Task 1:

a) Derive C -> B using stated FDs

C -> AD (FD2)
C -> A (Decompose FD2)
A -> BC (FD1)
A -> B (Decompose FD1)
C -> B (Transitivity)

b) Derive AE -> F

A -> BC(FD1)
A -> C (Decomposition FD1)

C -> AD(FD2)
C -> D (Decompose FD2)

A -> D (Transitivity)

AE -> F (Pseudo-transitivity, through A -> D and FD3)

Pseudo-transitivity explanation:
A -> D ~ X -> Y
DE -> F ~ YW - Z
AE -> F ~ XW - Z

Task 2:

a)
{A} (Add attribute set to result)
{A,B,C} (A -> BC, FD1, add B and C)
{A,B,C,D} (B adds nothing, C -> AD, FD2, add D)
X+ = {A,B,C,D}

b)
{C,E} (Add attribute set to result)
{C,E,A,D} (C->AD, FD2, add A and D)
{C,E,A,D,F} (DE->F, FD3, add F)
{C,E,A,D,F,B} (A -> BC, FD1, add B)
X+ = {C,E,A,D,F,B}

Task 3:

a) Find candidate keys

C and F only on RHS which means they cannot be a part of a CK

A is only on the LHS which means it has to be part of every CK

{A,B,D}+ = {A,B,D,C,E,F}
{A,B,E}+ = {A,B,E,C,D,F}
{A,D,E}+ = {A,D,E,F,B,C}

All are still superkeys so we can not eliminate any yet

{A,B}+ = {A,B,C,D,E,F} (Candidate key)
{A,D}+ ={A,D,B,C,E,F} (Candidate key)
{A,E}+ = {A,E,F} (not a superkey)

b) Which FD violate BCNF?

FD2 violates BCNF because E by itself is not a superkey of R
FD3 also violates BCNF because D by itself is not a superkey of R

c) Decompose R into BCNF

First we choose FD2 since it violates BCNF
{E} -> {F} (X -> Y)
R = {A,B,C,D,E,F} decompose into R1 and R2
R1 = {E,F} with FD2; candidate key is {E}
R2 = {A,B,C,D,E} with new FD(AB->CDE) and FD3; candidate keys are {A,B} and {A,D}

Using R2s FDs (FD(AB->CDE) and FD3) we can derive a FD that will be important later on
D -> B and AB -> CDE implies AD -> CDE using pseudo-transitivity
(X -> Y and WY -> Z, then WX -> Z) pseudo-transitivity

FD3 violates bcnf in R2 as well.
{D} -> {B} (X -> Y)
R2 = {A,B,C,D,E} decompose into R2A and R2B
R2A = {D,B} with FD3; candidate key is {D}
R2B = {A,C,D,E} with new FD (AD -> CDE); candidate key {A,D}

R after decomposition results in:
R1 = {E,F} with FD2; candidate key is {E}
R2A = {D,B} with FD3; candidate key is {D}
R2B = {A,C,D,E} with new FD (AD -> CDE); candidate key {A,D}

Task 4:

To determine if any FD is violating BCNF we must first find all the superkeys and candidate keys

E is only on RHS which means it cannot be a part of a CK
B and C are only on the LHS which means they have to be part of every CK

{B,C,A}+ = {B,C,A,D,E} (superkey)
{B,C,D}+ = {B,C,D,A,E} (superkey)
All are still superkeys so we can not eliminate any yet

{B,C}+ = {B,C, D, A, E} (candidate key)

a) Show why R is not BCNF

Since C is not a superkey FD3 violates BCNF

b) Decompose R into BCNF

FD1: {A,B,C} → {D,E}
FD2: {B,C,D} → {A,E}
FD3: {C} → {D}

Using FD2 and FD3 we can derive a FD that will be important later on
C -> D and BCD -> AE, implies BC -> AE using pseudo transitivity
(X -> Y and WY -> Z, then WX -> Z) pseudo-transitivity

FD3 violates BCNF
{C} -> {D} (X -> Y)
R = {A,B,C,D,E} decompose into R1 and R2
R1 = {C,D} with FD3; candidate key is {C}
R2 = {A,B,C,E} with new FDs (ABC -> E) and (BC -> AE); candidate key {B,C}