Logică matematică și computațională

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Grupa 143

$j=2$

$k=5$

$$\Sigma_{j,k} = \{\varphi_j, \psi_k\}$$

$$\Sigma_{2,5} = \{\gamma \to \varepsilon, \gamma \to \beta\}$$

$$\gamma = (p \to \neg\alpha) \vee (q \leftrightarrow (r \wedge \beta))$$

$$\delta = (\beta \to p) \to (\alpha \leftrightarrow (q \to r))$$

$$\varepsilon = p \to (q \to (r \to \neg(\alpha \wedge \beta)))$$

$$\xi = p \leftrightarrow ((\alpha \leftrightarrow q) \to \beta)$$

Fie $h: V \to L_2$ o interpretare arbitrară.

① $\beta \in T \Rightarrow \vdash \beta \overset{T.C.T.}{\Longrightarrow} \vDash \beta \Rightarrow \tilde{h}(\beta) = 1$,

unde $\tilde{h}: E \to L_2$ va fi unica prelungire a lui h la E care transformă conectorii logici în operații booleene.

$\alpha \in T \Rightarrow \vdash \alpha \overset{T.C.T.}{\Longrightarrow} \vDash \alpha \Rightarrow \tilde{h}(\alpha) = 1$

$$\gamma = (p \to \neg\alpha) \vee (q \leftrightarrow (r \wedge \beta))$$

$$\gamma = (p \to \neg\alpha) \vee ((q \to (r \wedge \beta)) \wedge ((r \wedge \beta) \to q))$$

$$\delta = (\beta \to p) \to ((\alpha \to (q \to r)) \wedge ((q \to r) \to \alpha))$$

$$\xi = p \leftrightarrow (((\alpha \to q) \wedge (q \to \alpha)) \to p)$$

$$\xi = (p \to (((\alpha \to q) \wedge (q \to \alpha)) \to \beta)) \wedge (((\alpha \to q) \wedge (q \to \alpha)) \to \beta) \to p)$$

$h(p)$	$h(q)$	$h(r)$	$\tilde{h}(p\to r)$	$\tilde{h}(r\wedge p)$	$\tilde{h}(q\to(r\wedge p))$	$\tilde{h}((r\wedge p)\to q)$	$\tilde{h}(q\leftrightarrow(r\wedge p))$	$\tilde{h}(\gamma)$
0	0	0	1	0	1	1	1	1
0	0	1	1	1	1	0	0	1
0	1	0	1	0	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1
1	0	1	0	1	1	0	0	0
1	1	0	0	0	0	1	0	0
1	1	1	0	1	1	1	1	1

$h(p)$	$h(q)$	$h(r)$	$\tilde{h}(q\to p)$	$\tilde{h}(q\to r)$	$\tilde{h}(\alpha\to(q\to r))$	$\tilde{h}((q\to r)\to\alpha)$	$\tilde{h}(\alpha\leftrightarrow(q\to r))$	$\tilde{h}(\delta)$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	0	0	1	0	1
0	1	1	0	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	1	0	0
1	1	1	1	1	1	1	1	1

$h(p)$	$h(q)$	$h(r)$	$\tilde{h}(\alpha\wedge\beta)$	$\tilde{h}(\neg(\alpha\wedge\beta))$	$\tilde{h}(r\to\neg(\alpha\wedge\beta))$	$\tilde{h}(q\to(r\to\neg(\alpha\wedge\beta)))$	$\tilde{h}(\varepsilon)$	$\tilde{h}(\gamma)$	$\tilde{h}(\gamma\to\varepsilon)$	$\tilde{h}(\gamma\to\beta)$
0	0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1
0	1	0	1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1
1	0	1	1	0	0	1	1	0	1	1
1	1	0	1	0	1	1	1	0	1	1
1	1	1	1	0	0	0	0	1	0	1

Fie h o interpretare a.î. $\begin{cases} h(p)=0 \\ h(q)=0 \\ h(r)=0 \end{cases}$

Avem $\begin{cases} \tilde{h}(\varphi_2) = 1 \\ \tilde{h}(\psi_5) = 1 \end{cases}$

$\Rightarrow \Sigma_{2,5}$ este satisfiabilă $\Rightarrow \Sigma_{2,5}$ este consistentă

② $h: V \to \mathcal{L}_2$ o interpretare arbitrară

$\alpha \in T \Rightarrow \vdash \alpha \overset{T.C.T.}{\Longrightarrow} \vDash \alpha \Rightarrow \tilde{h}(\alpha) = 1$, unde $\tilde{h}: E \to \mathcal{L}_2$ va fi unica prelungire a lui h la E care transformă conectorii logici în operații booleene.

$\beta \in V \setminus \{p, q, r\}$

Fixez $h(\beta) = 1$

Aleg $\left.\begin{cases} h(p)=0 \\ h(q)=0 \\ h(r)=0 \end{cases}\right\} \Rightarrow \begin{cases} \tilde{h}(\gamma) = 1 \\ \tilde{h}(\varepsilon) = 1 \end{cases}$ (conform subpunctului ①)

$\Rightarrow \begin{cases} \tilde{h}(\gamma \to \varepsilon) = 1 = \tilde{h}(\varphi_2) \\ \tilde{h}(\gamma \to \beta) = 1 = \tilde{h}(\psi_5) \end{cases}$

$\Rightarrow \Sigma_{2,5}$ satisfiabilă $\Rightarrow \Sigma_{2,5}$ consistentă