Fie funcția $f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)=e^{ax+by}$

$$\frac{\partial f}{\partial x}=a e^{ax+by} \quad \frac{\partial^2 f}{\partial x^2}=a^2 e^{ax+by} \quad \frac{\partial^n f}{\partial x^n}=a^n e^{ax+by}$$

$$\frac{\partial^{n+1} f}{\partial y\,\partial x^n}= a^n b\, e^{ax+by} \qquad \frac{\partial^{n+m} f}{\partial y^m\,\partial x^n}=\frac{\partial^{n+m} f}{\partial x^n\,\partial y^m}=a^n b^m e^{ax+by}.$$

$$df = a e^{ax+by} dx + b e^{ax+by} dy$$

$$f'(x,y)(u,v)= a e^{ax+by} u + b e^{ax+by} v$$

$$f''(x,y)\big((u_1,v_1),(u_2,v_2)\big)=$$

$$= e^{ax+by}\left(a^2 u_1 u_2 + ab\, u_1 v_2 + ab\, v_1 u_2 + b^2 v_1 v_2\right)$$

Alte exemple 1) $f:\mathbb{R}^3\to\mathbb{R}$ $f(x,y,z)=e^{x+2y+3z}$

2) $f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)=\sin(ax+by)$

3) $f:(0,\infty)^2\to\mathbb{R}$ $f(x,y)=\dfrac{1}{1+ax+by}$ $a>0$, $b>0$

4) Calculați f' și f'' pentru $f:\mathbb{R}^3\to\mathbb{R}$ $f(x,y,z)=x^2y+e^y\sin z$.

Să se determine extremele locale ale funcțiilor

a) $f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)=x^2-xy+y^2$

b) $f:(\mathbb{R}^+)^2\to\mathbb{R}$ $f(x,y,z)=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+xyz$

c) $f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)=(x+1)(y+1)(x+y)$

Rezolvări

a) $f'=\left(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right)=\left(2x-y,-x+2y\right)=0 \Rightarrow$

$$\begin{cases}2x-y=0\\-x+2y=0\end{cases}\qquad \Delta=\begin{vmatrix}2&-1\\-1&2\end{vmatrix}=3\neq0\Rightarrow x=y=0$$

$$f''=\begin{pmatrix}\frac{\partial^2 f}{\partial x^2}&\frac{\partial^2 f}{\partial y\partial x}\\ \frac{\partial^2 f}{\partial x\partial y}&\frac{\partial^2 f}{\partial y^2}\end{pmatrix}=\begin{pmatrix}2&-1\\-1&2\end{pmatrix}$$

$\Delta_1=2>0$

$\Delta_2=4-1=3>0$

$\Rightarrow (0,0)$ minim local.

b) $\frac{\partial f}{\partial x} = -\frac{1}{x^2} + yz = 0 \Rightarrow \quad 1 = x^2 yz$

$\frac{\partial f}{\partial y} = -\frac{1}{y^2} + xz = 0 \Rightarrow \quad 1 = xy^2 z$

$\frac{\partial f}{\partial z} = -\frac{1}{z^2} + xy \Rightarrow \quad 1 = xyz^2$

$\Rightarrow x = y = z \quad \left(1 = \frac{x^2yz}{xy^2z} = \frac{x}{y} \right) \Rightarrow$

$x^4 = 1 \quad \Rightarrow \begin{cases} x = y = z = 1 \\ x = y = z = -1 \end{cases}$

$$f'' = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial z \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial z \partial y} \\ \frac{\partial^2 f}{\partial x \partial z} & \frac{\partial^2 f}{\partial y \partial z} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{x^3} & z & y \\ z & \frac{2}{y^3} & x \\ y & x & \frac{2}{z^3} \end{pmatrix}$$

c) $f''(1,1,1) = \begin{array}{c} \Delta_1 \\ \Delta_2 \\ \\ \Delta_3 \end{array} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$\Delta_1 = 2 > 0 \quad \Delta_2 = 3 > 0 \quad \Delta_3 = 8 + 1 + 1 - 2 - 2 - 2 = 4 > 0$

$\Delta_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \qquad \Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$

minim local

$f''(-1,-1,-1) = \begin{pmatrix} -2 & -1 & -1 \\ -1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix} = -f''(1,1,1)$

$\Delta_1 = -2 \qquad \Delta_2 = 3 \qquad \Delta_3 = -4$

maxim local

$+ \quad ++++ \quad + \quad +++$ minim ~~maxim~~ local

$- \quad + - + - \quad + - + - +$ maxim local

det $\neq 0$ oricum punct șa

c)

$$\frac{\partial f}{\partial x} = (y+1)(x+y) + (x+1)(y+1) = (y+1)(2x+y+1)$$

$$\frac{\partial f}{\partial y} = (x+1)(x+2y+1)$$

$(y+1)(2x+y+1)=0 \quad \begin{cases} \nearrow y=-1 \\ \searrow 2x+y+1=0 \end{cases} \quad \Rightarrow$

$(x+1)(x+2y+1)=0 \quad \begin{cases} \nearrow x=-1 \\ \searrow x+2y+1=0 \end{cases}$

