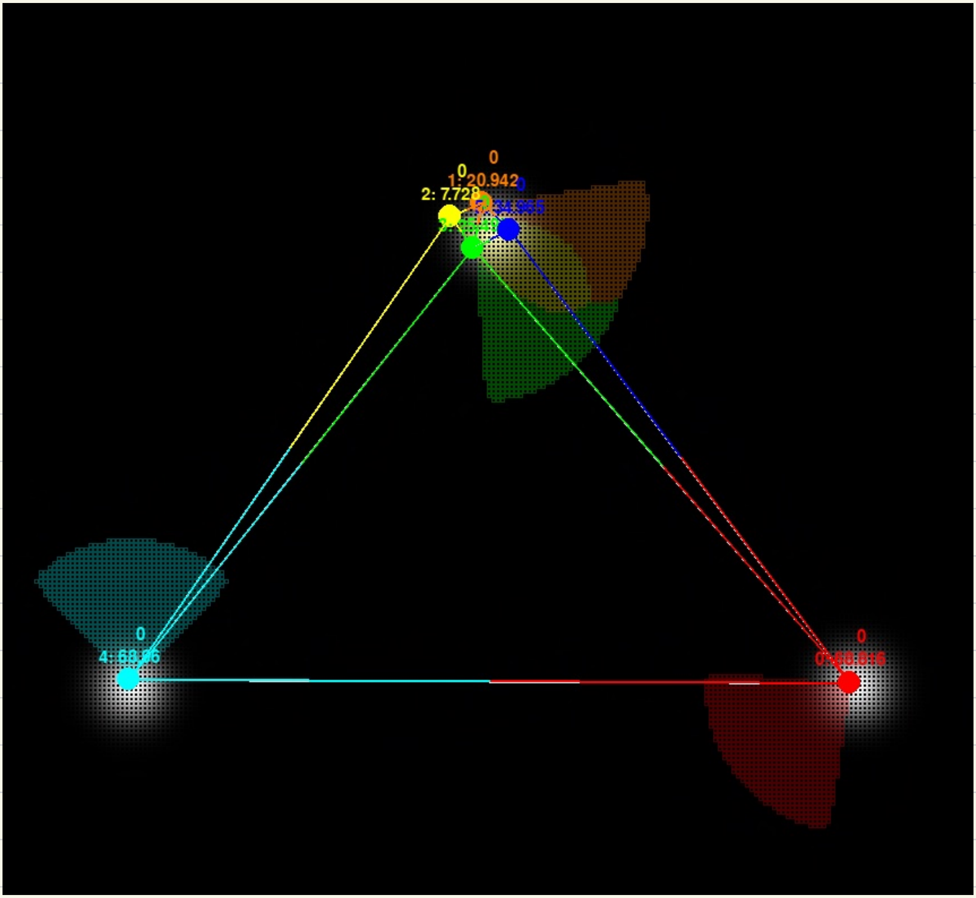
locational

new density function generated by target density function and each agent's sensing range

b. balancing term

2. 切換

已知 interceptor 位置



Can I evenly distribute the agents? through control law?

Research Topic I

Consensus-based load balancing and coverage control for

Motivation:

In previous implementation of the observer-interception CPS, I've found out that with only voronoi-based coverage and role assignment cannot efficiently accomplish the hybrid task. One main reason is that agents cannot be evenly distributed to each target without target assignment but voronoi-based control only. Therefore, my idea is that, can target assignment be partially integrated into the voronoi-based coverage control?

Schedule:

1. Design a method to balance the load of each sensor

→ area-constrained locational optimization Problem

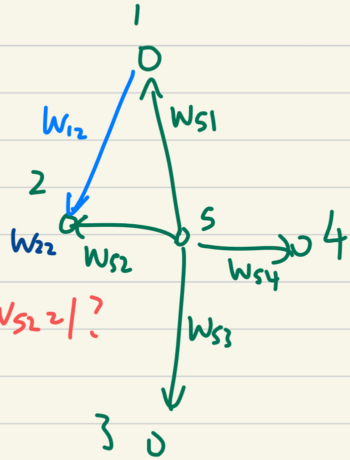
↳

$$\text{minimize } H(p_1, \dots, p_n, W_1, \dots, W_n),$$

$$\text{s.t } \int_{W_i} \phi(q)\,dq = a_i,\ i \in \{1, \dots, n\}$$

⇒ problems: a_i is predefined for every agents how to determine the weight?

$$f(||x-p||) + \sum w_{ij}$$



$w_{12}+w_{22}+w_{52}=1$?

different w for neighbors? Possible!

Coverage Optimization and Load Balancing for Heterogeneous Sensing Capability

Control Cost

inter-sensor awareness

role assignment index

intra-sensor ~~balance~~

$$H_{overall}(P,V) = \overbrace{\sigma \sum_{i=1}^{m} \sum_{j \in S} W_{i,j} \int_{V_i^j} \left(f^j(\| q - P_i \|) - \underline{M_{i,j}} \right) \phi_j(q)\, dq}^{\text{inter-sensor awareness}}$$
$$\underbrace{+ (1-\sigma) \sum_{i=1}^{m} \int_{V_i^0} \| q - P_i \|^2 \phi_{uni}(q)\, dq}_{\text{regularization term for collective awareness}}$$

load balancing term.

inter-sensor balance

Question: How to deal with W_k?

架構:

1. 把題目 formulate 成 跟 "Coverage Control for Multiple Event Types with Heterogeneous Robots" 和 "Coverage Control for Multirobot Teams With Heterogeneous Sensing Capabilities" 一樣

2. 定義好 不同 sensor 的 event density function 或是怎麼把 不同 $f(\|x-p\|)$ 考量進去

3. 參考 "Coverage Optimization and Spatial Load Balancing by Robotic Sensor Networks" 和 "Distributed Multi-robot Work Load Partition In Manufacturing Automation" 加進 Intra-sensor 的 load balancing

100 x 100

0.5 · 50 + 0.5 x 50

$$H_i = \sum_{s \in S_i} w_{i,s} \int_{V_{i,s}} \left(f(\|x - p_i\|) - M_{i,s}\right) \phi_s(q)\, dq$$

$$\int_{V_i} h(\|x - p_i\|)\, \phi(q)\, dq$$

$$H = \sum_{i \in N} \sum_{j \in \{1,2,3\}} w_{i,j} \cdot h(p_i)$$

P_i

Solve w (inter-modal) load balancing first

Method 1. Even Weight

difficult to solve

Should not based on joint utility

Method 2. Discrete Weight based on predefined reward function

use potential score

Method 3. Solve w through somesort of optimizor.

Objective:

$$H = \sigma H_c + (1-\sigma) H_a$$

$$= \overbrace{\sigma\left(\sum_{i \in N}\sum_{s \in S_i}\int_{V_{i,s}} f(\|q-p_i\|)\phi_s(q)\,dq\right)}^{\text{coordination with same sensor}}$$

$$\underbrace{+(1-\sigma)\left(\sum_{i \in N}\int_{V_i}\|q-p_i\|^2\phi_t(q)\,dq\right)}_{\text{agent avoidance term}}, \text{ where } \phi_t = \oplus\phi_{s\in S}$$

Problem:

Assuming all sensors are equally important for each agent. However, agent should coordinate to assign the sensors' distribution, i.e., each agent should rely differently on sensors.

New Objective:

$$H' = \sigma\left(\sum_{i \in N}\sum_{s \in S_i} w_{i,s}\int_{V_{i,s}} f(\|q-p_i\|)\phi_s(q)\,dq\right)$$

$$+(1-\sigma)\left(\sum_{i \in N}\int_{V_i}\|q-p_i\|^2\phi_t(q)\,dq\right), \text{ where } \phi_t = \oplus\phi_{s\in S}$$

New problem arises:

How to determine $w_{i,s}$?

What are the assumptions of $w_{i,s}$?

1. What's the assumption of w?

$$\sum_{i \in N} W_{i,s} = 1 \quad \forall s \in S$$

or

$$\sum_{s \in S_i} W_{i,s} = 1 \quad \forall i \in N$$ → reasonable for switching

2. How to solve for w?

Method I

Equally weighted (original method)

Method II → precise role assignment

Consensus based role assignment

Method III → fuzzy role assignment (weighted role assignment)

Numerical Solution

Control Law

$$H = \sigma H_c + (1-\sigma) H_a$$

$$= \sigma \sum_{i\in N}\sum_{s\in S_i} w_{i,s}\int_{V_i^s} f(\|x-p_i\|)\phi_s(q)dq$$

$$+(1-\sigma)\sum_{i\in N}\int_{V_i} f(\|x-p_i\|)\phi_t(q)dq$$

By gradient ascent,

$$\frac{\partial H}{\partial p_i} = \sigma\frac{\partial H_c}{\partial p_i} + (1-\sigma)\frac{\partial H_a}{\partial p_i}$$

$$\frac{\partial H_c}{\partial p_i} = \sum_{s\in S_i} w_{i,s}\int_{V_i^s}\frac{\partial f(\|q-p_i\|)}{\partial p_i}\phi_s(q)dq$$

$$+\sum_{l\in N_i^s} w_{l,s}\int_{V_i^s} \cancelto{0}{f^s(\|q-p_i\|)\, n_{il}^s(q)\frac{\partial(\delta V_{ik}^s)}{\partial p_i}(q)\phi_s(q)dq}$$

$\downarrow \times -1$

$$+\sum_{l\in N_i^s} w_{l,s}\int_{V_i^s} \cancelto{0}{f^s(\|q-p_i\|)\, n_{li}^s(q)\frac{\partial(\delta V_{ik}^s)}{\partial p_i}(q)\phi_s(q)dq}$$

$$= 2\sum_{s\in S_i} w_{i,s}\int_{V_i^s}(p_i-q)\frac{df^s(\|q-p_i\|)}{\partial(\|q-p_i\|^2)}\phi_s(q)dq$$

$$M_{V_i^s} = w_{i,s}\int_{V_i}\frac{\partial f^s(\|q-p_i\|)}{\partial(\|q-p_i\|^2)}\phi_s(q)dq$$

$$C_{V_i^s} = \frac{1}{M_{V_i^s}} w_{i,s}\int_{V_i^s} q\frac{df^s(\|q-p_i\|)}{\partial(\|q-p_i\|^2)}\phi_s(q)dq$$

$$\boxed{u_{c,i} = -k\frac{1}{\sum_{s\in S_i} M_{V_i^s}}\sum_{s\in S_i} M_{V_i^s}(p_i - C_{V_i^s}) \quad \forall i\in N}$$ → intra-modal control law

$$\frac{\partial H_a}{\partial P_i} = 2m_i(P_i - C_i)$$

$$m_i = \int_{V_i} \phi_t(q)\,dq$$

$$C_i = \frac{\int_{V_i} q\,\phi_t(q)\,dq}{m_i}$$

$$\boxed{u_{a,i} = -k\,m_i(P_i - C_i)}$$ → inter-modal control law

$$\boxed{\begin{aligned} u_{total,i} &= \sigma u_{c,i} + (1-\sigma)u_{a,i} \\ &= -2k\left(\sigma \frac{\sum_{s\in S_i} M_{V_i^s}(P_i - C_{V_i^s})}{\sum_{s\in S_i} M_{V_i^s}} + (1-\sigma)m_i(P_i - C_i)\right) \end{aligned}}$$

↳ total control law

sensor set $S = \{S_0 : \text{Capturer}, S_1 : \text{observer(camera)}, S_2 : \text{observer(lidar)}\}$

$$f^{S_0}(\|q-P_i\|) = e^{-\frac{\|q-P_i\|^2}{2V_{max}^2}}$$

$$\frac{\partial f^{S_0}(\|q-P_i\|)}{\partial(\|q-P_i\|^2)} = \frac{-1}{2V_{max}^2} e^{-\frac{\|q-P_i\|^2}{2V_{max}^2}}$$

$$f^{S_1}(\|q-P_i\|) = \frac{\|x-P_i\|^\lambda}{R^{\lambda+1}}\left(R\cos\alpha - \lambda(\|x-P_i\| - R\cos\alpha)\right)$$

$$\frac{\partial f^{s_1}(\|q-P_i\|)}{\partial(\|q-P_i\|^2)} = \frac{\partial\left(\frac{(\|q-P_i\|^2)^{\frac{\lambda}{2}}}{R^{\lambda+1}} R\cos\alpha - \frac{\lambda(\|q-P_i\|^2)^{\frac{\lambda+1}{2}}}{R^{\lambda+1}} + \frac{\lambda(\|q-P_i\|^2)^{\frac{\lambda}{2}} R\cos\alpha}{R^{\lambda+1}} \right)}{\partial(\|q-P_i\|^2)}$$

$$= \frac{\partial\left(\frac{A^{\frac{\lambda}{2}}}{R^{\lambda}}\cos\alpha - \frac{\lambda A^{\frac{\lambda+1}{2}}}{R^{\lambda+1}} + \frac{\lambda A^{\frac{\lambda}{2}}\cos\alpha}{R^{\lambda}} \right)}{\partial A}$$

$$= \frac{\lambda}{2} \cdot \frac{\cos\alpha}{R^{\lambda}} \cdot A^{\frac{\lambda-2}{2}} - \frac{\lambda+1}{2} \frac{\lambda}{R^{\lambda+1}} \cdot A^{\frac{\lambda-1}{2}} + \frac{\lambda}{2} \frac{\lambda\cos\alpha}{R^{\lambda}} A^{\frac{\lambda-2}{2}}$$

$$= (1+\lambda)\frac{\lambda\cos\alpha}{2R^{\lambda}} \|q-P_i\|^{\frac{\lambda-2}{2}} - (1+\lambda)\frac{\lambda}{2R^{\lambda+1}} \|q-P_i\|^{\frac{\lambda-1}{2}}$$

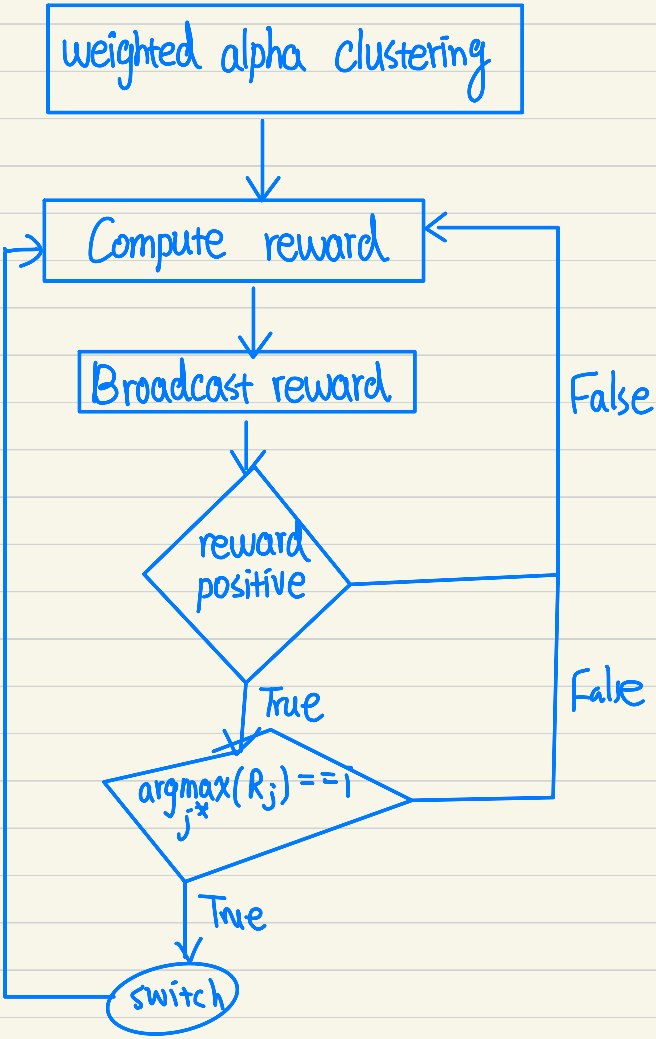
heading centroid for s_1: $G_i = \frac{\int_{V_i} q\phi(q)dq}{\int_{V_i} \phi(q)dq}$

$$f^{s_2}(\|q-P_i\|^2) = \|q-P_i\|^2$$

$$\frac{\partial f^{s_2}}{\partial(\|q-P_i\|^2)} = 1$$

Consensus based role assignment

for agent i



- Weighted α clustering

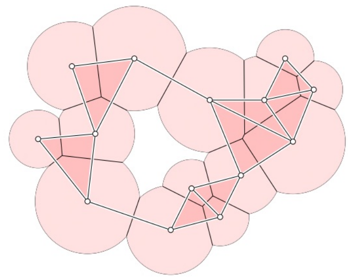


Figure III.16: Convex decomposition of a union of disks and the weighted alpha complex superimposed.

https://courses.cs.duke.edu/fall?06/cps296.1/Lectures/sec-III-4.pd

The weight is $(\text{sensing\_range})^2$, where the sensing range depends on the selected role.

