Problem 1. VAR model and Cointegration (40%)

For this problem, use the dataset **Dataset-InterestRate.txt**. The dataset consists the monthly U.S. 1-year r_{1t} (first column) and 3-year r_{3t} (second column) Treasury constant maturity rates from April 1953 to March 2004.

(a) (10%): Fit a VAR model on the two dimensional data (r_{1t}, r_{3t}).

(b) (10%): How do you check the adequacy of your fitted VAR model? Perform such check under 5 % significance level *(Hint: There is a multidimensional version LB test called Portmanteau test; check that out.)*

(c) (10%): Use the fitted VAR model to produce 1-step to 12-step ahead forecasts of the interest rates, assuming that the forecast origin is March 2004.

(d) (10%): We want to know whether the two interest rate series are cointegrated or not. What does "cointegrated" means mathematically? What kind of test can you test this? Use 5 % significance level to perform the test.

Problem 2. ARIMA model and Kalman Filter (40%)

For this problem, use the dataset **Dataset-Stock.txt**. The dataset consists the realized daily volatility series of Alcoa stock returns from January 2, 2003 to May 7, 2004. The volatility series is constructed using 20-minute intra-daily log returns.

(a) (10%): Estimate the local trend model in Equations (11.1) and (11.2) in the slide Week 8-2. Write down the exact equation of your estimated model.

(b) (10%): Obtain time plots for the filtered variables with pointwise 95 % confidence interval. What is the mathematical expression of these "filtered variables"?

(c) (10%): Obtain time plots for the smoothed variables with pointwise 95 % confidence interval. What is the mathematical expression of these "smoothed variables"?

Problem 3. Gibbs Sampling and Markov Switching Model (30%)

For this problem, use the dataset **Dataset-GE.txt**. The dataset consists GE stock monthly log return from 1926 to 1999. We would like to fit a Markov switching model as stated in Week 10-1, page 63, equation (12.55) and (12.56).

(a) (10%): Write down the corresponding Gibbs sampling algorithm. You would need to specified the (conditional) sampling distribution for each of the parameters.

(b) (10%): Write down an algorithm to sample α_{ij} (note that we only have partial likelihood for α_{ij}, as stated in page 66.)

(c) (10%): Combining (a) and (b) to finish the Bayesian inference of this model, and present the results as the graphs in Figure 12.16, page 70. You can choose your own prior parameters.