
Lecture 11:

TLT – 5606

Spread Spectrum techniques

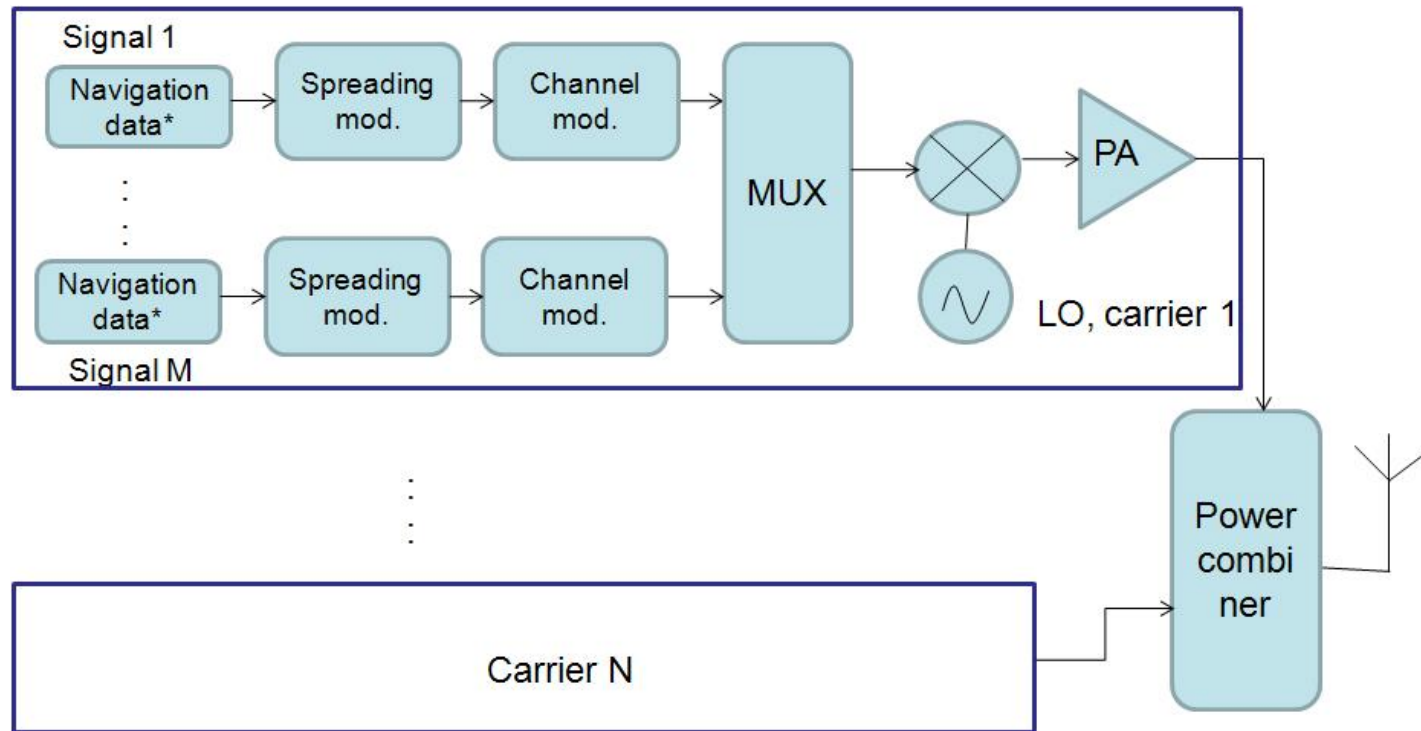
Lecturer: Simona Lohan

Satellite-based positioning (II)

Outline

- ❑ GNSS navigation signals&spectra: description and details
 - ❑ Basics: signal model, pilots, PRN sequences
 - ❑ GPS signals, including modernized GPS
 - ❑ Galileo Signals
 - ❑ Binary Offset Carrier (BOC) modulation
 - ❑ Multiplexed BOC (MBOC) modulation
 - ❑ Reasons for different codes and frequencies
 - ❑ Systems comparisons
 - ❑ GNSS challenges: multipaths & noise/interference effects
 - ❑ Summary & core content
 - ❑ References
-

