EXAMEN. ANUL I - MĂRIRE

STRUCTURI ALGEBRICE

ÎN INFORMATICĂ

BUHAI DARIUS

G. 135.

P1. $\sigma = (13)(24) \in S_4$

(1) $x^2 = \sigma$, $x \in S_4$, $x = ?$

$sgn(\sigma) = -1$ (perm. impară)

$\forall x \in S_4$, $sgn(x^2) = 1$ (perm. pară)

$\Rightarrow$ ecuația $x^2 = \sigma$ nu are soluții

(2) $x^3 = \sigma$, $x \in S_4$, $x = ?$

$x \in S_4$, x e de forma a 1 sau 2 cicli disjuncți

Cazul 1: $\left.\begin{array}{l} ord(x) = 2 \\ \sigma^3 = \sigma \end{array}\right| \Rightarrow$ ~~$x = \sigma$~~ $x = \sigma$ (sol. unică)

Cazul 2: $x = \zeta$, $\forall \zeta$ ciclu de lungime s

x^k se descompune în d cicli disjuncți

de lungime s/d, $\forall d \overset{cmmmc}{=} (k, s)$

$\Rightarrow \begin{array}{l} d = 2 \\ k = 3 \end{array}$, $s \leq 4$

$d =$ cmmmc$(k, s) \Rightarrow (3, s) = 2 \Rightarrow$

$\Rightarrow s = 6$ (contradicție)

(3) fie $H = \langle \sigma \rangle$

$\sigma = (13)(24)$

$\sigma^2 = e$

$\sigma^3 = \sigma \cdot e = \sigma$

C_2: ord $H = 2$

(4) ind $H = 2$

(5) $H \trianglelefteq S_4$?

Fie $H \trianglelefteq S_4 \Rightarrow xhx^{-1} \in H$, $\forall h \in H$, $x \in S_4$

Considerăm $x = (3\ 2\ 1)$

$(3\ 2\ 1)\, h\, (3\ 2\ 1)^{-1} \in H$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$= (1\ 2)(3\ 4) \notin H$

Cz: $H \ntrianglelefteq S_4$

(6) $K = \{ i, (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3) \} \trianglelefteq S_4$

Subgrupurile normale ale lui S_4 sunt K, $\{e\}$, S_4 și A_4

$H \in K$

P3 $\quad P(x) = x^3 + nx - 2,\ n \in \mathbb{Z}$

Q: $P(x) = 0 \Rightarrow x \mid 2 \Rightarrow x \in \{\pm 1, \pm 2\}$

$x = 1 \Rightarrow P(1) = 1 + n - 2 \Rightarrow n - 1 = 0 \Rightarrow n = 1$

$x = -1 \Rightarrow P(-1) = -1 - n - 2 \Rightarrow -n = 3 \Rightarrow n = -3$

$x = 2 \Rightarrow P(2) = 8 + 2n - 2 \Rightarrow 2n = -6 \Rightarrow n = -3$

$x = -2 \Rightarrow P(-2) = -8 - 2n - 2 \Rightarrow -2n = 10 \Rightarrow n = -5$

Caz 1: $n = 1 \Rightarrow P(x) = x^3 + x - 2$

$P(1) = 0 \Rightarrow P(x) = (x-1)(x^2 + x + 2)$

$\underline{P(x) = (x-1)(x^2 + x + 2)}$

Caz II $n=-3$

$P(x) = x^3 - 3x - 2$

$P(-1) = 0 \Rightarrow P(x) = (x+1)\ x^2 - x - 2)$

$$\begin{array}{l|l} x^3 - 3x - 2 & x+1 \\ \hline -x^3 - x^2 & x^2 - x - 2 \\ \hline -x^2 - 3x - 2 & \\ -x^2 + x & \\ \hline -2x - 2 & \\ 2x + 2 & \\ \hline 0 & \end{array}$$

$x^2 - x - 2 = 0$

$\Delta = 9 \Rightarrow x_{12} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

$P(x) = (x+1)^2 (x-2)$

Caz. III $n=-5 \Rightarrow P(x) = x^3 - 5x - 2$

$P(-2) = 0 \Rightarrow P(x) = (x+2)(x^2 - 2x - 1)$

$$\begin{array}{l|l} x^3 - 5x - 2 & x+2 \\ \hline -x^3 - 2x^2 & x^2 - 2x - 1 \\ \hline -2x^2 - 5x - 2 & \\ +2x^2 + 4x & \\ \hline -x - 2 & \end{array}$$

$x^2 - 2x - 1 = 0$

$\Delta = 4 + 4 = 8 \Rightarrow x_{12} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$P(x) = (x+2)\ (x^2 - 2x - 1)$

$\mathbb{Z}_2$:

$P(x) = x^3 + nx = x(x^2 + n) = 0$

$\underline{n=1} \Rightarrow x^2 + \hat{1} = 0 \Rightarrow x = \hat{1}$

$P(x) = x(x+\hat{1})^2$

$\underline{n=0} \Rightarrow x(x^2 + \hat{0}) = x^3$

$P(x) = x^3$

$\mathbb{Z}_3$: $P(x) = x^3 + nx + 1$

$P(0) = 1$ (ired)

$P(1) = 1 + n + 1 = n + 2$ (red, $n=1$)

$P(2) = 2n$ (red, $n=0$)

$n = 0$: $P(x) = x^3 + 1$

$P(2) = 0 \Rightarrow P(x) = (x - \hat{2})(x^2 - x + 1) = (x+1)(x^2 + \hat{2}x + 1)$

$= (x+1)^3$

$n = 1 \Rightarrow P(x) = x^3 + x + 1$

$P(1) = 0 \Rightarrow P(x) = (x+2)(x^2 + x + 2)$ ired. în $\mathbb{Z}_3$

$P(x) = (x+2)(x^2 + x + 2)$