- Compute reward

$$R_i(\iota) = \Pi_r(\Sigma_n^N \mathbb{1}_{r,n} Q_{r,n} + \mathbb{1}_{\iota,r} Q_{r,i})$$
$$r \in \{interceptor, observer\}$$
$$\mathbb{1}_{r,n} = \begin{cases} 1, & \text{if role(n)} = \text{r} \\ 0, & \text{otherwise} \end{cases}$$
$$\mathbb{1}_{\iota,r} = \begin{cases} 1, & \text{if } \iota = r \\ 0, & \text{otherwise} \end{cases}$$

=> Previous reward depends on the joint utility function with its neighbors, which has no direct relationship nor explanation toward the new utility function.

Therefore, a new reward based on potential capacity is proposed.

$$R_i(w) = \frac{Q_{w,i}}{1 + \Sigma_n^{N_i} e^{\frac{-\|P_i - P_n\|^2}{2}} \mathbb{1}(w, \text{role}(n)) Q_{w,n}}$$

, where N_i is the neighbor set of agent i obtain from weighted α-complex which is the subset of delaunay graph

Numerical Solution

Research Topic II

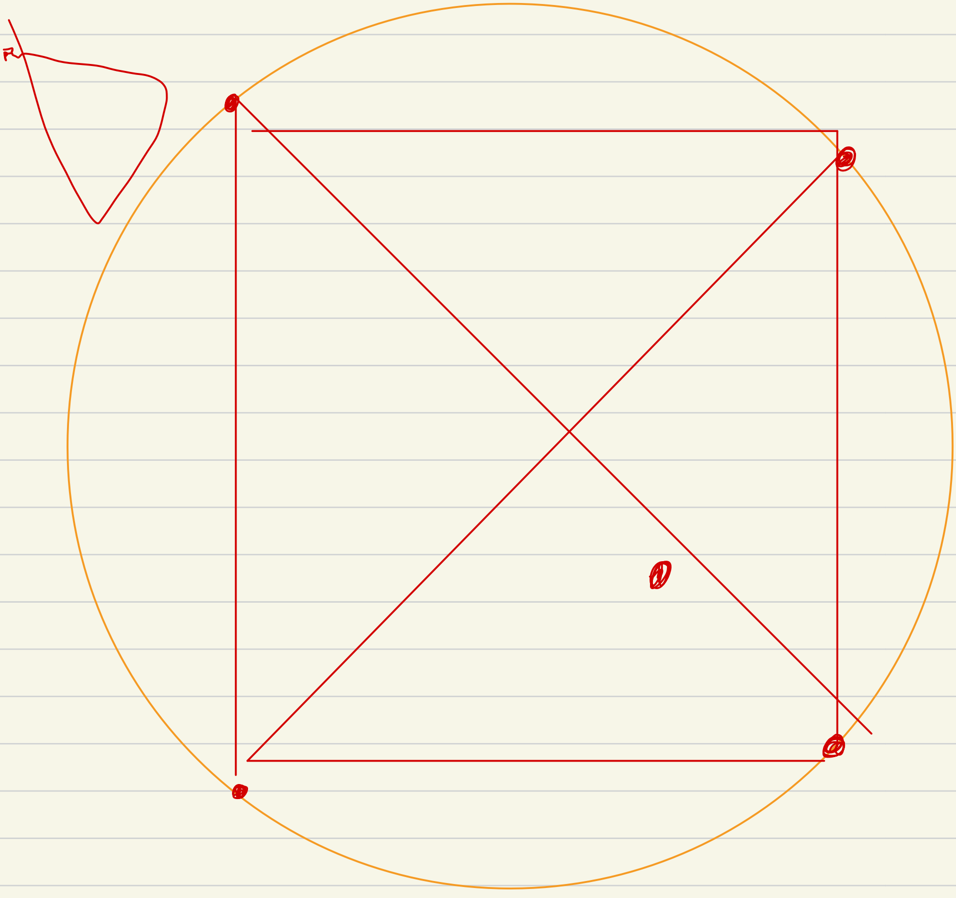
Cluster-based role assignment algorithm for voronoi-based coverage control of heterogeneous sensor network

Motivation:

Role assignment is important in multi-agent cooperative tasks. Especially in tasks where the environment is dynamic (i.e. the target will move or agent's characteristic is time varying). Therefore, a dynamic role assignment approach is crucial. However, in some of the literature, the algorithm assume the agent persists the same role after assignment, which is not suitable in dynamic environment. Thus, a dynamic leader-election-based switching algorithm is proposed as a heuristic solution to fast and converging approach. Moreover, a delaunay graph is used to perform role assignment since voronoi tessellation is broadly used ~~in~~ as multi-agent system's control law.

Scenario:

Given n agents who can arbitrarily select any number i sensors from a sensor set $S = \{S_0, S_1, \ldots, S_m\}$. Each sensor provide a different density function.



Problem:

Coverage Control of multiple events isnt designed for cooperation different type of sensors, but to maximize agents' self interest. Therefor, a control law with cooperation between different type of sensor need to be designed.

Control Scenario

Each agent is equipped with a sensor chosen from a set S. It is ensured that at least one agent possesses each type of sensor. The coverage task is designed as covering a composite event which consists of multiple features. Only when all the features are detected will the event be observed. Thus, the utility will be marked as the joint detection between different type of sensor, which we described as the exploitation of agents. To furthur avoid local minima and let the agent seek for the best configuration for its type of sensor, a regularization term for what we called exploration was added.

Control Objective

$$\underset{p_i}{\text{maximize}} \int_{V_i^s \cap V_j^s} \left(1+f^i(\|q-p_i\|)\right) \prod_{j \in N_i} \left(1+f^j(\|q-p_j\|)\right) \phi(q)\, dq$$

$$V_i = \{ q \mid \|q-p_i\| < \|q-p_j\| \ \forall j \in D_i^{c_i} ,\ \forall q \in Q \}$$

Related work

1. Composite Event Coverage in wireless Sensor Networks with Heterogeneous Sensors
2. Coverage Control for Multiple Event Types with Heterogeneous Robots
3. Coverage Control for Multirobot Teams with Heterogeneous Sensing Capabilities
4. The Impact of Diversity on Optimal Control Policies for Heterogeneous Robot Swarms

Control Constraint

Spatial Avoidance of agent with varying capabilities

=> potential force or another voronoi

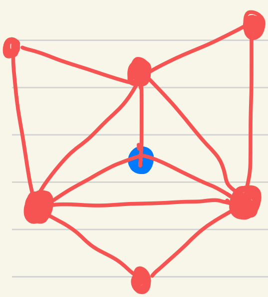
$$\underset{p_i}{\operatorname{argmax}} \ \sigma \int_{V_i^{c_i} \cap V_j^{c_j}} (1+f^i(\|q-p_i\|)) \prod_{j \in N_i} (1+f^j(\|q-p_j\|)) \phi(q)\,dq$$

$$-c(1-\sigma) \int_{V_i} \|q-p_i\|^2 \underbrace{\alpha(q)}_{\text{uniform distribution}} dq$$

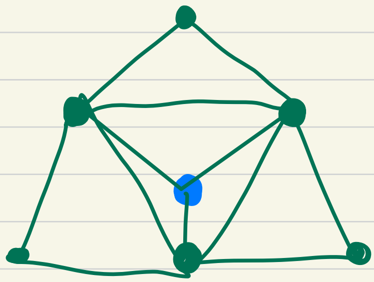
Cooperative Neighbor Graph

$$N_i = \bigcup_{k \in C \setminus C_i} D_i$$

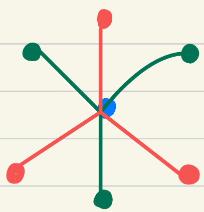
$D_{i,1}$ $\cup$ $D_{i,2}$ $=$ N_i



$+$



$=$



●: agent i with C_0

●: agents with C_1

●: agents with C_2

Task allocation scenario

Assume agents are equipped with multiple capabilities, and they are expected to finish a compound task cooperatively by choosing the role from their available capabilities. The compound task should be tackled simultaneously by several specified capabilities. The question lies with how does each agent choose their distribution (confidence) of the available capabilities. The agents should take the relationship between neighbors and the environment into consideration.

Related work

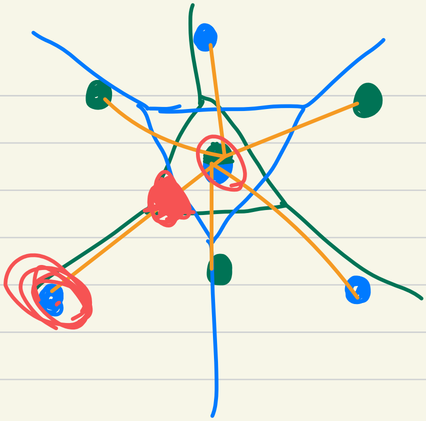
1. Resilient Task Allocation in Heterogeneous Multi-robot System
2. The Impact of Diversity on Optimal Control Policies for Heterogeneous Robot Swarms
3. Formalizing the Impact of Diversity on Performance in a Heterogeneous Swarm of Robots
4. Adaptive Task Allocation for Heterogeneous Multi-Robot Teams with Evolving and Unknown Robot Capabilities
5. Multi-robot Coordination and Cooperation with Task Precedence Relationship
6. A consensus based grouping algorithm for multi-agent cooperative task allocation with complex requirements.
7. Adaptation to Team Composition Changes for Heterogeneous Multi-robot Sensor Coverage
8. Interaction modeling with multiplex attention

Control Objective

$$\underset{w_i}{\text{maximize}} \sum_{c \in C_i} w_{ic} \int_{V_i^c \cap V_j^{c_j}} \left(1+f^c(\|q-p_i\|)\right) \prod_{j \in N_i^c} \left(1+f^{c_j}(\|q-p_j\|)\right) \phi(q)\, dq$$

$$V_i^c = \{ q \mid \|q-p_i\| - w_{ic} < \|q-p_j\| - w_{jc} \quad \forall j \in D_i^c, \forall q \in Q \}$$

$$\text{s.t.} \quad \sum_{c \in C_i} w_{ic} = 1,$$



use iterative method to solve for w Decentralized Linearized Alternating Direction Method of Multipliers

$$\sum_{e\in E} \sum_{c\in C_i} W_{ice} \int_{V_i^c \cap V_j^{c_j}} (1+f^c(\|q-P_i\|)) \prod_{j\in N_{ic}^e} (1+f^{c_j}(\|q-P_j\|)) \phi_e(q)\,dq$$

=> Multiple events with different requirements for sensing capabilities are needed to be covered with a team of heterogeneous sensing capabilities agents

$$W_{ic}^e = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{\text{agent's capability}} \odot \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{event's requirement}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

N_{ic}^e is the cooperative graph for sensor c of i with event e

$$\sum_{e\in E} \sum_{c\in C_i} C_c^i E_c^e \int_{V_i^c} (1+f^c(\|q-P_i\|)) \prod_{k\in}$$

$$\prod_{k\in C\setminus c}\sum_{j\in N_i}\mathbb{I}(k,q,j)f^k(\|q-p_j\|)$$

$$H_i=\sum_{e\in E}\sum_{c=1}^{M}\mathbb{C}_c^i\mathbb{E}_c^e\int_{V_i^c}\left(1+f^c(\|q-p_i\|)\right)\overbrace{\prod_{k\in C\setminus c}\left(1+\mathbb{E}_c^e f^k(\|q-p_{\mathbb{I}(k,q)}\|)\right)}\phi_e(q)dq$$

$$H_i=\sum_{e\in E}\sum_{c=1}^{M}\mathbb{C}_c^i\mathbb{E}_c^e\int_{V_i^c}\left(1+f^c(\|q-p_i\|)\right)\prod_{k\in C\setminus c}\left(1+\mathbb{E}\sum_{j\in N_i}\mathbb{1}_j(k,q)f^k(\|q-p_j\|)\right)\phi_e(q)dq\ ,\quad \mathbb{1}_j(k,q)=\begin{cases}0, \text{if } q\notin V_j^k\\ 1, \text{if } q\in V_j^k\end{cases}$$

$$H_i=\sum_{e\in E}\sum_{c=1}^{M}\mathbb{C}_c^i\mathbb{E}_c^e\int_{V_i^c}\left(1+f^c(\|q-p_i\|)\right)\prod_{k\in C\setminus c}\left(1+\mathbb{E}_k^e\sum_{j\in N_i}\mathbb{1}_j(k,q)f^k(\|q-p_j\|)\right)\phi_e(q)dq$$

