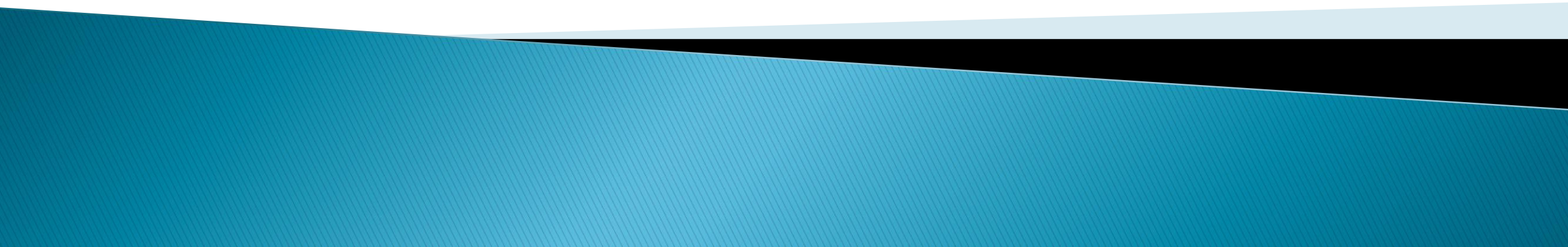


Mediana a doi vectori sortați



Mediana a doi vectori sortați



Se dau doi vectori a și b **de lungime n** , cu elementele **ordonate crescător**. Să se determine mediana vectorului obținut prin interclasarea celor doi vectori.

Mediana a doi vectori sortați

Exemplu: $n = 5$

1 12 15 16 38

2 13 17 30 45

Mediana a doi vectori sortați

Exemplu: $n = 5$

1 12 15 16 38

2 13 17 30 45



1 2 12 13 15 16 17 30 38 45

Mediana $(15+16)/2 = 15,5$

Mediana a doi vectori sortați

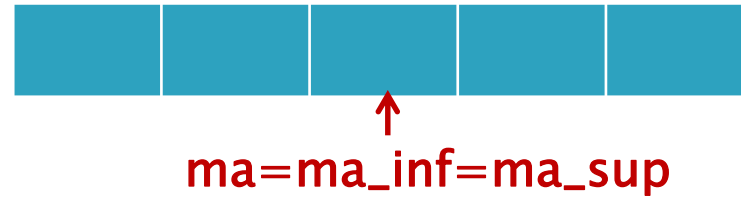
- **Algoritm $O(n)$** – interclasăm vectorii și apoi aflăm mediana în timp constant (din elementele de la mijlocul vectorului, conform definiției)

Mediana a doi vectori sortați

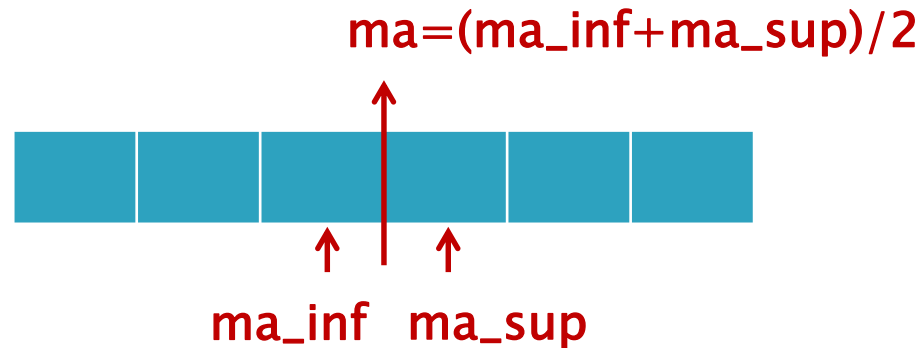
- Algoritm $O(\log n)$

- ▶ Fie ma_inf , ma_sup , ma mediana inferioară, superioară, respectiv mediana vectorului a
- ▶ mb_inf , mb_sup , mb – similar pentru vectorul b
- ▶ c – vectorul obținut prin interclasare

- ▶ n impar:

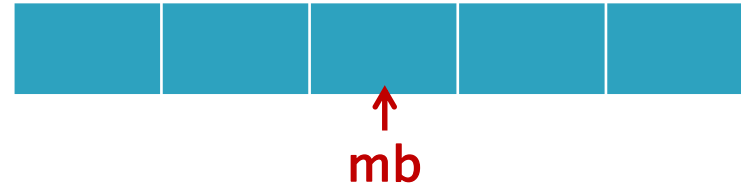
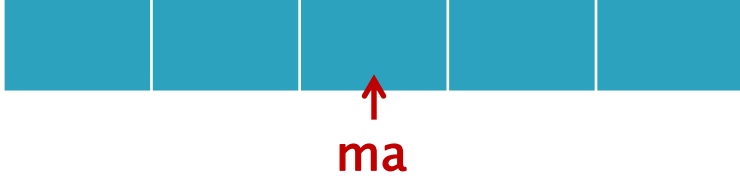


- ▶ n par:





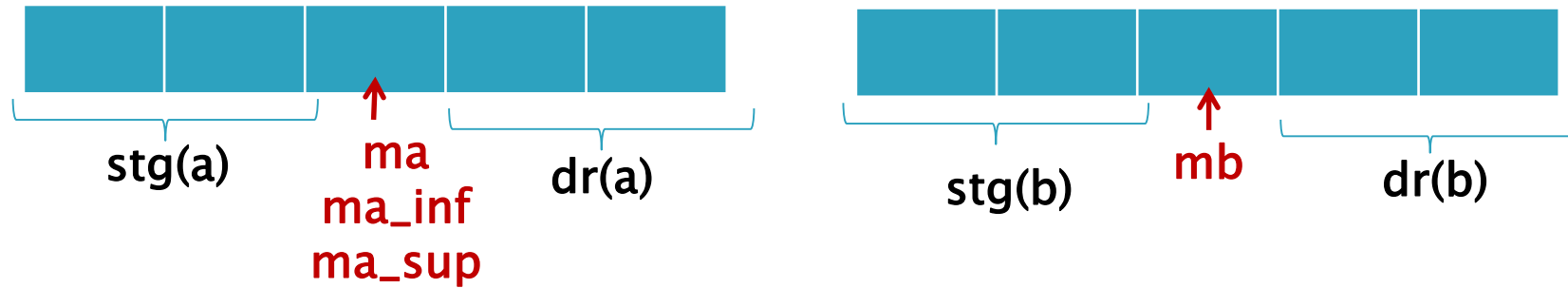
► Comparăm ma și mb



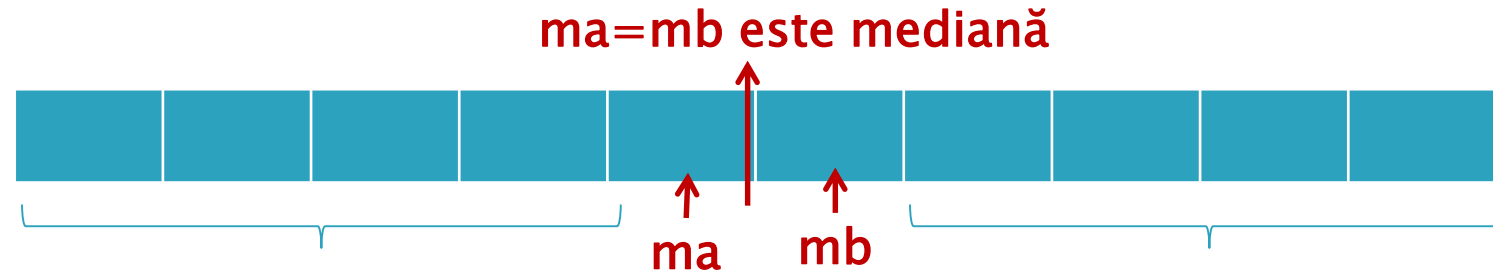
▶ $m_a = m_b \Rightarrow m_c = m_a = m_b$?

