

Bayesian Gompertz models for mortality

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1 Overview

This Quarto document illustrates how to fit a Gompertz mortality model in a Bayesian framework using Stan, with a couple of extensions. We will be using data from the Canadian HMD, and some simulated data, as an example.

2 Packages required

To follow along and execute the code on your own computer, you will need the packages below installed and loaded. `rstan` can be a bit tricky to get working; detailed instructions on how to install can be found [here](#).

```
library(rstan)
library(rstanarm)
library(tidyverse)
library(tidybayes)
library(janitor)
```

3 What is a Gompertz model?

In 1825 Benjamin Gompertz, an actuary in London, proposed the following two-parameter model for mortality:

$$\mu(x) = \alpha e^{\beta x}$$

where $\mu(x)$ is the instantaneous mortality rate (hazard) at age x . This model assumes that mortality increases exponentially with age, which is a pretty good assumption for adult mortality (above age 40 or so, with the exception of older ages).

4 Data

For the following two examples we're going to use death and population counts by age and sex for Ontario, sourced from the Canadian Human Mortality Database project. We can read the data files in directly from the URLs:

```
dd <- read_table("https://www.prdh.umontreal.ca/BDLC/data/ont/Deaths_1x1.txt", skip = 1)
dp <- read_table("https://www.prdh.umontreal.ca/BDLC/data/ont/Population.txt", skip = 1)
```

These data files are in 'wide' format. For our purposes it's going to be easier to work with in 'long format'. So let's do that and also clean some other stuff up:

```
dd <- dd |>
  clean_names() |>
  pivot_longer(-(year:age), names_to = "sex", values_to = "deaths") |>
  mutate(deaths = as.numeric(deaths), age = as.numeric(age)) |>
  mutate(age = ifelse(is.na(age), 110, age))
```

```

dp <- dp |>
  clean_names() |>
  pivot_longer(-(year:age), names_to = "sex", values_to = "pop") |>
  mutate(pop = as.numeric(pop), age = as.numeric(age)) |>
  mutate(age = ifelse(is.na(age), 110, age))

d <- dd |>
  left_join(dp) |>
  mutate(mx = deaths/pop,
         log_mx = log(mx))

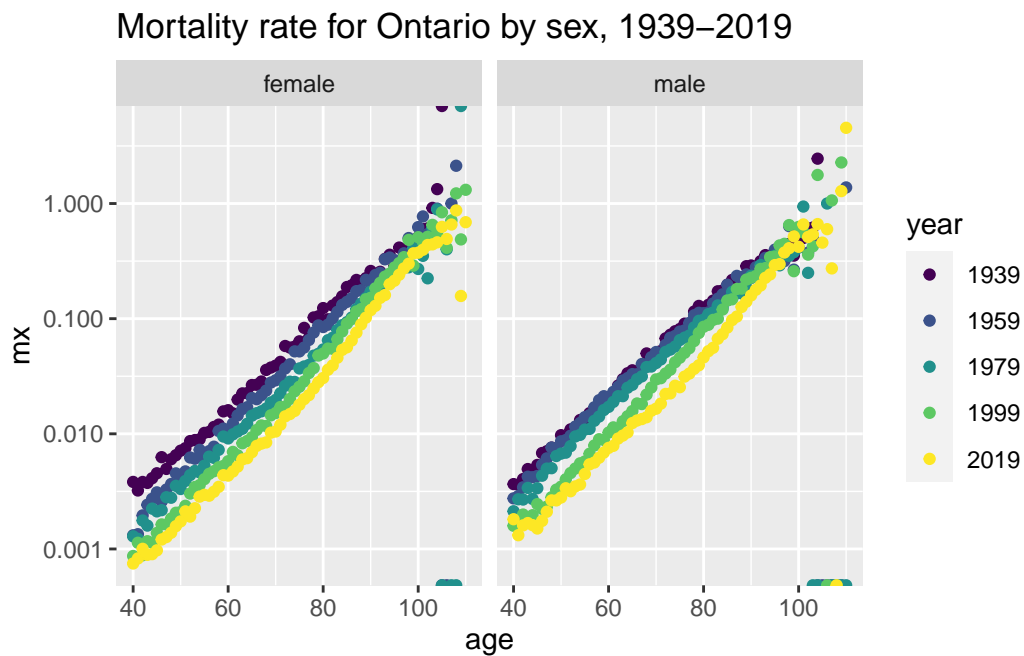
```

Do some quick plots

```

d |>
  filter(age>39, year %in% seq(1939, 2019, by = 20), sex!="total") |>
  ggplot(aes(age, mx, color = factor(year))) +
  geom_point() +
  facet_wrap(~sex)+
  labs(title = "Mortality rate for Ontario by sex, 1939-2019")+
  scale_y_log10()+
  scale_color_viridis_d(name = "year")

```



5 Base model

First let's fit a Gompertz model to males aged 40+ in 2019. Note that Gompertz models have the form

$$\mu(x) = \alpha e^{\beta x}$$

So we can write

$$\log \mu(x) = \log \alpha + \beta x$$

More notes:

- To make this fully Bayesian we need to specify the likelihood and priors. The full model here is

$$y_x \sim \text{Poisson}(P_x \cdot m_x)$$

$$\log m_x = \log \alpha + \beta x$$

$$\log \alpha \sim N(0, 10^2)$$

$$\beta \sim N(0, 0.1^2)$$

where y_x is deaths at age x and P_x is population

- Could have used a normal likelihood (c.f. using `lm`) but nice to account for population size
- We are fitting not on age, but on a centered version (why?)

