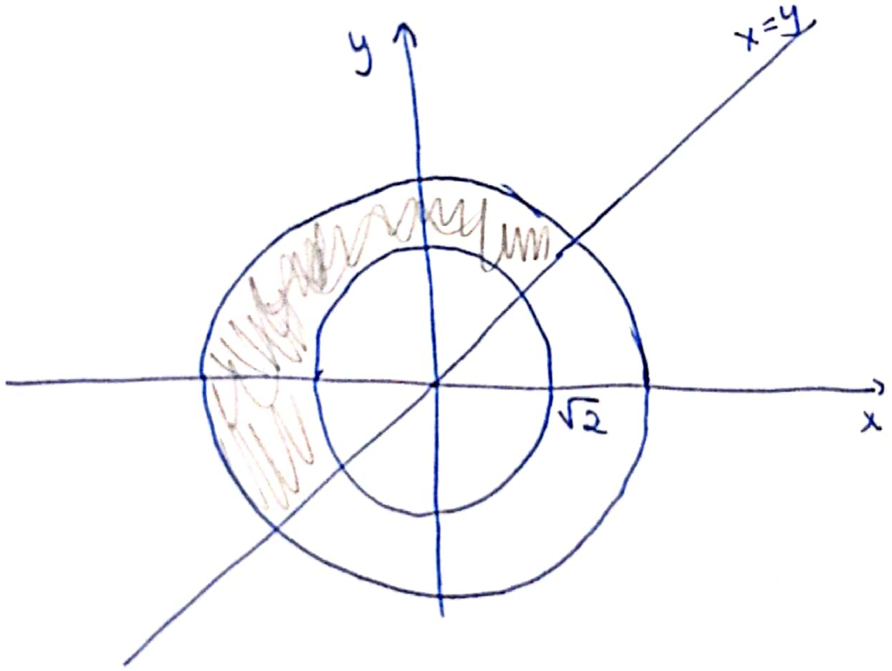
$$I = \int_D \sqrt{x^2+y^2}\, dx\, dy$$

$$D = \{ 2 \le x^2+y^2 \le 4,\ x \le y \}$$

$$\begin{cases} x^2+y^2 \ge (\sqrt{2})^2 \Rightarrow \text{exterior} \\ x^2+y^2 \le 2^2 \Rightarrow \text{interior} \\ x \le y \Rightarrow \text{deasupra primei bisectoare} \end{cases}$$



$f: \mathbb{R}^2 \to \mathbb{R}^2 \Rightarrow f(R, \theta) = (R\cos\theta, R\sin\theta)$

$$\left.\begin{array}{l} R^2 \ge (\sqrt{2})^2 \\ R^2 \le 2^2 \\ R \ge 0 \end{array}\right\} \Rightarrow R \in [\sqrt{2}, 2]$$

$x \le y \Rightarrow R\cos\theta \le R\sin\theta \Rightarrow \cos\theta \le \sin\theta \Rightarrow 1 \le tg\,\theta$

$$\left.\begin{array}{l} tg\,\theta \ge 1 \\ \theta \in [0, 2\pi] \text{ sau } \theta \in [-\pi, \pi] \end{array}\right\} \Rightarrow \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

$$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\int_{\sqrt{2}}^{2} \sqrt{R^2 \cos^2\theta + R^2 \sin^2\theta}\ |R| \, dR \right) d\theta$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\int_{\sqrt{2}}^{2} \sqrt{R^2} \cdot |R| \, dR \right) d\theta$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\int_{\sqrt{2}}^{2} |R|^2 \, dR \right) d\theta$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{R^3}{3} \Big|_{\sqrt{2}}^{2} \, d\theta$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{3}(8 - 2\sqrt{2}) \, d\theta$$

$$I = \frac{1}{3}(8 - 2\sqrt{2}) \cdot \theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$I = \frac{1}{3}(8 - 2\sqrt{2}) \cdot \left(\frac{\pi}{2} - \frac{\pi}{4} \right)$$

$$I = \frac{1}{3}(8 - 2\sqrt{2}) \cdot \frac{\pi}{4}$$

$$I = \frac{(4 - \sqrt{2})\pi}{3 \cdot 2}$$

$$I = \frac{(4 - \sqrt{2})\pi}{6}$$