► $ma = mb \Rightarrow mc = ma = mb$

n impar

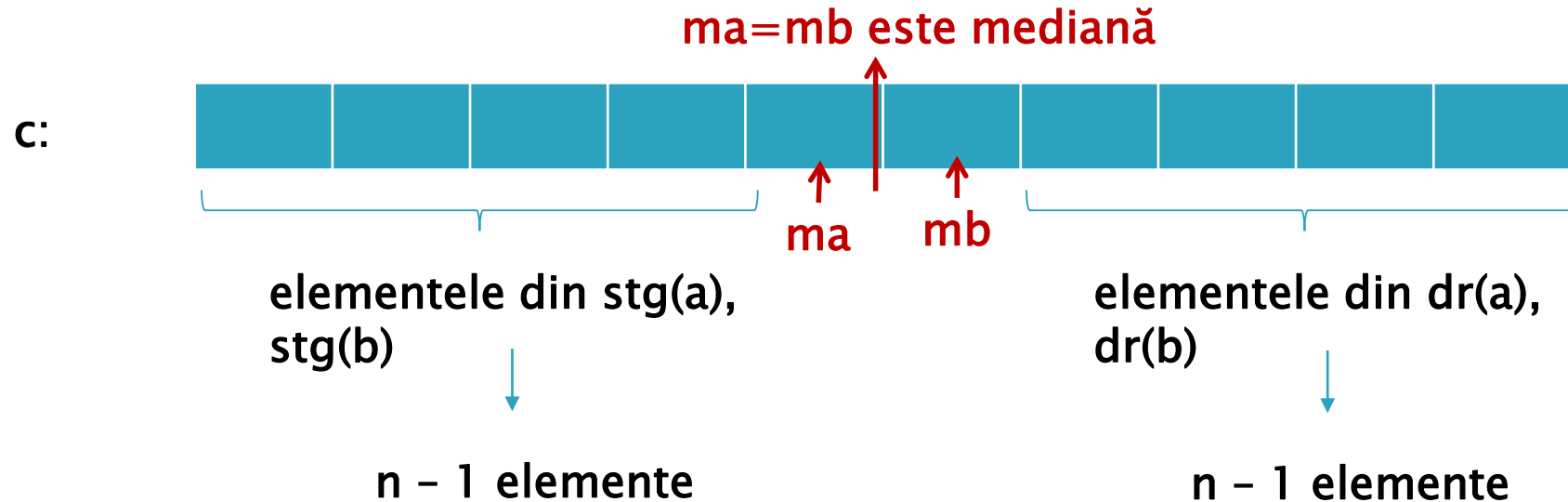
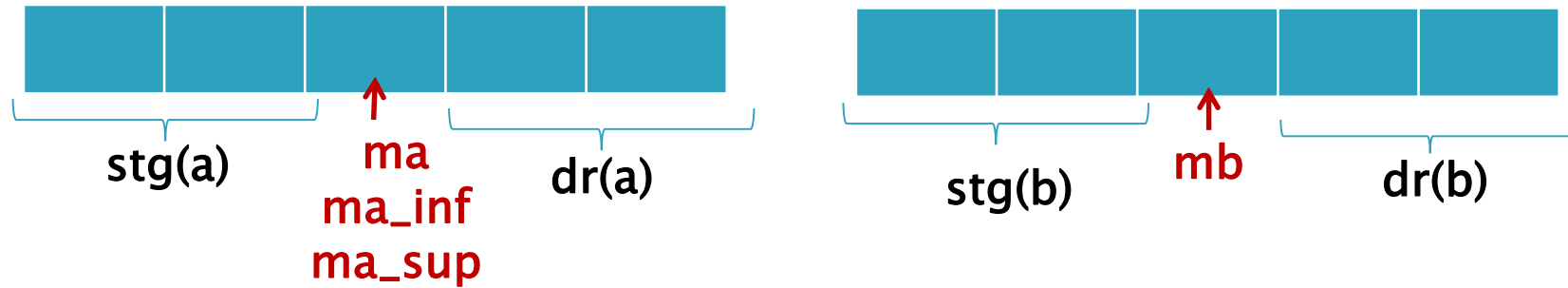


c:



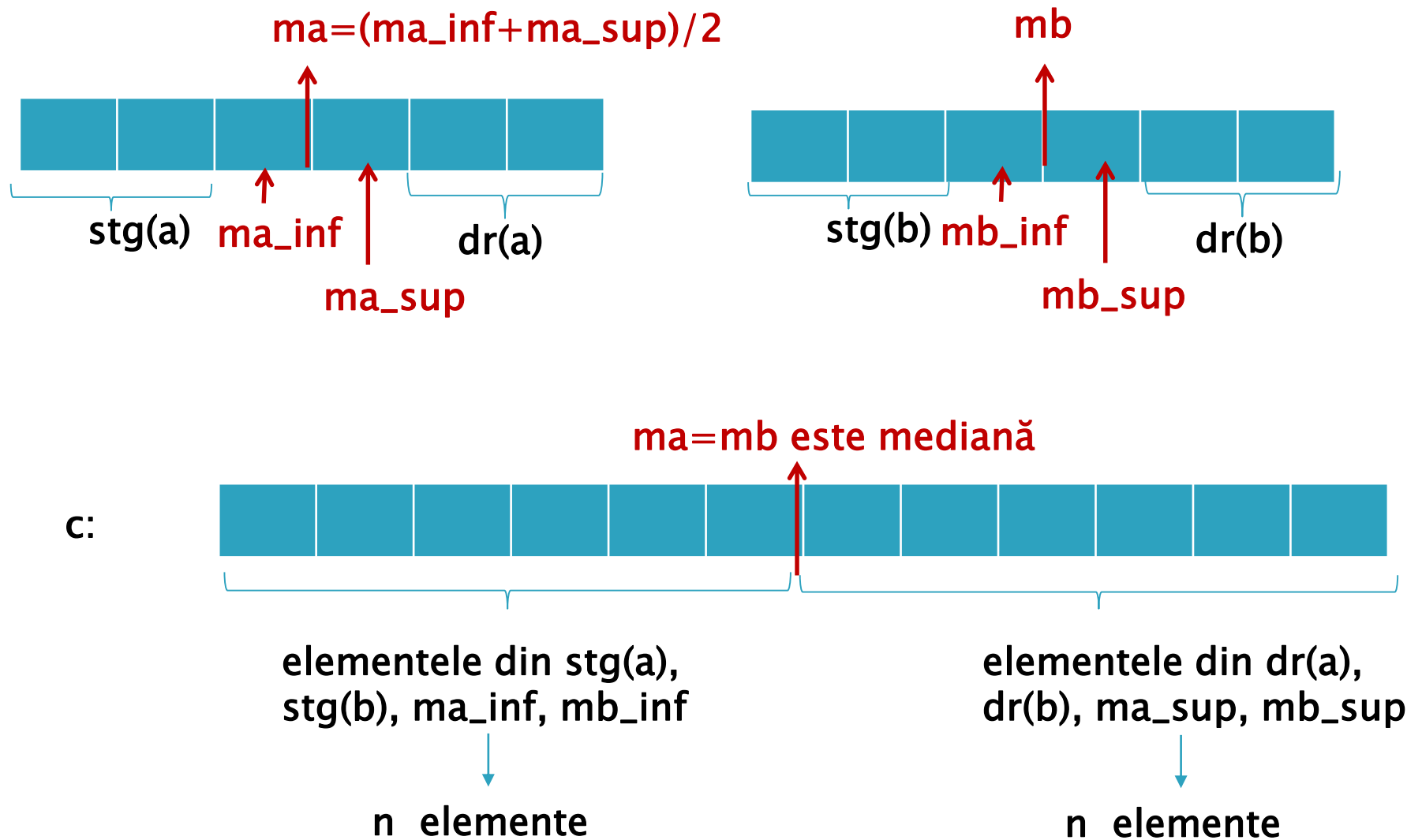
► $ma = mb \Rightarrow mc = ma = mb$

n impar

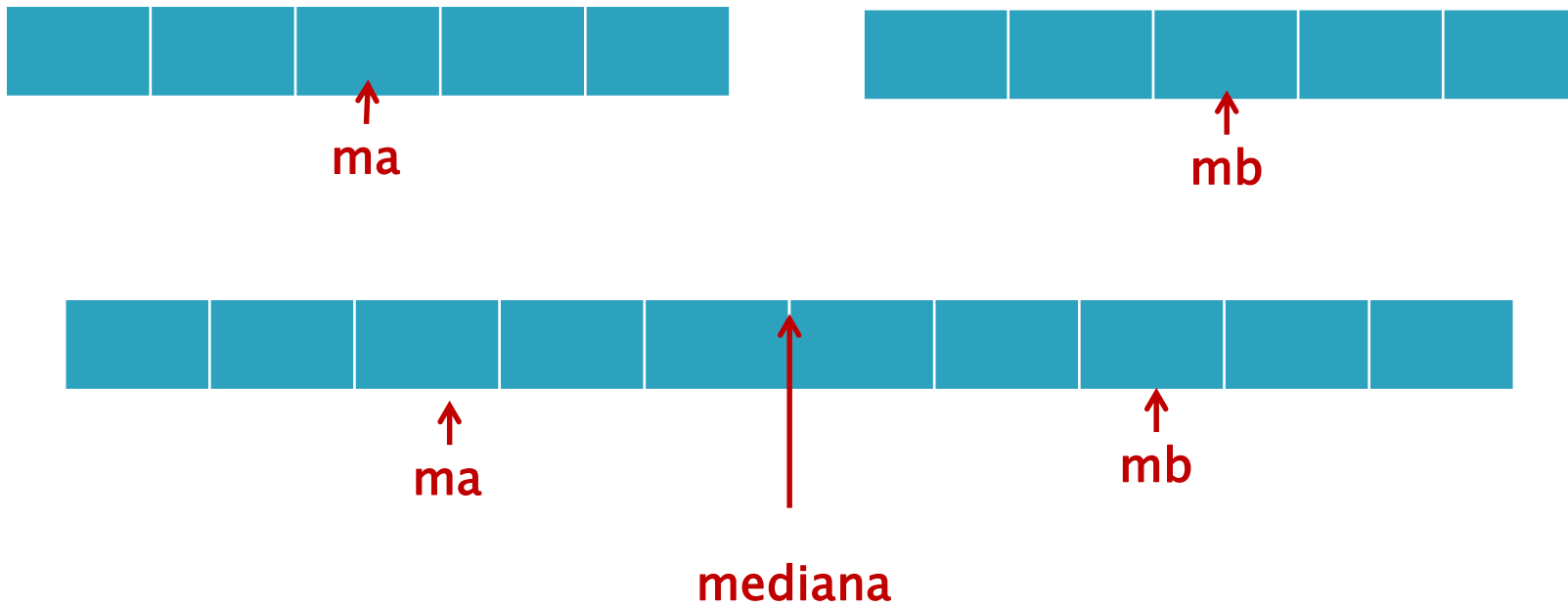


► $ma = mb \Rightarrow mc = ma = mb$

n par

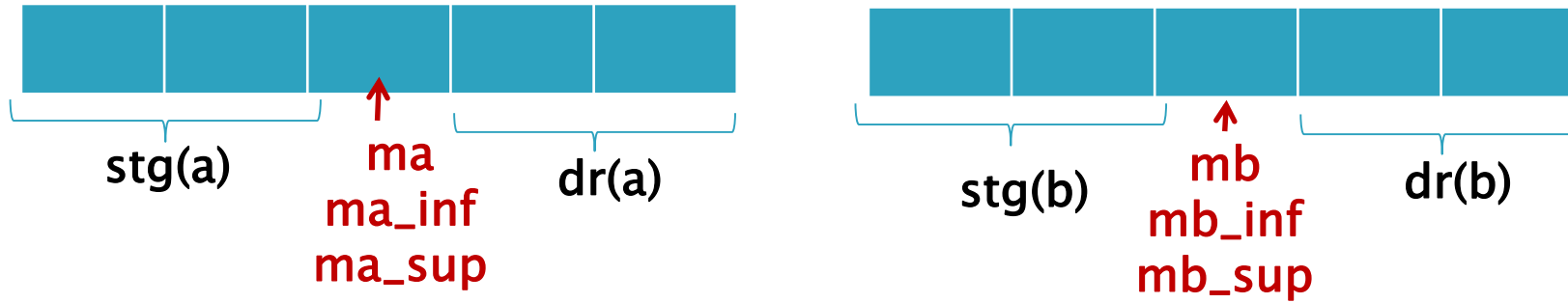


► $ma < mb \Rightarrow mc \in [ma, mb]$?



- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

n impar

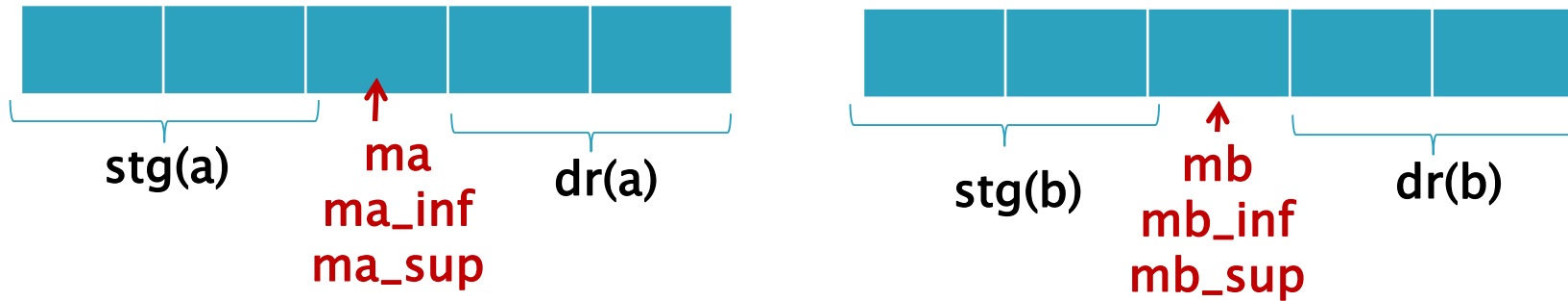


c:



- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

n impar

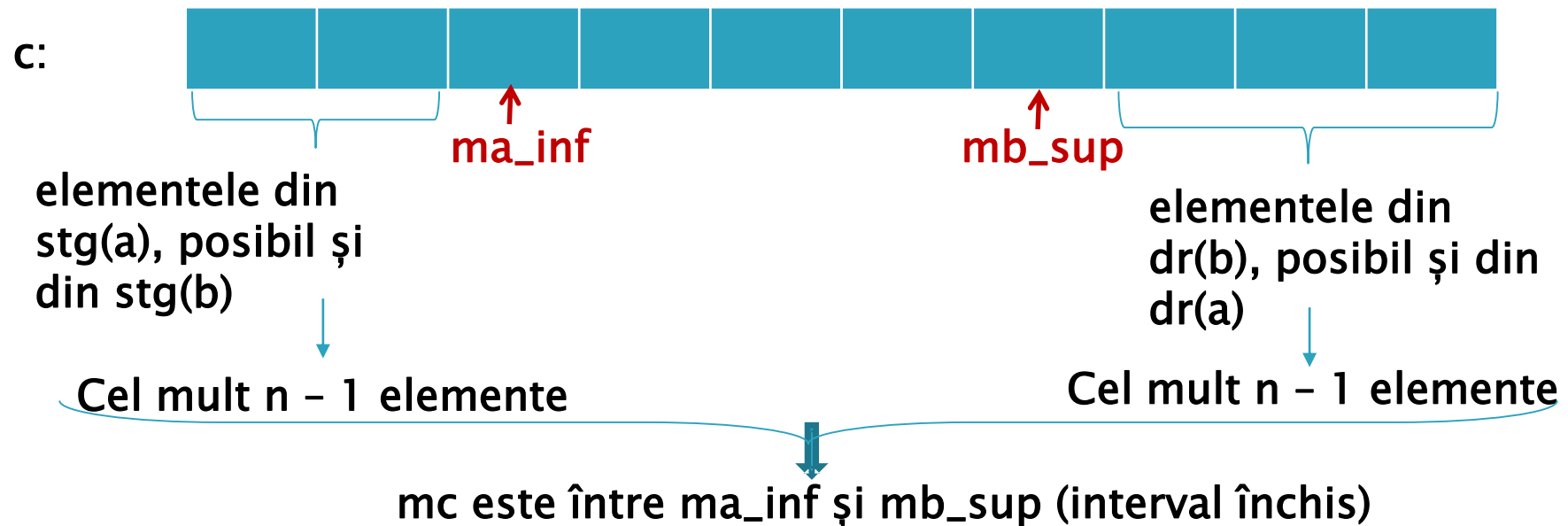
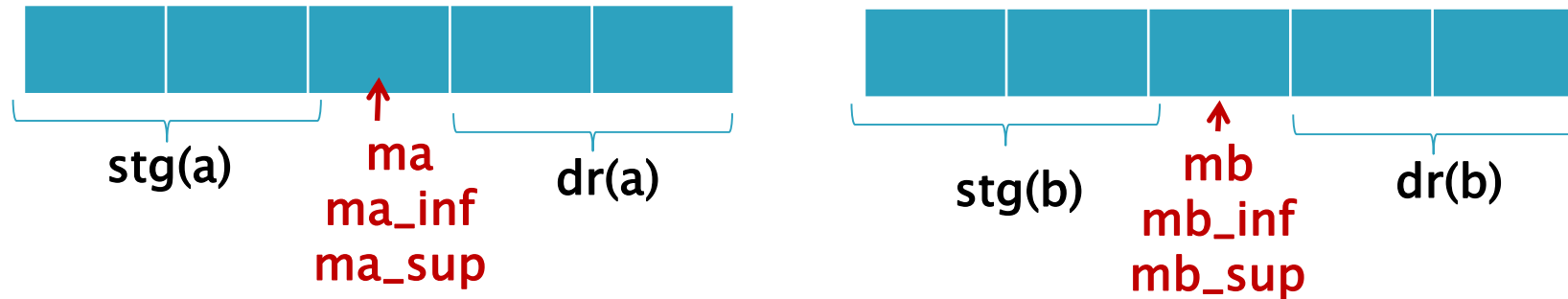


c:



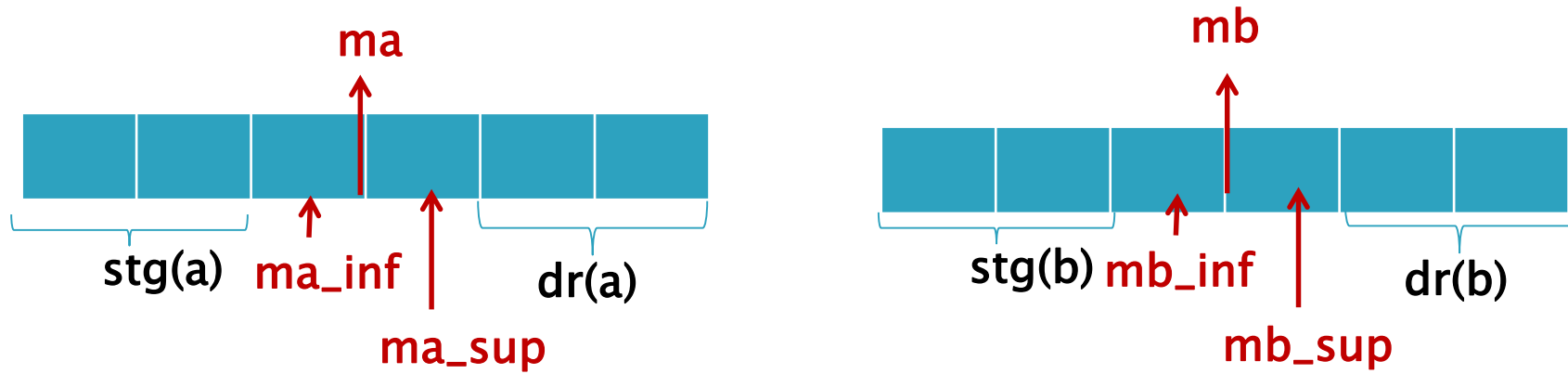
- $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

n impar



- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

n par

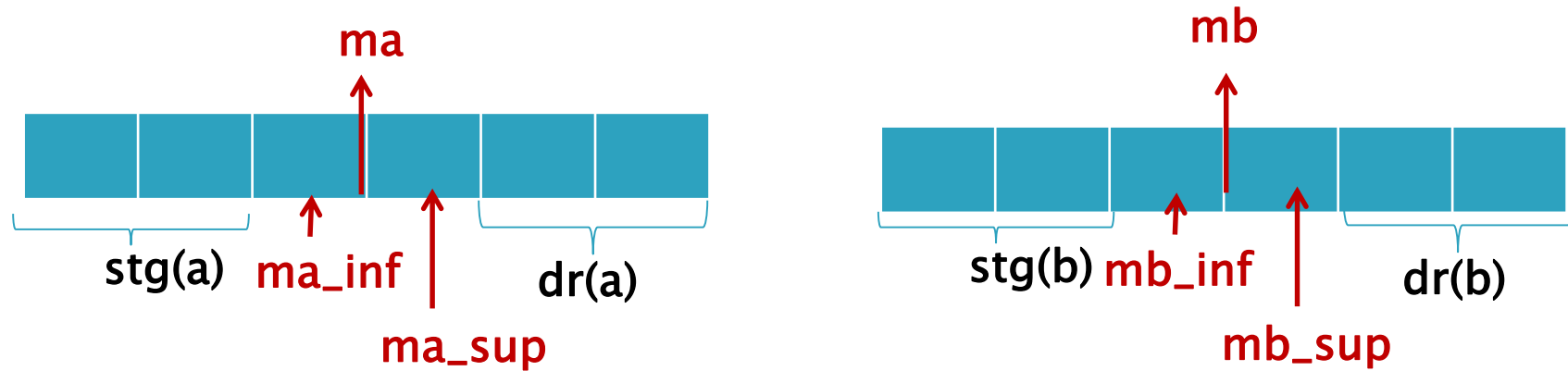


c:

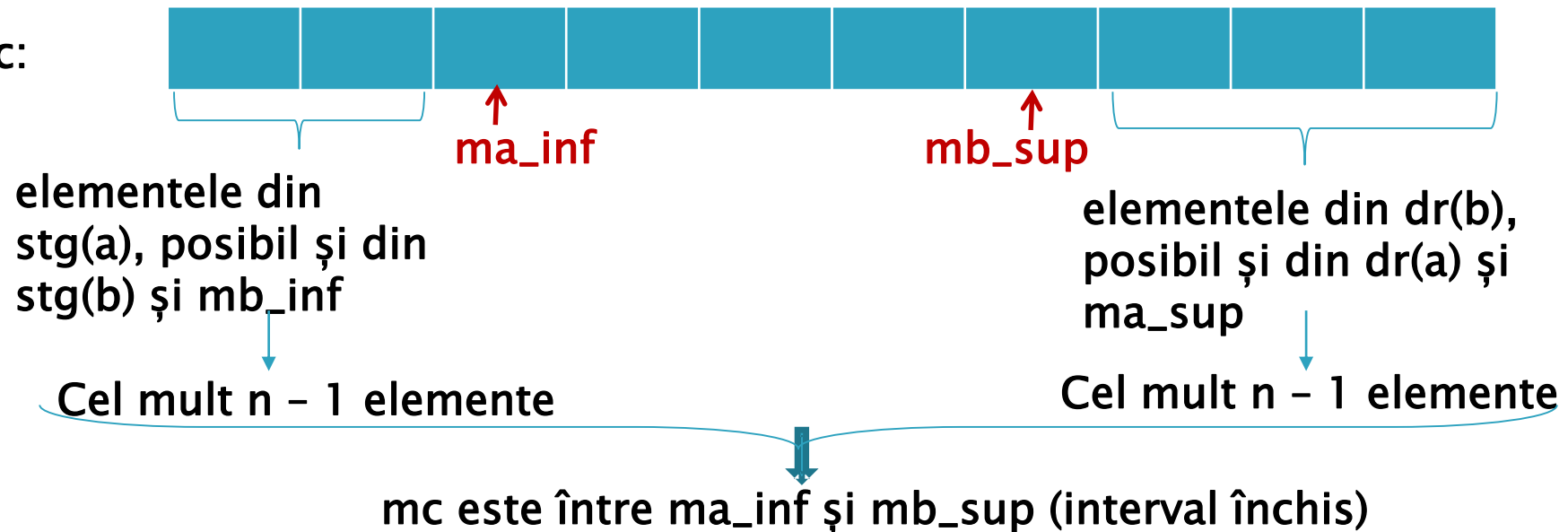


- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

n par



c:



- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

Rezultă:

Pentru a determina mediana este suficient să considerăm:

- Subvectorul drept din primul vector (inclusiv mediana inferioară)
- Subvectorul stâng din al doilea vector (inclusiv mediana superioară)

- ▶ $ma < mb \Rightarrow mc, mc_inf, mc_sup \in [ma_inf, mb_sup]$

Rezultă:

Pentru a determina mediana este suficient să considerăm:

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- Subvectorul stâng din al doilea vector (inclusiv mediana superioară)

Astfel

- din vectorul a renunțăm la $[(n-1)/2]$ elemente care sunt înaintea lui ma_inf (deci și a lui mc_inf) în c
- din vectorul b renunțăm **tot la** $[(n-1)/2]$ elemente care sunt după lui mb_sup (deci și după mc_sup) în c

deci **media noilor vectori este egală cu mediana lui c**

▶ $m_a > m_b$ - Similar

Mediana a doi vectori sortați

Corectitudine:

mediana noii probleme = mediana problemei
inițiale

Pseudocod

- ▶ Fie m_a mediana vectorului a și m_b mediana vectorului b
 - Dacă $m_a = m_b$ atunci această valoare este mediana

- ▶ Fie m_a mediana vectorului a și m_b mediana vectorului b
 - Dacă $m_a = m_b$ atunci această valoare este mediana
 - Dacă $m_a > m_b$ atunci mediana = mediana subvectorilor
 $a[0.. \lfloor n/2 \rfloor]$, $b[\lfloor (n-1)/2 \rfloor.. n-1]$
 - Dacă $m_a < m_b$ atunci mediana = mediana subvectorilor
 $a[\lfloor (n-1)/2 \rfloor.. n-1]$, $b[0.. \lfloor n/2 \rfloor]$