目前的code

假設每一台 agent 都只有一台sensor 則 N_i 就不包含 i 就可以使用上面的式

之後

假設每一台 agent 可以有多台 sensor，則 N_i 包含 i ⇒ 怎麼微分 ex: $M=3$

$$H_i=\sum_{e\in E}\sum_{c=1}^{M}\mathbb{C}_c^i\mathbb{E}_c^e\int_{V_i^c}F^c(p_i)$$

$$\sum_{e\in E}\Big(\mathbb{C}_1^i\mathbb{E}_1^e\int_{V_i^1}F^1(p_i)G^2(p_i)G^3(p_i)\phi_e(q)dq$$

$$+\mathbb{C}_2^i\mathbb{E}_2^e\int_{V_i^2}F^2(p_i)G^1(p_i)G^3(p_i)\phi_e(q)dq$$

$$+\mathbb{C}_3^i\mathbb{E}_2^e\int_{V_i^3}F^3(p_i)G^1(p_i)G^2(p_i)\phi_e(q)dq$$

sensing model

$$f^1=\frac{1}{1+e^{-2k(R-\|q-p_i\|)}}=(1+e^{-2k(R-\|q-p_i\|)})^{-1}=(1+e^{-2kR+2k\|q-p_i\|})^{-1}$$

$$\frac{\partial f^1}{\partial\|q-p_i\|}=-1\cdot(1+e^{-2k(R-\|q-p_i\|)})^{-2}\left(-2ke^{-2k(R-\|q-p_i\|)}\cdot(-1)\right)=\frac{-2ke^{-2k(R-\|q-p_i\|)}}{(1+e^{-2k(R-\|q-p_i\|)})^2}$$

$$\frac{\partial f^1}{\partial p_{ix}}=\frac{\partial f^1}{\partial\|x-p_i\|}\frac{\partial\|x-p_i\|}{\partial p_{ix}}$$

$$\frac{\partial f^1}{\partial p_{i,y}}=\frac{\partial f^1}{\partial\|x-p_i\|}\frac{\partial\|x-p_i\|}{\partial p_{i,y}}$$

$$f^2=e^{\frac{-(\|q-p_i\|)^2}{2\sigma^2}}$$

$$\frac{\partial f^2}{\partial\|q-p_i\|}=\frac{-\|q-p_i\|e^{-\frac{\|q-p_i\|^2}{2\sigma^2}}}{\sigma^2}$$

$$\|x-p_i\|=\sqrt{(x_x-p_{ix})^2+(x_y-p_{iy})^2}$$

$$f^3=e^{\frac{-(\|x-p_i\|-R)^2}{2\sigma^2}}$$

$$\frac{\partial\|x-p_i\|}{\partial p_{i,x}}=2(p_{i,x}-p_x)$$

$$\boxed{\frac{\partial f^3}{\partial\|q-p_i\|}=\frac{-(\|q-p_i\|-R)e^{\frac{-(\|q-p_i\|-R)^2}{2\sigma^2}}}{\sigma^2}}$$

$$H_i = \sum_{e\in E}\sum_{c=1}^{M} \mathbb{C}_c^i \,\mathbb{E}_c^e \int_{V_i^c} F^c(p_i)$$

$$\sum_{e\in E}\Big(\mathbb{C}_1^i \,\mathbb{E}_1^e \int_{V_i^1} F^1(p_i)\, G^2(p_i)\, G^3(p_i)\, \phi_e(q)\, dq$$

$$+\mathbb{C}_2^i \,\mathbb{E}_2^e \int_{V_i^2} F^2(p_i)\, G^1(p_i)\, G^3(p_i)\, \phi_e(q)\, dq$$

$$+\mathbb{C}_3^i \,\mathbb{E}_3^e \int_{V_i^3} F^3(p_i)\, G^1(p_i)\, G^2(p_i)\, \phi_e(q)\, dq$$

$$G^c = 1 + \mathbb{E}_c^e \sum_{j\in\{N,i\}} \mathbb{I}_j(c,q)\, f^c(\|q-p_j\|)$$

$$G'^c = \mathbb{E}_c^e\, \mathbb{I}_i(c,q)\, f^{c\prime}(\|q-p_i\|)$$

$$f^1 = \frac{1}{1+e^{-2k(R-\|q-p\|)}}, \quad f^2 = e^{\frac{-(q-p)^2}{2\sigma^2}}, \quad f^3 = e^{\frac{-(\|q-p\|-R)^2}{2\sigma^2}}$$

$$\frac{\partial f^1}{\partial P_i} = \frac{\partial f^1}{\partial \|q-P_i\|}\frac{\partial \|q-P_i\|}{\partial P_i} = \frac{2ke^{-2k(R-\|q-P_i\|)}}{(1+e^{-2k(R-\|q-P_i\|)})^2}\cdot\frac{\partial\|q-P_i\|}{\partial P_i}$$

$$\frac{\partial f^2}{\partial P_i} = \frac{\partial f^2}{\partial \|q-P_i\|}\frac{\partial \|q-P_i\|}{\partial P_i} = \frac{-\|q-P_i\|e^{\frac{-(q-P_i)^2}{2\sigma^2}}}{\sigma^2}\cdot\frac{\partial\|q-P_i\|}{\partial P_i}$$

$$\frac{\partial f^3}{\partial P_i} = \frac{\partial f^3}{\partial \|q-P_i\|}\frac{\partial \|q-P_i\|}{\partial P_i} = \frac{-(\|q-P_i\|-R)e^{\frac{-(\|q-P_i\|-R)^2}{2\sigma^2}}}{\sigma^2}\frac{\partial\|q-P_i\|}{\partial P_i}$$

$$\frac{\partial\|q-P_i\|}{\partial P_{i,x}} = \frac{q_x-P_{i,x}}{\|q-P_i\|}, \quad \frac{\partial\|q-P_i\|}{\partial P_{i,y}} = \frac{q_y-P_{iy}}{\|q-P_i\|}$$

$$\frac{\partial f^1}{\partial P_{i,x}} = \frac{-2ke^{-2k(R-\|q-P_i\|)}}{(1+e^{-2k(R-\|q-P_i\|)})^2}\cdot\frac{q_x-P_{ix}}{\|q-P_i\|}$$

$$\frac{\partial f^2}{\partial P_{i,x}} = \frac{-\|q-P_i\|e^{\frac{-(q-P_i)^2}{2\sigma^2}}}{\sigma^2},\frac{q_x-P_{ix}}{\|q-P_i\|}$$

$$\frac{\partial f^2}{\partial P_{i,x}} = -\|q-P_i\|e^{\frac{-(q-P_i)^2}{2\sigma^2}}\cdot\frac{q_y-P_{iy}}{\|q-P_i\|}$$

$$\frac{\partial f^3}{\partial P_{i,x}} = \frac{-(\|q-P_i\|-R)e^{\frac{-(\|q-P_i\|-R)^2}{2\sigma^2}}}{\sigma^2}\frac{q_x-P_{ix}}{\|q-P_i\|}$$

$$\frac{\partial f^3}{\partial P_{ix}} = \frac{-(\|q-P_i\|-R)e^{\frac{-(\|q-P_i\|-R)^2}{2\sigma^2}}}{\sigma^2}\frac{q_y-P_{iy}}{\|q-P_i\|}$$

$$(1-\cos\alpha)\left(\frac{(x-p)^T v}{\|x-p\|}\right) \quad (1-\cos\alpha)\frac{A_1V_x+A_2V_y}{\|AP\|} \quad \frac{\partial}{\partial V_x} = \frac{(1-\cos\alpha)A_1}{\|x-p\|}$$

$$\frac{\partial}{\partial V_y} = \frac{(1-\cos\alpha)A_2}{\|x-p\|}$$

$$H_i = \sum_{e\in E}\sum_{c=1}^{3} \mathbb{C}^i_c \mathbb{E}^e_c \int_{V_i^c} (1+f^c(\|q-p_i\|)) \prod_{k\in\{1,2,3\}|c} (1+\mathbb{E}^e_k \sum_{j\in\{N_i,i\}} \mathbb{I}_j(k,q) f^k(\|q-p_j\|)) \phi_e(q)\,dq$$

Let

$F^c = (1+f^c(\|q-p_i\|))$, which calculate agent i's score with sensor c.

Let

$G^c = (1+\mathbb{E}^e_k \sum_{j\in\{N_i,i\}} \mathbb{I}_j(k,q) f^k(\|q-p_j\|))$, which is the cooperate score of agent i's sensor c's cooperative partners.

$\frac{\partial G^c}{\partial p_i} = 0$ if $\mathbb{C}^i_c = 0$ else $\mathbb{E}^e_c \mathbb{I}_j(k,q) f^{c'}(\|q-p_j\|)$ $\quad \frac{\partial F^c}{\partial p_i} = f^{c'}(\|q-p_i\|)$

$$H_i = \sum_{e\in E}\sum_{c=1}^{3} \mathbb{C}^i_c \mathbb{E}^e_c \int_{V_i^c} F^c \prod_{k\in\{1,2,3\}|c} G^k \phi_e(q)\,dq$$

$$= \sum_{e\in E}\Big(\mathbb{C}^i_1 \mathbb{E}^e_1 \int_{V_i^1} F^1 G^2 G^3 \phi_e(q)\,dq + \mathbb{C}^i_2 \mathbb{E}^e_2 \int_{V_i^2} F^2 G^1 G^3 \phi_e(q)\,dq + \mathbb{C}^i_3 \mathbb{E}^e_3 \int_{V_i^3} F^3 G^1 G^2 \phi_e(q)\,dq\Big)$$

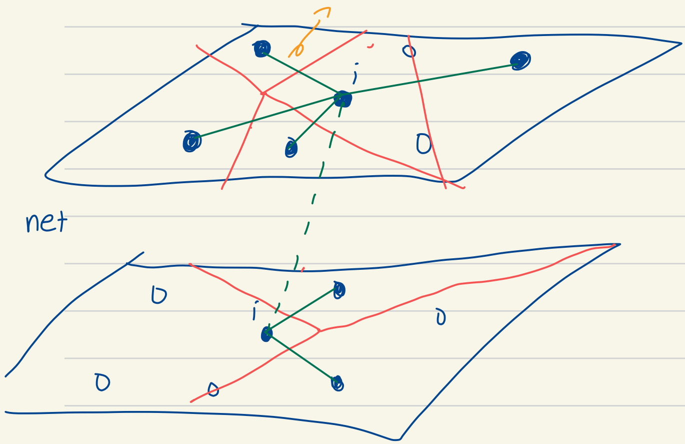
$$\frac{\partial H_i}{\partial p_i} = \sum_{e\in E}\Big(\mathbb{C}^i_1 \mathbb{E}^e_1 \int_{V_i^1} (F^{1'} G^2 G^3 + F^1 G^{2'} G^3 + F^1 G^2 G^{3'})\phi\,dq$$

$$+ \mathbb{C}^i_2 \mathbb{E}^e_1 \int_{V_i^1} (F^{2'} G^1 G^3 + F^2 G^{1'} G^3 + F^2 G^1 G^{3'})\phi\,dq$$

$$+ \mathbb{C}^i_3 \mathbb{E}^e_1 \int_{V_i^1} (F^{3'} G^1 G^2 + F^3 G^{1'} G^2 + F^3 G^1 G^{2'})\phi\,dq$$

camera, e

agent's i's camera is cooperating with any other neighbor with different sensor



$$\frac{f_i^c}{f_i^c + \sum_{j \in N_i} f_j^c}$$

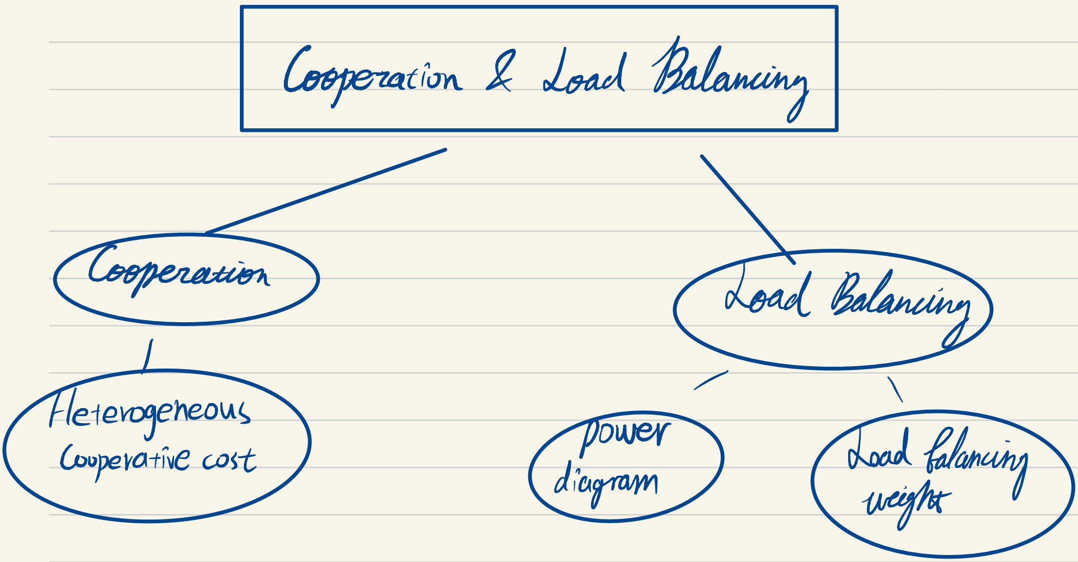
$$\frac{f_i^{c,e}}{A_c^e} \qquad \frac{\dfrac{F_i^e}{A_c^e}}{\sum_e \sum_c \dfrac{F_c^e}{A_c^e}}$$

$$\frac{\left(\dfrac{f_i^c}{f_i^c + \sum_{j \in N_i} f_j^c}\right)(e)}{\sum_{e \in E} \sum_{c \in C_i} \left(\dfrac{f_i^c}{f_i^c + \sum_{j \in N_i} f_j^c}\right)(e)}$$

$$\frac{\int f_i^c \phi_e}{\sum_e \sum_c \dfrac{\int f_i^c \phi_e}{\int f_i^c \phi_e + \sum_{j \in N_i} \int f_j^c \phi_e}}$$: weight for agent with sensor c to event e

$$H_i = \sum_e \sum_c w \, CE \int \cdots$$

Paper Structure



- Abstract

- Introduction
  - Motivation

    Coverage control mostly focus on coordination

  - Related work
    - → composite event coverage
    - → voronoi-based coverage
      - conventional
      - heterogeneous sensing range
      - heterogeneous sensing capabilities
  - The gap between related works & my work
  - Summary of Contribution

- Problem formulation
  - What is composite event? What is the definition?
  - What is the cooperative coverage scenario?
    - → State the environment (N events, m agents ......)
      - Events, agents, sensing modality, density function, voronoi cell
    - → concept of cooperation

Introduction

- 带出 Heterogeneous robot，swarm robots
- 应用场景 → coordination，cooperation → coverage control，transportation
- 传统 coverage control 属于 coordination
  为什么要加入 cooperation → 提出 composite event
- 总结加了 cooperation 的好处
- contribution
- outline

$$H_i = \sum_{e\in E}\sum_{c=1}^{M} \mathrm{w}_{c,e}\mathbb{C}_c^i\mathbb{E}_c^e \int_{V_i^{c,e}} \Big((1+f_c(\|q-p_i\|)) \prod_{k\in C\setminus c} (1+\mathbf{w}_{\mathbf{coop}}\mathbb{E}_k^e \sum_{j\in\{N_i,i\}} \mathbb{I}_j(k,q) f_k(\|q-p_j\|))\Big)\phi_e(q)dq$$

$$\frac{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq}{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq + \sum_{j\in D_i}\int_{V_i^{c,e}} f_c(\|q-p_j\|)\phi_e(q)dq}$$

$$\mathrm{w}_{c,e} = \frac{\frac{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq}{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq+\sum_{j\in D_i}\int_{V_i^{c,e}} f_c(\|q-p_j\|)\phi_e(q)dq}}{\sum_{e\in E}\sum_{c\in C}\frac{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq}{\int_{V_i^{c,e}} f_c(\|q-p_i\|)\phi_e(q)dq+\sum_{j\in D_i}\int_{V_i^{c,e}} f_c(\|q-p_j\|)\phi_e(q)dq}}$$

$$P_e = \int_Q \prod_{k\in C_e}\sum_{j\in N} \mathbb{I}_j(k,q) f_k(\|q-P_j\|)\phi_e(q)\,dq$$

$$(1+f_\alpha)\prod(1+f_\beta)$$

$$H_i = \sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E}\int_{V_i^{\alpha E}(P_i)} 1+f_\alpha(d(q,P))\prod_{\beta\in S_E\setminus\alpha}\Big(1+\underline{W_{coop}}\sum_{j\in N_i^{Group}}\mathbb{X}^{\beta j}(q) f_\beta(d(q,P_j))\Big)\phi_E(q)dq$$

? How to define the boundary

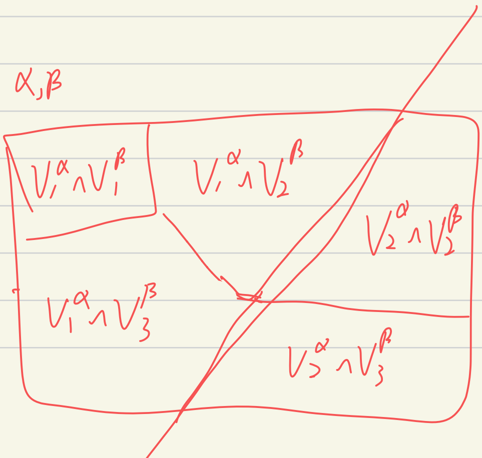
How to diminate the boundary

$$H = \sum_{E\in\mathcal{E}}\sum_{i\in N}\quad\sum_{\alpha\in S_i\cap S_E}$$

$E \quad \alpha,\beta$

on boundary

$$d(q,P_1) = d(q,P_2) = d(q,P_3)$$



$$H=\sum_{i\in N} H_i$$

$$H_i=\sum_{E\in\mathcal{E}}\ \sum_{\alpha\in S_i\cap S_E} W_i^{\alpha E}\int_{V_i^{\alpha E}(P_i)} f_\alpha(d(q,P_i,E))\prod_{\beta\in S_E\setminus\alpha}\left(1+W_{coop}G_i^{\beta E}(q)\right)\phi_E(q)\,dq$$

$$G_i^{\beta E}(q)=\sum_{j\in N_i^{g_{coop}}}\mathbb{X}(j,\beta,q)\,f_\beta(d(q,P_j,E))$$

$$H=\sum_{i\in N}\ \sum_{E\in\mathcal{E}}\ \sum_{\alpha\in S_i\cap S_E} W_i^{\alpha E}\int_{V_i^{\alpha E}(P_i)} f_\alpha(\|q-P_i\|)\prod_{\beta\in S_E\setminus\alpha}\Big(1+W_{coop}\sum_{j\in N_i^{g_{coop}}}\mathbb{X}(j,\beta,q)f_\beta(\|q-P_j\|)\Big)\phi_E(q)\,dq$$

