Bayesian Analysis of Macroeconomic Influences on Inflation and Gold Price

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Abstract

In this analysis, we seek to analyze and better understand the intricate behavior of inflation and gold price and how they relate to their influential factors. In the first part of this work, we conduct a Bayesian analysis of several important influential factors on inflation, including Money Supply, Prime Interest Rate, GDP growth, S&P 500 stock index performance, and Oil price change, using multiple linear regression models informed by the pertinent Directed Acyclic Graph of these parameters. In our expanded approach, we employed multiple regression analyses, including multiple nonlinear regression, to further explore the complexity of these relationships. Notably, our findings on time-delayed inflation highlight a consistent 12-month lag in the central bank's response to inflation surges, emphasizing the critical role of time-delayed effects in economic policymaking. The second part of this work includes a Bayesian analysis of gold price increase versus CPI inflation, S&P 500 stock index performance, and GDP growth as influential factors, confirming the effectiveness of gold as an inflation-hedging asset. This analysis helps to further develop our understanding of the complex impact of inflation on economic indicators and the effectiveness of gold as an inflation hedge.

1 Introduction

The study of inflation and its myriad influential factors has grown in importance in recent years, largely as a result of the unprecedented rate of government expenditures and the attendant problems of persistent inflation [6]. This topic has also fostered significant study and theorizing in previous decades; Crowder and Hoffman (1996) found a notable correlation between the nominal interest rate and inflation [4], which Ascari and Ropele (2013) further confirmed by pointing out how the inflation rate can be controlled by both the money supply and the interest rate, although controlling the interest rate was likely the more effective option [1]. Other potentially significant factors are oil price and economic growth, since Hunt (2006) points out that the 'Stagflation' of the 1970's was largely the result of rapid oil price increases, combined with excessive economic growth in the preceding period [7]. Overall, these sources indicate that there exists a complex relationship between inflation, interest rate, money supply, oil price, economic growth, and (by extension) the stock market, which merits further analysis and quantification.

1.1 Previous Work

Previously, Andrei compiled a dataset of United States monthly data from January 1960 to June 2023, with the percentage changes (with annual compounding) in CPI-U (Consumer Price Index for all Urban Consumers) [12] as the response variable, and Andrei took the Prime Interest Rate and monthly percentage changes in Money Supply, GDP, S&P 500 stock index, and West Texas Intermediate Oil Price [10, 11, 9, 5, 13] as the explanatory variables of interest.

In obtaining this data, Andrei also performed some pre-processing to convert it into a more useful form. Firstly, the CPI-U (Consumer Price Index for all Urban Consumers) was adapted from the data given by the Bureau of Labor Statistics [12], using annual compounding of the monthly change by the equation $((1 + \Delta CPI/100)^{12} - 1) * 100$. Furthermore, since the analysis used monthly data but the GDP data was recorded only quarterly, Andrei interpolated the GDP monthly percentage change using the equation $((GDP(Q_{n+1})/GDP(Q_n))^{1/3} - 1) * 100$.

In a previous Multiple Linear Regression analysis of this data, Andrei observed a statistically significant relationship between Inflation, Interest Rate, GDP, S&P 500, and Oil Price, but not the Money Supply. However, given the complexity of market and economic forces, a Bayesian analysis of the data (informed by a Directed Acyclic Graph) is needed to more accurately analyze the directional nature of this relationship and avoid any potential confounding errors and spurious correlations in the data.

The outputs of the analysis (conducted in R) took the form given in Figure 1.

Figure 1: Frequentist Analysis of Inflation (CPI-U) data

```
Call:
lm(formula = CPI.U ~ Money.Base.Change + DPRIME + GDP + SPIndex +
    Oil)

Residuals:
    Min      1Q  Median      3Q     Max
-9.0302 -1.9292 -0.3537  1.6259 15.8998

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        -0.53405    0.29791  -1.793   0.0734 .
Money.Base.Change  -0.07665    0.07492  -1.023   0.3066
DPRIME              0.46845    0.03436  13.633  < 2e-16 ***
GDP                 2.35970    0.32079   7.356 4.99e-13 ***
SPIndex           -16.81471    3.22664  -5.211 2.43e-07 ***
Oil                 0.11646    0.01298   8.974  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.074 on 749 degrees of freedom
Multiple R-squared:  0.3401,    Adjusted R-squared:  0.3356
F-statistic: 77.19 on 5 and 749 DF,  p-value: < 2.2e-16
```

Interest rate, GDP, S&P 500, and Oil Price were significant. Money Supply was not.

In the Google Colab file in the GitHub repository, Ziqiu Wan replicated and expanded on this frequentist analysis using Python, in which she separated the data into training and testing data and then performed Multiple Linear Regression analysis. She obtained results very similar to the ones obtained in Figure 1.

1.2 Study Scope Expansion

To develop an even more in-depth understanding of the nature of market conditions surrounding inflation, we wish to study not only how a variety of important factors relate to inflation, but also how inflation influences the price of prominent inflation-hedging assets (particularly gold); this is especially important because, as Oktay et al. (2016) point out, gold is viewed as a crucial asset to help ensure monetary stability and protect against the scourges of inflation [14].

The data for gold was obtained in two parts: from January 1971 to December 2000, and from January 2001 to June 2023. The data from 1971-2000 was obtained from *SD Bullion* [3], while the data from 2001-2023 was obtained from *IndexMundi* [8]. The gold data for our analysis is recorded as the monthly percentage change in the gold price. Prior to 1971, the US dollar was subject to a fixed dollar-gold conversion rate [15], making any data prior to that year effectively meaningless; hence, we only recorded gold prices for 1971 on.

1.3 Study Goals & Questions

An in-depth analysis of this data would give us valuable insights into the nature of the relationship between inflation, money supply, prime interest rate, GDP growth, WTI oil price, and S&P 500 stock index, as well as the way in which gold price and inflation are interconnected. As such, a Bayesian analysis of this data has great potential and applicability in further developing our understanding of the complex market and economic mechanisms relating to inflation.

In particular, a notable weakness of our previous analysis of this data is that it failed to consider the multifaceted and layered ways in which the different market forces interacted with and influenced each other. As such, to more thoroughly address this problem, we will use Directed Acyclic Graphs to inform our expectation of these relationships and will conduct Bayesian analysis using Markov Chain Monte Carlo (MCMC) based on these DAGs to observe the significance of these relationships.

Overall, using Multiple Regression MCMC, this project seeks to answer two very important questions about the nature of inflation and how to protect against it:

1. **What is the relationship between inflation, money supply, interest rate, oil price, GDP growth, and S&P 500 stock index?**

2. **How does gold price relate to inflation, S&P 500 stock index, and GDP?**

2 Formulation of the Mathematical Model

In this project, we aim to investigate two separate (but equally vital) questions surrounding inflation. First, we wish to investigate the nature of the relationship between inflation (as measured by CPI-U, or Consumer Price Index for all Urban Consumers), money supply change, prime interest rate, GDP growth, oil price change, and S&P 500 stock index growth. Second, we wish to investigate the nature of the relationship between changes in gold price (as the response variable) and inflation, GDP growth, and S&P 500 stock index as predictor variables.

2.1 Part 1 Outline

In the first part of this study, we wish to analyze and find the significance and nature of the relationships between inflation, money supply, interest rate, GDP growth, S&P 500 stock index, and West Texas Intermediate Oil price. As an initial guess of how we expect our data to behave, we construct a Directed Acyclic Graph to replicate the following anticipated behaviors:

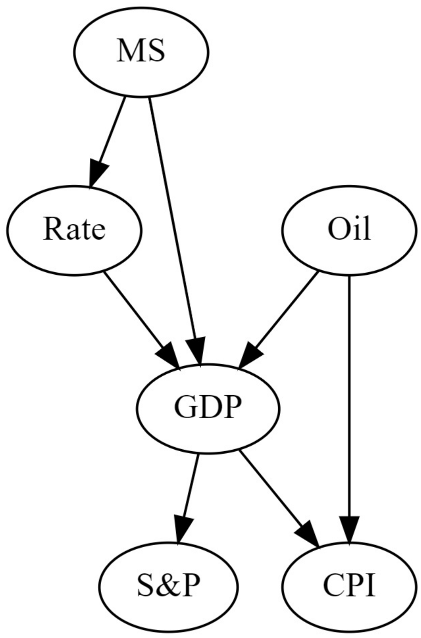
- The Money Supply (MS) directly influences the Interest Rate and economic growth (GDP)

- The Interest Rate, in turn, influences the GDP growth.

- Oil price also influences economic well-being, and hence affects GDP growth.
- The GDP growth then influences investor optimism/pessimism and hence influences the stock market (including the S&P 500 stock index).
- GDP growth also provokes price growth in consumer items, leading to CPI growth (inflation).
- Oil price increases further cause the producer cost of consumer items to rise, thereby bringing CPI growth (inflation).
- Since the stock market does not directly affect inflation, we anticipate that any correlation between S&P 500 stock index and CPI inflation is fundamentally spurious in nature.