1	12	15	16	38
---	----	----	----	----



2	13	17	30	45
---	----	----	----	----



1	12	15	16	38
---	----	----	----	----



15	16	38
----	----	----

2	13	17	30	45
---	----	----	----	----



2	13	17
---	----	----

► **Știm să rezolvăm direct:**

- $n = 1$: $(a[1] + b[1]) / 2$
- $n = 2$: mediana lui $\{a[1], b[1], a[2], b[2]\}$
 $= (\max\{a[1], b[1]\} + \min\{a[2], b[2]\}) / 2$

Exemplu

1	12	15	16	38
---	----	----	----	----

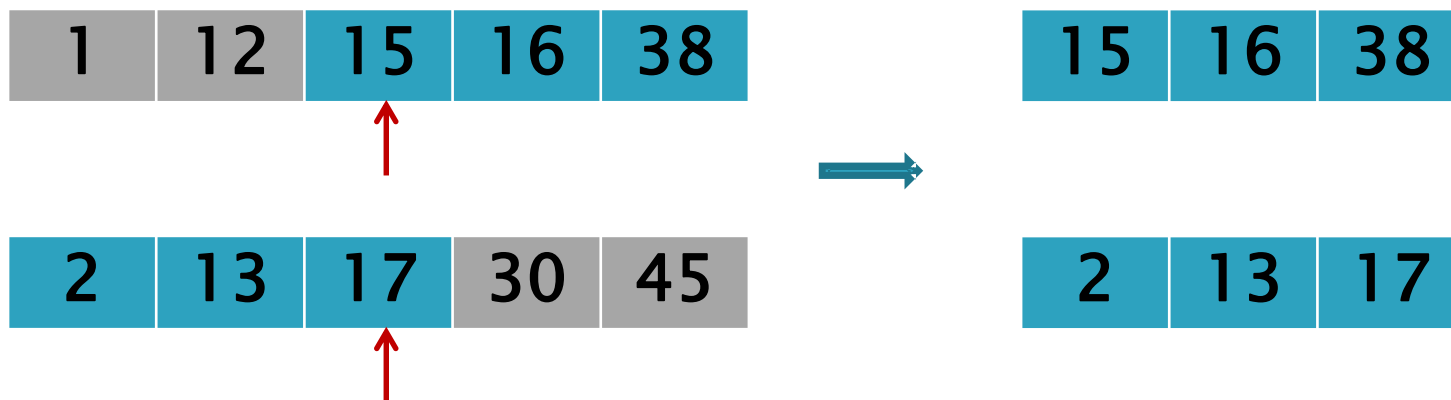
2	13	17	30	45
---	----	----	----	----

1	12	15	16	38
---	----	----	----	----



2	13	17	30	45
---	----	----	----	----





1	12	15	16	38
---	----	----	----	----



15	16	38
----	----	----

2	13	17	30	45
---	----	----	----	----



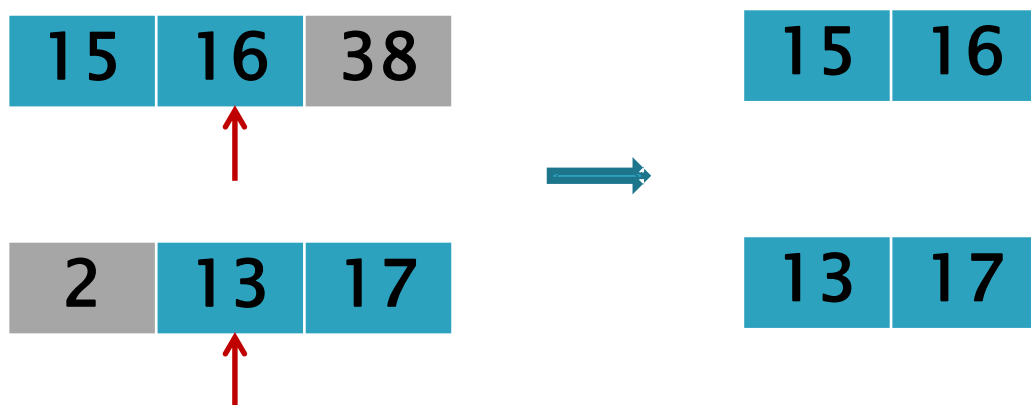
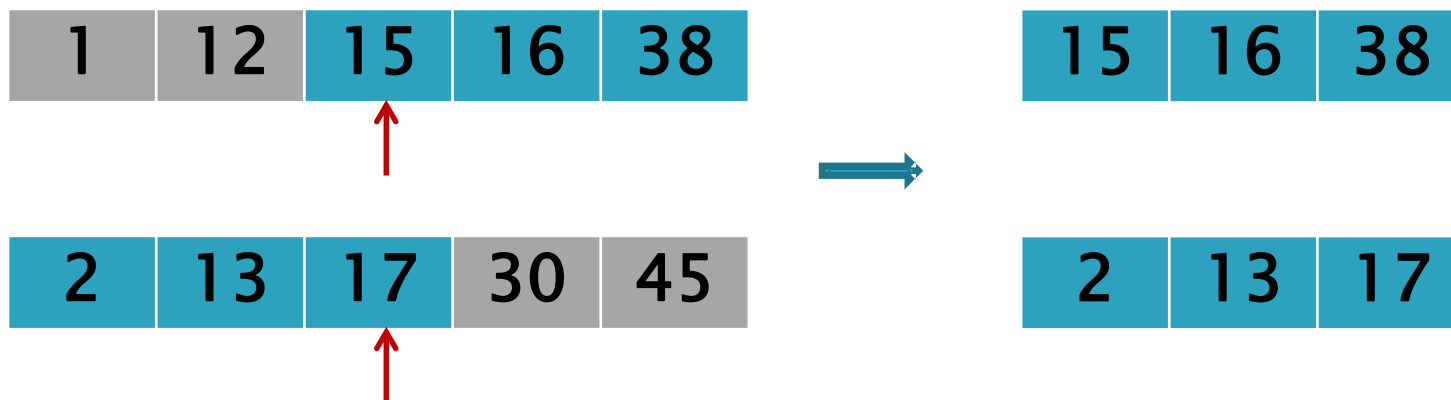
2	13	17
---	----	----

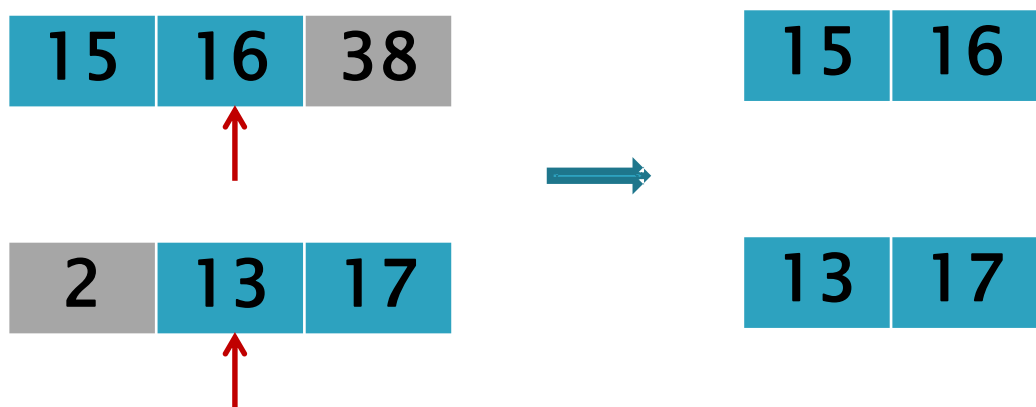
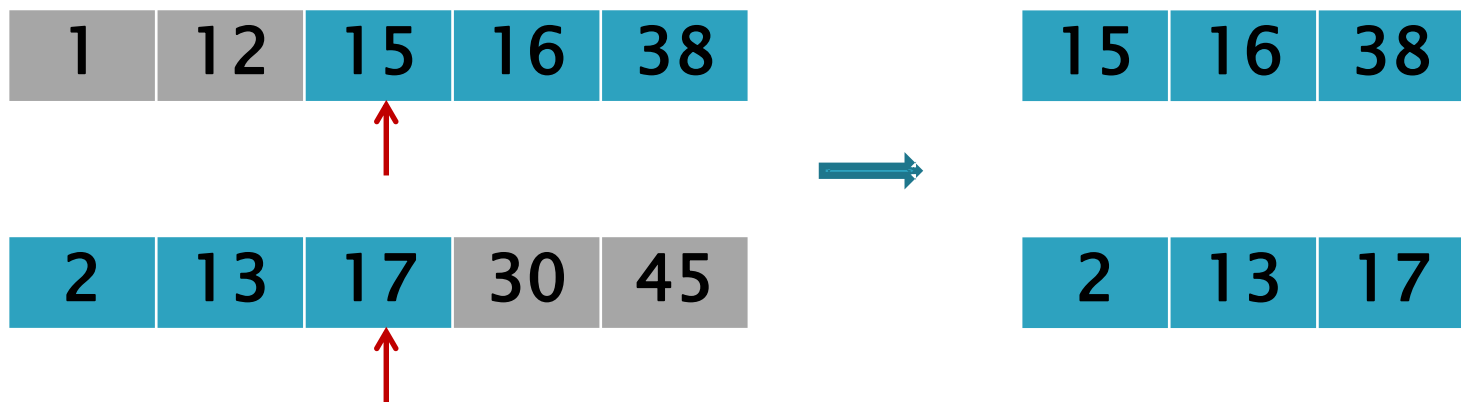
15	16	38
----	----	----