把 $V_i^{\alpha E}$ 再拆成很多小塊把 indicator function 取代

$\Uparrow$

但兩個的積分項裡的東西不一樣

Method 1

$$\frac{\partial H}{\partial P_i}=\sum_{E\in\mathcal{E}}\Big(\frac{\partial}{\partial P_i}\sum_{\alpha\in S_i\cap S_E}\int_{V_i^{\alpha E}(P_i)} f_\alpha(\|q-P_i\|)\prod_{\beta\in S_E\setminus\alpha}\big(1+W_{coop}\sum_{j\in N_i^{g_{coop}}}\mathbb{X}(j,\beta,q)f_\beta(\|q-P_j\|)\big)\phi_E(q)\,dq \rightarrow \text{robot } i$$

$$+\frac{\partial}{\partial P_i}\sum_{\alpha\in S_i\cap S_E}\ \sum_{\substack{k\in N_i^{D_\alpha}\\ k\in N_i^{g_{coop}}}}\int_{V_k^{\alpha E}(P_k)} f_\alpha(\|q-P_k\|)\prod_{\beta\in S_E\setminus\alpha}\big(1+W_{coop}\sum_{j\in N_k^{g_{coop}}}\mathbb{X}(j,\beta,q)f_\beta(\|q-P_j\|)\big)\phi_E(q)\,dq$$

$\rightarrow$ 共邊 & 包含 P_i

$$+\frac{\partial}{\partial P_i}\sum_{\alpha\in S_i\cap S_E}\ \sum_{\substack{k\in N_i^{D_\alpha}\\ k\notin N_i^{g_{coop}}}}\int_{V_k^{\alpha E}(P_k)} f_\alpha(\|q-P_k\|)\prod_{\beta\in S_E\setminus\alpha}\cdots$$

$\rightarrow$ 共邊

$$+\frac{\partial}{\partial P_i}\sum_{\alpha\in S_i\cap S_E}\ \sum_{\substack{k\notin\{N_i^{D_\alpha},i\}\\ k\in N_i^{g_{coop}}}}$$

$\rightarrow$ 不共邊但包含 P_i $\rightarrow$ 不用 Leibniz

$$+\frac{\partial}{\partial P_i}\sum_{\alpha\in S_i\cap S_E}\ \sum_{\substack{k\notin\{N_i^{D_\alpha},i\}\\ k\notin N_i^{g_{coop}}}}$$

$\rightarrow$ 不共邊不包含 P_i $\rightarrow 0$

$$+\frac{\partial}{\partial P_i}\sum_{\substack{\alpha\notin S_i\\ \alpha\in S_E}}\ \sum_{k\in N_i^{g_{coop}}}$$

$\rightarrow$ 不共邊包含 P_i $\rightarrow$ 不用 Leibniz

$$+\frac{\partial}{\partial P_i}\sum_{\substack{\alpha\notin S_i\\ \alpha\in S_E}}\ \sum_{k\notin N_i^{g_{coop}}}$$

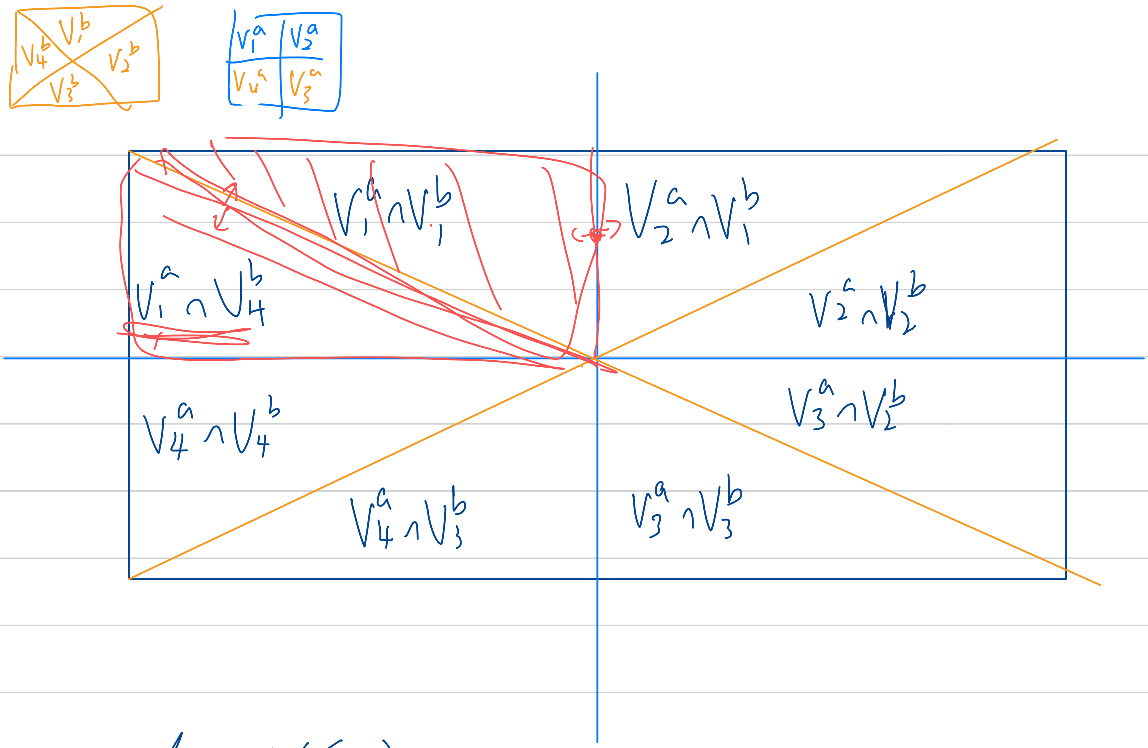
$\rightarrow$ 無關項

$$\sum_{i\in N}\ \sum_{\alpha\in S_i}$$

$$\sum_{\alpha\in S_i\cap S_E}\int_{V_i^a\cap V_i^b}$$

$$\int_{V_i^a\cap V_i^b(P_i)} f_a(d(q,P_i))(1+f_b(d(q,P_i)))n \qquad f_a(d(q,P_i))$$

$$\int_{V_i^a\cap V_4^b(P_i)} f_a(d(q,P_i))(1+f_b(d(q,P_4)))n$$



$n = length(S_E)$

$$H_i = \sum_{\alpha \in S_i \cap S_E} \sum_{j_0 \in N_i^{gwp}} \sum_{j_1 \in N_i^{gwp}} \cdots \sum_{j_n \in N_i^{gwp}} \int_{V_{j_0}^a \cap V_{j_1}^b \cdots}$$

$\frac{\partial H}{\partial P_1}$

for agent 1. a

$$\int_{V_1^a \cap V_1^b} f_a(d(q,P_1))(1+f_b(d(q,P_1)))$$

$$\int_{V_1^a \cap V_4^b} f_a(d(q,P_1))(1+f_b(d(q,P_4)))$$

for agent 4 b

$$\int_{V_4^b \cap V_1^a} f_b(d(q,P_4))(1+f_a(d(q,P_1)))$$

for agent 1 b

$$\int_{V_1^b \cap V_1^a} f_b(d(q,P_1))(1+f_a(d(q,P_1)))$$

$\rightarrow$ self

for agent 1a (3 boundaries to eliminate)

$$\int_{\partial V_1^a \cap V_j^b} \frac{\partial(f_a(d(q,P_1))(1+f_b(d(q,P_1))))}{\partial P_1}$$

$$\int_{\partial V_1^a \cap V_4^b} \frac{\partial(f_a(d(q,P_1))(1+f_b(d(q,P_4))))}{\partial P_1}$$

$$\int_{V_1^a \cap \partial V_1^b} \frac{\partial(f_a(d(q,P_1))(1+f_b(d(q,P_1))))}{\partial P_1}$$

$$\int_{V_1^a \cap \partial V_4^b} \frac{\partial(f_a(d(q,P_1))(1+f_b(d(q,P_4))))}{\partial P_1}$$

for agent 1b $\rightarrow$

$$\int_{V_1^b \cap \partial V_1^a} \frac{\partial(f_b(d(q,P_1))(1+f_a(d(q,P_1))))}{\partial P_1}$$

$$\int_{V_1^b \cap \partial V_2^a} \frac{\partial(f_b(d(q,P_1))(1+f_a(d(q,P_2))))}{\partial P_1}$$

for agent 4b

$$\int_{V_4^b \cap \partial V_1^a} \frac{\partial(f_b(d(q,P_4))(1+f_a(d(q,P_1))))}{\partial P_4}$$

$$\int_{V_4^b \cap \partial V_4^a} \frac{\partial(f_b(d(q,P_4))(1+f_a(d(q,P_4))))}{\partial P_1}$$

$\frac{\partial H_i}{\partial p_i}$

agent 1's 0 sensor

$f_1 \; f_0 \to f_0$

$$\partial \int_{V_1^0 \cap V_4^1} \overline{f_0(d(q, p_1))(1 + f_1(d(q, p_4)))}$$

$+$

$$\partial \int_{V_4^1 \cap V_1^0} \overline{f_1(d(q, p_4))(1 + f_0(d(q, p_1)))}$$

$f_0 \; f_1$

$$H_1 = \sum_{\alpha=0} \int_{V_1^\alpha \cap V_4^\beta} (1 + f_0(d(q, p_1)))(1 + f_1(d(q, p_4)))\, dq$$

$\parallel$

$$H_4 = \sum_{\alpha=1} \int_{V_4^\alpha \cap V_1^\beta} (1 + f_1(d(q, p_4)))(1 + f_0(d(q, p_1)))\, dq$$

$\int \frac{\partial F}{\partial p_i}$ $\quad \frac{\partial H_i}{\partial p_i}$ $\quad \sum_{\alpha \in \text{sinse}} 2 \cdot \int \overline{\partial F(\cdot)}$

$H = H_1 + H_4$

$\frac{\partial H}{\partial p_i} = \frac{\partial H_1}{\partial p_1} + \frac{\partial H_4}{\partial p_1} = 2 \cancel{\frac{\partial H_1}{\partial p_1}}$

∂

$C_E = \mathrm{len}(S_E)$

$\uparrow$

$$\frac{\partial H}{\partial p_i} = \sum_{E \in \mathcal{E}} \sum_{\alpha \in S_{in} S_E} C_E \int_{V_i^\alpha} \frac{\partial F}{\partial p_i} \phi_E(\xi) d\xi$$

load balancing

$\Rightarrow$ ~~C_E~~ <u>Weight</u> + power diagram

$\because$ $d(q, p_i) \Rightarrow$

Dynamic Coalition Formation and task allocation

When multiple compound events are introduced to the scenario, we can see that the performance of the coverage quality varies with different initial conditions, which will cause uneven coverage. Therefore, the concept of coalition is adopted. The entire team will form into coalitions based on agents' spatial, capabilities, and inter-task relationships. The control law will then be disassemble into coalition controller and agent controller. The coalition controller will follow power diagram and agent controller will follow the result of first research.

Related Works

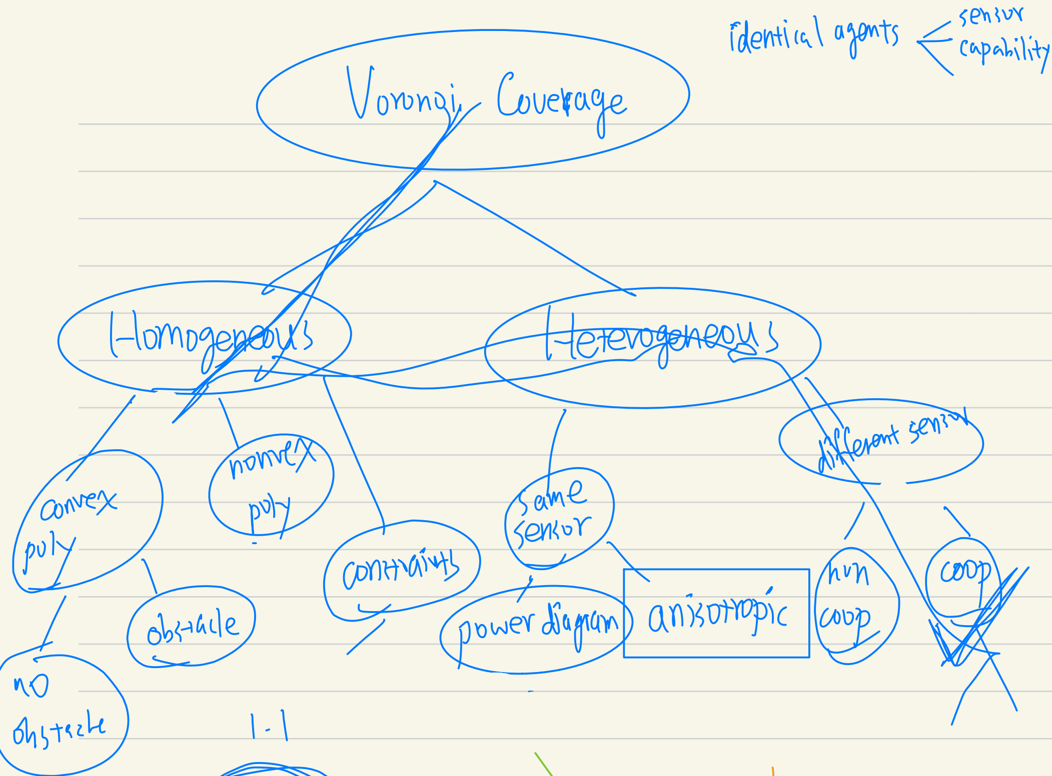
1. Team Assignment for Heterogeneous Multi-robot Sensor Coverage through Graph Representation Learning

Objective

Learn the graph representation of the robot team using DRL + GNN

↔ multiplex graph → GNN → RL

⇒ Fuzzy Clustering (Soft Clustering)



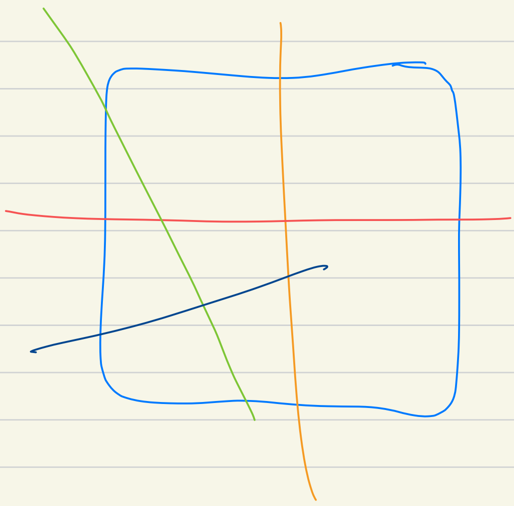
$f(\|q-p_i\|)$

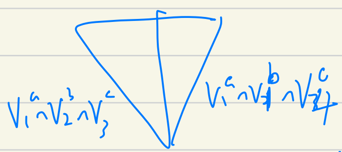
$\min \int \|q-p_i\| \phi(q)\, dq$

(1,1)

