

Problema rămasă Teoriă 2

$$X_n = \frac{\sqrt{1 - \cos \frac{\pi}{n}}}{n \cdot \ln(n+1)}$$

$$1 - \cos \frac{\pi}{n} = 1 - \cos \frac{2\pi}{2n} = 2 \sin^2 \frac{\pi}{2n}$$

$$X_n = \frac{\sqrt{2 \sin^2 \frac{\pi}{2n}}}{n \cdot \ln(n+1)} = \frac{\sqrt{2} \cdot \sin \frac{\pi}{2n}}{n \cdot \ln(n+1)}$$

$$\sin \frac{\pi}{2n} < \frac{\pi}{2n} \Rightarrow \frac{\sqrt{2} \cdot \sin \frac{\pi}{2n}}{n \cdot \ln(n+1)} < \frac{\sqrt{2} \cdot \frac{\pi}{2n}}{n \cdot \ln(n+1)} = \frac{\pi \sqrt{2}}{2n^2 \ln(n+1)}$$

$$\text{Pentru } n \geq e^3 - 1 \Rightarrow \ln(n+1) \geq 3 > \frac{\sqrt{2}}{2} \cdot \pi$$

$$\Rightarrow \frac{\pi \sqrt{2}}{2n^2 \ln(n+1)} < \frac{\ln(n+1)}{n^2 \cdot \ln(n+1)} = \frac{1}{n^2}$$

$$\Rightarrow X_n < \frac{1}{n^2}$$

$\frac{1}{n^2}$ serie convergentă

C. COMPARAȚIEI $\sum_n X_n$ converge