$$\sum \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n+1)}{1\cdot 4\cdot 7\cdot \ldots \cdot (3n+1)} \cdot x^n, \quad x>0$$

$$a_n = \frac{1\cdot 3\cdot 5\cdot \ldots (2n+1)}{1\cdot 4\cdot 7\cdot \ldots (3n+1)} \cdot x^n$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{1\cdot 3\cdot 5\cdot \ldots (2n+1)}\cdot (2n+3)}{\cancel{1\cdot 4\cdot 7\cdot \ldots (3n+1)}\,(3n+4)} \cdot x^{n+1} \cdot \frac{1}{x^n} \cdot \frac{\cancel{1\cdot 4\cdot 7\cdot \ldots (3n+1)}}{\cancel{1\cdot 3\cdot 5\cdot \ldots (2n+1)}}$$

$$= x \cdot \frac{2n+3}{3n+4}$$

$$L = \lim_{n\to\infty} x\cdot \frac{2n+3}{3n+4} = \frac{2}{3}x$$

1° Dacă $\frac{2}{3}x < 1 \iff x < \frac{3}{2} \iff x \in \left(0, \frac{3}{2}\right)$

$\Rightarrow L < 1 \Rightarrow$ Conf crit raportului, seria este convergentă

2° Dacă $\frac{2}{3}x > 1 \Rightarrow x \in \left(\frac{3}{2}, \infty\right)$

$\Rightarrow L > 1 \Rightarrow$ Conf. crit. raportului, seria este divergentă

3° Dacă $\frac{2}{3}x = 1 \to x = \frac{3}{2}$

$$\frac{a_{n+1}}{a_n} = \frac{2n+3}{3n+4} \cdot \frac{3}{2} = \frac{6n+9}{6n+8} \Rightarrow \frac{a_n}{a_{n+1}} = \frac{6n+8}{6n+9}$$

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = \lim_{n\to\infty} n\cdot\left(\frac{6n+8}{6n+9} - 1\right) =$$

$= \lim_{n\to\infty} n \cdot \frac{-1}{6n+9} = -\frac{1}{6} < 1 \Rightarrow$ Conform crit. Raabe Duhamel, seria este divergentă