+

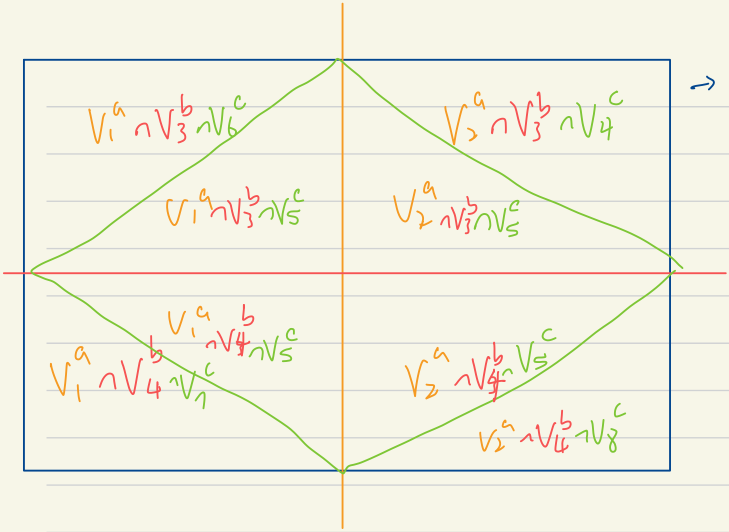
(2,1)





$(1+f^{a}(\|q-p_1\|))(1+f^{b}(\|q-p_2\|))(1+f^{c}(\|q-p_3\|))$

$(1+f^{a}(\|q-p_1\|))(1+f^{b}(\|q-p_1\|))(1+f^{c}(\|q-p_4\|))$



→ two adjacent region will have same value on each boundary since every boundaries are from voronoi diagram

$$F_i^{\alpha E}(q) = \left(1+f^{\alpha}(\|q-p_i\|)\right)\prod_{\beta\in S_E\setminus\alpha}\left(1+w_{coop}G_i^{\beta E}(q)\right)$$

$$G_i^{\beta E}(q) = \sum_{j\in N_i^{group}} \mathbb{I}(j,\beta,q)\, f^{\beta}(\|q-p_j\|)$$

$$H_i = \sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E} w_i^{\alpha E}\int_{V_i^{\alpha}} F_i^{\alpha E}(q)\,\phi_E(q)\,dq \quad , V_i^{\alpha E} = \left\{q\in Q \,\middle|\, (\|q-p_i\|^2-(w_i^{\alpha E})^2) < (\|q-p_k\|^2-(w_k^{\alpha E})^2), \forall k\in N_i^{\alpha}\right\}$$

$$H = \sum_{i\in N} H_i$$

$$= \sum_{i\in N}\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E} w_i^{\alpha E}\int_{V_i^{\alpha}} \left(1+f^{\alpha}(\|q-p_i\|)\right)\prod_{\beta\in S_E\setminus\alpha}\left(1+w_{coop}\sum_{j\in N_i^{group}}\mathbb{I}(j,\beta,q)\,f^{\beta}(\|q-p_j\|)\right)\phi_E(q)\,dq$$

To simplify the computation, we can break down V_i^{α} into several subregion $V_i^{\alpha}\cap V_j^{\beta}\cdots\cap V_k^{\gamma}$ in which the node (i,α), (j,β), ..., (k,γ) are cooperating within the area. The set $\{\alpha,\beta,\ldots,\gamma\}$ is equal to S_E. We define the set $\{(i,\alpha),(j,\beta),\ldots,(k,\gamma)\}$ as n_{Ei}^{α}.

H can be rewritten as:

$$\sum_{i\in N}\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E}\sum_{V\in n_i^{\alpha E}} w_i^{\alpha E}\int_{V}\left(1+f^{\alpha}(\|q-p_i\|)\right)\prod_{\beta\in S_E\setminus\alpha}\left(1+w_{coop}f^{\beta}(\|q-p_j\|)\right)\phi_E(q)\,dq$$

$$H = \sum_{i \in N} \sum_{E \in \mathcal{E}} \sum_{\alpha \in S_i \cap S_E} \int_{V_i^\alpha} \left(1 + f^\alpha(\|q - p_i\|) \prod \left(1 + w_{coop} \sum_{j \in N_i^{group}} \mathbb{X}(j, \beta, q) f^\beta(\|q - p_j\|)\right)\right) \phi_E \, dq$$

expand ↓

$$= \sum_{E \in \mathcal{E}} \Big(\sum_{\alpha \in S_i \cap S_E} \int_{V_i^\alpha} F_i^{\alpha E}(q) \phi_E(q) \, dq \quad \rightarrow \text{self term + out \& inner boundary}$$

expand ↓

$$+ \sum_{j \in N_i^{coop}} \sum_{\beta \in S_j \cap S_E} \int_{V_j^\beta} F_j^{\beta E}(q) \phi_E(q) \, dq \rightarrow \text{coop-term + inner boundary}$$

$$+ \sum_{\substack{j \notin N_i^{coop} \\ k \in N_i^{V^0} \cap N_i^{V^1} \cap V_i^{V_2} \cdots \cap V_i^{V_n}}} \sum_{\gamma \in S_k \cap S_E} \int_{V_k^\gamma} F_k^{\gamma E}(q) \phi_E(q) \, dq \rightarrow \text{outer boundary}$$

$$+ \sum_{j \notin N_i^{coop} \cap V_i^{V^0} \cdots \cap V_i^{V_n}}$$

$$H = \sum_{i \in N} \sum_{E \in \mathcal{E}} \sum_{\alpha \in S_i \cap S_E} \int_{V_i^\alpha} F_i^\alpha(q) \phi_E(q) dq$$

$$= \sum_{E \in \mathcal{E}} \sum_{i \in N} \sum_{\alpha \in S_i \cap S_E} \int_{V_i^\alpha} F_i^\alpha(q) \phi_E(q) dq$$

nudes on the same layer (self, voronoi neighbor, irrelevant)

$$\frac{\partial H}{\partial P_i} = \sum_{E \in \mathcal{E}} \Big(\sum_{\alpha \in S_i \cap S_E} \Big(\underbrace{\int_{V_i^\alpha} F_i^\alpha(q)\phi_E(q) dq + \sum_{j \in N_i^\alpha} \int_{V_j^\alpha} F_j^\alpha(q) \phi_E(q) dq + \sum_{j \notin \{N_i^\alpha, i\}} \int_{V_j^\alpha} F_j^\alpha(q)\phi_E(q) dq}_{\text{nudes on the same layer (self, voronoi neighbor, irrelevant)}} \quad \Rightarrow \text{self}$$

$$+ \sum_{j \in N_{cup}} \sum_{\beta \in S_j \cap S_E \setminus \alpha}$$

$$\sum_{i \neq j} \begin{cases} \lambda \\ \neq \end{cases}$$

$$\sum_{j \in N_i^{cup}} \begin{cases} \lambda \\ \neq \end{cases}$$

$$\sum_{j \cancel{\in} N_i^{cup}} \begin{cases} \neq \\ \cancel{=} \end{cases}$$

Voronoi-Based Cooperative Composite Event Coverage Control for Heterogeneous Sensor Network Via Multi-layer Network

Yen-Cheng Hsu and Teng-Hu Cheng

Put more effort on interactive graph & multi-layer graph.

***Abstract*—Hello**

***Index Terms*—Multi-robot systems, Cooperative systems, Heterogeneous networks, Robot sensing systems, Gradient methods**

I. Introduction

COOPERATION and coordination are essential in multi-robot systems (MRS), especially with the surge of swarm robotics and rapid deployment in setups like multi-UAV (Unmanned Aerial Vehicle) configurations. MRS is harnessed across diverse tasks that embody both coordination and cooperation, including reconnaissance, surveillance, environmental monitoring, and target pursuit [1] [2] [3]. Cooperation involves diverse robots working together, such as using a camera and manipulator for object manipulation. In contrast, coordination aligns homogeneous robots towards a collective goal, like camera-based surveillance. These tasks share a central concern: optimizing robot positions for optimal area coverage, and ensuring the best observation or pursuit outcomes. This pivotal query drives extensive research in coverage control.

Among the primary strategies to address the aforementioned coverage challenge, the Voronoi-based Method for homogeneous robot teams [4] has garnered significant attention. This method employs Voronoi Tessellation to partition a large environment into distinct sub-regions, which are then allocated to individual agents. Subsequently, each agent performs gradient descent within its designated area, aiming to minimize a cost function relative to distance or time.

Research on Voronoi-based coverage control can be broadly categorized into homogeneity and heterogeneity. In homogeneous MRS, agents share identical sensor equipment and capabilities. These studies often emphasize constraints such as energy consumption [5], and connectivity [6]. In contrast, heterogeneous MRS can be further divided into two categories: 1) agents possessing the same type of sensors but with quantitative differences, such as distinct sensing ranges [7], mobility levels [8], or varying body sizes [9]; 2) agents endowed with qualitatively distinct sensing capabilities, capturing multiple event types detectable exclusively by unique sensor modalities [10] [11] [12] [13]. The second category of heterogeneous MRS possesses the capability to undertake collaborative tasks

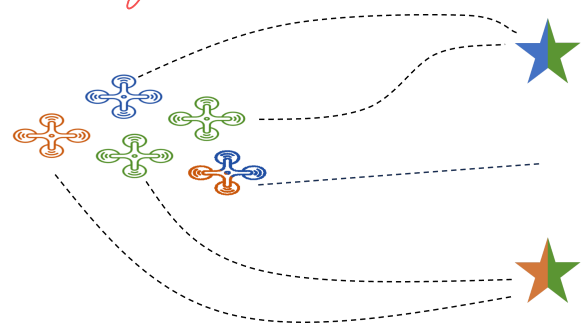


Fig. 1. Heterogeneous Cooperative Scenario Illustration: Two composite events are depicted as star icons with different color combinations: blue-green indicates the event requires both a manipulator and a camera, while orange-green indicates the event requires both a manipulator and a camera. Five heterogeneous agents are present, each with distinct modalities: orange UAV (carrier), blue UAV (manipulator), green UAV (camera), and mixed orange-blue UAV (manipulator and carrier).

such as search and rescue (SAR) missions. These missions entail the cooperation of cameras for searching and carriers for rescuing, a scenario which we identify as a *composite event* [14]. Notably, earlier studies in the heterogeneous MRS have overlooked the potential of this inter-modal cooperative model but predominantly centered on inter-agent coordinating tasks. For instance, an MRS may contain optical sensors for mineral composition in agriculture [15] and nanoelectrode arrays for water quality measurement [16]. However, these two modalities remain distinct and independent. Thus, minerals' composition measurement and water quality measurement are referred to as *atomic events*.

This letter explores a cooperative coverage control scenario where multiple composite events occur in a convex environment, including SAR and excavation events. The heterogeneous MRS features varied modalities—manipulators for excavation, carriers for rescuing, and cameras for visual capture—collaborating seamlessly across different scenes. This scenario underpins our proposed cooperative composite event coverage control problem. It involves inter-agent coordination within identical modalities, like agent grouping with cameras, and inter-modal cooperation between complementary modalities, such as cameras working with manipulators for excavation and carriers for SAR missions. The visual representation of this scenario is depicted in Figure X. Moreover, the inter-agent spatial load balancing [17], and the inter-modal load balancing [18] problems are also considered by introducing a common normalized weight for both the power diagram and modals'

Department of Mechanical Engineering, National Yang Ming Chiao Tung University, Hsinchu 30010, Taiwan. Email: andrew9504089@gmail.com, tenghu@nycu.edu.tw

This research is supported by the Ministry of Science and Technology, Taiwan (Grant Number MOST 111-2221-E-A49 -175 -MY2).

weight in controller design.

In this letter, we introduce a distributed cooperative controller for composite event coverage control, addressing both inter-agent coordination and inter-modality cooperation. Furthermore, we conduct thorough simulations of diverse cooperative scenarios involving multiple modalities while considering practical sensor footprints.

The main contributions of this study are summarized as follows:

1) Introduction of a Novel Heterogeneous Cooperative Utility Function: We propose a novel utility function that effectively models the cooperative relationship between agents with varying modalities.
2) Heterogeneous Cooperative Graph: A multi-layer graph was proposed to model the cooperative behavior between agents' sensing modalities.
3) Load Balancing for Enhanced Performance: We address load balancing by incorporating a balancing weight for both the power diagram and distinct sensing modalities.

The letter is structured as follows: In Section II, we present the scenario for heterogeneous cooperative coverage control and formulate the composite event model. Section III-B focuses on constructing the heterogeneous cooperative utility, which is designed to facilitate both inter-agent coordination and inter-modal cooperation. The derivation of the gradient ascent controller, aimed at maximizing the cooperative utility function, is discussed in Section III-C. To assess the effectiveness of the proposed cooperative controller, we conduct experiments in Section IV. Finally, in Section V, we summarize our findings and conclude our research.

II. Problem Formulation

In this section, we focus on establishing the problem statement for the heterogeneous cooperative coverage control of composite events, which requires joint observation from multiple sensing modalities. We will first introduce the concept of composite events, followed by the exploration of the cooperative scenario.

A. Composite Event Model

change the indicator function for the event sensor to a sensor set

An event is defined as an occurrence of a phenomenon at a location with certain characteristics. When an event comprises solely a single characteristic, indicating that only one sensor modality is necessary for its observation, it is referred to as an atomic event. Conversely, if an event requires the involvement of multiple sensing modalities, it is categorized as a composite event [14].

Consider a set of sensor modalities $\mathcal{S}$ with a cardinality of $|\mathcal{S}| = m$, indicating the existence of m distinct types of sensors. Each sensor type $\alpha \in \mathcal{S}$ corresponds to observing a unique characteristic.

Let $e(\mu, \gamma_e, s_e), 0 \leq \gamma_e \leq 1$ denote an atomic event with γ_e being the confidence value of the occurrence of event e located at μ, where s_e indicates whether the characteristic of the atomic event e can be captured by sensor type s_e.

Now, consider several atomic events $e_i(\mu, \Gamma_E, s_{e_i})$ occurring at the same location μ for $0 \leq i \leq k$. We can represent a composite event consisting of $\{e_0, e_1, \ldots, e_k\}$ as $E(\mu, s_E)$, whose indicator vector and confidence value is given by

$$s_E = \{s_{e_0}, s_{e_1}, \ldots, s_{e_k}\}$$

$$\Gamma_E = \min_{0 \leq i \leq k} \gamma_{e_i} \ , 0 \leq \Gamma_E \leq 1$$

where $s_E$is a set of sensor types necessary for observing the composite event E, and Γ_E represents the level of assurance regarding the event occurring at μ.

