Lab 07

Monday, November 15, 2021 11:57 AM

Lab 7 – Proprietăți de închidere

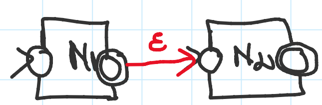
O proprietate P este închisă în rap. cu mulțimea limbajelor regulate dacă este adev. rel.: $\mathcal{L} \in LR \Rightarrow P(\mathcal{L}) \in LR$.

Limbajele regulate sunt închise peste:
- concatenare
- reuniune
- Kleene Star
- intersecție
- complement
- diferență
- reversal
- sufix
- prefix
- homomorfism

1) Concatenarea $(\mathcal{L}_1, \mathcal{L}_2)$:

I → găsim NFA-urile N_1 și N_2 a.î. $\mathcal{L}(N_1) = \mathcal{L}_1$ și $\mathcal{L}(N_2) = \mathcal{L}_2$

→ construim NFA care acceptă $\mathcal{L}_1\mathcal{L}_2$:



II → găsim E_1, E_2 a.î. $\mathcal{L}(E_1) = \mathcal{L}_1$ și $\mathcal{L}(E_2) = \mathcal{L}_2$

→ E_1E_2 generează $\mathcal{L}_1\mathcal{L}_2$

2) Reuniune $(\mathcal{L}_1, \mathcal{L}_2)$:

— || —

3) Kleene Star $(\mathcal{L})$:

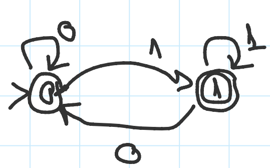
— || —

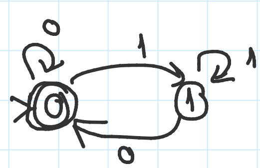
4) Complement $(\mathcal{L})$:

$\mathcal{L} \in LR \Rightarrow \overline{\mathcal{L}} \in LR$

→ găsim DFA-ul A a.î. $\mathcal{L}(A) = \mathcal{L}$: $A = (K, \Sigma, \delta, q_0, F)$

→ construim A' a.î. $\mathcal{L}(A') = \overline{\mathcal{L}}$: $A' = (K, \Sigma, \delta, q_0, K \setminus F)$





$\mathcal{L} = \ldots 1$ $\qquad$ $\bar{\mathcal{L}} = \ldots 0 \cup \varepsilon$

4) Intersecția $(\mathcal{L}_1, \mathcal{L}_2)$:

→ DFA-urile $A_1 = (K_1, \Sigma, \delta_1, q_{01}, F_1)$, $A_2 = (K_2, \Sigma, \delta_2, q_{02}, F_2)$

$\mathcal{L}(A_1) = \mathcal{L}_1$ $\qquad$ $\mathcal{L}(A_2) = \mathcal{L}_2$

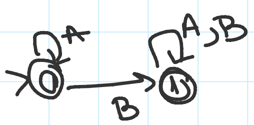
→ construim DFA-ul A a.î. $\mathcal{L}(A) = \mathcal{L}_1 \cap \mathcal{L}_2$

$$A = (K_1 \times K_2, \Sigma, \delta, (q_{01}, q_{02}), F_1 \times F_2)$$

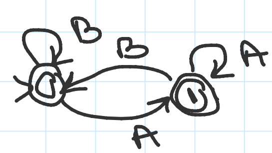
$$\delta((q_{i_1}, q_{i_2}), c) = (q_{j_1}, q_{j_2}) \text{ dacă } \begin{cases} \delta_1(q_{i_1}, c) = q_{j_1} \\ \delta_2(q_{i_2}, c) = q_{j_2} \end{cases}$$

Ex:

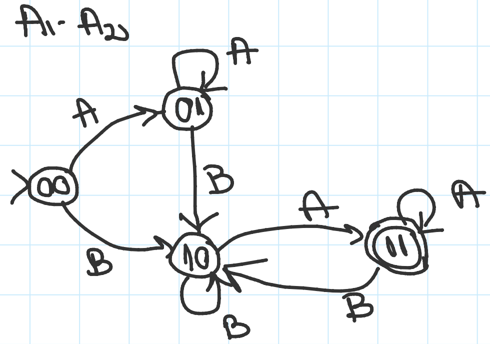
A_1



A_2



$A_1 \cdot A_2$



6) Reversal $(\mathcal{L})$: $\quad rev(\mathcal{L}) = \{ c_n c_{n-1} \ldots c_1 \in \Sigma^* \mid c_1 c_2 \ldots c_{n-1} c_n \in \mathcal{L} \}$