Now we need to get the data in the right format to read into Stan (this required a named list):

```
d_male_19 <- d |>
  filter(year==2019, sex == "male", age>39, age<105) |>
  mutate(age_c = age - 40)

stan_data <- list(y = round(d_male_19$deaths),
                 pop = round(d_male_19$pop),
                 N = nrow(d_male_19),
                 age_c = d_male_19$age_c)
```

Run the model and look at some output:

```
mod <- stan(file = "../models/gomp.stan",
            data = stan_data,
            seed = 123,
            refresh = 0)
```

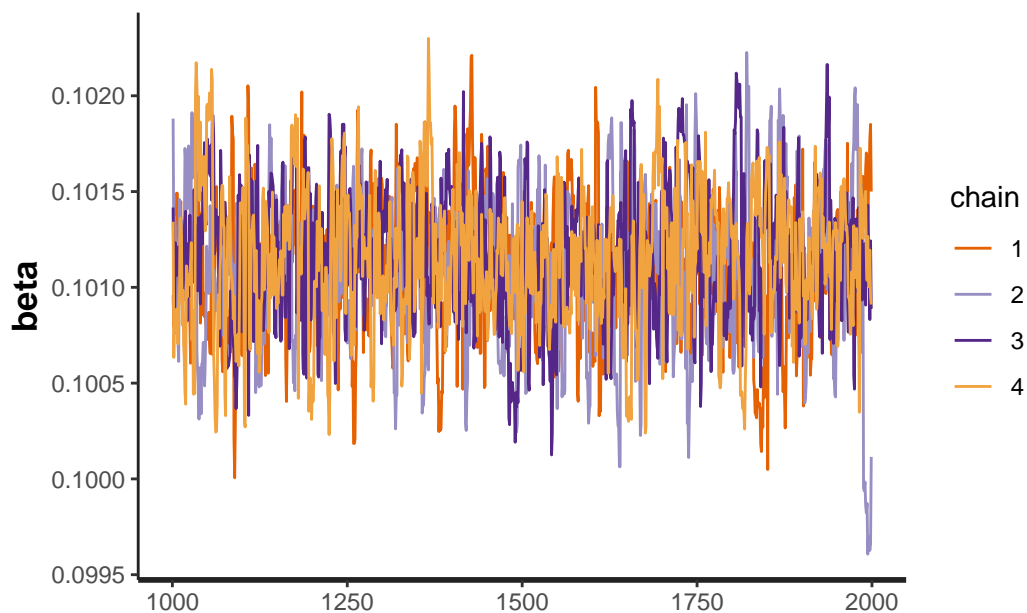
```
summary(mod)$summary[c("log_alpha", "beta"),]
```

	mean	se_mean	sd	2.5%	25%	50%
log_alpha	-7.0080473	5.314773e-04	0.0139930761	-7.0360337	-7.0174794	-7.0082219
beta	0.1011261	1.365872e-05	0.0003643962	0.1004042	0.1008886	0.1011334

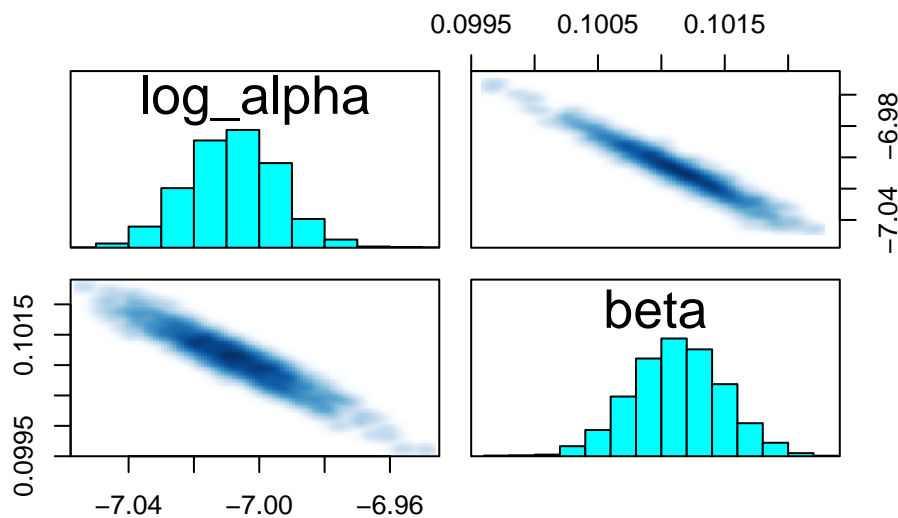
	75%	97.5%	n_eff	Rhat
log_alpha	-6.998623	-6.9804263	693.1973	1.007995
beta	0.101374	0.1018417	711.7498	1.007239

Some quick model checks, looking at the traceplot and pairwise densities

```
pars <- c("log_alpha", "beta")  
traceplot(mod, pars = c("beta"))
```



```
pairs(mod, pars = pars)
```



5.1 Extract the parameter estimates using tidyverse

```
mod |>
  gather_draws(log_alpha, beta) |>
  median_qi()
```

```
# A tibble: 2 x 7
  .variable .value .lower .upper .width .point .interval
  <chr>      <dbl> <dbl> <dbl> <dbl> <chr> <chr>
1 beta      0.101  0.100  0.102  0.95 median qi
2 log_alpha -7.01  -7.04  -6.98  0.95 median qi
```

5.2 Side note: rstanarm is good for these simpler models

Above, we wrote our own Stan model to fit a Gompertz model to one year. This is probably a bit of overkill (although good to see). The `rstanarm` and `brms` packages are very useful for standard models (if you've used `lme4`, the syntax is similar, including for multilevel models). For example here's the same model fit in `rstanarm`:

```
mod_pois_rsarm <- stan_glm(deaths ~ age_c + offset(log(pop)),
  data = d_male_19,
  family = poisson,
  refresh = 0)

summary(mod_pois_rsarm)
```