These anticipated relationships are summarized in a Directed Acyclic Graph (DAG) given in Figure 2.

Figure 2: DAG for Inflation and Influential Factors



Money supply is 'MS', Interest Rate is 'Rate', and S&P 500 is 'S&P'

As such, based on the DAG in Figure 2, we determine that there is little point in studying the relationship between the S&P 500 stock index and CPI inflation. Hence, we break down our analysis into several different parts, each corresponding to a particular group of influential factors. In the first part (in Section 3.1.1), we analyze the relationship between inflation and WTI oil price. In the second part (in Section 3.1.2), we analyze the relationship between inflation, interest rate and GDP growth. In this case, we observe that cyclicality poses a problem for using the DAG in Figure 2. Therefore, in the third part of this analysis (in Section 3.1.3), we then introduce a time delay component to inflation to deal with this cyclicality, and analyze the prime interest rate versus time-delayed inflation and the money supply change, including (as will be seen) using multiple *nonlinear* regression.

2.2 Part 2 Outline

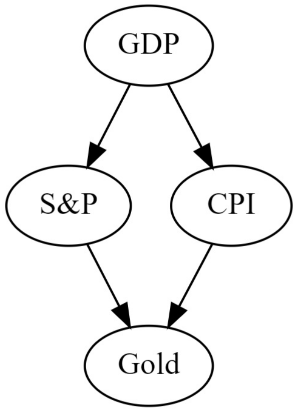
In the second part of this project, we examine the nature of the relationship between gold price change (as the predictor variable) and inflation, GDP growth, and S&P 500 stock index as predictor variables.

From Part 1, we know that there is a spurious correlation between S&P 500 stock index growth and inflation, and both are influenced by GDP growth. Furthermore, we expect to observe the following behaviors:

- The performance of the stock market (exemplified by the S&P 500 stock index) has a direct effect on the performance of different (hedging) assets, including gold
- An increase in inflation will also bring an increase in demand (and hence increase in price) of hedging assets, especially gold.
- Because GDP growth is economic in nature and asset price (including gold) is a market phenomenon, there is no direct impact of GDP on gold price increase.

With these relationships established, we proceed to summarize them in a Directed Acyclic Graph (DAG) given in Figure 3.

Figure 3: DAG for Gold Price and Influential Factors



Inflation is ‘CPI’, and S&P 500 is ‘S&P’

2.3 Mathematical Model

In analyzing the data in both Parts 1 and 2, we will begin with the (multiple) linear regression model centered at the mean of each predictor variable. As such, our (initial) model takes the form:

$$Y = \alpha + \sum_{i=1}^{n} \beta_{X_i}(X_i - \bar{X}_i) \tag{1}$$

Where Y denotes the response variable in the analysis, X_i denotes the i^{th} predictor variable used in the analysis, and $\bar{X}_i$ denotes the mean value of X_i.

In this analysis, we are interested in observing and finding the estimates of the β-parameters β_{X_i}, to determine which of the predictor variables has a significant relationship with the response variable. For this purpose, we will use Markov Chain Monte Carlo (MCMC) using priors based on the model in Equation 1.

We chose the multiple linear regression model because it reflects the causal relationships we expect to encounter in this model based on the DAGs above. We elected to center the regression of each component at 0 using $(X_i - \bar{X}_i)$ so as to simplify the MCMC parameter results and the credibility interval plots being obtained. Furthermore, we elected to use *linear* regression in this case because it serves to depict the simplest regression relationship between influential and response variables. Our linearity assumption arises from our initial lack of information on the specificity of the relationship between influential and response variables, so we elected to perform the simplest type of regression first in this analysis.

Nevertheless, we wish to further investigate whether the data actually *does* exhibit this linear relationship as we assumed. For this, we plot the credibility interval plots, and examine whether a linear model can appropriately be fit to the line. As will be seen below, some of our analyses do not fit our linearity assumption. In that case, we will reject our initial linearity assumption for the regression model, and will perform further analysis using **multiple *nonlinear* regression** to obtain a more appropriate model fit to the data. Therefore, we make our linearity assumption only because of our initial lack of information, and we will use a different (nonlinear) regression analysis if our results indicate it necessary to do so.

3 Solution of the Problem

3.1 Part 1: The Influential Factors of Inflation

3.1.1 Oil Price Growth

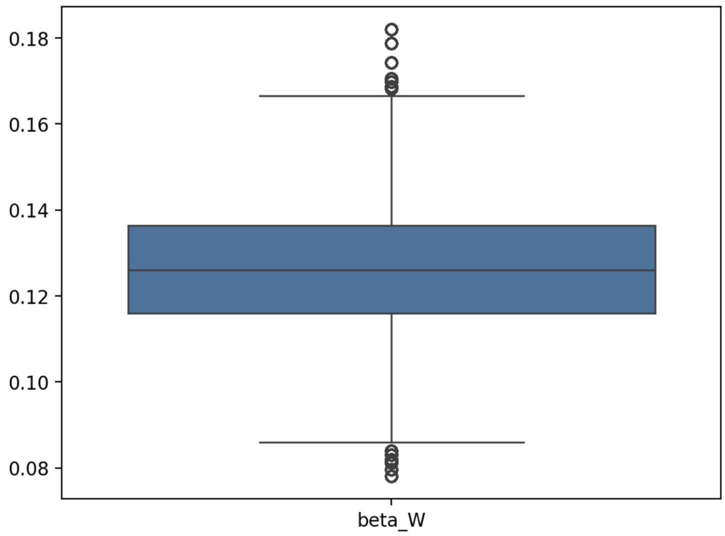
Firstly, we examine the relationship between inflation and oil price growth as a predictor variable. Examining the DAG in Figure 2, we determine that we must condition on GDP growth in order to avoid any confounding error brought by secondary paths from Oil price to CPI inflation. As such, we take the following prior:

$$
\begin{aligned}
CPI_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_W (W_i - \bar{W}) \\
\alpha &\sim \text{Normal}(4, 2) \\
\beta_W &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where W_i denotes the West Texas Intermediate oil price growth rate. Again, as before, we center the data in this prior, although we do not standardize it (as that would distort the magnitude of the relationship).

Conducting MCMC analysis using this prior yields the boxplot for β_W given in Figure 4.

Figure 4: MCMC Boxplot for Oil Price Growth



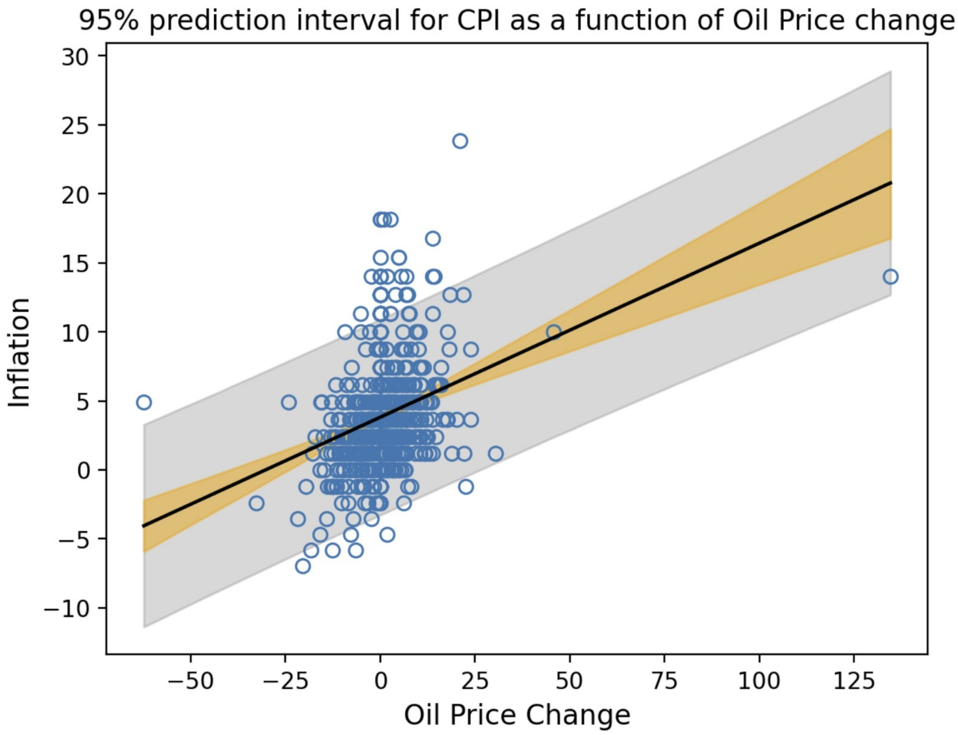
As can be seen, the slope of β_W is clearly positive, indicating that **there is a strong positive correlation between inflation and oil price growth**.