I

→ NFA-ul A a.î. $\mathcal{L}(A) = \mathcal{L}$

→ construim A^R a.î. $\mathcal{L}(A^R) = rev(\mathcal{L})$

astfel:

- q_0' stare inițială
- ε-tranz. de la q_0' la toate stările finale din A
- inversăm toate tranz.

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— inversăm toate tranz.:

$$A: \; q_i \xrightarrow{c} q_j \;\Rightarrow\; A^R: \; q_j \xrightarrow{c} q_i$$

— starea finală din A^R este starea inițială din A

VI → găsim E a.î. $\mathcal{L}(E)=\mathcal{L}$
→ construim E^R care gen. $rev(\mathcal{L})$, inductiv

$E = c \in \Sigma \rightarrow E^R = c$
$E = \varepsilon \rightarrow E^R = \varepsilon$
$E = \emptyset \rightarrow E^R = \emptyset$
$E = E_1E_2 \rightarrow E^R = E_2^R E_1^R$
$E = E_1 \cup E_2 \rightarrow E^R = E_1^R \cup E_2^R$
$E = E_1^* \rightarrow E^R = (E_1^R)^*$

7) Diferența $(\mathcal{L}_1, \mathcal{L}_2)$

$\mathcal{L}_1 \setminus \mathcal{L}_2 = \mathcal{L}_1 \cap complement(\mathcal{L}_2)$

Exerciții

7.1.

$E_1 = (1 0 \cup 0)^* (1 \cup \varepsilon)$
$E_2 = (1 \cup \varepsilon)(0 0^* 1)^* 0^*$

E_1 eg E_2?

I. $E_1 \rightarrow NFA \rightarrow DFA \rightarrow$ min DFA D_1
$E_2 \rightarrow$ - - - - $\rightarrow$ D_2
$D_1 = D_2$?

II. $E_1 \rightarrow NFA \rightarrow DFA \; A_1$ - - q_{01}
$E_2 \rightarrow$ — " — A_2 - - - q_{02}
verif. dacă q_{01} și q_{02} se disting între ele?

III.

$\mathcal{L}(E_1) = \mathcal{L}(E_2)$? $\Leftrightarrow$ $\mathcal{L}(E_1) \subseteq \mathcal{L}(E_2)$ și $\mathcal{L}(E_2) \subseteq \mathcal{L}(E_1)$ $\Leftrightarrow$

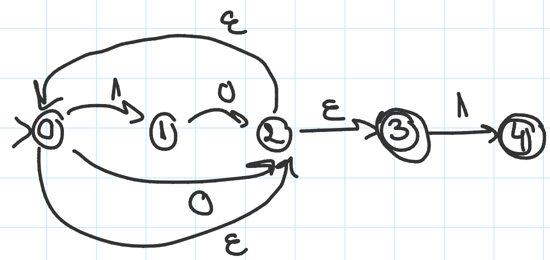
$\Leftrightarrow$ $\mathcal{L}(E_1) \setminus \mathcal{L}(E_2) = \emptyset$ și $\mathcal{L}(E_2) \setminus \mathcal{L}(E_1) = \emptyset$ $\Leftrightarrow$

$\Leftrightarrow \mathcal{L}(E_1) \cap \text{complement}(\mathcal{L}(E_2)) = \emptyset$ și $\mathcal{L}(E_2) \cap \overline{\mathcal{L}(E_1)} = \emptyset$

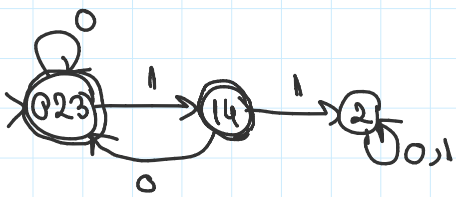
$E_1 \Rightarrow$ NFA $A_1 \to \overline{A_1}$
$E_2 \to$ NFA $A_2 \to \overline{A_2}$