Model Info:

```
function:    stan_glm
family:      poisson [log]
formula:     deaths ~ age_c + offset(log(pop))
algorithm:   sampling
sample:      4000 (posterior sample size)
priors:       see help('prior_summary')
observations: 65
predictors:  2
```

Estimates:

	mean	sd	10%	50%	90%
(Intercept)	-7.0	0.0	-7.0	-7.0	-7.0
age_c	0.1	0.0	0.1	0.1	0.1

Fit Diagnostics:

	mean	sd	10%	50%	90%
mean_PPD	808.8	4.9	802.5	808.8	815.2

The mean_ppd is the sample average posterior predictive distribution of the outcome variable

MCMC diagnostics

	mcse	Rhat	n_eff
(Intercept)	0.0	1.0	1891
age_c	0.0	1.0	2239
mean_PPD	0.1	1.0	2657
log-posterior	0.0	1.0	1646

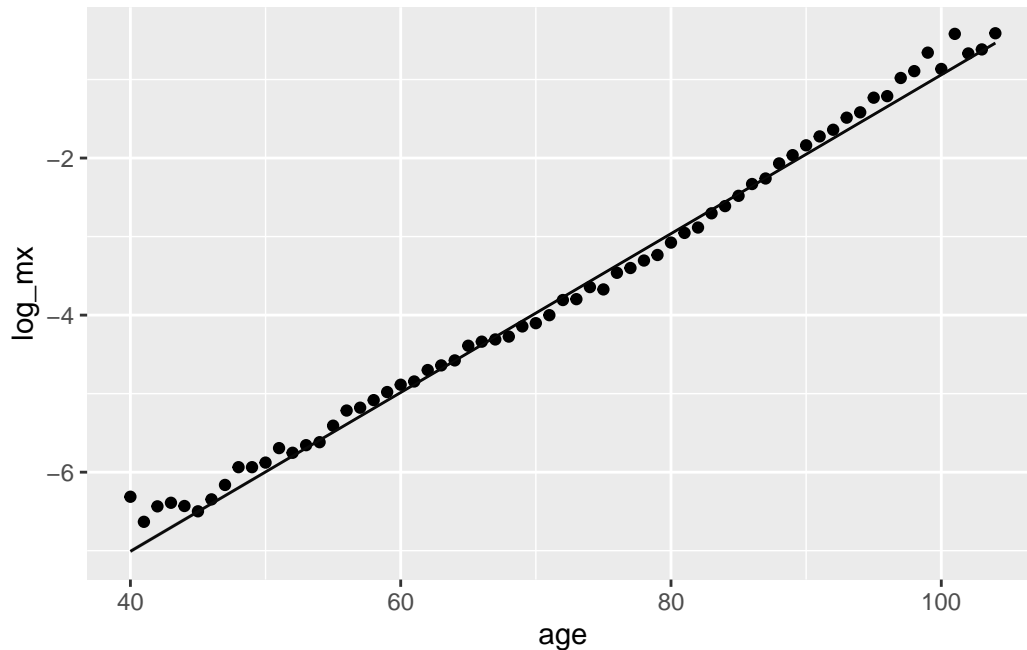
For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective

5.3 Calculate and plot some results

Now we can combine the `tidybayes` syntax with `ggplot` to plot some results. For example, here's the data versus the estimates for the linear predictor:

```
mod |>
  gather_draws(mu[x]) |>
  median_qi() |>
  mutate(age_c = x - 1) |>
  left_join(d_male_19) |>
  ggplot(aes(age, log_mx)) +
```

```
geom_point() +
geom_line(aes(age, .value)) +
geom_ribbon(aes(x = age, ymin = .lower, ymax = .upper), alpha = 0.2)
```



Let's calculate the modal age at death (which for Gompertz mortality is a function of α and β , see [here](#)).

```
mod |>
  spread_draws(log_alpha, beta) |>
  mutate(mode_age = 1/beta*log(beta/exp(log_alpha))) |>
  median_qi()
```

```
# A tibble: 1 x 12
  log_alpha log_alpha.lower log_alpha.upper  beta beta.lower beta.upper mode_age
  <dbl>         <dbl>         <dbl> <dbl>    <dbl>    <dbl>    <dbl>
1    -7.01         -7.04         -6.98 0.101    0.100    0.102    46.6
# i 5 more variables: mode_age.lower <dbl>, mode_age.upper <dbl>, .width <dbl>,
#   .point <chr>, .interval <chr>
```


6 Model over time

Let's fit a slightly more complicated model, for multiple years, where the coefficients themselves are modeled as a random walk over time, i.e.

$$\beta_t \sim N(\beta_{t-1}, \sigma_\beta^2)$$

That is, a different set of Gompertz parameters are fit to every year, but we are assuming that the values in the current year are related to those in the previous year. This is a form of dynamic linear regression. We need to put priors on the first time point, and also on the variance terms:

$$\log \alpha_1 \sim N(-6, 1)$$

$$\beta_1 \sim N(0.1, 0.1^2)$$

$$\sigma^\alpha, \sigma_\beta \sim N^+(0, 1)$$

Note that the likelihood and model on mortality rates are as before, we just have an additional subscript for time:

$$y_{x,t} \sim \text{Poisson}(P_{x,t} \cdot m_{x,t})$$

$$\log m_{x,t} = \log \alpha_t + \beta_t x$$

Now to fit the model. First get the data in the right format:

```
years <- 1969:2019
d_male <- d |> filter(year>=years[1], sex == "male", age>39, age<105) |>
  mutate(age_c = age - 40)

y <- d_male |>
  select(age, year, deaths) |>
  pivot_wider(names_from = "year", values_from = "deaths") |>
  select(-age) |>
  as.matrix()

pop <- d_male |>
  select(age, year, pop) |>
  pivot_wider(names_from = "year", values_from = "pop") |>
  select(-age) |>
  as.matrix()

stan_data <- list(y = y,
                  pop = pop,
                  N = nrow(d_male_19),
                  age_c = d_male_19$age_c,
                  T = ncol(y))
```