Unlike the relationship with interest rate studied subsequently, in the case of oil we do not expect any meaningful cyclicality of the data, as oil price changes are often driven by international events outside the scope of domestic monetary policy, such as the 1973 Oil crisis [2]. For this reason, we expect that the results of this oil-price MCMC analysis are accurate.

As such, we now plot the estimated mean of inflation versus oil price growth, as well as the 95% credibility interval for the mean, and the 95% credibility interval for the data. This is given in Figure 5.

In this case, the presence of several influential datapoints make it difficult to judge whether our linearity assumption for oil price growth in the prior is reasonable. However, if we drop these influential datapoints and perform the MCMC analysis again for the

Figure 5: 95% Credibility Interval Plot for Oil Price Growth



reduced dataset, the corresponding 95% credibility interval plot takes the form given in Figure 6.

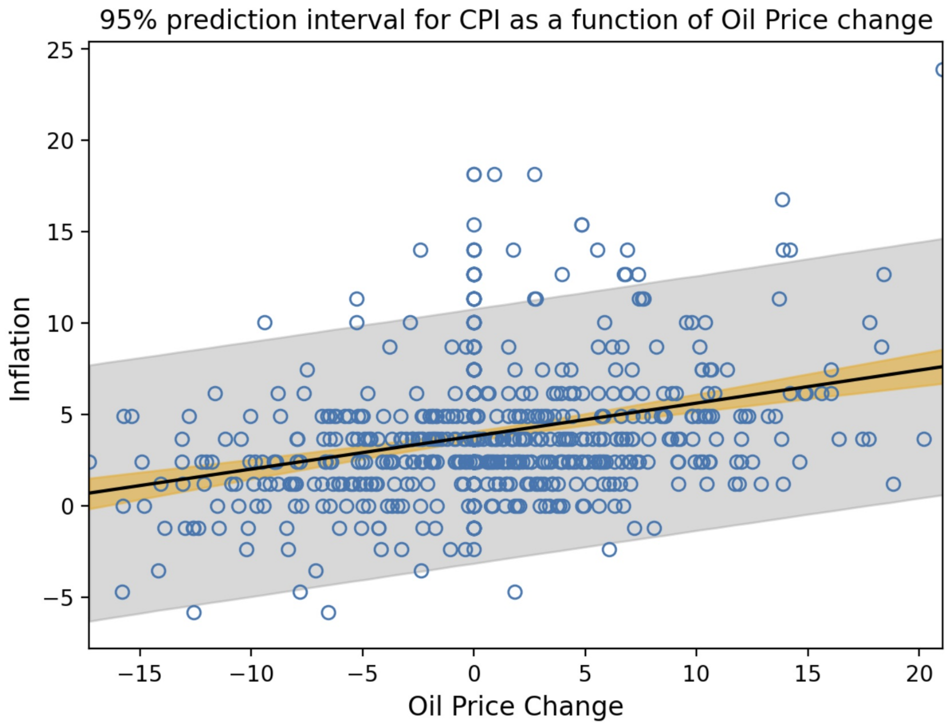
As can be observed from Figure 6, the data appears to follow the linear regression curve relatively well, indicating that our linearity assumption for the oil price growth in the regression prior is reasonable.

3.1.2 Interest Rate and GDP

Secondly, we are interested in analyzing the effect of interest rate and GDP on CPI inflation. Looking at the DAG in Figure 2, we observe that there should be a conditional independence of CPI inflation versus Interest Rate when including GDP growth, so we predict our model will yield a corresponding slope estimate $\beta_R \approx 0$.

From our data, we also observe that the mean of CPI inflation is approximately 3.90%, so we take the intercept α to have a mean of 4 and standard deviation of 2. Furthermore, we take a slightly wider distribution of σ. Because we do not know the nature of the relationship between inflation and interest rate and GDP growth, we take a prior of the

Figure 6: 95% Credibility Interval Plot for GDP Growth (w/o influential datapoints)



β-estimates to be centered at 0, with standard deviation at 1. Although standardizing the data would distort the magnitude of the relationship between inflation and the influential factors, we still elect to center the data by setting the prior proportional to the value of the factors minus their mean.

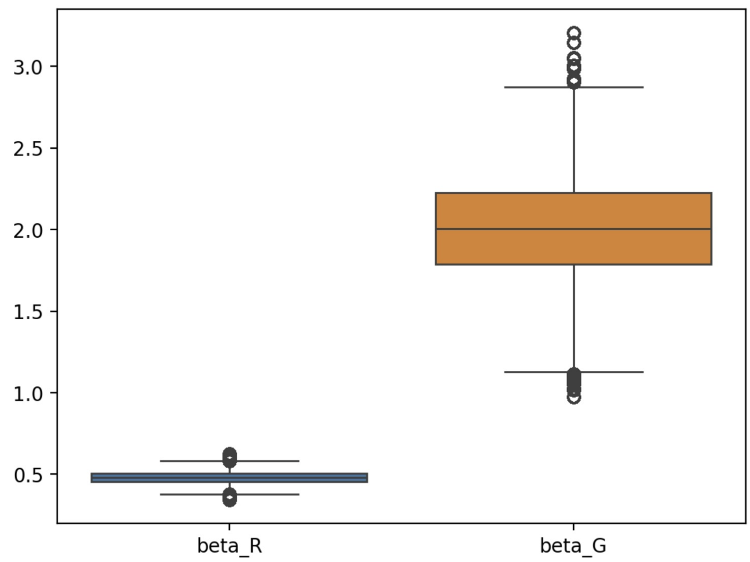
As such, our prior for this analysis takes the form:

$$
\begin{aligned}
CPI_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_R(R_i - \bar{R}) + \beta_G(G_i - \bar{G}) \\
\alpha &\sim \text{Normal}(4, 2) \\
\beta_R &\sim \text{Normal}(0, 1) \\
\beta_G &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where G_i denotes the GDP growth.

We conduct MCMC analysis of this prior in Python, taking 4 chains and training it on the CPI inflation data in our dataset. We then plot the resultant boxplot for the two β-parameters β_R and β_G in Figure 7.

Figure 7: MCMC Boxplots for Interest Rate (R) and GDP Growth (G)



As can be seen from Figure 7, both β_R and β_G are significantly above 0. Nevertheless, the estimate for β_R is considerably smaller than for β_G, and is reduced only slightly by including GDP growth; this indicates that these two variables are only slightly related with each other. This outcome contradicts our prediction of conditional independence of inflation versus interest rate.

An important possible reason for this nonzero slope of β_R is the cyclical relationship it has with CPI inflation; namely, while low interest rates (indirectly) bring about higher inflation, the increase in inflation induces the government to raise interest rates (as has happened in recent time). As such, we will need to study this relationship in such a way as to eliminate this cyclicality from the data.

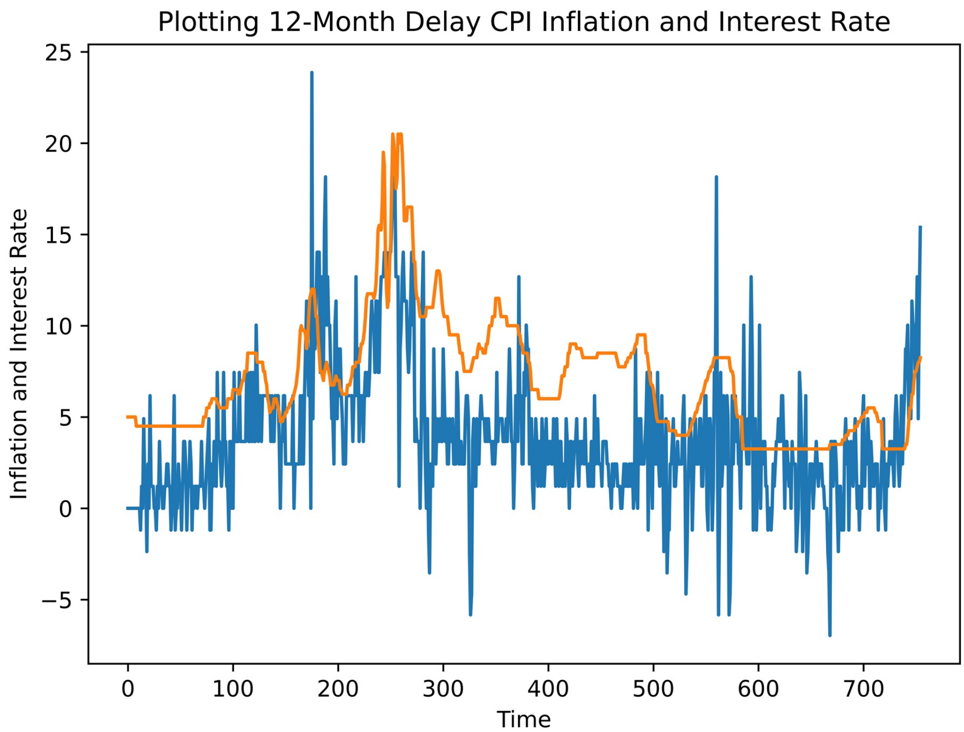
3.1.3 Time-Delayed Inflation

In order to avoid cyclicality and the confounding error that arises from it, we strive to include another component to the model: that of time. For this purpose, we choose to take the CPI inflation values after several months' delay and compare them to the interest rate, in order to observe how past inflation trends influence current interest rates selected by the Federal Reserve. In this way, analyzing interest rates as the response variable versus past inflation rates should eliminate cyclicality and help establish a more accurate directional relationship between inflation and prime interest rate.