Transmitter (satellite) simplified model



Mod. =
modulation

MUX=
multiplexing

LO =Local
Oscillator

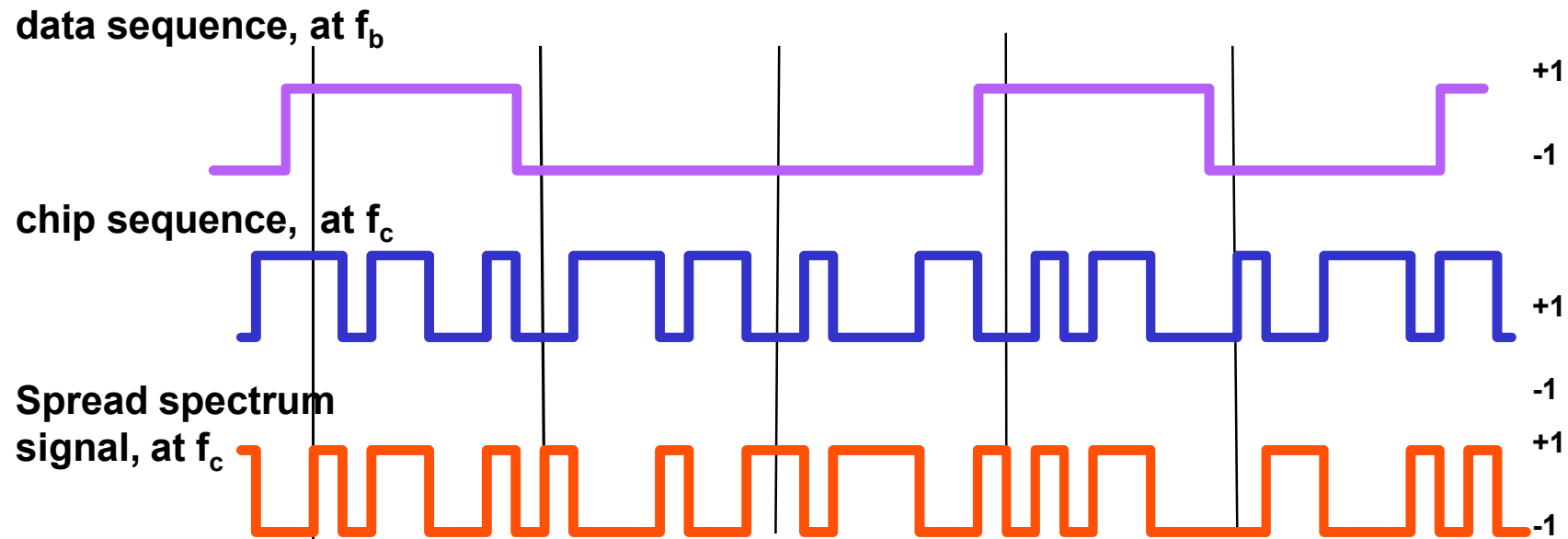
PA= Power
Amplifier

*Navigation
data can be
absent (e.g,
pilot
channels)

- In this lecture, we focus on the **spreading and digital channel modulation parts**, and on their modeling

Spreading modulation: DS-CDMA

Spreading factor $S_F = \frac{f_c}{f_b}$



Signal model for DS-SS in GNSS – spreading sequence

- ❑ The spreading sequence properties establish the shape of the autocorrelation function
 - ❑ The **spreading sequence together with the modulation waveform** are responsible for:
 - Code tracking accuracy in noise and interference and in multipaths
 - Stability of code tracking
 - Susceptibility to channel distortions
 - Complexity of the transmitter and receiver, which depends (more specifically) on:
 - 2-level (e.g., BPSK) versus multi-level (e.g., QPSK, 16-QAM, etc) spreading symbols
 - Clock rate
 - Sample rate for initial synchronization processing
-