Now fit the model (note: takes a while):

```
mod <- stan(file = "../models/gomp_time.stan",
            data = stan_data,
            seed = 852,
            refresh = 0)
```

```
summary(mod)$summary[paste0("beta[", 1:(length(years)), "]"),]
```

	mean	se_mean	sd	2.5%	25%	50%
beta[1]	0.08637726	5.408155e-06	0.0003980909	0.08558443	0.08611616	0.08637743
beta[2]	0.08609944	5.382601e-06	0.0003651452	0.08538432	0.08585654	0.08609603
beta[3]	0.08652996	5.348717e-06	0.0003544698	0.08583556	0.08629793	0.08653449
beta[4]	0.08684064	5.400215e-06	0.0003564779	0.08612462	0.08659735	0.08684537
beta[5]	0.08741744	5.199602e-06	0.0003509300	0.08673113	0.08718470	0.08740712
beta[6]	0.08748773	4.949059e-06	0.0003538722	0.08677683	0.08724562	0.08748556
beta[7]	0.08774906	5.227438e-06	0.0003470942	0.08707205	0.08752175	0.08774641
beta[8]	0.08775315	4.605522e-06	0.0003488372	0.08706722	0.08751955	0.08775056
beta[9]	0.08714442	4.909790e-06	0.0003637462	0.08642407	0.08690333	0.08715067
beta[10]	0.08750291	5.067639e-06	0.0003493777	0.08683635	0.08726701	0.08749992
beta[11]	0.08849544	5.278865e-06	0.0003562111	0.08778669	0.08825092	0.08850578
beta[12]	0.08985115	4.954444e-06	0.0003511657	0.08917759	0.08960734	0.08985032
beta[13]	0.09052507	4.938765e-06	0.0003514422	0.08981549	0.09029087	0.09052456
beta[14]	0.09140559	5.018458e-06	0.0003498535	0.09074796	0.09116877	0.09140231
beta[15]	0.09215278	5.256351e-06	0.0003468065	0.09148419	0.09192024	0.09214732
beta[16]	0.09236948	5.350142e-06	0.0003474065	0.09170140	0.09213687	0.09236870
beta[17]	0.09318506	4.934216e-06	0.0003427723	0.09250845	0.09296097	0.09318540
beta[18]	0.09362172	4.827602e-06	0.0003407646	0.09294258	0.09338867	0.09362186
beta[19]	0.09448456	5.031244e-06	0.0003450523	0.09380537	0.09425529	0.09449205
beta[20]	0.09589743	4.736623e-06	0.0003380852	0.09523916	0.09567167	0.09589801
beta[21]	0.09570937	4.632706e-06	0.0003402578	0.09504408	0.09547927	0.09570833
beta[22]	0.09559315	5.000297e-06	0.0003473280	0.09490788	0.09536296	0.09558930
beta[23]	0.09665824	4.950162e-06	0.0003413273	0.09601284	0.09641574	0.09665355
beta[24]	0.09687286	5.376057e-06	0.0003385324	0.09621242	0.09664052	0.09687314
beta[25]	0.09707877	4.906067e-06	0.0003410519	0.09639510	0.09685583	0.09708573
beta[26]	0.09803144	4.597040e-06	0.0003342023	0.09736212	0.09780229	0.09803443
beta[27]	0.09873372	4.623289e-06	0.0003342355	0.09808297	0.09849884	0.09873116
beta[28]	0.09982051	4.129549e-06	0.0003178325	0.09921858	0.09960067	0.09981570
beta[29]	0.10069637	4.539651e-06	0.0003259581	0.10004710	0.10047692	0.10069030
beta[30]	0.10201285	4.426542e-06	0.0003303988	0.10138374	0.10179314	0.10200705
beta[31]	0.10278899	4.890553e-06	0.0003238709	0.10216854	0.10257216	0.10278989
beta[32]	0.10218301	4.836046e-06	0.0003368970	0.10151708	0.10195512	0.10218451

beta[33]	0.10154374	4.535083e-06	0.0003290753	0.10088974	0.10132732	0.10154684
beta[34]	0.10109115	4.577365e-06	0.0003332883	0.10044712	0.10086770	0.10109366
beta[35]	0.10134945	4.449425e-06	0.0003206309	0.10071706	0.10113792	0.10134655
beta[36]	0.10127201	4.370720e-06	0.0003215180	0.10062505	0.10105768	0.10127816
beta[37]	0.10170418	4.901921e-06	0.0003188131	0.10109436	0.10148031	0.10170576
beta[38]	0.10161387	4.433367e-06	0.0003270773	0.10096578	0.10139755	0.10161475
beta[39]	0.10157482	4.326682e-06	0.0003197609	0.10092946	0.10136405	0.10157236
beta[40]	0.10205767	4.174372e-06	0.0003099799	0.10145796	0.10184443	0.10206079
beta[41]	0.10201268	4.166429e-06	0.0003111773	0.10139494	0.10180401	0.10200843
beta[42]	0.10192130	4.156107e-06	0.0003146041	0.10131034	0.10170127	0.10192421
beta[43]	0.10195703	4.263020e-06	0.0003134395	0.10136025	0.10174919	0.10195307
beta[44]	0.10183627	4.423804e-06	0.0003147932	0.10123076	0.10161419	0.10182999
beta[45]	0.10213364	4.067378e-06	0.0003045724	0.10153586	0.10192837	0.10213084
beta[46]	0.10230145	4.138734e-06	0.0002977424	0.10170537	0.10210635	0.10229703
beta[47]	0.10218170	4.094610e-06	0.0003122484	0.10158453	0.10197153	0.10217805
beta[48]	0.10136463	4.190264e-06	0.0003045539	0.10076529	0.10115875	0.10137030
beta[49]	0.10180627	4.051448e-06	0.0002999508	0.10121524	0.10160704	0.10180346
beta[50]	0.10137754	4.156122e-06	0.0002967507	0.10078747	0.10116896	0.10138211
beta[51]	0.10076741	4.196141e-06	0.0002926109	0.10020392	0.10056956	0.10076230