For this purpose, we created a new dataframe, which contained the prime interest rate data in one column and contained several other columns with the CPI inflation values taken from n months previously (taking a variety of values of n). In order to determine which time delay yielded the most instructive information, we then calculated the correlation between the prime interest rate and the time-delayed inflation for each n-value. In this case, we observed that the largest correlation occurred for $n = 12$ (with a value of 0.526239), indicating that there is (on average) a 12-month delay between a rise in inflation and the resulting governmental interest rate response.

To verify this, we also plot the interest rate versus this 12-month-delayed inflation, which takes the form given in Figure 8.

Figure 8: Plotting 12-Month-Delay Inflation and Prime Interest Rate

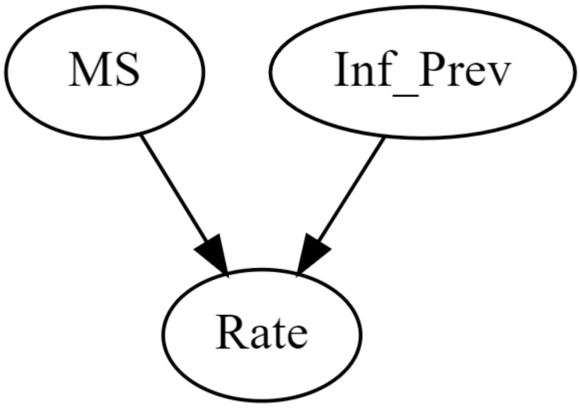


The plot in Figure 8 shows that the changes in interest rate (orange plot) match the 12-month-previous inflation data very well, indicating that there is an almost consistent

12-month delay from the onset of inflation to the beginning of interest rate changes. This is very interesting and (perhaps) unexpected, as we would expect government reaction times to vary considerably more than we observe here.

As such, we now run an MCMC analysis of the prime interest rate as the response variable versus the time-delayed inflation rate and the money supply change as the predictor variables, based on the DAG in Figure 9.

Figure 9: DAG for Time-delayed Inflation and Money Supply



Time-delayed Inflation is 'Inf_Prev', and money supply change is 'MS'

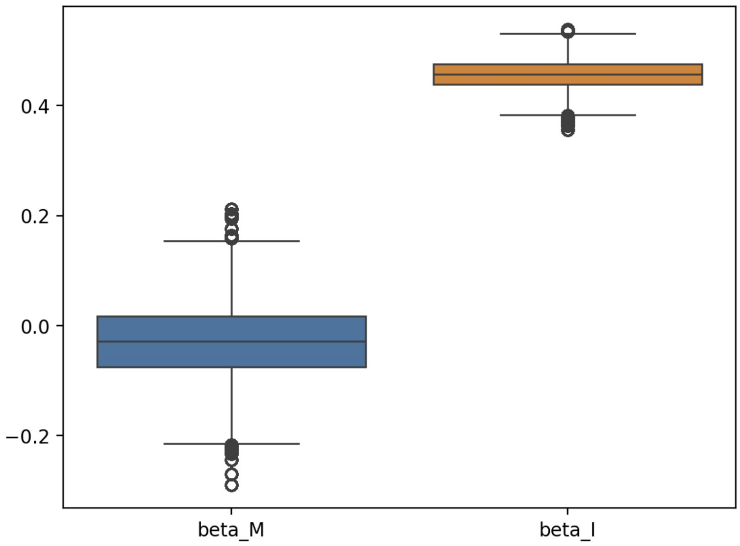
For this, we observe that the mean of the prime interest rate is approximately 7.0, so we take the intercept α prior to have a mean of 7 and standard deviation of 2. Because we do not know the nature of the relationship between interest rate, time-delayed inflation, and money supply, we take a prior of the β estimates to be centered at 0, with standard deviation at 1. As such, our prior takes the form:

$$
\begin{aligned}
DPRIME_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_I(I_i - \bar{I}) + \beta_M(M_i - \bar{M}) \\
\alpha &\sim \text{Normal}(7, 2) \\
\beta_I &\sim \text{Normal}(0, 1) \\
\beta_M &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where I_i denotes the time-delayed inflation rate, and M_i is the money supply change.

Running the MCMC analysis for this prior, we obtain the boxplots for β_M and β_I in Figure 10.

Figure 10: MCMC Boxplots for Money Supply (M) and Time-Delayed Inflation (I)



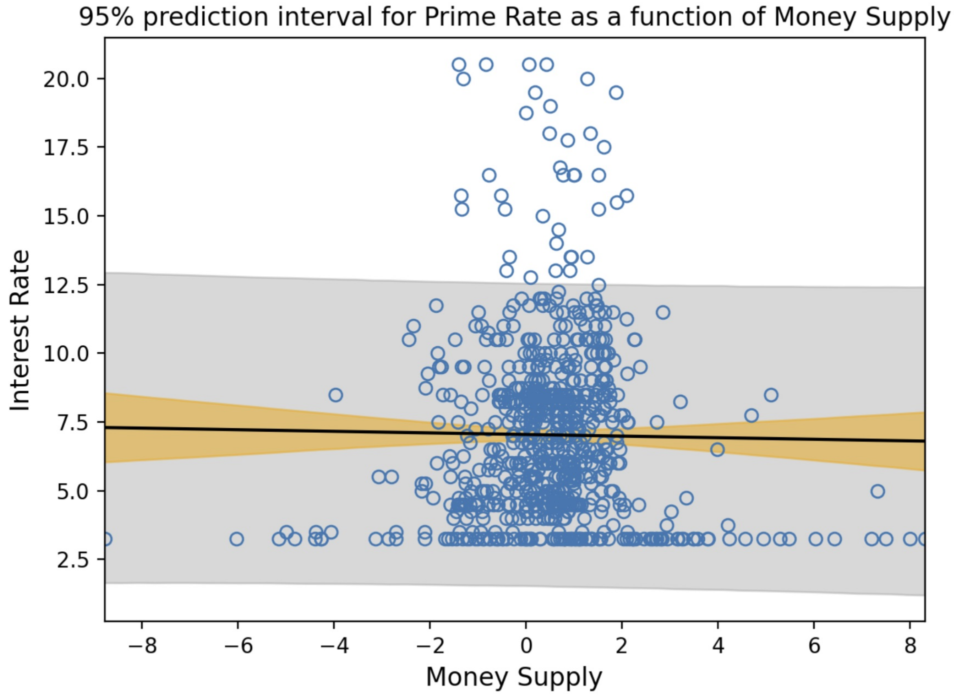
As can be seen, there is a *strongly* significant correlation between time-delayed inflation and the prime interest rate, but there is no significant (linear) correlation between prime interest rate and the money supply change. Hence, we conclude that the 12-month-delayed inflation plays a critical role in influencing central bank interest rate policy. However, in order to better understand the relationship between interest rate and money supply (given our estimate of $\beta_M \approx 0$), we must plot the credibility interval and check whether our linearity assumption is valid.

As such, we now plot the credibility interval plot for interest rate versus money supply change (holding time-delayed inflation at its mean), which is given in Figure 11.