2	13	17
---	----	----







$$\begin{aligned} \text{Mediana} &= \frac{\max\{13, 15\} + \min\{16, 17\}}{2} = \frac{15 + 16}{2} \\ &= 15,5 \end{aligned}$$

Exemplul 2

1	12	15	16	38	40
---	----	----	----	----	----

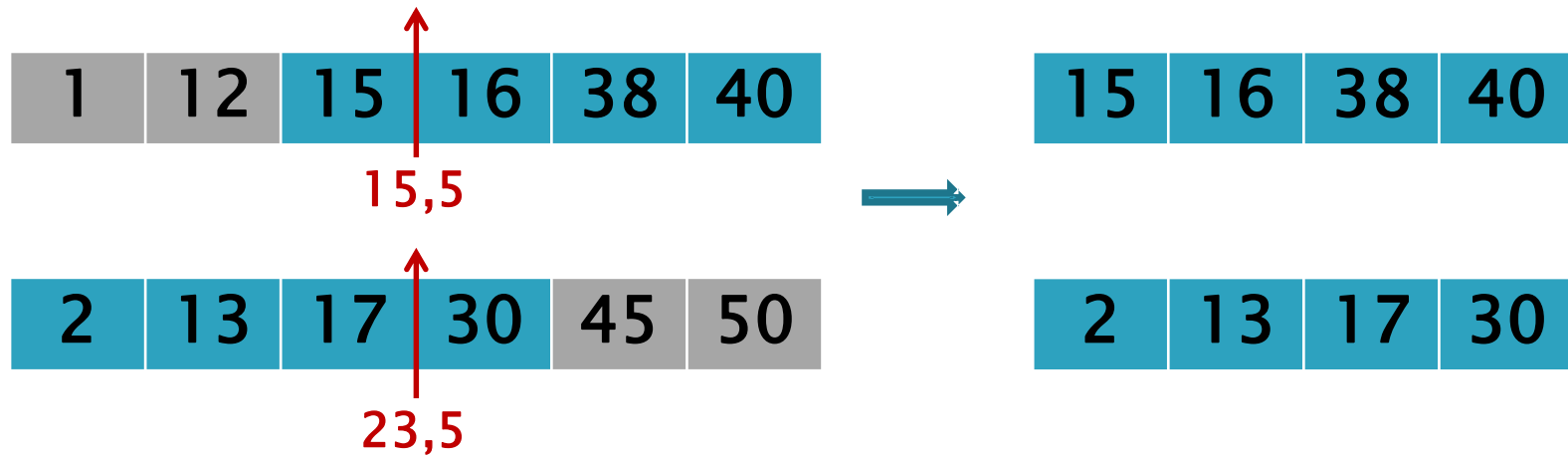
2	13	17	30	45	50
---	----	----	----	----	----

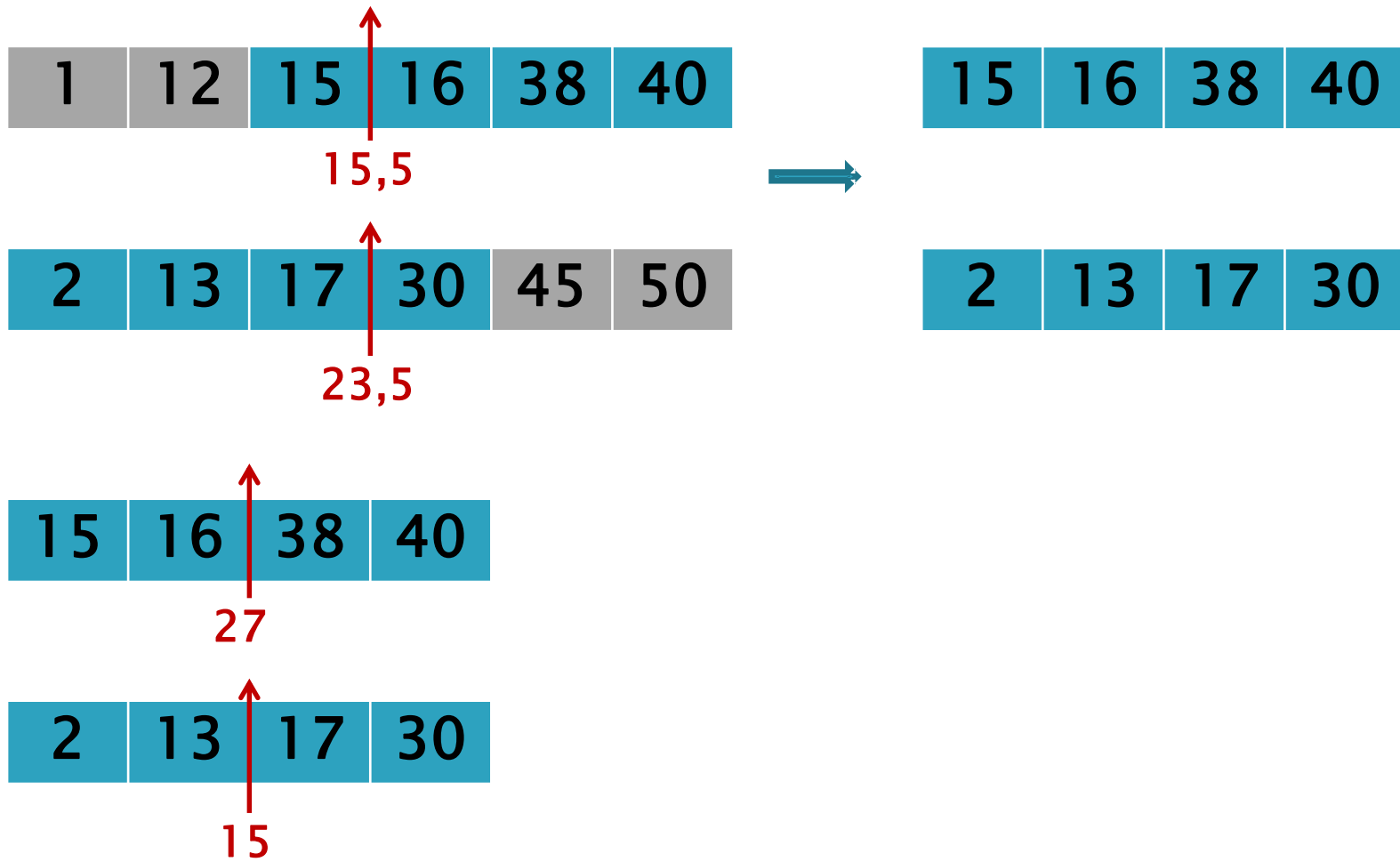
1	12	15	16	38	40
---	----	----	----	----	----