$\underline{\underline{c1}}$ $\quad x=y=-1$

$\underline{\underline{c2}}$ $\quad x=-1 \quad y\neq -1 \Rightarrow y=1$

$\underline{c3}$ $\quad y=-1 \quad x\neq -1 \Rightarrow x=1$

$\underline{\underline{c4}}$ $\quad x\neq -1,\ y\neq -1 \Rightarrow \begin{cases} 2x+y+1=0 \\ x+2y+1=0 \end{cases} \Rightarrow x=y=-\frac{1}{3}$

$$\frac{\partial^2 f}{\partial x^2} = 2(y+1) \qquad \frac{\partial^2 f}{\partial y \partial x} = 2x+y+1+y+1 = 2x+2y+2$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}.$$

$$\frac{\partial^2 f}{\partial y^2} = 2(x+1)$$

C1

$$f''(-1,-1) = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \qquad \Delta_1 = 0 \qquad \Delta_2 = -4 < 0$$ punct șa

$$f''(-1, 1) = \begin{pmatrix} 4 & -2 \\ -2 & 0 \end{pmatrix} \qquad \Delta_1 = 4 > 0 \qquad \Delta_2 = -4 < 0$$ punct șa

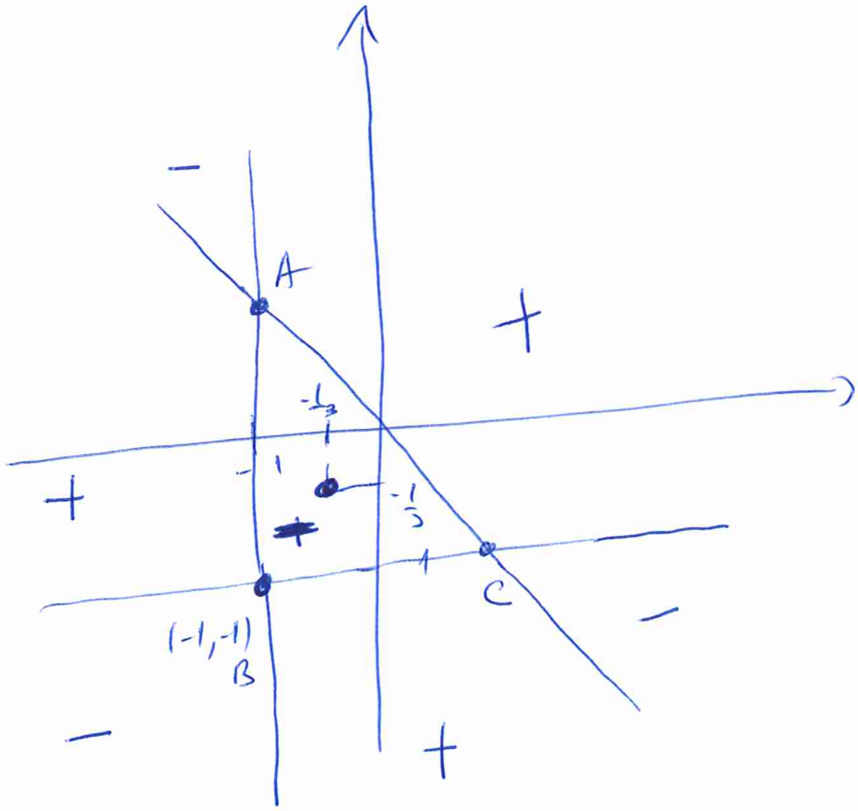
$$f''(1,-1) = \begin{pmatrix} 0 & -2 \\ -2 & 4 \end{pmatrix} \qquad \Delta_2 = -4 < 0$$ punct șa

$$f''\left(-\frac{1}{3}, -\frac{1}{3}\right) = \begin{pmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{pmatrix} \qquad \Delta_1 = \frac{4}{3} > 0$$

$$\Delta_2 = \frac{16}{9} - \frac{4}{9} = \frac{12}{9} = \frac{4}{3} > 0$$

punct de minim

$x = -1 \quad y = -1 \quad x + y = 0$



• - punctele de extrem

$f(1,1) = 8 > 0$

negativă în interiorul ΔABC

Să se determine extremele locale ale funcțiilor:

1) $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z) = x^2 - xy + y^2 - 2x - y + z^2$

2) $f: (\mathbb{R}_+^*)^2 \to \mathbb{R}$ $\quad f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$

3) $f: \mathbb{R}_+^3 \to \mathbb{R}$ $\quad f(x,y,z) = \frac{1}{x} + \frac{x}{y} + \frac{y}{z} + z$

4) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^2 - 3xy + y^2$.

5). $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^2 - 3xy^2 + 2y^4$

6). $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z) = xy^2z^3(5 - x - 2y - 3z)$

7) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = (x+1)^4 + (y-3)^6$

8) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = (x^2+y^2)\, e^{x+2y}$

9) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^4 + y^4 - x^2 - y^2 - xy$.

10) $f: (\mathbb{R}^*_+)^3 \to \mathbb{R}$ $\quad f(x,y,z) = x + \frac{y^2}{4x} + \frac{z^2}{y} + \frac{3}{z}$

11) $f: \{x^2+y^2<1\}$ $\quad f(x,y) = xy\sqrt{1-x^2-y^2}$