B. Heterogeneous Cooperation Scenario

Consider an environment $\mathcal{Q} \subset \mathbb{R}^2$ with a heterogeneous MRS consisting of N agents, where each agent i has sensor set $s_i \subseteq \mathcal{S}$, and a set of composite events $\mathcal{E}$, for every $E \in \mathcal{E}$, its location $\mu \in \mathcal{Q}$ is known in prior. Moreover, for each $E \in \mathcal{E}$, we can define the probabilistic density function for composite event E as a multivariate Gaussian distribution $\mathcal{N}(\mu, \Sigma)$:

$$\phi_E(q) \sim \mathcal{N}(\mu, \Sigma) = \frac{1}{2\pi|\Sigma|^{\frac{1}{2}}} e^{\frac{-1}{2}(q-\mu)^T \Sigma^-1(q-\mu)} \ , q \in \mathcal{Q} \tag{1}$$

where

$$\Sigma = \begin{pmatrix} \Gamma_E^2 & 0 \\ 0 & \Gamma_E^2 \end{pmatrix} \tag{2}$$

Our objective is to find an optimal configuration that each agent can eventually reach a local optimal position satisfying the coverage of the composite events which involve both the coordination and cooperation relationships. A common approach for heterogeneous coverage control is a Voronoi-based technique, where an objective function H_i^{het} is designed as:

$$H_i^{het} = \sum_{E \in \mathcal{E}} \sum_{\alpha \in s_i \cap s_E} \int_{V_i^\alpha} f^\alpha(||q - p_i||) \phi_E(q) dq \tag{3}$$

, where $f^\alpha(||q - p_i||)$ represents the performance of sensing modal α , and V_i^α demonstrated in Fig.2a is the Voronoi cell [19] assigned to agent i among the robots within the neighborhood $\mathcal{N}_i^\alpha$: name the Voronoi cell for sensor type

$$V_i^\alpha = \{q \in \mathcal{Q} \mid \|q - p_i\| < \|q - p_k\|, \forall k \in \mathcal{N}_i^\alpha\} \tag{4}$$

However, the above formulation (3) & (4) only take coordination among the identical sensing modality into consideration which is often not the case in a real-world application that requires multiple distinct modalities to cooperate. Therefore, we will formulate a cooperative form of the objective function in the next section. Add lemma 1 from santos 2018

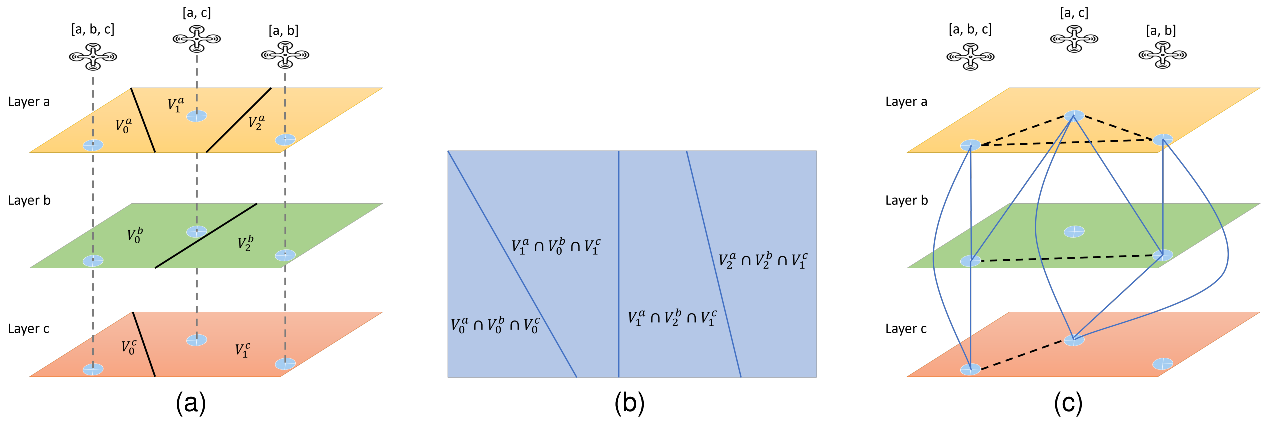


Fig. 2. (a)Multiplex Voronoi Diagram: Agents with sensor suites [a,b,c], [a,c], and [a,b] (where "a", "b", and "c" are sensor types) are shown in separate layers with corresponding Voronoi tessellations. Agent i's cell in the layer for sensor type "a" is denoted as V_i^a. (b)Multi-layer Cooperative Graph: The black dotted line depicts the Delaunay Triangulation Graph, indicating coordination among agents with the same sensing modality. The blue solid line represents the inter-layer cooperative connections between agents with different sensing modalities.

III. Methodology

In this section, formulation for heterogeneous cooperative coverage control will be introduced. First, the definition of cooperation will be formulated by constructing a multi-layer graph. Second, a heterogeneous cooperative utility function will be proposed based on the cooperative graph. Lastly, we utilize the gradient ascent method to design a velocity controller that maximizes the overall utility function to control agents.

A. Cooperative Multi-layer Network

Given a heterogeneous Multi-Robot System (MRS) comprising N agents, each equipped with distinct sensor suites, we introduce a multi-layer graph [20] denoted as $\mathcal{G} = (\mathcal{D}, \mathcal{C})$, depicted in Fig.2c. The notation $\mathcal{D} = \{\mathcal{D}_\alpha; \alpha \in \mathcal{S}\}$ defines distinct layers, where α signifies a layer corresponding to a unique sensing modality α. Each $\mathcal{D}_\alpha = (\mathcal{V}, \mathcal{L}_\alpha)$ represents the Delaunay Triangulation for agents with sensor type α, where the set $\mathcal{V}$ encompasses all N agents as vertices, and $\mathcal{C}$ captures inter-layer cooperative connections defined as:

$$\mathcal{C} = \mathcal{L}_{\alpha\beta} \subseteq \mathcal{V}_\alpha \times \mathcal{V}_\beta;\ \alpha, \beta \in \mathcal{S}, \alpha \neq \beta$$

These cooperative edges $\mathcal{L}_{\alpha\beta}$ link node v_i^α with node v_k^β only when agent i of sensor type α collaborates with agent k of sensor type β. This collaboration is indicated by the condition that their Voronoi cells overlap, i.e., $V_i^\alpha \cap V_k^\beta \neq \emptyset$, where $\alpha \neq \beta$.

The adjacency matrix of the multi-layer graph $\mathcal{G}$ is represented by the block matrix A:

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1M} \\ A_{21} & A_{22} & \cdots & A_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} & A_{M2} & \cdots & A_{MM} \end{bmatrix}$$

Each $A_{\alpha\beta} \in \mathbb{R}^{N\times N}$ represents the intra-layer adjacency matrix of the Delaunay triangulation within layer α if $\alpha = \beta$. If $\alpha \neq \beta$, it serves as the inter-layer adjacency matrix between layers α and β. The entry $a_{ik}^{\alpha\beta}$ indicates the existence of the cooperative relationship between node v_i^α and node v_k^β is defined as follows:

$$a_{ik}^{\alpha\beta} = \begin{cases} 1, & \text{if } V_i^\alpha \cap V_k^\beta \neq \emptyset, \alpha \neq \beta \\ 0, & \text{otherwise} \end{cases}$$

To emphasize the cooperative model, we extract the cooperative graph $\mathcal{G}_{coop} \subset \mathcal{G}$ to isolate the interactions between heterogeneous sensing capabilities. $\mathcal{G}_{coop}$ is illustrated using the adjacency matrix $\bar{A}$, obtained by replacing the diagonal blocks, which represent the Delaunay graph within layers, with zero matrices.

$$\bar{A} = \begin{bmatrix} O & A_{12} & \cdots & A_{1M} \\ A_{21} & O & \cdots & A_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} & A_{M2} & \cdots & O \end{bmatrix}$$

The projection network of $\mathcal{G}_{coop}$ is the network $proj(\mathcal{G}_{coop} = (\mathcal{V}, \mathcal{L})$ where

$$\mathcal{L} = \bigcup_{\substack{\alpha,\beta\in\mathcal{S} \\ \alpha\neq\beta}} \mathcal{L}_{\alpha\beta} \tag{5}$$

the cooperative network can be visualized in a generalized single-layer projection network which indicates all cooperative partners of each agent. Furthermore, we can obtain a projected Voronoi graph from the multi-layer Voronoi graph, where each region is the intersection of multiple cooperating agents' distinct sensors, as shown in Fig. 2b. For example, in the region $V_1^a \cap V_2^b \cap V_1^c$, agent 1's sensor a is cooperating with 2b and 1c. In other words, from the aspect of agent i's sensing modality α, we can break down the Voronoi Cell $V_i^{\alpha E}$ into sub-cells where C_E nodes are cooperating.

B. Heterogeneous Cooperative Utility

In section III-A, a cooperative graph $\mathcal{G}_{coop}$ that describes the cooperative relationships between heterogeneous sensing modalities was proposed, and a cooperative neighborhood of agent i ($\mathcal{N}_i^{\mathcal{G}_{coop}}$) is obtained. In this section, we will focus on establishing a unified utility function that can evaluate the performance of the cooperation for composite events between varying sensing modalities for each agent.

To depict the cooperation between varying sensing modalities, we design a performance function describing the cooperative performance at location q for agent i's sensor type α on composite event E as follows:

$$F_i^{\alpha E}(q) = \big(1 + f^{\alpha}(||q-p_i||)\big) \times \prod_{\beta \in s_E \setminus \alpha} \Big(\overbrace{1}^{\text{ego term}} + \underbrace{w_{coop} G_i^{\beta E}(q)\Big)}_{\text{cooperatve term}} \tag{6}$$

Eq 6 was designed as a joint probability-like function to emphasize that every required sensing modality plays an important role in cooperative coverage control. $f^{\alpha}(||q-p_i||)$ is the sensor function describing the performance of a single sensing type α that agent i possess. The product terms are composed of all other possible sensing modalities $\beta \in \mathcal{S}$ except α which is comprised of two terms, *ego term*, and *cooperative term*. The ego term is simply a constant 1 which preserves the sensor function of α even if the cooperative term is 0. The cooperative term is designed as an arbitrary cooperative constant weight w_{coop} times a function $G_i^{\beta E}(q)$ that describes how well the cooperation is between type α and type β on event E at location q as follows:

$$G_i^{\beta E}(q) = \sum_{j \in \mathcal{N}_i^{\mathcal{G}_{coop}}} \mathbb{X}(j, \beta, q) f^{\beta}(||q-p_j||) \tag{7}$$

, where $\mathbb{I}_E(\beta)$ indicates the necessity of sensing modality β to event E, and $f^{\beta}(||q-p_j||)$ is the sensor function for agent j's sensor type β. To define the cooperative partner at location q between type α and β, an indicator function is designed to tell who is responsible for location q as follows:

$$\mathbb{X}(j, \alpha, q) = \begin{cases} 1, \text{ if } q \in V_j^{\alpha} \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

If location q belongs to agent j 's Voronoi cell of sensing modality α, then $\mathbb{X}(j, \alpha, q)$ will be 1. Note that, since each location q can only be assigned to a single cell, the summation over $\mathcal{N}_i^{\mathcal{G}_{coop}}$ will be 1.

$$\sum_{j \in \mathcal{N}_i^{\mathcal{G}_{coop}}} \mathbb{X}(j, \alpha, q) = 1$$

Eq.6 is confined to a single location q. Now we extend to the entire environment $\mathcal{Q}$. The total coverage performance can be depicted as the integral of eq. 6 times the event density function in eq. 1 as follows:

$$H = \sum_{i \in N} H_i \tag{9}$$

$$H_i = \sum_{E \in \mathcal{E}} \sum_{\alpha \in s_i \cap s_E} \int_{V_i^{\alpha}} F_i^{\alpha E}(q) \phi_E(q) dq \tag{10}$$

Eq. 10 describes the performance of agent i's cooperative coverage performance which we want to maximize, where

$$\begin{aligned} V_i^{\alpha E} = &\{q \in \mathcal{Q} \mid (\|q-p_i\|^2 - (w_i^{\alpha E})^2) \\ &< (\|q-p_k\|^2 - (w_k^{\alpha E})^2), \forall k \in \mathcal{N}_i^{\mathcal{D}_\alpha}\} \end{aligned} \tag{11}$$

is the power diagram [21] as a variant for the Voronoi cell. This power diagram is used to balance the load among varying agents and sensing modalities. If an agent has multiple sensing modalities, and its neighbor has a duplicated sensor type, the Power cell of that agent's sensor type will be smaller. The adaptive weight $w_i^{\alpha E}$ is designed as follows:

$$w_i^{\alpha E} = \frac{Q_i^{\alpha E}}{\sum_{E \in \mathcal{E}} \sum_{\beta \in \mathcal{S}} Q_i^{\alpha E}} \tag{12}$$

where

$$Q_i^{\alpha E} = \frac{\int_{V_i^{\alpha E}} f^{\alpha}(||q-p_i||)\phi_E(q)dq}{\sum_{j \in \mathcal{N}_i^{\mathcal{G}_{coop}}} \int_{V_i^{\beta E}} f^{\beta}(||q-p_j||)\phi_E(q)dq} \tag{13}$$

This weight is designed as eq. 12 which is a normalization of the current sensor type. If all agents have overlapping sensor types, the weight of that sensor type will decrease, and on the other hand, the other sensor types that an agent possesses will increase.

move the load balancing to Adaptive Load balancing section

C. Gradient Ascent-based controller

In this section, we'll derive a gradient ascent-based controller. We will follow the common gradient ascent approach to drive the robots to the optimal configuration that maximizes the utility function in (10), which is the gradient ascent method. The controller for agent i can be designed as follows:

$$u_i = \kappa \frac{\partial H(p_i)}{\partial p_i} \tag{14}$$

Lemma 1: (Leibiniz Integral Rule [19]). Let $\Omega = \Omega(p)$ be a region that depends smoothly on p and that has a well-defined boundary. If $F = \int_{\Omega(p)} f(q)dq$, then

$$\frac{dF}{dp} = \int_{\partial\Omega(p)} f(q) n(q)^T \frac{\partial q}{\partial p} dq \tag{15}$$

where $\int_{\partial\Omega(p)}$ represents the line integral over the boundary of $\Omega(p)$.

By applying Lemma 1, we can formulate the partial derivative of (**??**)

$$\tag{16}$$

where the first term is the contribution of agent i itself, the second term is the neighboring node in projected cooperative voronoi graph, the third term is the neighboring node on any layer of voronoi diagram but not in the projected voronoi cooperative graph, and the last term are the nodes regardless of agent i. For the first term we can expand it into multiple sub-cells to ease the computation of the indicator function.

IV. Experimental Results

A. Environment Setup

asd

B. Sensor Model

asd

V. Conclusions & Future Works

References

[1] C. Wang, S. Zhu, W. Yu, L. Song, and X. Guan, “Minimum norm coverage control of auvs for underwater surveillance with communication constraints,” in *2022 American Control Conference (ACC)*, pp. 1222–1229, 2022.

[2] A. Pierson, A. Ataei, I. C. Paschalidis, and M. Schwager, “Cooperative multi-quadrotor pursuit of an evader in an environment with no-fly zones,” in *2016 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 320–326, 2016.

[3] M. Argany, M. A. Mostafavi, and F. Karimipour, “Voronoi-based approaches for geosensor networks coverage determination and optimisation: A survey,” in *2010 International Symposium on Voronoi Diagrams in Science and Engineering*, pp. 115–123, 2010.

[4] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, “Coverage control for mobile sensing networks,” *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.

[5] K. R. Jensen-Nau, T. Hermans, and K. K. Leang, “Near-optimal area-coverage path planning of energy-constrained aerial robots with application in autonomous environmental monitoring,” *IEEE Transactions on Automation Science and Engineering*, vol. 18, no. 3, pp. 1453–1468, 2021.

[6] W. Luo and K. Sycara, “Voronoi-based coverage control with connectivity maintenance for robotic sensor networks,” in *2019 International Symposium on Multi-Robot and Multi-Agent Systems (MRS)*, pp. 148–154, 2019.

[7] H. Mahboubi, K. Moezzi, A. G. Aghdam, K. Sayrafian-Pour, and V. Marbukh, “Self-deployment algorithms for coverage problem in a network of mobile sensors with unidentical sensing ranges,” in *2010 IEEE Global Telecommunications Conference GLOBECOM 2010*, pp. 1–6, 2010.

[8] S. Kim, M. Santos, L. Guerrero-Bonilla, A. Yezzi, and M. Egerstedt, “Coverage control of mobile robots with different maximum speeds for time-sensitive applications,” *IEEE Robotics and Automation Letters*, vol. 7, no. 2, pp. 3001–3007, 2022.

[9] O. Arslan and D. E. Koditschek, “Voronoi-based coverage control of heterogeneous disk-shaped robots,” in *2016 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 4259–4266, 2016.

[10] F. Abbasi, A. Mesbahi, and J. M. Velni, “Coverage control of moving sensor networks with multiple regions of interest,” in *2017 American Control Conference (ACC)*, pp. 3587–3592, 2017.

[11] M. Santos and M. Egerstedt, “Coverage control for multi-robot teams with heterogeneous sensing capabilities using limited communications,” in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 5313–5319, 2018.