Quadratic Regression

From this Credibility Interval plot for Interest Rate vs Money Supply above, we observe a clear nonlinear (in fact, possibly downward parabolic) trend to the data. As such, we proceed to conduct a **nonlinear MCMC analysis** of the interest rate data versus time-delayed inflation and money supply. Note that our credibility interval plot for

Figure 11: 95% Credibility Interval Plot for Interest Rate vs. Money Supply Change (Linear)



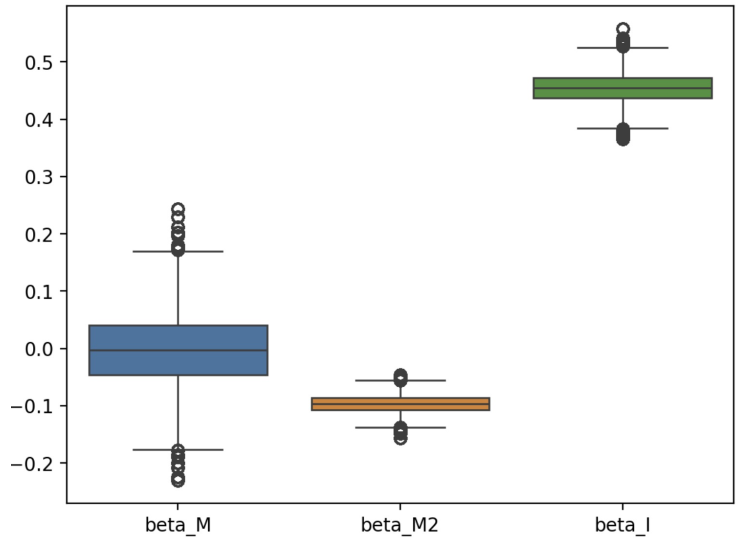
interest rate versus time-delayed inflation from the linear prior (not given here) shows that our linearity assumption for inflation is reasonable. Nevertheless, we first elect to model the relationship between money supply and interest rate using a quadratic equation, since the data appears to follow an approximately symmetric distribution. As such, our prior now takes the form:

$$
\begin{aligned}
DPRIME_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_I(I_i - \bar{I}) + \beta_M(M_i - \bar{M}) + \beta_{M2}(M_i - \bar{M})^2 \\
\alpha &\sim \text{Normal}(7, 2) \\
\beta_I &\sim \text{Normal}(0, 1) \\
\beta_M &\sim \text{Normal}(0, 1) \\
\beta_{M2} &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where I_i denotes the time-delayed inflation rate, and M_i is the money supply change.

Running this in MCMC analysis, we obtain the boxplots given in Figure 12.

Figure 12: Boxplots for Quadratic Money Supply (M, M2) & Time-Delayed Inflation (I)



Interestingly, as can be seen, the β_M parameter is extremely well-centered on 0, but the β_{M2} parameter (for the quadratic component) is significantly negative. This indicates that there is a significant quadratic correlation between money supply and interest rate. There was very little observable change in the β_I (inflation) parameter estimate, which remained significant. This reinforces the indication that inflation operates relatively independently of the money supply change month-on-month, as there is no indication of conditional independence in our analysis results.

As such, we now verify that our linearity assumption for interest rate versus time-delayed inflation is reasonable. Plotting the 95% credibility interval plot yields Figure 13. As can be observed, the data appears to follow the linear regression curve relatively well, indicating that our linearity assumption for the time-delayed inflation in the regression prior is reasonable.

Now, we turn to plot the 95% credibility interval plot for the quadratic (nonlinear) regression of interest rate versus money supply change. This is obtained in Figure 14.

As can be seen, the quadratic (nonlinear) regression model has a much better fit to the data than does the linear regression model, although it is somewhat skewed toward the endpoints of the interval because of several influential datapoints.

Figure 13: 95% Credibility Interval Plot for Interest Rate vs. Time-Delayed Inflation

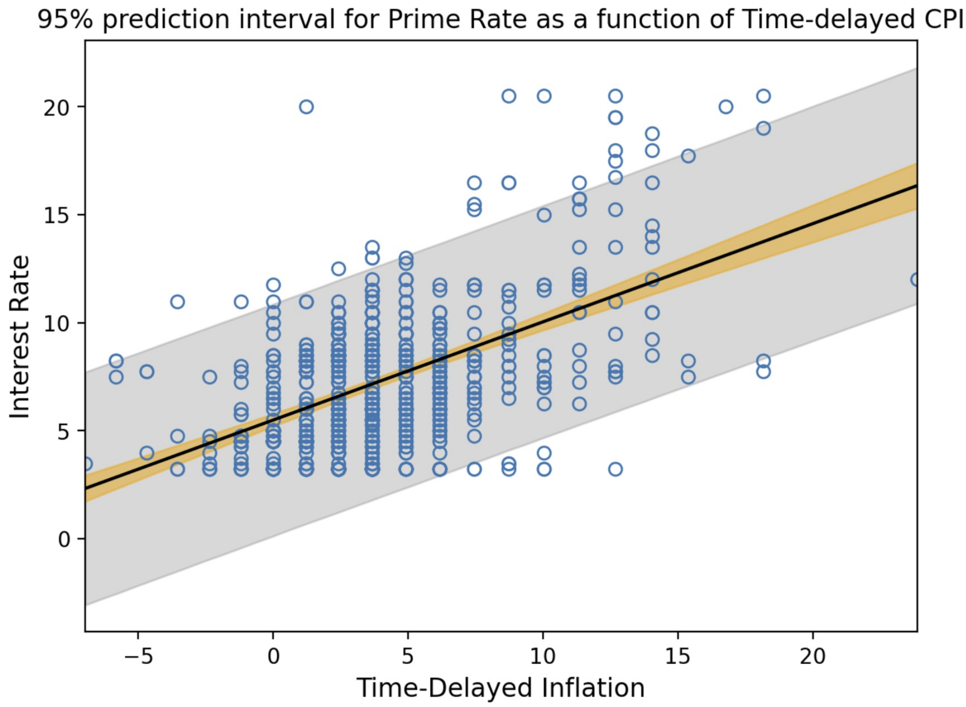
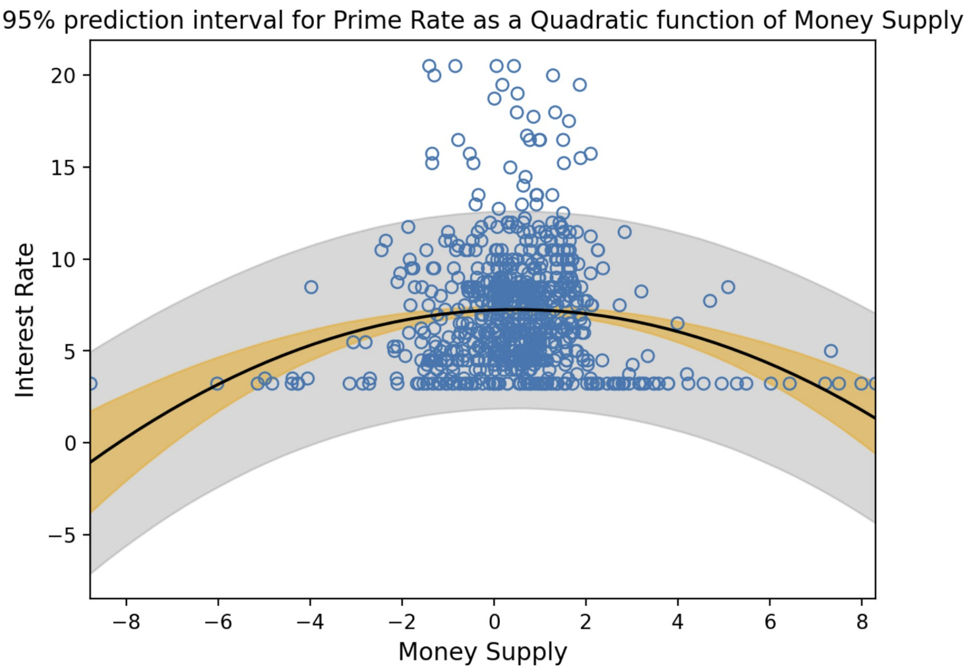


Figure 14: 95% Credibility Interval Plot for Interest Rate vs. Quadratic Money Supply



Overall, this multiple nonlinear regression fit yields a very interesting behavior. Namely, we anticipated a negative relationship between interest rate and money supply growth in the model, so the positive relationship of interest rate and money supply below the mean is somewhat unexpected. Our posterior distribution results indicate that interest rates are low even in times of significant money supply contraction, indicating that other factors (such as economic stagnation) are likely contributing to the low interest rates in extreme cases of money supply reduction.

Exponential Regression

Although the quadratic regression analysis of interest rate versus money supply had an acceptable fit, the endpoints did not perform very well because the interest rate is always kept positive, whereas the quadratic model reached negative values. As such, we now wish to investigate another possible nonlinear (exponential) fit of money supply to interest rate, which should approximate a bell curve with a better fit to the data. For this purpose, we take a new nonlinear (exponential) regression model and fit it to the data using the following prior:

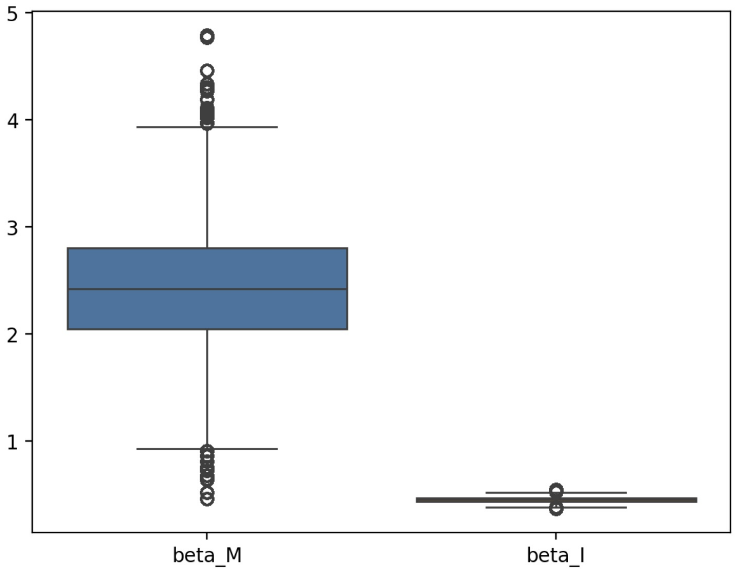
$$
\begin{aligned}
DPRIME_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_I(I_i - \bar{I}) + \beta_M e^{-S|M_i - \bar{M}|} \\
\alpha &\sim \text{Normal}(7, 2) \\
\beta_I &\sim \text{Normal}(0, 1) \\
\beta_M &\sim \text{Normal}(0, 1) \\
S &\sim \text{Exponential}(1.0) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where I_i denotes the time-delayed inflation rate, and M_i is the money supply change.