$\to A_1 \cdot \overline{A_2}$
$A_2 \cdot \overline{A_1}$

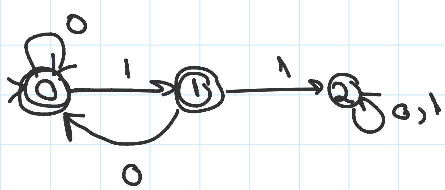
I $E_1 \to N_1$:



$\to D_1$:



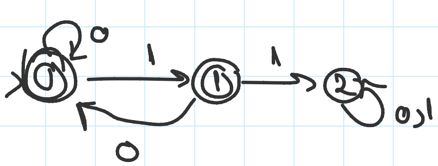
$E_2 \to D_2$:



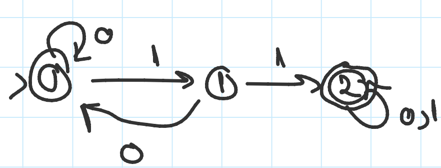
$D_1 = D_2 \to E_1$ eq E_2

f.2. $E = (10 \cup 0)^*(1 \cup \varepsilon)$
complement(E) = ?

DFA pt. $\mathcal{L}(E)$:



$\to$ DFA pt. $\overline{\mathcal{L}(E)}$:



$\to$ conversie la expr. reg. $\overline{E}$:

eliminăm starea 1:

0,10

eliminăm starea 1 :

$E = (0 \cup 10)^* \, 11 \, (0 \cup 1)^*$

7.4. coeficient: $L/c = \{ w \in \Sigma^* \mid wc \in L \}$

$c \in \Sigma$

Ex: $abcd \in L \Rightarrow abc \in L/d$

① $L = L((aaa \cup ba)^* (ab)^*)$

$L/a = ?$

I. $wa = (aaa \cup ba)^* (ab)^+ \Rightarrow$ niciun w posibil

II. $wa = \varepsilon \Rightarrow$ niciun w

III. $wa = (aaa \cup ba)^+ = (aaa \cup ba)^* (aaa \cup ba) \Rightarrow$

$\Rightarrow w = (aaa \cup ba)^* (aa \cup b)$

② $L = L(a^*) \Rightarrow L/a = L(a^*)$

$aaa \in L \Rightarrow aa \in L/a$

$aaa{-}{-}a$

③ + coef e o prop. de închidere

$L \in LR \Rightarrow coef(L) \in LR$

$L \in LR \Rightarrow L/c \in LR \quad \forall c \in \Sigma$

$L \setminus (L \cap L(c^*))$

a^*/a

$a^* \setminus (\underbrace{a^* \cap a^*})$

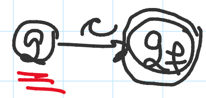
$a^* \setminus a^* = \emptyset$

→ DFA A a.î. $\mathcal{L}(A) = \mathcal{L}$

→ DFA A/c a.î. $\mathcal{L}(A/c) = \mathcal{L}/c$

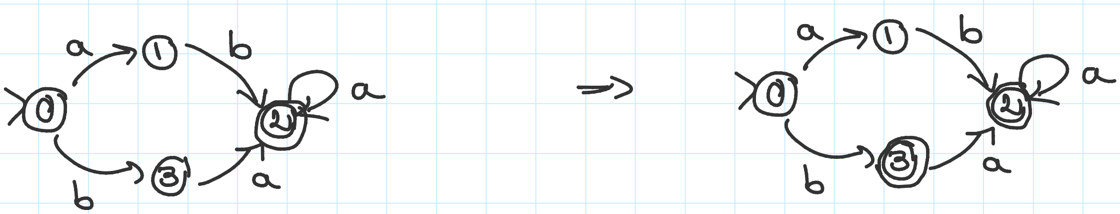
astfel:

- → toate tranz. sunt la fel ca în A
- → starea inițială rămâne cea din A
- → stările finale = stările q din A pt. care avem o tr.:



Exemplu

$\mathcal{L} = \mathcal{L}((ab \cup ba)\, a^*)$ $\qquad\qquad \mathcal{L}/a$



(7.5) $c/\mathcal{L} = \{ w \in \Sigma^* \mid cw \in \mathcal{L} \}$

$\mathcal{L} = \{abc\} \to rev(\mathcal{L}) = \{cba\}$

$a/\mathcal{L}$

$\Downarrow$

$rev(\mathcal{L})/a = \{cb\}$

$\Downarrow$

$= rev(\ldots)$

\# este proprietate de închidere peste LR

$c/\mathcal{L} = rev\big(rev(\mathcal{L})/c\big)$ → OK

rev, coef sunt închise peste LR

(7.6) $suffix(\mathcal{L}) = \{ w \in \Sigma^* \mid \exists x \in \Sigma^* \text{ a.î. } xw \in \mathcal{L} \}$

\# suffix e proprietate de închidere peste LR

$$suffix(\mathcal{L}) = \mathcal{L} \cup \bigcup_{c \in \Sigma} c/\mathcal{L} \cup \bigcup_{c \in \Sigma} c/c/\mathcal{L} \cup \ldots$$

DFA:

→ stări accesibile: q pt. care $\exists w$ $(q_0, w) \vdash^* (q, w')$

→ stări productive: q pt. care $\exists w$ $(q, w) \vdash^* (\underset{\in F}{q_f}, \varepsilon)$

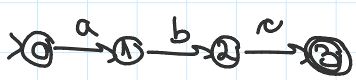
$\mathcal{L} \in LR \Rightarrow suffix(\mathcal{L}) \in LR$

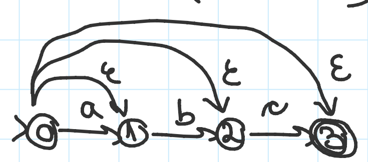
→ DFA A a.î. $\mathcal{L}(A) = \mathcal{L}$

→ construim NFA A' a.î. $\mathcal{L}(A') = suffix(\mathcal{L})$

$A' = A +$ ε-tranziții de la q_0 la toate stările accesibile

$\mathcal{L} = \{abc\}$ $\Rightarrow$ $suffix(\mathcal{L}) = \{abc, bc, c, \varepsilon\}$





7.7. $min(\mathcal{L}) = \{w \in \mathcal{L} \mid \nexists x \in \mathcal{L}, y \in \Sigma^* \setminus \{\varepsilon\} \text{ a.î. } xy = w\}$

Ex: $\mathcal{L} = \{\cancel{aab}, \underline{bab}, \underline{\underline{aa}}\} \Rightarrow min(\mathcal{L}) = \{bab, aa\}$

$w = aab$
$x = aa$
$y = b$
$x \in \mathcal{L}$

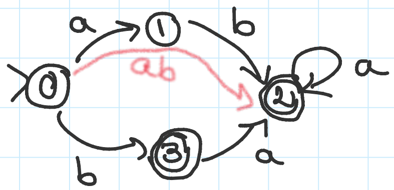
① $min(\mathcal{L}(a^*)) = \{\varepsilon\}$

$w = a$ ✗
$x = \varepsilon$
$y = a$ $\mid x \in \mathcal{L}(a^*)$

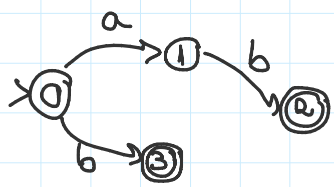
② $min(\mathcal{L}(a^*b)) = \mathcal{L}(a^*b)$

$a \ldots ab \in \mathcal{L}$
$a \ldots a \notin \mathcal{L}$

③ # min este proprietate de închidere peste LR



→



$\mathcal{L} = (ab \cup b)\, a^*$

$\min(\mathcal{L}) = \mathcal{L}(ab \cup b)$
$= \{ab, b\}$

$\boxed{ab}\, aa^*$

$\boxed{b}\, aa^*$

aba

→ DFA A care acceptă $\mathcal{L}$

→ construim DFA-ul A' care acceptă $\min(\mathcal{L})$, pornind de la A și ștergând toate tranzițiile care pornesc din stări finale