	75%	97.5%	n_eff	Rhat
beta[1]	0.08663952	0.08715981	5418.341	0.9997651
beta[2]	0.08634806	0.08681256	4602.004	0.9996388
beta[3]	0.08676876	0.08721581	4391.971	0.9993949
beta[4]	0.08708340	0.08750984	4357.558	0.9994744
beta[5]	0.08765920	0.08811718	4555.130	0.9994284
beta[6]	0.08772717	0.08817612	5112.668	0.9998507
beta[7]	0.08799186	0.08841994	4408.764	1.0005681
beta[8]	0.08798697	0.08844275	5737.038	0.9997367
beta[9]	0.08739520	0.08783403	5488.722	0.9995110
beta[10]	0.08773899	0.08819625	4753.123	0.9992677
beta[11]	0.08874090	0.08917115	4553.379	0.9997617
beta[12]	0.09009148	0.09054310	5023.823	0.9998000
beta[13]	0.09075470	0.09121383	5063.736	0.9994911
beta[14]	0.09164203	0.09211567	4859.950	1.0000228
beta[15]	0.09239278	0.09281552	4353.171	1.0000687
beta[16]	0.09260002	0.09306442	4216.434	1.0008383
beta[17]	0.09341342	0.09384493	4825.865	1.0005281
beta[18]	0.09385106	0.09430672	4982.486	0.9995901
beta[19]	0.09471497	0.09516511	4703.477	0.9994943
beta[20]	0.09612373	0.09655150	5094.655	0.9994698
beta[21]	0.09594115	0.09638838	5394.442	1.0008745
beta[22]	0.09582528	0.09627444	4824.898	0.9998200
beta[23]	0.09689225	0.09734074	4754.480	0.9996771

```

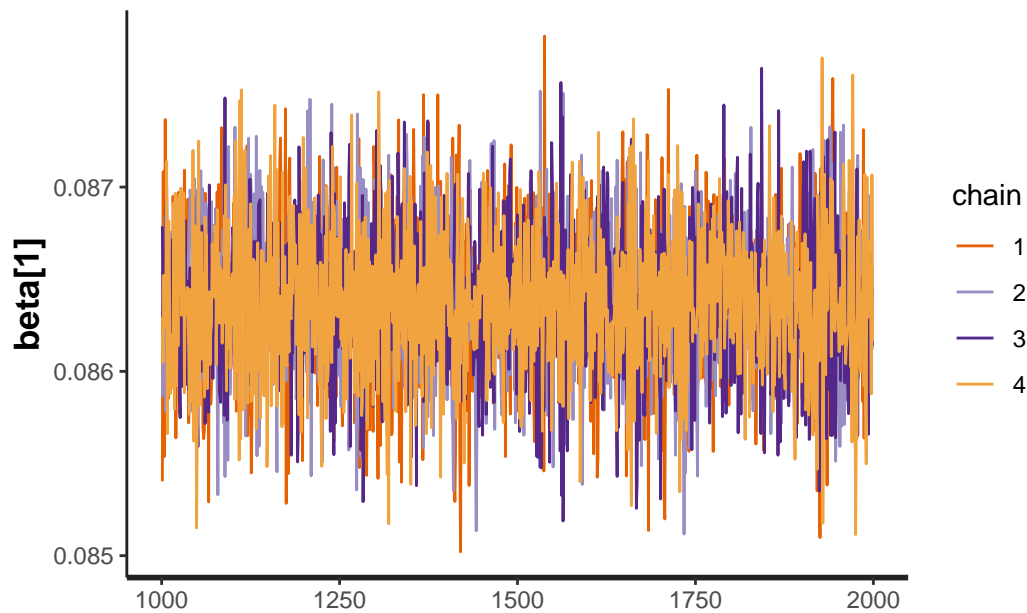
beta[24] 0.09710082 0.09754250 3965.270 0.9997944
beta[25] 0.09730369 0.09774547 4832.524 0.9990396
beta[26] 0.09824938 0.09870363 5285.211 0.9997612
beta[27] 0.09897322 0.09939630 5226.407 0.9992504
beta[28] 0.10003862 0.10044991 5923.678 0.9998480
beta[29] 0.10091535 0.10134433 5155.595 0.9998185
beta[30] 0.10223717 0.10266737 5571.187 0.9995604
beta[31] 0.10299817 0.10343151 4385.591 0.9995600
beta[32] 0.10240877 0.10284245 4853.035 0.9993742
beta[33] 0.10176144 0.10218559 5265.265 0.9993428
beta[34] 0.10130965 0.10174405 5301.625 0.9995127
beta[35] 0.10156371 0.10196972 5192.815 0.9994740
beta[36] 0.10149341 0.10189053 5411.338 1.0003346
beta[37] 0.10192093 0.10233315 4229.993 1.0003420
beta[38] 0.10184554 0.10223206 5442.936 1.0001457
beta[39] 0.10179104 0.10218890 5461.861 0.9993711
beta[40] 0.10226643 0.10267457 5514.229 1.0000402
beta[41] 0.10222812 0.10261621 5578.119 0.9997010
beta[42] 0.10213817 0.10253942 5730.006 0.9996566
beta[43] 0.10216582 0.10258639 5405.959 0.9996635
beta[44] 0.10205022 0.10244746 5063.596 1.0001324
beta[45] 0.10233665 0.10272931 5607.279 0.9998480
beta[46] 0.10250437 0.10287845 5175.427 0.9991479
beta[47] 0.10238781 0.10281103 5815.343 0.9996321
beta[48] 0.10157612 0.10194778 5282.576 1.0005244
beta[49] 0.10200775 0.10240731 5481.251 1.0001391
beta[50] 0.10158540 0.10195414 5098.084 1.0003962
beta[51] 0.10096640 0.10134141 4862.738 0.9994851