Note that, in this case, when the money supply change is at the mean ($M_i = \bar{M}$), the intercept of DPRIME mean versus time-delayed inflation is actually $\alpha + \beta_M$.

As well, we chose the exponential form $\mu_i = \alpha + \beta_I(I_i - \bar{I}) + \beta_M e^{-S|M_i - \bar{M}|}$ instead of $\mu_i = \alpha + \beta_I(I_i - \bar{I}) + \beta_M e^{-S(M_i - \bar{M})^2}$ (the normal distribution form) because the former is computationally less intensive when running MCMC. Performing MCMC analysis of this prior yields the boxplot given in Figure 15

Figure 15: Boxplots for Exponential Money Supply (M) & Time-Delayed Inflation (I)



As can be seen, both β_M and β_I are significantly positive in this boxplot, indicating that there is a significant positive linear relationship between time-delayed inflation and interest rate and a significant exponential/bell curve relationship between money supply change and interest rate.

As such, we now verify that our linearity assumption for interest rate versus time-delayed inflation is reasonable. Plotting the 95% credibility interval plot yields Figure 16. As can be observed, the data appears to follow the linear regression curve relatively well, indicating that our linearity assumption for the time-delayed inflation in the regression prior is reasonable.

Now, we turn to plot the 95% credibility interval plot for the exponential (nonlinear) regression of interest rate versus money supply change. This is obtained in Figure 17.

As can be seen, the exponential (nonlinear) regression model has a better fit to the data than does the linear regression model, although its fit to the data at the endpoints

Figure 16: 95% Credibility Interval Plot for Interest Rate vs. Time-Delayed Inflation

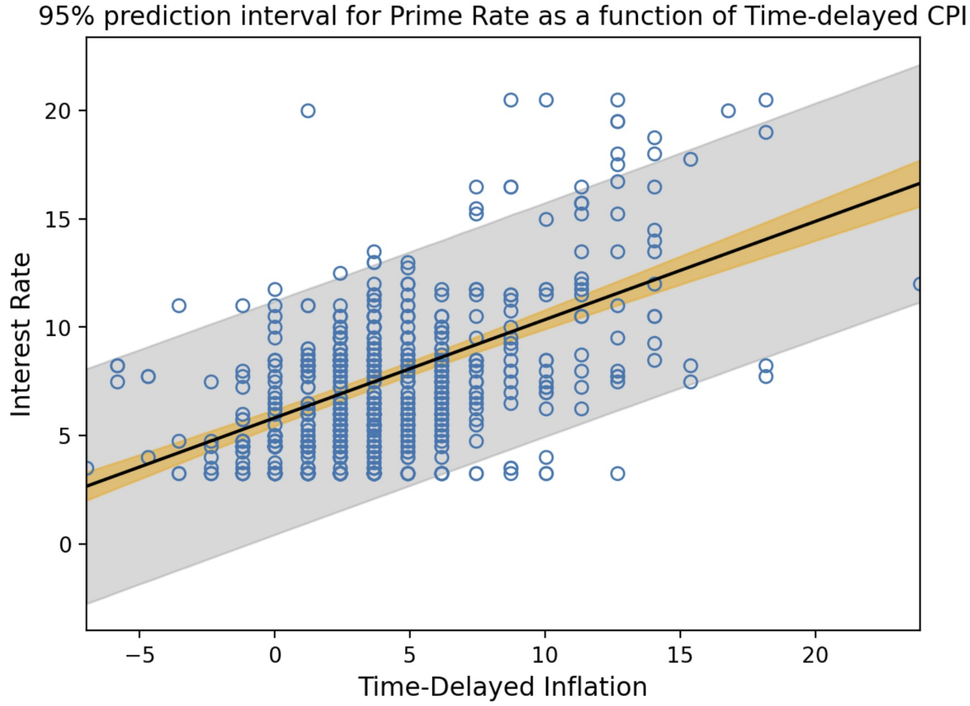
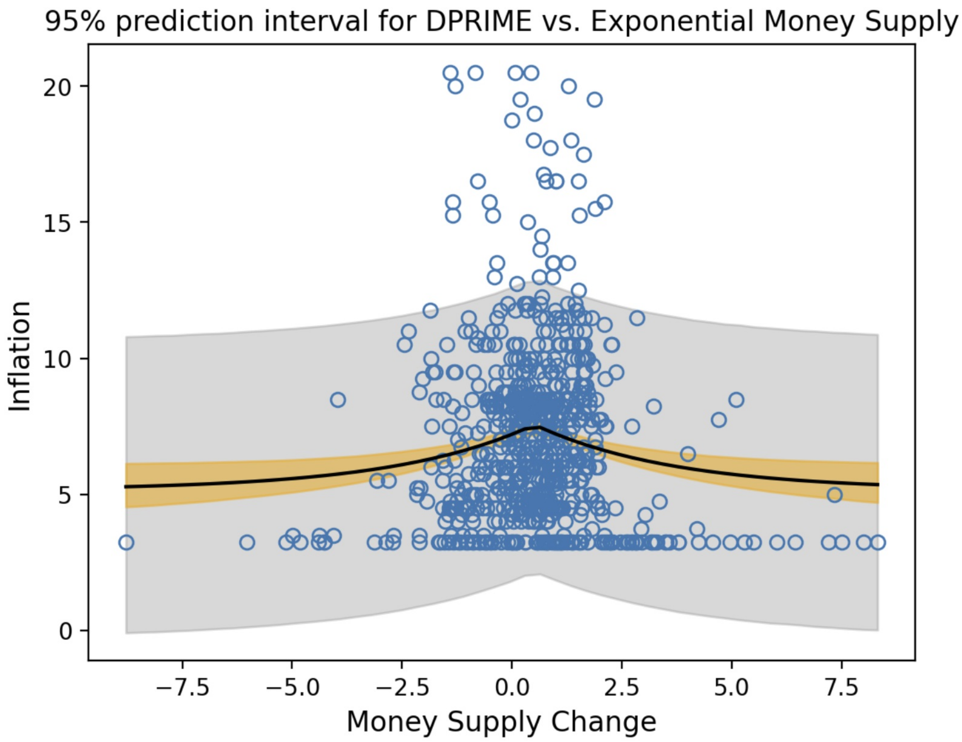


Figure 17: 95% Credibility Interval Plot for Interest Rate vs. Quadratic Money Supply



is not necessarily better than was obtained by the quadratic fit.

3.1.4 Results & Interpretation

In conducting this analysis, we made the following observations:

- There is a significant (linear) correlation between oil price increase in a particular month and CPI inflation in that month.
- This confirms that oil prices play a critical role in driving inflation, as was the case with the 1973 oil embargo [2].
- Meanwhile, conducting MCMC analysis of inflation versus GDP and Interest Rate did not yield the expected conditional independence.
- This likely arises from cyclical influences between inflation and the interest rate/ central bank response.
- To deal with this cyclicality, we took inflation data with time delay, and found an almost consistent year-long delay from the onset of inflation until central banks react.
- Using MCMC analysis, we observed a significant linear correlation between time-delayed inflation and the interest rate.
- This suggests that past inflation strongly influences central banks' interest rate policy.
- Unexpectedly, we also observed a significant nonlinear relationship between money supply change and interest rate, and analyzed it using both quadratic and exponential regression.

3.2 Part 2: Gold Price and Inflation

In this part, we investigate the nature of the relationship between gold price change as response variable and (non-delayed) inflation, GDP growth, and S&P 500 stock index as predictor variables. The expected relationship between these variables is depicted in the Directed Acyclic Graph in Figure 3.

Note that, because our recorded data for gold price only begins in 1971, we first preprocess our data to remove all datapoints prior to that time. As such, for this part, we analyze the monthly data from January 1971 to June 2023.

