Broscoteanu Daria-Mihaela

Grupa 143

$f_n : [5,6] \to \mathbb{R}$

$f_n(x) = \frac{(x+4)^3}{(n+2)^4}$

Convergența simplă

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{(x+4)^3}{(n+2)^4} = 0$$

$$\Rightarrow f_n \xrightarrow{s} 0 \quad , f(x) = 0 \, , f : [5,6] \to \mathbb{R}$$

Convergența uniformă

$$\sup_{x\in[5,6]} |f_n(x) - f(x)| = \sup_{x\in[5,6]} |f_n(x)|$$

$g(x) = |f_n(x)| \, , g : [5,6] \to \mathbb{R}$

$g(x) = \frac{(x+4)^3}{(n+2)^4} = \frac{1}{(n+2)^4} \cdot (x+4)^3$, g continuă și derivabilă pe $[5,6]$

$g'(x) = \frac{1}{(n+2)^4} \cdot 3(x+4)^2$

$g'(x) = 0 \Rightarrow 3(x+4)^2 = 0 \Rightarrow (x+4)^2 = 0 \Rightarrow x+4 = 0 \Rightarrow x = -4 \notin [5,6]$

x	5		6
$g'(x)$	+ + + + + + + + +		
$g(x)$	$g(5)$	↗	$g(6)$

$g'(5) = \frac{3 \cdot g^2}{(n+2)^4} > 0$

$\Rightarrow \sup_{x \in [5,6]} g(x) = g(6)$

$$g(6) = \frac{1}{(n+2)^4} \cdot \cdot 10^3$$

$$\lim_{n \to \infty} \sup_{x \in [5,6)} |f_n(x)| = \lim_{n \to \infty} \frac{10^3}{(n+2)^4} = 0$$

$\Rightarrow f_n \xrightarrow{u} f, \quad f(x) = 0, \quad \forall x \in [5,6]$