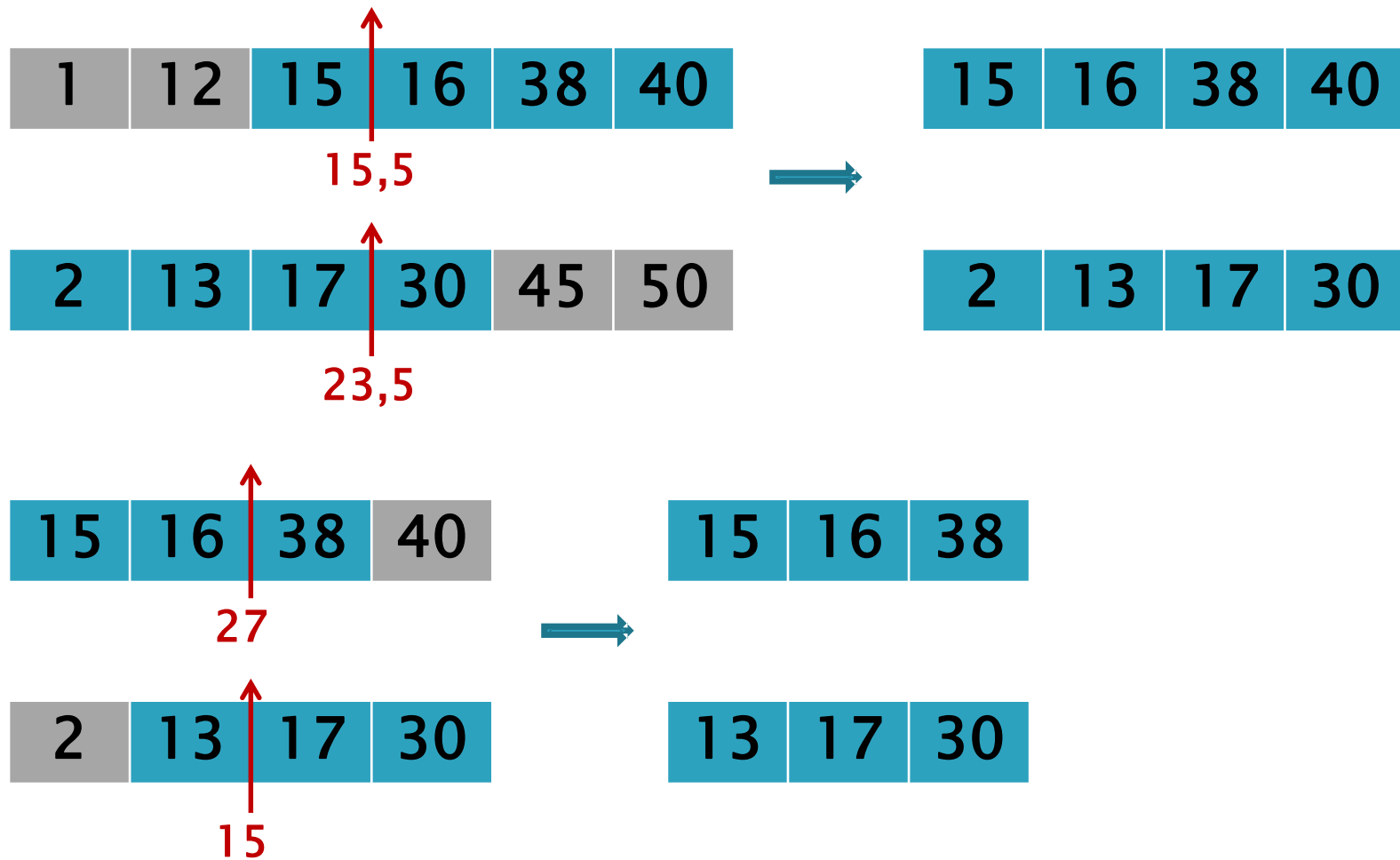
15,5

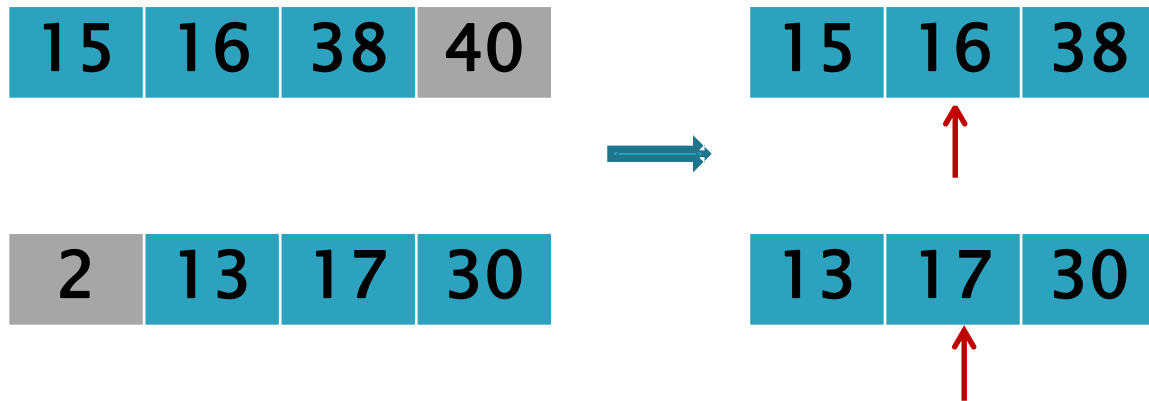
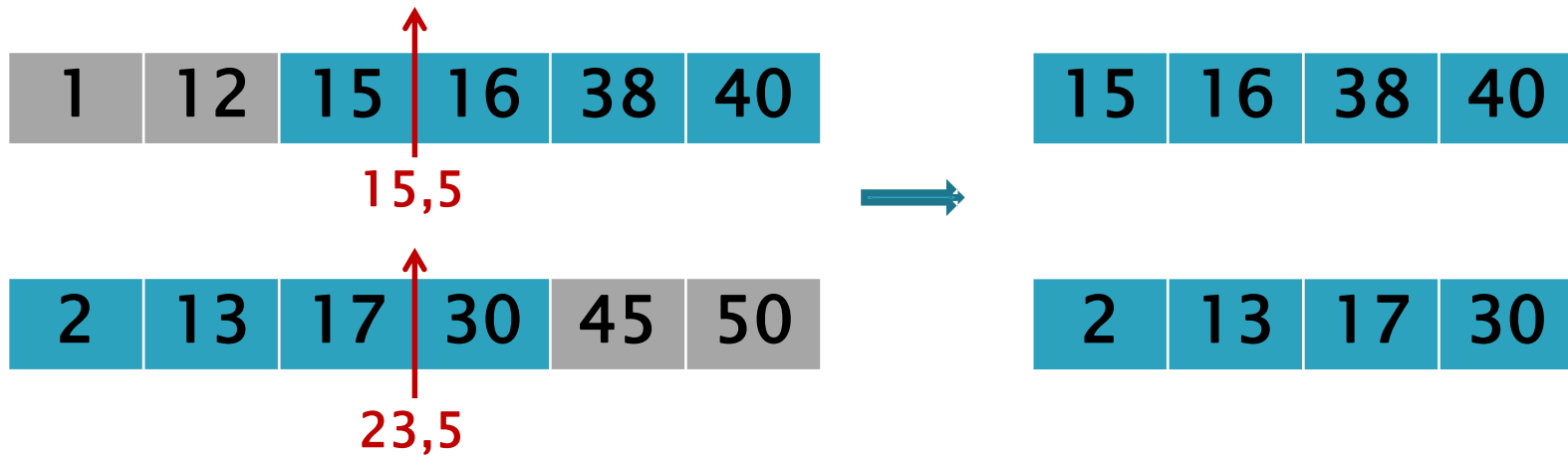
2	13	17	30	45	50
---	----	----	----	----	----