Pilot channels

- ❑ Some GNSS channels are transmitting the PRN codes without any navigation data (without data modulation)
 - ❑ These so-called '**pilot channels**' are useful in the acquisition and tracking process:
 - ❑ Absence of data bit transitions allows for longer integration times => better performance in noisy environments
 - ❑ Spreading time series is known at the receiver, except the delay, but navigation data (when present) is not known. Without data modulation => less parameters to be estimated
 - ❑ However, **channels with navigation data message are also necessary, because the information carried by the navigation messages allows us to convert the pseudoranges into the final receiver position: (x,y,z) coordinates**
-

Pseudorandom (PRN) sequences in GNSS

- ❑ Pseudo Random Noise (PRN) codes are essential element in every GNSS
- ❑ All existing codes are based on Linear Feedback Shift Registers (LFSR) of length $n \Rightarrow$ code length $= 2^n - 1$
- ❑ The choice of PRN codes is motivated by their **auto- and cross-correlation properties**.
- ❑ Example below for GPS:

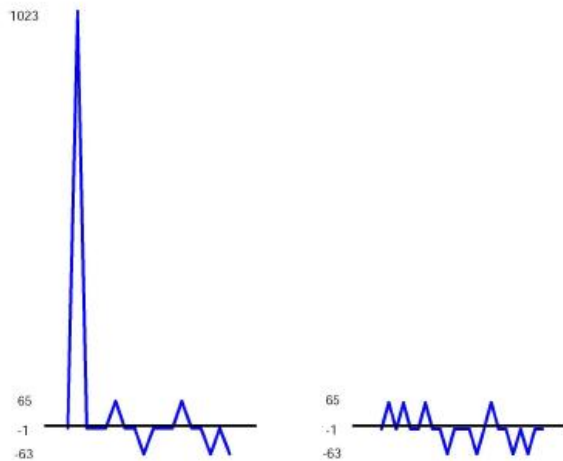
- **Autocorrelation peak (n=10 for GPS):**