As such, our analysis comprises two parts. Firstly, in order to close unwanted secondary paths between inflation and S&P 500 stock index, we condition on GDP growth and therefore conduct MCMC analysis of gold price change versus inflation and S&P 500 stock index. Secondly, in order to analyze the relationship between GDP and gold price increase, we must condition on the intermediate parameters (in this case, inflation and S&P 500 stock index), so we will perform MCMC analysis of gold price change versus GDP growth.

3.2.1 Gold vs. Inflation and S&P 500

In this first part, we are primarily interested in studying how inflation affects changes in gold price. Examining the DAG in Figure 3, we observe that there are two paths between CPI and Gold price; to close the second (unwanted) path, we condition on the GDP variable above, and hence write a model that includes the S&P 500 stock index and CPI versus the Gold price.

In this case, we observe that the mean for the price change of gold is approximately 0.76%, so we take the prior α to have a mean of 0.75 and standard deviation of 0.2.

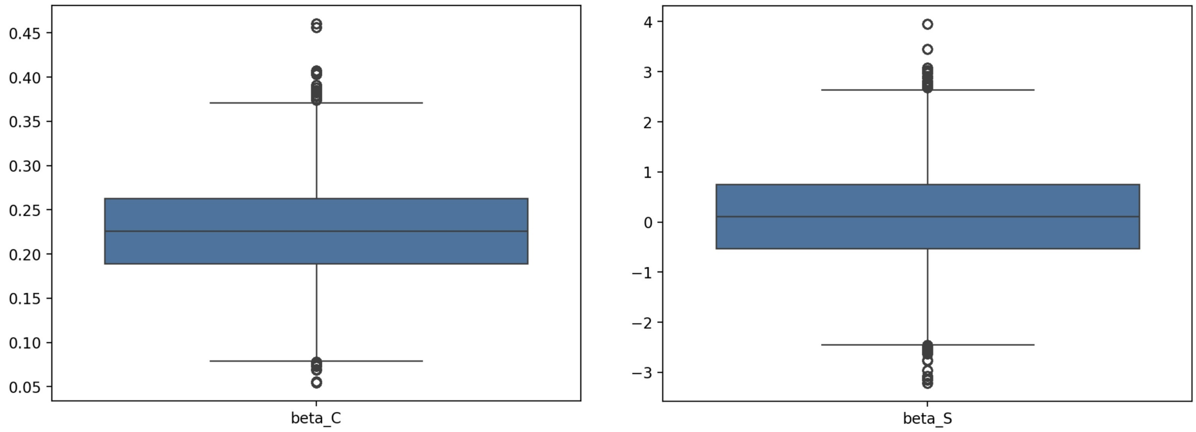
As such, we take the following prior:

$$
\begin{aligned}
Gold_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_C(C_i - \bar{C}) + \beta_S(S_i - \bar{S}) \\
\alpha &\sim \text{Normal}(0.75, 0.2) \\
\beta_C &\sim \text{Normal}(0, 1) \\
\beta_S &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where C_i denotes the CPI inflation rate and S_i denotes the S&P 500 growth rate.

Conducting MCMC analysis using this prior and the Gold data from our dataset, we obtain the boxplots in Figure 18 for β_C and β_S, respectively.

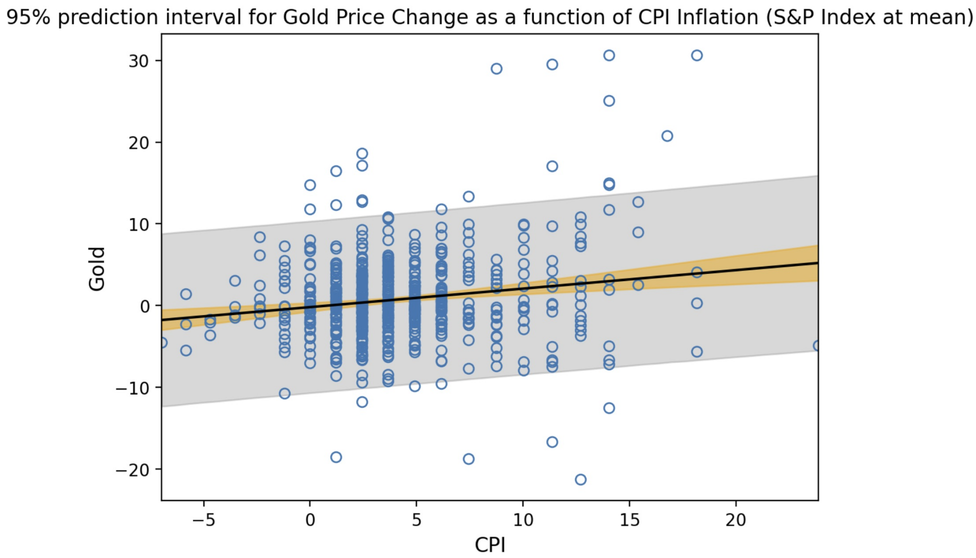
Figure 18: MCMC Boxplot for Inflation (β_C) and S&P 500 Index (β_S)



As can be seen, the two boxplots differ significantly in their variance. However, we also observe from Figure 18 that the inflation slope estimate β_C is significantly positive, while the S&P 500 slope estimate β_S is very closely centered to 0. This indicates that **there is a statistically significant (positive) relationship between inflation and gold price change, but not between stock market (S&P 500) performance and gold price change**.

Plotting the 95% credibility interval plots for the mean and data for gold price versus inflation (holding S&P 500 performance at the mean) yields the credibility interval plot in Figure 19.

Figure 19: 95% Credibility Interval Plot for Gold vs. Inflation



As can be seen, there is a clearly upward slope to the relationship between gold price change and inflation. Furthermore, we observe that our linearity assumption in the model is reasonable.

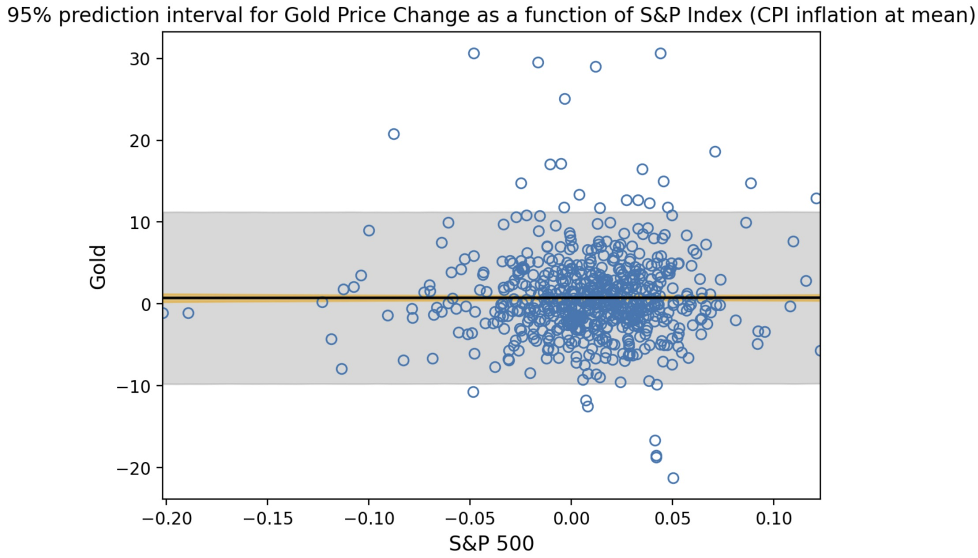
Meanwhile, plotting the 95% credibility interval plots for the mean and data for gold price versus stock index (S&P 500) performance, holding inflation at the mean, yields the credibility interval plot in Figure 20.

As can be seen, the mean relationship is very clearly horizontal, indicating that there is no notable relationship between gold price change and S&P 500 stock index performance. Furthermore, we conclude that our linearity assumption in the model is reasonable.

3.2.2 Gold vs. GDP

In this second part, we wish to observe whether there is any relationship between gold price and GDP growth. Examining the DAG in Figure 3, we observe that there is no *direct* relationship between GDP growth (economic performance) and gold price change (a market phenomenon). As such, to conduct this analysis, we condition on the other

Figure 20: 95% Credibility Interval Plot for Gold vs. S&P 500



two parameters (CPI inflation and S&P 500 stock index), and our prior (based on our multiple regression model) takes the form

$$
\begin{aligned}
Gold_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_G(G_i - \bar{G}) \\
\alpha &\sim \text{Normal}(0.75, 0.2) \\
\beta_G &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(0.5)
\end{aligned}
$$

Where G_i denotes GDP growth.

Conducting MCMC analysis of this prior using the Gold price data in our dataset, we get the boxplot for the slope estimate β_G given in Figure 21.

As can be seen, the boxplot for β_G is centered relatively close to 0, indicating that there is likely not a notable relationship between GDP growth and Gold price changes.