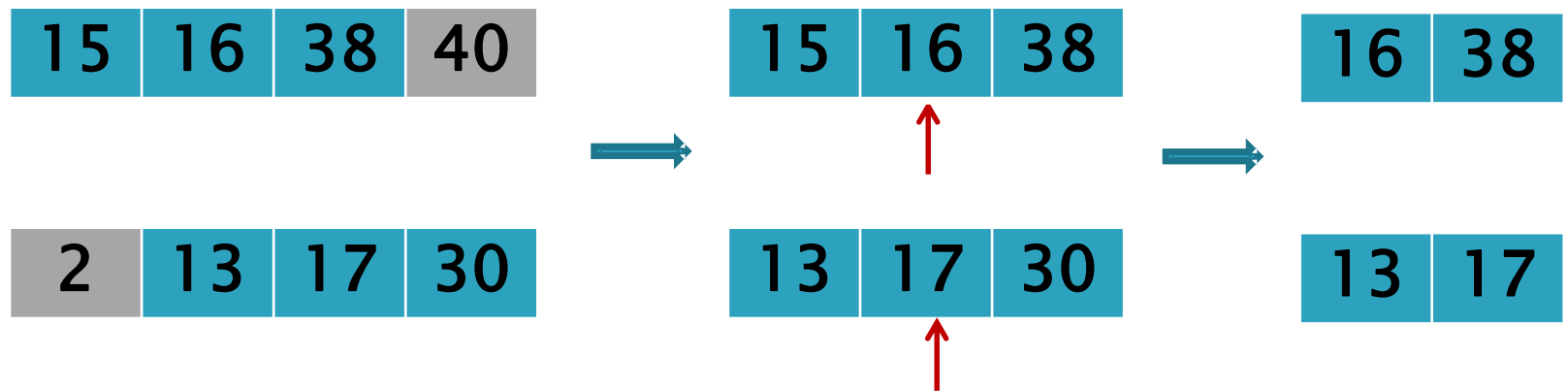
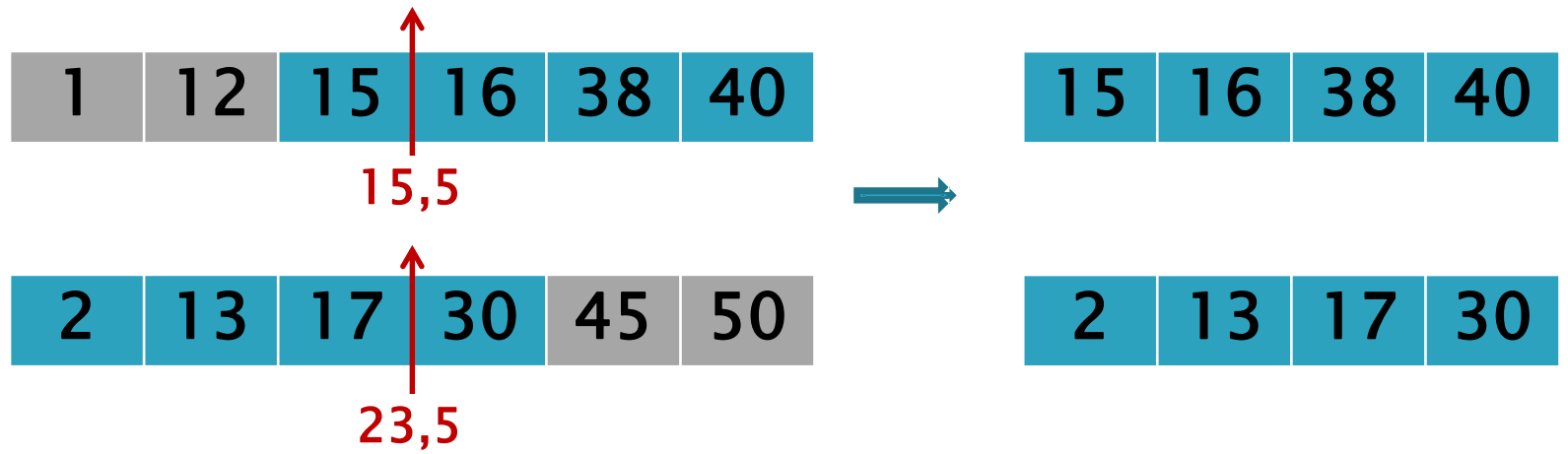
23,5

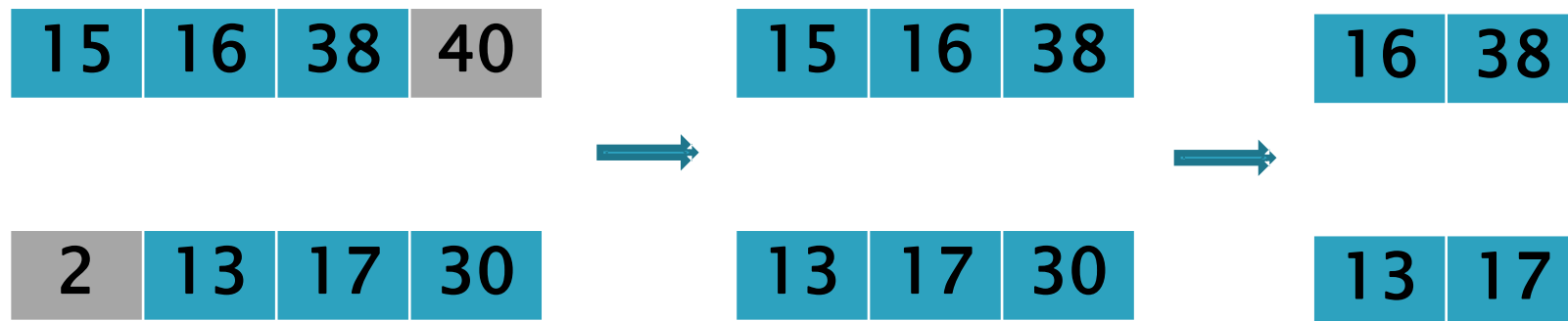
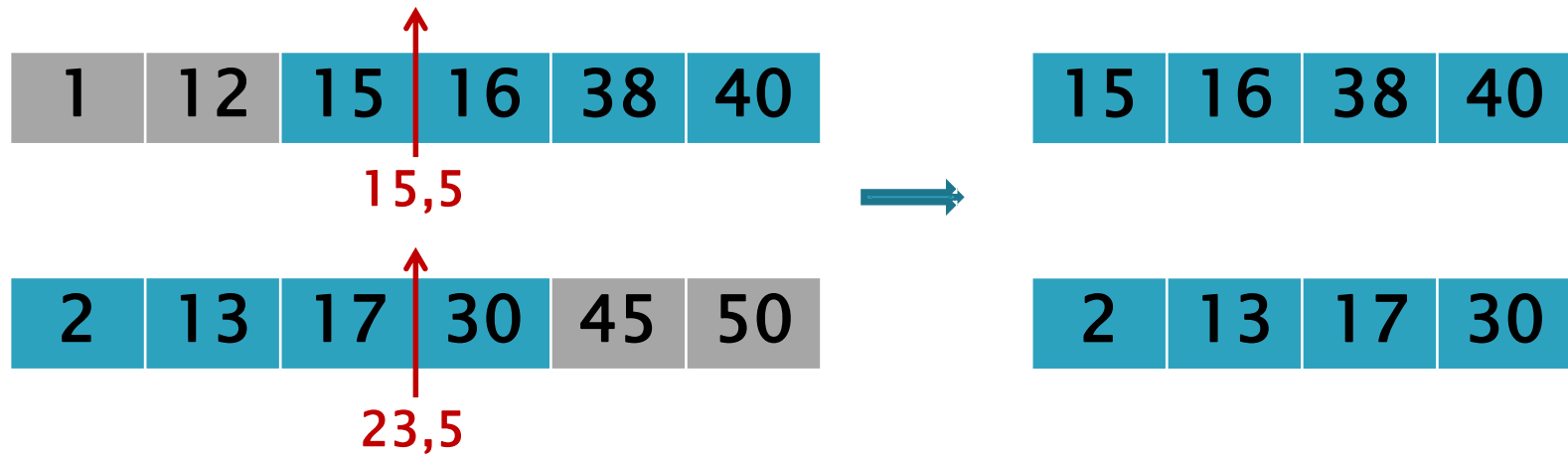












$$\begin{aligned} \text{Mediana} &= \frac{\max\{13, 16\} + \min\{17, 38\}}{2} = \frac{16 + 17}{2} \\ &= 16,5 \end{aligned}$$

Algoritm

```
double mediana(double[] v, int pv, int uv){  
    int n=uv-pv+1;  
    int m=(uv+pv)/2;  
    if (n%2==0)  
        return (v[m]+v[m+1])/2.0;  
    else  
        return v[m];  
}
```

```
double calculMediana(int pa, int ua,int pb, int ub) {  
    int n = ua-pa+1;  
    if (n<=2) //rezolv direct  
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2.0;  
  
    double ma=mediana(a,pa,ua); //mediana lui a[pa..ua]  
    double mb=mediana(b,pb,ub); //mediana lui b[pb..ub]
```

```
double calculMediana(int pa, int ua,int pb, int ub) {  
    int n = ua-pa+1;  
    if (n<=2) //rezolv direct  
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2.0;  
  
    double ma=mediana(a,pa,ua); //mediana lui a[pa..ua]  
    double mb=mediana(b,pb,ub); //mediana lui b[pb..ub]  
  
    if (ma==mb) return ma;  
    if (ma>mb)  
        return calculMediana(pa, pa+n/2, pb+(n-1)/2,ub);  
    else  
        return calculMediana(pa+(n-1)/2, ua, pb,pb+n/2);  
}
```



```

double calculMediana(int pa, int ua,int pb, int ub) {
    int n = ua-pa+1;
    if (n<=2) //rezolv direct
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2.0;

    double ma=mediana(a,pa,ua);//mediana lui a[pa..ua]
    double mb=mediana(b,pb,ub);//mediana lui b[pb..ub]

    if(ma==mb) return ma;
    if (ma>mb)
        return calculMediana(pa, pa+n/2, pb+(n-1)/2,ub);
    else
        return calculMediana(pa+(n-1)/2, ua, pb,pb+n/2);
}

```

```

double calculMediana() {
    return calculMediana(0,n-1,0,n-1);
}

```

Mediana a doi vectori sortați

- ▶ Complexitate: $O(\log n)$

Mediana a doi vectori sortați



Mai este valabilă ideea pentru vectori de lungimi diferite (reducem problema la o problemă de același tip păstrând jumătate din fiecare vector)?