```

```

traceplot(mod, pars = c("beta[1]"))

```

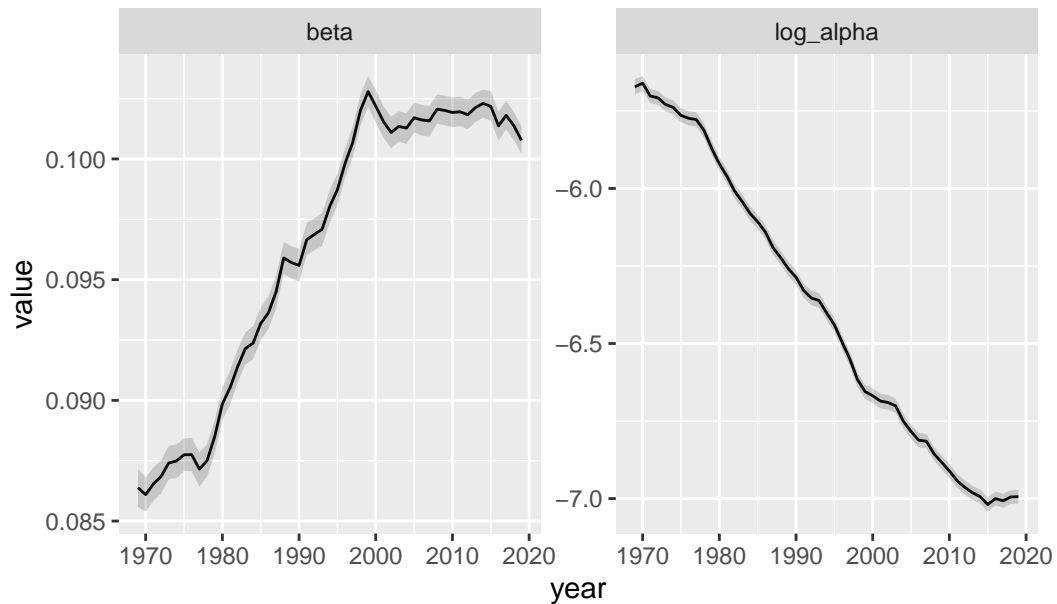


6.1 Plot parameter estimates over time

Now we can plot the parameter estimates over time:

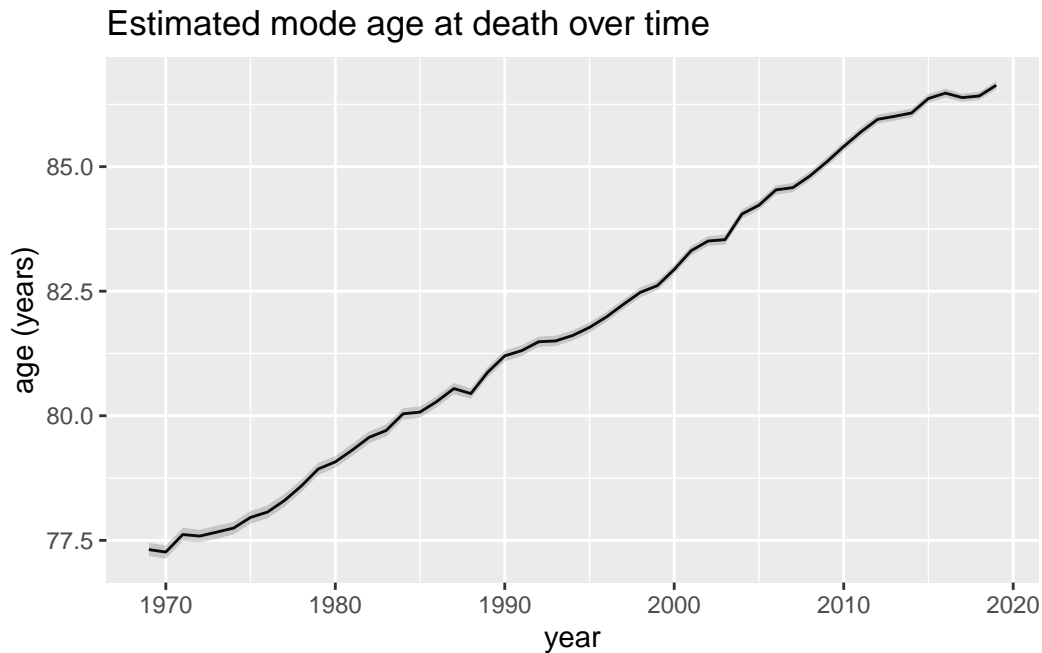
```
mod |>
  gather_draws(log_alpha[i], beta[i]) |>
  median_qi() |>
  mutate(year = years[i]) |>
  ggplot(aes(year, .value)) + geom_line() +
  facet_wrap(~.variable, scales = "free_y") +
  geom_ribbon(aes(ymin = .lower, ymax = .upper), alpha = 0.2)+
  labs(y = "value", title = "Estimates of Gompertz parameters over time")
```

Estimates of Gompertz parameters over time



...and also the mode age over time:

```
mod |>
  spread_draws(log_alpha[i], beta[i]) |>
  mutate(mode_age = 1/beta*log(beta/exp(log_alpha))+40) |>
  median_qi() |>
  mutate(year = years[i]) |>
  ggplot(aes(year, mode_age)) + geom_line() +
  geom_ribbon(aes(ymin = mode_age.lower, ymax = mode_age.upper), alpha = 0.2) +
  labs(title = "Estimated mode age at death over time", y = "age (years)")
```



6.2 Question for you

Can you forecast mortality rates with this model? If so, how? What are the assumptions behind the forecasts?