[12] M. Santos, Y. Diaz-Mercado, and M. Egerstedt, “Coverage control for multirobot teams with heterogeneous sensing capabilities,” *IEEE Robotics and Automation Letters*, vol. 3, no. 2, pp. 919–925, 2018.

[13] A. adeghi and S. L. Smith, “Coverage control for multiple event types with heterogeneous robots,” in *2019 International Conference on Robotics and Automation (ICRA)*, pp. 3377–3383, 2019.

[14] J. Gao, J. Li, Z. Cai, and H. Gao, “Composite event coverage in wireless sensor networks with heterogeneous sensors,” in *2015 IEEE Conference on Computer Communications (INFOCOM)*, pp. 217–225, 2015.

[15] M. Pyingkodi, K. Thenmozhi, K. Nanthini, M. Karthikeyan, S. Palarimath, V. Erajavignesh, and G. A. Kumar, “Sensor based smart agriculture with iot technologies: A review,” in *2022 International Conference on Computer Communication and Informatics (ICCCI)*, pp. 1–7, 2022.

[16] C. K. Ho, A. Robinson, D. R. Miller, and M. J. Davis, “Overview of sensors and needs for environmental monitoring,” *Sensors*, vol. 5, no. 1, pp. 4–37, 2005.

[17] J. Cortes, “Coverage optimization and spatial load balancing by robotic sensor networks,” *IEEE Transactions on Automatic Control*, vol. 55, no. 3, pp. 749–754, 2010.

[18] B. Reily, T. Mott, and H. Zhang, “Adaptation to team composition changes for heterogeneous multi-robot sensor coverage,” in *2021 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 9051–9057, 2021.

[19] Q. Du, V. Faber, and M. Gunzburger, “Centroidal voronoi tessellations: Applications and algorithms,” *SIAM Review*, vol. 41, no. 4, pp. 637–676, 1999.

[20] S. Boccaletti, G. Bianconi, R. Criado, C. del Genio, J. Gómez-Gardeñes, M. Romance, I. Sendiña-Nadal, Z. Wang, and M. Zanin, “The structure and dynamics of multilayer networks,” *Physics Reports*, vol. 544, pp. 1–122, nov 2014.

[21] F. Aurenhammer, “Power diagrams: Properties, algorithms and applications,” *SIAM Journal on Computing*, vol. 16, no. 1, pp. 78–96, 1987.

Given N agents, we use the subscript to denote agent's unique id and superscript to denote different sensor type throughout this letter. Therefore, for an agent i with a sensor set S_i, we define a set of nodes to represent its different sensing modalities, $A_i = \{a_i^j \mid j \in S_i\}$.

Then we define the corresponding voronoi diagram of a sensor j as

$V_i^j = \{ q \mid d(q, p_i) \leq d(q, p_j), \forall k \in N^j \}$, where N^j is all agents' ids whose sensor set S_i includes sensor j...

Each layer is a diagram of corr. sensor V^j

set V =

把多層 Voronoi 投影到 single layer 算 union of voronoi diagrams 嗎? 還是 projection.

In other words, we divide V_i^j into multiple subregion $\hat{V}_i^j = \{ V_i^j \cap$

Then we define a multi-layer cooperative graph which serve as the ~~dual graph~~ of the projected voronoi (2-b)

wrong

$$\sum_{i\in\{0,1,2\}} \sum_{\alpha\in S_i}$$

$$= \sum_{\alpha\in S_0} + \sum_{\alpha\in S_1} + \sum_{\alpha\in S_2}$$

$$= \sum_{\alpha\in\{a,b,c\}} + \sum_{\alpha\in\{a,c\}} + \sum_{\alpha\in\{a,b\}}$$

$$= \int_{V_0^a}+\int_{V_0^b}+\int_{V_0^c}+\int_{V_1^a}+\int_{V_1^c}+\int_{V_2^a}+\int_{V_2^b}$$

$$= \left(\int_{V_0^a\cap V_0^b\cap V_0^c}\right) + \left(\int_{V_0^a\cap V_0^b\cap V_0^c} + \int_{V_1^a\cap V_0^b\cap V_1^c}\right) + \left(\int V_0^a\cap V_0^b\cap V_0^c\right)$$

a outer

inner a, c

inner b

outer c

$$+ \left(\left(\int_{V_1^a\cap V_0^b\cap V_1^c} + \int_{V_1^a\cap V_2^b\cap V_1^c}\right) + \left(\int_{V_1^a\cap V_0^b\cap V_1^c} + V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c\right)\right.$$

(1, a)

inner b

(1, c)

$$\left.+\left(V_2^a\cap V_2^b\cap V_1^c\right) + \left(V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c\right)\right)$$

outer b

✓ case 1. Voronoi 邻居 + 合作

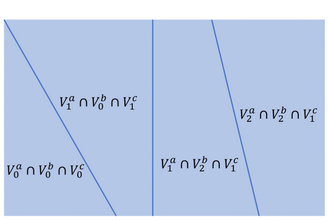
✓ case 2. Voronoi 邻居 + 不合作

✓ case 3. 非 Voronoi 邻居 + 合作

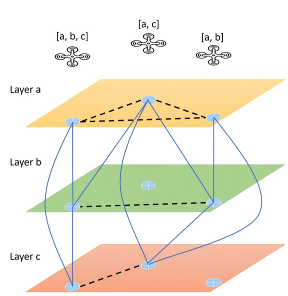
case 4. 非 Voronoi 邻居 + 不合作

on layer α

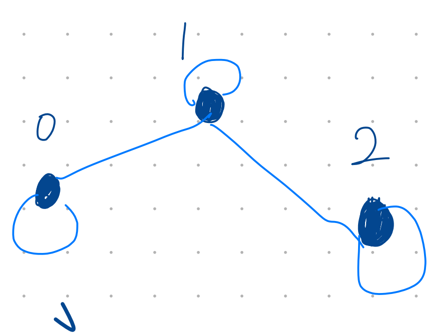
$$\sum_{\alpha\in S_i}\left(\int_{V_i^\alpha} I_i^\alpha(q)\phi_E(q)\,dq + \left(\sum_{j\in N_i^{coop}\cap N_i^\alpha}\int_{V_j^\alpha}dq + \sum_{\substack{j\notin N_i^{coop}\\ j\in N_i^\alpha}}\int_{V_j^\alpha}dq + \sum_{\substack{j\in N_i^{coop}\\ j\notin N_i^\alpha}}\int_{V_j^\alpha}dq\right)\right)$$



(b)



(c)



$$\sum_{i\in\{0,1,2\}} \sum_{\alpha\in S_i} \qquad \sum_{\alpha\in S_i} + \sum_{\alpha\in S\setminus S_i}$$

$$= \sum_{\alpha\in S_0} + \sum_{\alpha\in S_1} + \sum_{\alpha\in S_2}$$

$$= \underbrace{\sum_{\alpha\in\{a,b,c\}} + \sum_{\alpha\in\{a,c\}} + \sum_{\alpha\in\{a,b\}}}_{} \rightarrow \sum_{i\in N^a} + \sum_{i\in N^b} + \sum_{i\in N^c}$$

$$= \int_{V_0^a} + \int_{V_0^b} + \int_{V_0^c} + \int_{V_1^a} + \int_{V_1^c} + \int_{V_2^a} + \int_{V_2^b}$$

$$= \left(\int_{V_0^a\cap V_0^b\cap V_0^c}\right) + \left(\int_{V_0^a\cap V_0^b\cap V_0^c} + \int_{V_1^a\cap V_0^b\cap V_1^c}\right) + \left(\int V_0^a\cap V_0^b\cap V_0^c\right)$$

$$+ \left(\int_{V_1^a\cap V_0^b\cap V_1^c} + \int V_1^a\cap V_2^b\cap V_1^c\right) + \left(\int V_1^a\cap V_0^b\cap V_1^c + V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c\right)$$

$$+ \left(V_2^a\cap V_2^b\cap V_1^c\right) + \left(V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c\right)$$

不同 layer

$$\sum_{i\in N^a} + \sum_{i\in N^b} + \sum_{i\in N^c}$$

$$= \int_{V_0^a} + \int_{V_1^a} + \int_{V_2^a} + \int_{V_0^b} + \int_{V_2^b} + \int_{V_0^c} + \int_{V_1^c}$$

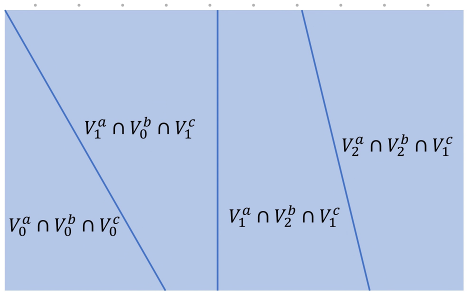
$$= (V_0^a\cap V_0^b\cap V_0^c) + (V_1^a\cap V_0^b\cap V_1^c + V_1^a\cap V_2^b\cap V_1^c) + (V_2^a\cap V_2^b\cap V_1^c)$$

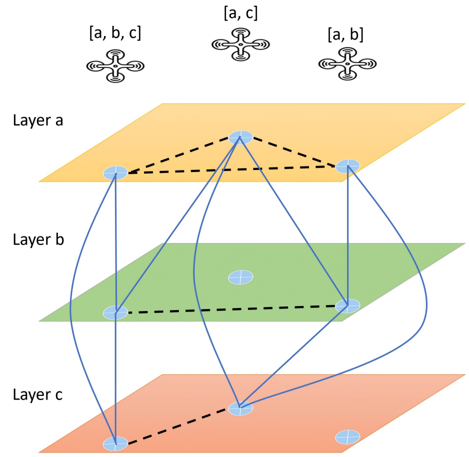
$$+ (V_0^a\cap V_0^b\cap V_0^c + V_1^a\cap V_0^b\cap V_1^c) + (V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c)$$

$$+ (V_0^a\cap V_0^b\cap V_0^c) + (V_1^a\cap V_0^b\cap V_1^c + V_1^a\cap V_2^b\cap V_1^c + V_2^a\cap V_2^b\cap V_1^c)$$

$$H=\sum_{i\in N}\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E}\int_{V_i^\alpha}\overbrace{\left(1+W_{coop}f^\alpha(\|q-p_i\|)\right)}^{\text{ego-term}}\prod_{\beta\in S_E\setminus\alpha}\overbrace{\left(1+W_{coop}\sum_{j\in N_i^{g_{coop}}}\chi(j,\beta,q)f^\beta(\|q-p_j\|)\right)}^{\text{coop-term}}\phi_E(q)\,dq$$

Take





$$\frac{\partial H}{\partial p_i}=\sum_{E\in\mathcal{E}}\sum_{i\in N}\sum_{\alpha\in S_i\cap S_E}=\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_E}\sum_{i\in N^\alpha}$$

for $i=1$

$$\sum_{\alpha\in\{a,c\}}\begin{cases}\int_{V_0^a}+\int_{V_1^a}+\int_{V_2^a} & (2,a)\text{ is }(1,c)\text{ coop}\\ \int_{V_0^c}+\int_{V_1^c}\end{cases}$$

$$\sum_{i\in N^a}+\sum_{i\in N^b}+\sum_{i\in N^c}+\sum_{\alpha\in\{b\}}\int_{V_0^b}+\int_{V_2^b}$$

$$=\int_{V_0^a}+\int_{V_1^a}+\int_{V_2^a}+\int_{V_0^b}+\int_{V_2^b}+\int_{V_0^c}+\int_{V_1^c}$$

$$=(V_0^a\cap V_0^b\cap V_0^c)+(V_1^a\cap V_0^b\cap V_1^c+V_1^a\cap V_2^b\cap V_1^c)+(V_2^a\cap V_2^b\cap V_1^c)$$

$$+(V_0^a\cap V_0^b\cap V_0^c+V_1^a\cap V_0^b\cap V_1^c)+(V_1^a\cap V_2^b\cap V_1^c+V_2^a\cap V_2^b\cap V_1^c)$$

$$+(V_0^a\cap V_0^b\cap V_0^c)+(V_1^a\cap V_0^b\cap V_1^c+V_1^a\cap V_2^b\cap V_1^c+V_2^a\cap V_2^b\cap V_1^c)$$

$\Rightarrow$ from above we can observe that the subregions of V_i^α will cancelled out themselves & V_i^α can be cancelled by finding the delaunay neighbors on layer α

$\Rightarrow$ Thus, we can categorize all the nodes (i, j) denoting agent i's j sensing modality

case 1 voronoi neighbor & coop
case 2 voronoi neighbor & non-coop
case 3 non-neighbor & coop
case 4 non-neighbor & non-coop

where case 1 & 2 provide the conjugate boundary to cancel out and case 1 & 3 provide the cooperating term

問題是我要如何表達 subregions

By observation we can see that the same subregion is considered C^E times where C^E is the number of sensing modalities event E needs. Thus, we can rewrite our partial derivative as

$$\frac{\partial H}{\partial P_i} = \sum_{E \in \mathcal{E}} \sum_{\alpha \in S_i \cap S_E} C^E \int_{V_i^\alpha} \frac{\partial \left(1 + W_{coop} f^i(\|q - P_i\|)\right) \prod_{\beta \in S_E \setminus \alpha} \left(1 + W_{coop} \sum_{j \in N_i^{coop}} \mathbb{1}(j, \beta, q) f(\|P_j\|) \ldots\right)}{\partial P_i}$$

$$\sum_{\alpha \in S_i \cap S_E} \left(\text{agent } i + \text{cooperative partner} + \text{others} \right)$$

$$+ \sum_{\alpha \in S_E \setminus S_i} \left(\text{cooperative} + \text{others} \right)$$

證明思路

1. 定義

a. (i,α) 為在 layer α 的 agent i, i.e agent i 的 sensing modality α

b. $\mathcal{N}$ 為所有的 agents, $|N| = n$

$\mathcal{N}^\alpha$ 為所有擁有 modality α 的 agents, $|N^\alpha| \le n$

c. $\mathcal{D}_i^\alpha$ 為 agent i 在 layer α 的 delaunay neighbor

doesn't include i

from projected coop graph

d. $N_{(i,\alpha)}$ 為 node (i,α) 在 cooperative graph 中所有的 neighboring nodes, N_i is agent i coop neighbor (agent)

e. $N_{(i,\alpha)}^\beta$ 為 node (i,α) 在 cooperative graph 的 layer β 中所有的 neighboring nodes

f. C_E is the cardinality of the set S_E, i.e. the number of sensing modalities that event E requires

g. $P^{i\alpha} = \{ P_1^{i\alpha}, P_2^{i\alpha}, \dots, P_k^{i\alpha} \}$, $\Sigma N^{(C_E-1)} \ge k \ge 1$, where $P_k^{i\alpha}$ is the k^{th} path in the cooperative graph that starts from node (i,α) traversing all layers exactly once, that contains all traversed nodes with cardinality $|P_k^{i\alpha}| = C_E$ → coop-path, $P_k^{i\alpha} = \{(i,\alpha), \dots, (j,\beta)\}$ → propose the concept to multilayer hamiltonian cycle

h. A is the supra-adjacency matrix to describle the multi-layer cooperative graph

i. $\hat{A}$ is the supra-adjacency matrix that coserve only the off-diagonal matrices (i.e the inter-layer relationships)

j. From the projected voronoi diagram, we can see that each voronoi cell V_i^α can be divided into several sub-regions which we refer to as coop-cell formed by node (i,α) and other nodes within the coop-path, i.e each coop-path corresponds to a unique coop-cell. Note that the coop-cell doesn't correspond to an unique coop-path. Since multiple coop-paths with same set of traversed share the same coop-cell. Each coop-cell can be written as $\bigcap_{(j,\beta)\in P_k^{i\alpha}} V_j^\beta$

k.
$$\int_{V_i^\alpha} F_i^{\alpha E}(q)\phi_E(q)dq = \int_{V_i^\alpha} (1+W_{coop} f^\alpha(\|q-P_i\|)) \prod_{\beta\in S_E\setminus\alpha} \Big(1+W_{coop} \sum_{j\in N_{(i,\alpha)}^\beta} \mathbb{I}(j,\beta,q) f^\beta(\|q-P_j\|)\Big)\phi_E(q)dq$$

$$= \sum_{P_k^{i\alpha}\in P^{i\alpha}} \int_{\bigcap_{(j,\beta)\in P_k^{i\alpha}} V_j^\beta} \prod_{(j,\beta)\in P_k^{i\alpha}} (1+W_{coop} f^\beta(\|q-P_j\|))\phi_E(q)dq$$

H:

⇒ 先寫出 coop-cells 的形式再證相加後会变成 F 的樣子 (generalized form)

2. formulation

$$H=\sum_{i\in N}\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E}\int_{V_i^\alpha}F_i^{\alpha E}(q)\phi_E(q)dq$$

From the definition k, we can expand the integral of V_i^α to the summation of multiple coop-cells.

$$H=\sum_{i\in\mathcal{N}}\sum_{E\in\mathcal{E}}\sum_{\alpha\in S_i\cap S_E}\sum_{p_k^{i\alpha}\in p^{i\alpha}}\int_{\bigcap_{(j,\beta)\in p_k^{i\alpha}}V_j^\beta}\prod_{(j,\beta)\in p_k^{i\alpha}}\left(1+W_{coop}f^\beta(\|q-p_j\|)\right)\phi_E(q)dq$$

$$=\sum_{E\in\mathcal{E}}\sum_{i\in\mathcal{N}}\sum_{\alpha\in S_i\cap S_E}\sum_{p_k^{i\alpha}\in p^{i}}\int_{\cap_{(j,\beta)\in p_k^{i\alpha}}V_j^\beta}\prod_{(j,\beta)\in p_k^{i\alpha}}\left(1+W_{coop}f^\beta(\|q-p_j\|)\right)\phi_E(q)dq$$

考慮 $p_k^{i\alpha}$ 的定義，不包含 (i,α) 好像会比較清楚?