$$r_{kk,peak} = 2^n - 1 = 1023$$

- **Cross correlation max:**

$$|r_{kk}| \leq 2^{(n+2)/2} + 1$$

$$|r_{kk}| \leq 65$$

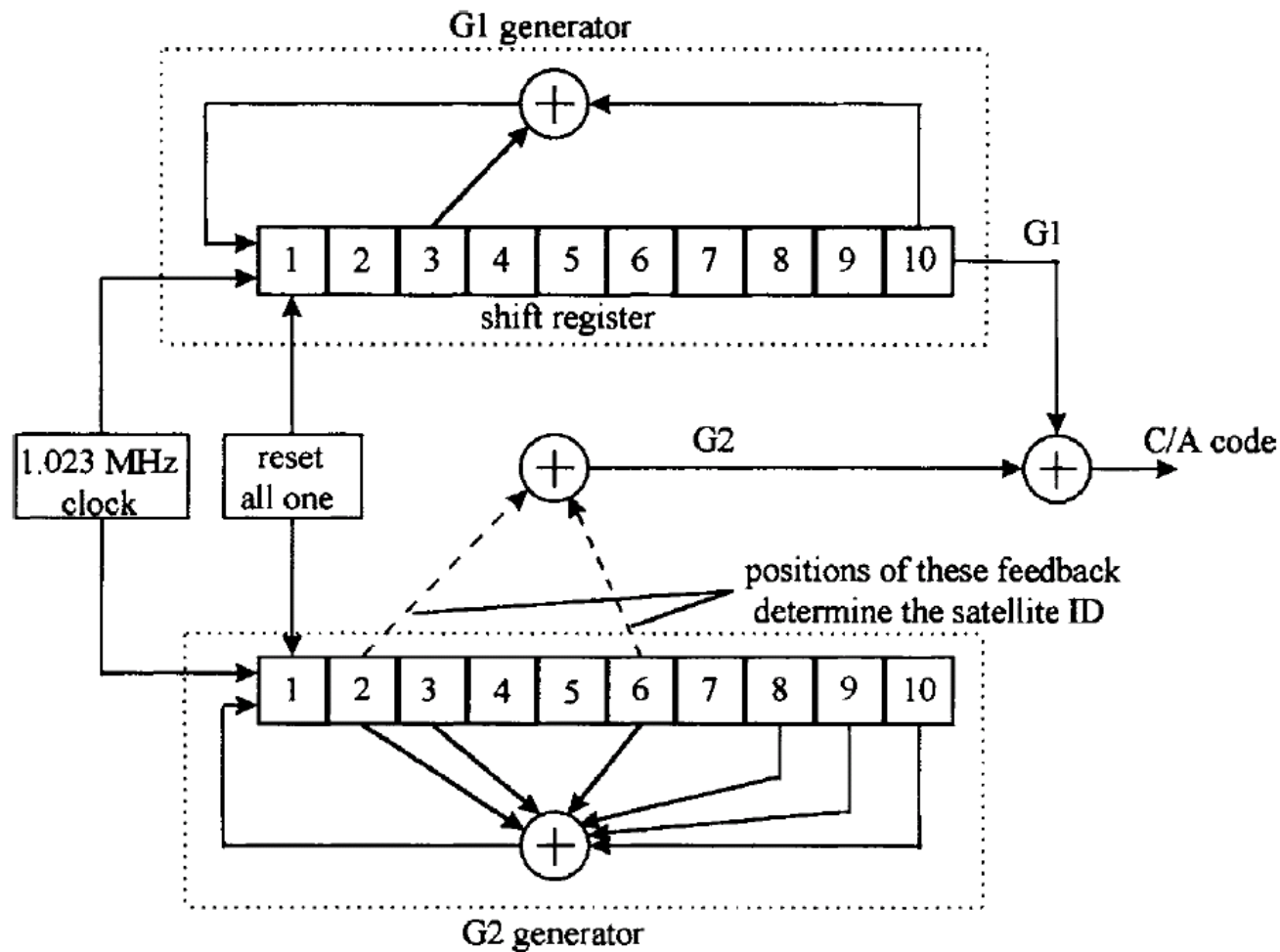


Pseudorandom (PRN) sequences in GNSS (II)

- ❑ Code families used in GNSS [Hein2006]:
 - Maximal-length sequences (or m-codes): basis for Gold codes; GPS L5 and L2 signals are using truncated m-sequences
 - Gold codes (GPS, Galileo)
 - Random codes- memory codes (Galileo E1)
 - Weill codes (used for GPS L1C): similar with Gold codes in a way, but based on prime number and Legendre polynomials

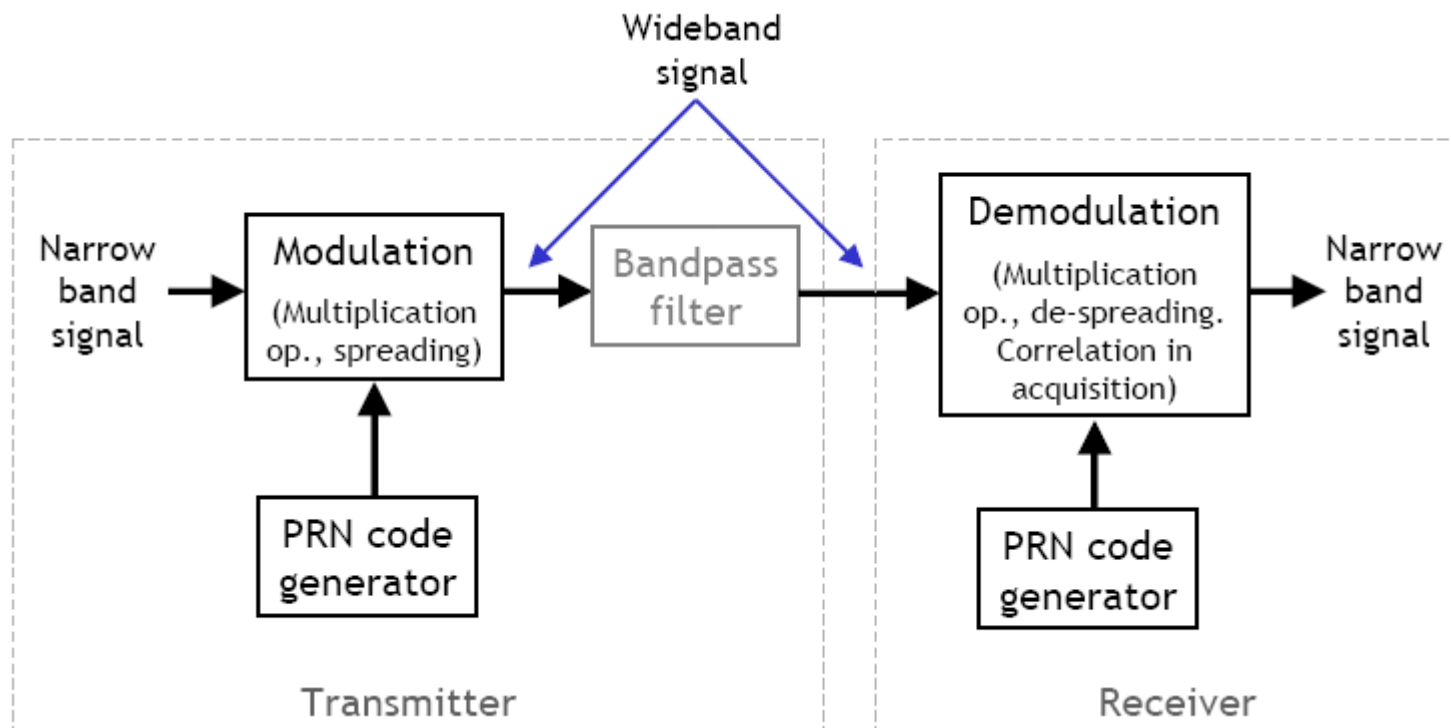
Choosing and optimizing the code family is not an easy task; there is a lot of on-going work regarding the optimal code sequences in GNSS.