7 Partial data observed

Now we're going to switch gears a bit and have a look at the situation where we have mortality rates for five geographic areas, but in one area we only have partial information. This is based on simulated data (you can have a look to see how I generated it based on the `simulated_data.R` script).

For 4 areas, we have deaths and population counts from ages 40 up to 60:

```
df <- read_rds("../data/sim.rds")
df
```

```
# A tibble: 80 x 4
# Groups:   area [4]
   age area deaths pop
<int> <int> <int> <dbl>
1    40     1     1 5000
```

```

2    41    1    1  5000
3    42    1    1  5000
4    43    1    2  5000
5    44    1    1  5000
6    45    1    2  5000
7    46    1    0  5000
8    47    1    3  5000
9    48    1    5  5000
10   49    1    3  5000
# i 70 more rows

```

For one region, we just have ${}_{20}q_{40}$, that is, the probability of dying between ages 40 and 60:

```

q40 <- read_rds("../data/q40.rds")
q40

```

```
[1] 0.07262754
```

7.1 Relating ${}_{20}q_{40}$ to the Gompertz model

We want to fit a Gompertz model to each of the five areas (even the one with just a summary indicator). How to do this? Well, if we tell Stan how ${}_{20}q_{40}$ relates to μ_x , then the model has at least some information to estimate α and β . In particular, for each area, we're assuming:

$${}_1p_x = e^{-\mu_x}$$

where $\mu_x = \alpha e^{\beta x}$ and

$${}_{20}q_{40} = 1 - \prod_{x=40}^{60} {}_1p_x$$

Check the Stan file `gomp_partial.stan` to see this translated into code.

7.2 Fit the model

Get the data in the right format. Note that everything is a matrix now because we have more than one area:


```

y <- as.matrix(df |>
  pivot_wider(names_from = "area", values_from = "deaths") |>
  select(-age, -pop))
pop <- as.matrix(df |>
  select(-deaths) |>
  pivot_wider(names_from = "area", values_from = "pop") |>
  select(-age))

stan_data <- list(N = length(40:59),
  M = 4,
  K = 5,
  y = y,
  pop = pop,
  z = q40,
  age_c = (40:59)-40)

```

Fit the model:

```

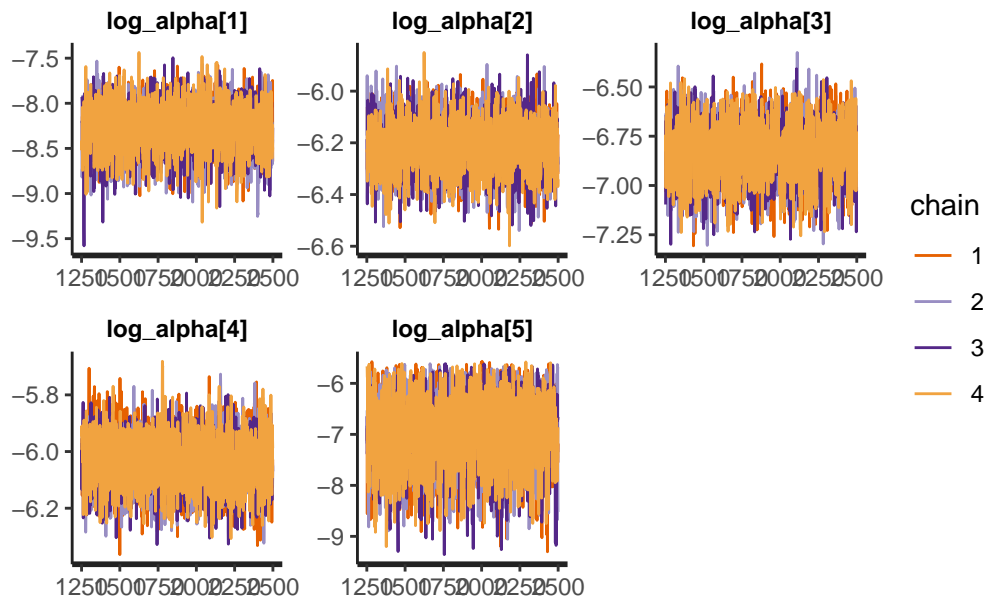
mod <- stan(file = "../models/gomp_partial_data.stan",
  data = stan_data,
  seed = 15,
  control = list(adapt_delta = 0.96),
  iter = 2500,
  refresh = 0)