3. Derivative

$$\frac{\partial H}{\partial p_i}=\frac{\partial}{\partial p_i}\sum_{E\in\mathcal{E}}\Bigg(\sum_{\alpha\in S_E\cap S_i}\Bigg(\underbrace{\int_{V_i^\alpha}F_i^{\alpha E}(q)\phi_E(q)dq}_{\text{self}}+\underbrace{\sum_{j\in\mathcal{N}^\alpha\cap N_i^\alpha}\int_{V_j^\alpha}F_j^{\alpha E}(q)\phi(q)dq}_{\text{same layer coop with other self layer}}+\underbrace{\sum_{j\in\mathcal{N}^\alpha\setminus N_i^\alpha}\int_{V_j^\alpha}F_j^{\alpha E}(q)\phi(q)dq}_{\text{samelayer with out any coop with } i}\Bigg)$$

$$+\sum_{\beta\in S_E\setminus S_i}\Bigg(\underbrace{\sum_{j\in\mathcal{N}^\beta\cap N_i^\alpha}\int_{V_j^\beta}F_j^{\alpha E}(q)\phi(q)dq}_{\text{doesn't share layer but coop}}+\underbrace{\sum_{j\in\mathcal{N}^\beta\setminus N_i^\alpha}\int_{V_j^\beta}F_j^{\alpha E}(q)\phi(q)dq}_{\text{doesn't share layer and not coop.}}\Bigg)\Bigg)$$

Thm.1

All voronoi cell's boundary can found a conjugate partner that has a identical Leibniz boundary term with an opposite direction normal vector. Therefore, voronoi cell's boundary term can be eliminated.

Corollary 1.

Voronoi Cell 被分割為多個 coop-cells 會有以下兩種情況

1. Voronoi Cell = coop-cells → coop-cells equals to Voronoi cell

2. coop-cells ⊂ Voronoi-cell

For each boundary of two coop-cells within the voronoi cell, it's a voronoi boundary at some layer β. Therefore, by applying Thm 1., the boundary term stem from Lemma 1 can be eliminate

For $\frac{\partial H}{\partial p_i}$'s derivative, we organized all nodes into five categories.

Case 1. Agent i itself

$$\sum_{\alpha \in S_E \cap S_i} \frac{\partial}{\partial p_i} \int_{V_i^\alpha} F_i^{\alpha E}(q) \phi_E(q) dq$$

Case 2. Nodes on the same layer (share the same sensing modalities) & coop-neighbors

$$\sum_{\alpha \in S_E \cap S_i} \sum_{j \in N^\alpha \cap N^i} \frac{\partial}{\partial p_i} \int_{V_j^\alpha} F_j^{\alpha E}(q) \phi_E(q) dq$$

Case 3 nodes on the same layer but not coop-neighbors

$$\sum_{\alpha \in S_E \cap S_i} \sum_{j \in N^\alpha \setminus N^i} \frac{\partial}{\partial p_i} \int_{V_j^\alpha} F_j^{\alpha E}(q) \phi_E(q) dq$$

Case 4 nodes not on the same layer but cooperate with any node involving agent i

$$\sum_{\beta \in S_E \setminus S_i} \sum_{j \in N^\beta \cap N_i} \frac{\partial}{\partial P_i} \int_{V_j^\beta} F_j^{\beta E}(q) \phi_E(q) dq$$

Case 5 nodes not on the same layer and not cooperating with any node involving agent i

$$\sum_{\beta \in S_E \setminus S_i} \sum_{j \in N^\beta \setminus N_i} \frac{\partial}{\partial P_i} \int_{V_j^\beta} F_j^{\beta E}(q) \phi_E(q) dq$$

Case 1, 2, 4 contain terms involving agent i's position P_i. Therefore, we first expand case 1, 2, 4

Case 1

$$\sum_{\alpha \in S_E \cap S_i} \frac{\partial}{\partial P_i} \int_{V_i^\alpha} F_i^{\alpha E}(q) \phi_E(q) dq = \sum_{\alpha \in S_E \cap S_i} \int_{V_i^\alpha} \frac{\partial}{\partial P_i} \left(F_i^{\alpha E}(q) \phi_E(q) \right) dq$$

$$+ \sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha} \int_{\partial V_{ij}^\alpha (P_i)} F_i^{\alpha E}(q) \phi_E(q) n_{ij}^\alpha(q)^T \frac{\partial q}{\partial P_i} dq$$

Case 2

$$\sum_{\alpha \in S_E \cap S_i} \sum_{j \in N^\alpha \cap N^i} \frac{\partial}{\partial P_i} \int_{V_j^\alpha} F_j^{\alpha E}(q) \phi_E(q) dq = \sum_{\alpha \in S_E \cap S_i} \sum_{j \in N^\alpha \cap N_i} \int_{V_j^\alpha} \frac{\partial}{\partial P_i} \left(F_j^{\alpha E}(q) \phi_E(q) \right) dq$$

$$+ \sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha \cap N_i} \int_{\partial V_{ji}^\alpha (P_i)} F_j^{\alpha E}(q) \phi_E(q) n_{ji}^\alpha(q)^T \frac{\partial q}{\partial P_i} dq$$

$\Rightarrow$ The boundary term only take summation from $\mathcal{D}_i^\alpha \cap N_i$ instead of $N^\alpha \cap N_i$ is that from corollary 1 we can know that the boundary of the coop-cells will merge as the voronoi cell's boundary. Thus, the nodes that are not (i,α)'s delaunay neighbur's cells don't depend on p_i.

case 3

$$\sum_{\alpha\in S_E\cap S_i}\sum_{j\in N^\alpha\setminus N_i}\frac{\partial}{\partial p_i}\int_{V_j^\alpha}F_j^{\alpha E}(q)\phi_E(q)dq=\sum_{\alpha\in S_E\cap S_i}\sum_{j\in \mathcal{D}_i^\alpha\setminus N_i}\int_{\partial V_{ji}^\alpha(p_i)}F_j^{\alpha E}(q)\phi_E(q)n_{ji}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq$$

$\to$ since for nodes in $N^\alpha\setminus N_i$, $F_j^{\alpha E}(q)$ is regardless of p_i

Case 4

$$\sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\cap N_i}\frac{\partial}{\partial p_i}\int_{V_j^\beta}F_j^{\beta E}(q)\phi_E(q)dq=\sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j^{\alpha E}(q)\phi_E(q)\right)dq$$

case 5

$$\sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\setminus N_i}\frac{\partial}{\partial p_i}\int_{V_j^\beta}F_j^{\beta E}(q)\phi_E(q)dq$$ $\to$ regardless of p_i $\to 0$ after taking gradient

First we sum up case 1,2,3

$$\sum_{\alpha\in S_E\cap S_i}\int_{V_i^\alpha}\frac{\partial}{\partial p_i}\left(F_i^{\alpha E}(q)\phi_E(q)\right)dq \qquad (1)$$

$$+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in\mathcal{D}_i^\alpha}\int_{\partial V_{ij}^\alpha(p_i)}F_j^{\alpha E}(q)\phi_E(q)n_{ij}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq \qquad (2)$$

$$+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in N^\alpha\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j^{\alpha E}(q)\phi_E(q)\right)dq \qquad (3)$$

$$+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in\mathcal{D}_i^\alpha\cap N_i}\int_{\partial V_{ji}^\alpha(p_i)}F_j^{\alpha E}(q)\phi_E(q)n_{ji}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq \qquad (4)$$

$$+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in\mathcal{D}_i^\alpha\setminus N_i}\int_{\partial V_{ji}^\alpha(p_i)}F_j^{\alpha E}(q)\phi_E(q)n_{ji}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq \qquad (5)$$

First, we'll show that the boundary terms can be eliminated

$$\sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha} \int_{\partial V_{ij}^\alpha (P_i)} F_i^{\alpha E}(q) \phi_E(q) n_{ij}^\alpha (q)^T \frac{\partial q}{\partial p_i} dq$$

<u>Proposition</u>

On the boundary of arbitrary two coop-cells, the coop utility function $F_i^{\alpha E}(q)$ and $F_j^{\alpha E}(q)$ are identical.

<proof>

$$\sum_{k \in \{1, \dots, C_E\}} \int_{\bigcap_{(j,\beta) \in P_k^{i\alpha}} V_j^\beta} \prod_{(j,\beta) \in P_k^{i\alpha}} (1 + W_{coop} f^\beta (\underbrace{\|q - P_j\|}_{d(q, P_j)})) \phi_E(q) dq$$

On any boundary, the distance function $d(q, P_i)$ equals to $d(q, P_j)$. Also, for every coop-cells, the utility function composed of the same set of C_E sensing function. Thus, for any point on the boundary $F_i^{\alpha E}$ equals to $F_j^{\alpha E}$

$$\sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha \cap N_i} \int_{\partial V_{ji}^\alpha (P_i)} F_j^{\alpha E}(q) \phi_E(q) n_{ji}^\alpha (q)^T \frac{\partial q}{\partial p_i} dq$$

$$+ \sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha \setminus N_i} \int_{\partial V_{ji}^\alpha (P_i)} F_j^{\alpha E}(q) \phi_E(q) n_{ji}^\alpha (q)^T \frac{\partial q}{\partial p_i} dq$$

$$= \sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha} \int_{\partial V_{ji}^\alpha (P_i)} F_j^{\alpha E}(q) \phi_E(q) n_{ji}^\alpha (q)^T \frac{\partial q}{\partial p_i} dq$$

$n_{ji}^\alpha = -n_{ij}^\alpha$

$$= - \sum_{\alpha \in S_E \cap S_i} \sum_{j \in \mathcal{D}_i^\alpha} \int_{\partial V_{ji}^\alpha (P_i)} F_j^{\alpha E}(q) \phi_E(q) n_{ij}^\alpha (q)^T \frac{\partial q}{\partial p_i} dq$$

$$\sum_{\alpha\in S_E\cap S_i}\sum_{j\in\mathcal{D}_i^\alpha}\int_{\partial V_{ij}^\alpha(P_i)} F_i^{\alpha E}(q)\phi_E(q) n_{ij}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq$$

$$-\sum_{\alpha\in S_E\cap S_i}\sum_{j\in\mathcal{D}_i^\alpha}\int_{\partial V_{ji}^\alpha(P_i)} F_j^{\alpha E}(q)\phi_E(q) n_{ij}^\alpha(q)^T\frac{\partial q}{\partial p_i}dq$$

$$=0$$

So the summation of case 1,2,3 equals to

$$\sum_{\alpha\in S_E\cap S_i}\int_{V_i^\alpha}\frac{\partial}{\partial p_i}\left(F_i^{\alpha E}(q)\phi_E(q)\right)dq+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in N^\alpha\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j^{\alpha E}(q)\phi_E(q)\right)dq$$

since case 4 doesn't have boundary term according to corollary 1 and case 5 is zero, we can add up them directly

$$\sum_{\alpha\in S_E\cap S_i}\int_{V_i^\alpha}\frac{\partial}{\partial p_i}\left(F_i^{\alpha E}(q)\phi_E(q)\right)dq+\sum_{\alpha\in S_E\cap S_i}\sum_{j\in N^\alpha\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j^{\alpha E}(q)\phi_E(q)\right)dq$$

$$+\sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j(q)\phi_E(q)\right)dq \quad \sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\cap N_i}\int_{V_j^\alpha}\frac{\partial}{\partial p_i}\left(F_j^{\alpha E}(q)\phi_E(q)\right)dq$$

$$=\sum_{\alpha\in S_{E}\cap S_i}\left(\sum_{P_k^{i\alpha}\in P^{i\alpha}}\int_{\bigcap_{(j,\beta)\in P_k^{i\alpha}}V_j^\beta}\frac{\partial}{\partial p_i}\left(\prod_{(j,\beta)\in P_k^{i\alpha}}\left(1+W_{coop}f^\beta(d(q,p_i))\right)\right)\phi_E(q)dq\,+\right.$$

$$\left.\sum_{j\in N^\alpha\cap N_i}\sum_{P_k^{j\alpha}\in P^{j\alpha}}\int_{\bigcap_{(l,\beta)\in P_k^{j\alpha}}V_l^\beta}\frac{\partial}{\partial p_i}\left(\prod_{(l,\beta)\in P_k^{j\alpha}}\left(1+W_{coop}f^\beta(d(q,p_l))\right)\right)\phi_E(q)dq\right)$$

$$+\sum_{\beta\in S_E\setminus S_i}\sum_{j\in N^\beta\cap N_i}\sum_{P_k^{j\beta}\in P^{j\beta}}\int_{\bigcap_{(l,\xi)\in P_k^{j\beta}}V_l^\xi}\frac{\partial}{\partial p_i}\left(\prod_{(l,\xi)\in P_k^{i\alpha}}\left(1+W_{coop}f^\beta(d(q,p_l))\right)\right)\phi_E(q)dq$$

For each multilayer hamiltonian cycle starting from node (i,α), we can find other (C_E-1) hamiltonian cycles that traverses same sets of nodes starting at node (j,β) $j\in N\backslash i$, $\beta\in S_E\backslash\alpha$, we view $P_k^{i\alpha}=P_t^{j\beta}$.

Thus, we can write the above summation as

$$\sum_{\alpha\in S_E\cap S_i} C_E \sum_{P_k^{i\alpha}\in P^{i\alpha}} \int_{\bigcap_{(j,\beta)\in P_k^{i\alpha}} V_j^{\beta}} \frac{\partial}{\partial P_i}\Big(\prod_{(j,\beta)\in P_k^{i\alpha}} (1+W_{coop} f^{\beta}(d(q,P_i))\Big)\phi_E(q)dq$$

$$=\sum_{\alpha\in S_E\cap S_i} \int_{V_i^{\alpha}} \frac{\partial}{\partial P_i}\Big(\prod_{(j,\beta)\in P_k^{i\alpha}} (1+W_{coop} f^{\beta}(d(q,P_i))\Big)\phi_E(q)dq$$