As such, we now make the 95% Credibility Interval plot for Gold price change versus GDP growth, and check the linearity assumption by dropping some of the influential datapoints, yielding the right-hand graph. This is provided in Figure 22.

Figure 21: MCMC Boxplot for GDP (β_G)

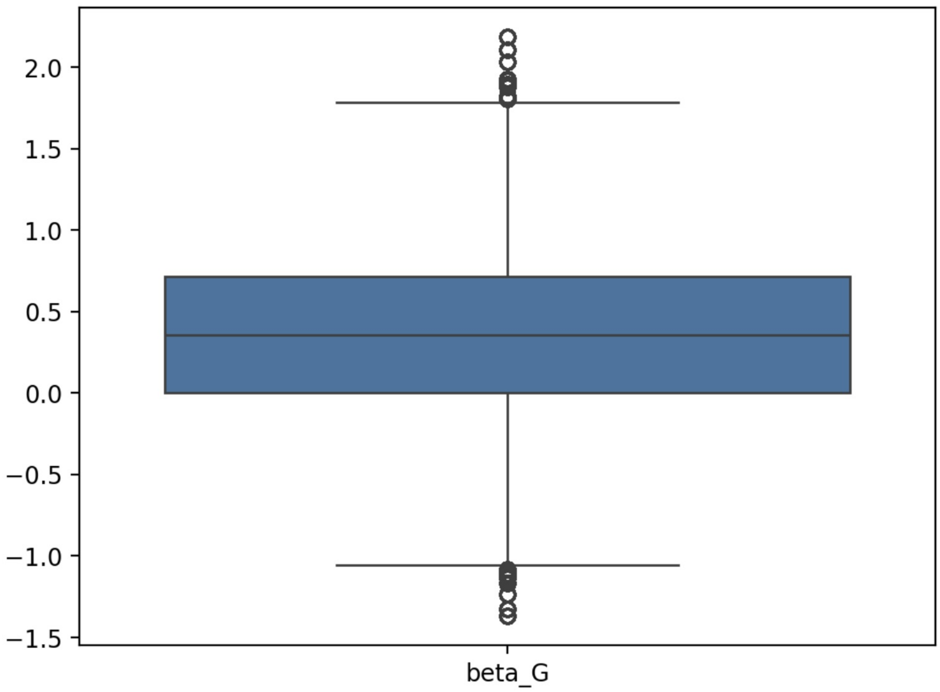
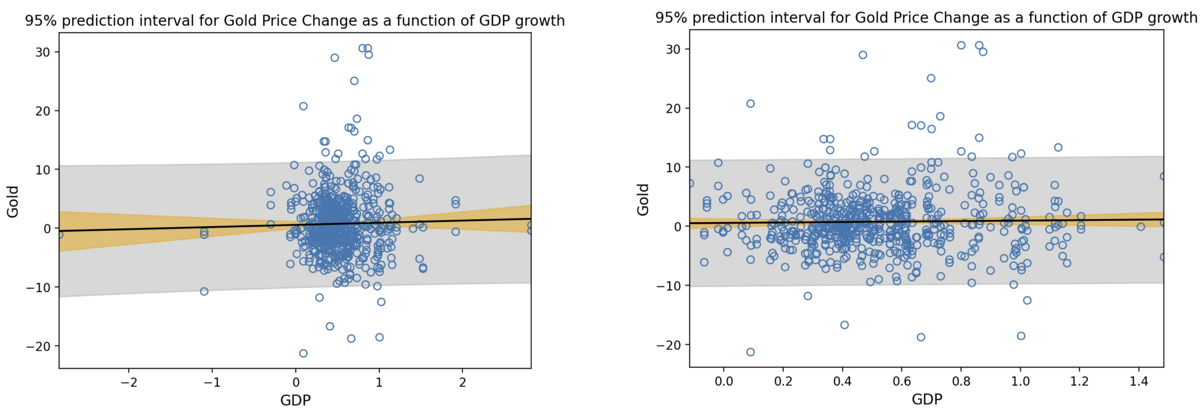


Figure 22: 95% CI plot, Gold Price vs. GDP Growth



As can be observed from Figure 22, the data appears to follow the linear regression curve relatively well, indicating that our linearity assumption for the GDP growth versus Gold price change in the regression prior is reasonable.

4 Interpretation of Results

Overall, our work has provided valuable insights into our questions from the Introduction 1.3 regarding the intricacies surrounding the nature of inflation, and has also provided some unexpected insights in the topic of protecting against inflation using hedging assets (namely gold).

In our analysis of Part 1 (finding the relationships between CPI inflation and various important factors), we made the following observations relating to the corresponding Study Question posed in the Introduction:

- We observed a significant positive linear relationship between **oil price change** and inflation in Figure 4.
- We observed a (nearly) consistent **12-month delay** between inflation and the central bank response in interest rate policy.
- There exists a **significant positive linear relationship** between time-delayed inflation and interest rate.
- There exists a significant **nonlinear relationship** between money supply change and interest rate, which we studied using quadratic and exponential regression.

Meanwhile, in our analysis of Part 2 (finding the significant influential factors on gold price change), we made the following observations relating to the corresponding Study Question posed in the Introduction:

- In analyzing **inflation** and **S&P 500 stock index performance** on gold price change in Figure 18, we observed a statistically significant positive correlation between inflation and gold price increase.

- However, we did not observe a significant correlation between S&P 500 stock index performance and gold price increase.

- This indicates that **gold is an even more effective inflation hedging asset than expected**, as it strongly correlates to inflation but is (generally) resilient with regard to stock market trends.

- In analysing **GDP growth** versus gold price change, we did not observe any significant correlation between gold price growth and GDP growth.

- This further confirms that gold is resilient to market and macroeconomic trends, making it a valuable source of stability and an effective inflation hedge.

Overall, we made three very interesting (and unexpected) observations in this study. The first is the clear 12-month delay between the onset of inflation and the central bank reaction to it in interest rate policy; this reaction time is considerably longer and more consistent than we anticipated.

Second is the clearly symmetric and nonlinear relationship exhibited between money supply change and interest rate; this shows that the relationship between central banks' monetary policies and the prime interest rate is not as clear-cut as we anticipated.

Third is the degree to which gold price change is strongly correlated to inflation but is insignificantly correlated to stock index performance and macroeconomic growth. This result was quite unexpected, and indicates that gold price is more stable to large-scale fluctuations than anticipated. This makes gold an even more valuable inflation hedging asset than we expected.

5 Model Critique

Overall, the results of this study yield some very interesting and insightful conclusions regarding the nature of inflation, how it relates and influences (or is influenced by) a variety of factors, and how inflation relates to gold price. As such, we determine that our analysis has contributed valuable information to answering the study questions posed in the Introduction. Nevertheless, there are still limitations to this work that can serve as valuable directions of future study.

Firstly, in this analysis we concluded from the Directed Acyclic Graph in Figure 2 that the correlation between inflation and S&P 500 stock index performance was most likely spurious. However, we did not investigate this further due to time constraints. As such, given more time on this project, it would be interesting to analyze whether the expected spurious correlation between inflation and S&P 500 is observed, or if not, whether there is a significant correlation between time-delayed inflation and S&P 500 performance that would indicate the presence of a causal relationship. This would further open a direction of study as to how this causal relationship arises, if at all.

Secondly, as indicated in the Results Interpretation in Section 4, we observed an (almost) consistent 12-month delay between the onset of heightened inflation and the central bank interest rate response. However, we did not investigate the precise details of this delay further in this project. As such, a valuable direction of future analysis would be to examine the exact time delay exhibited between inflation and interest rate increases, and determine whether more accentuated spikes in inflation bring about a swifter policy response from the central bank.

Thirdly, our analysis of money supply versus interest rate yielded no significant linear relationship, but a significant nonlinear relationship, which we analysed using quadratic and exponential/bell curve regression. However, we did not investigate why this unexpected relationship was observed. As such, it would be greatly instructive to further examine the reasons behind this nonlinear relationship, and to discover what other factors (not studied in this project) play a significant role in central bank interest rate policy.

Fourthly, in analysing gold price versus the stock market and inflation, we observed a statistically significant positive relationship between gold price change and inflation, but did not observe a significant relationship between gold price change and S&P 500 stock performance. As such, it would be interesting to investigate whether this lack of a relationship between gold price and stock performance is consistent throughout the time period studied, or if it varies with the well-being of the stock market. For instance, would we observe different (significant) relationships between gold price and the stock market in times of economic recession than in times of great economic growth and prosperity?

Overall, this analysis has provided some very interesting and (at times unexpected) insights into the complex topic of inflation and gold price, and has opened to us a diverse array of possible directions for future study and investigation of this fascinating topic.

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Appendix: GitHub Repository for the Math 509 Project Code

The code used in this project, along with the CSV file with the data, can be found in the GitHub repository at the URL: `https://github.com/AndreiAf02/Math509Project_Public/tree/main`