```

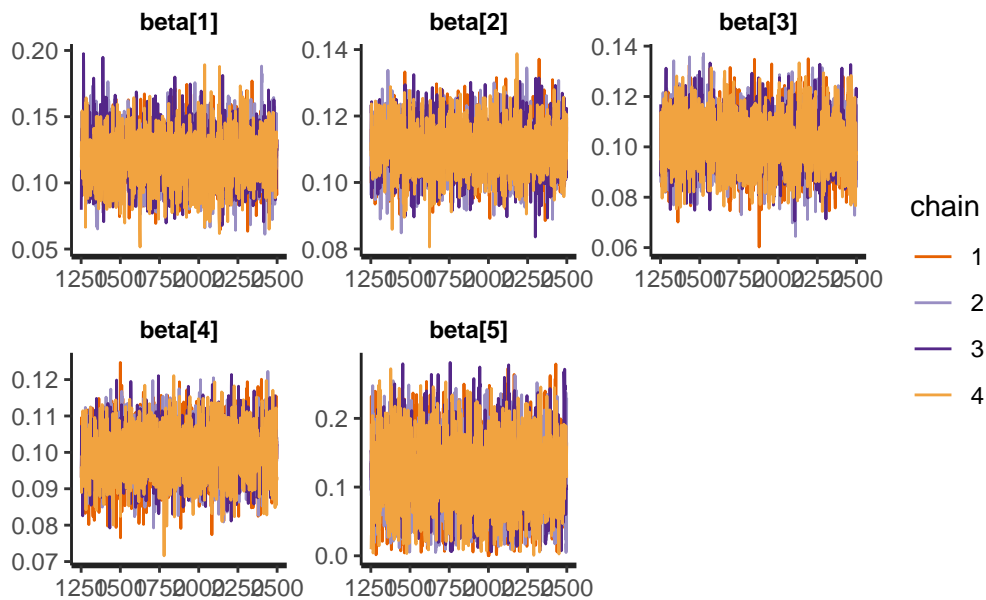
```

traceplot(mod, c("log_alpha"))

```



```
traceplot(mod, c("beta"))
```



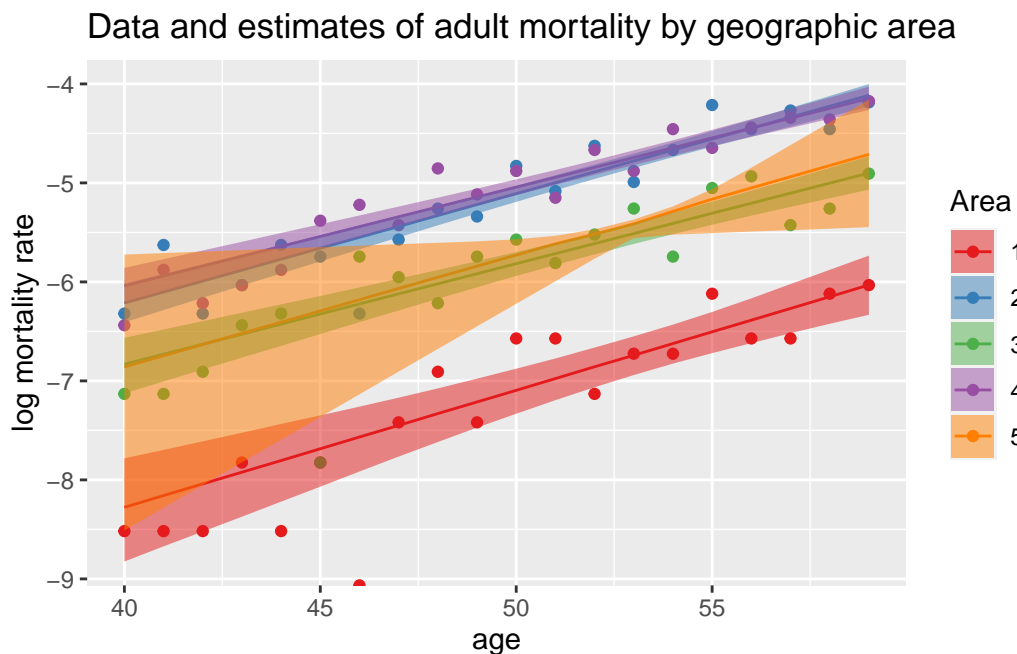
Let's plot the fitted lines (with uncertainty) with the observed data for each region. Notice the difference in uncertainty around area 5!

```
mod |>
  gather_draws(mu[i,j]) |>
  median_qi() |>
```

```

mutate(age = i-1+40) |>
rename(area = j) |>
left_join(df) |>
mutate(log_mx = log(deaths/pop)) |>
ggplot(aes(age, log_mx))+
  geom_point(aes(color = factor(area)))+
  geom_line(aes(age, .value, color = factor(area)))+
  geom_ribbon(aes(age, ymin = .lower, ymax = .upper, fill = factor(area)), alpha = 0.5)+
  scale_color_brewer(name = "Area", palette = "Set1")+
  scale_fill_brewer(name = "Area", palette = "Set1")+
  labs(title = "Data and estimates of adult mortality by geographic area", y = "log mortality")

```



7.3 Compare parameter estimates against the truth

Here's what we estimated:

```

mod |>
gather_draws(log_alpha[i], beta[i]) |>
median_qi()

```

```

# A tibble: 10 x 8
  i .variable .value .lower .upper .width .point .interval

```

	<int>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<chr>	<chr>
1	1	beta	0.118	0.0823	0.158	0.95	median	qi
2	1	log_alpha	-8.28	-8.82	-7.78	0.95	median	qi
3	2	beta	0.110	0.0959	0.125	0.95	median	qi
4	2	log_alpha	-6.21	-6.41	-6.02	0.95	median	qi
5	3	beta	0.102	0.0821	0.123	0.95	median	qi
6	3	log_alpha	-6.83	-7.13	-6.57	0.95	median	qi
7	4	beta	0.0995	0.0861	0.113	0.95	median	qi
8	4	log_alpha	-6.04	-6.22	-5.86	0.95	median	qi
9	5	beta	0.113	0.0152	0.229	0.95	median	qi
10	5	log_alpha	-6.86	-8.51	-5.72	0.95	median	qi

And here's the truth (the values underlying the simulation):

```
read_rds("../data/true_params.rds")
```

```
# A tibble: 5 x 3
  are log_alpha beta
<int>   <dbl> <dbl>
1     1    -7.98 0.0985
2     2    -6.38 0.125
3     3    -6.88 0.110
4     4    -5.93 0.0924
5     5    -7.48 0.154
```