Example: GPS C/A code generation (Gold codes)



Spreading modulation/despreading modulation

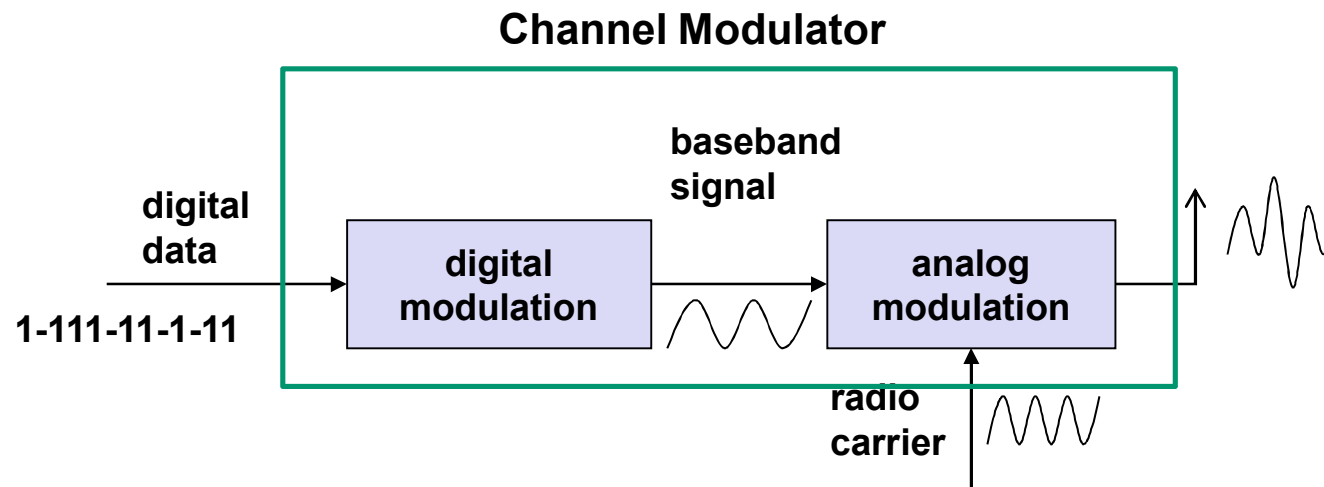
Same principle as for all DS-SS systems



Channel modulation



Modulation task: some characteristics (amplitude, phase or frequency) of a carrier wave are varied in accordance with an information bearing signal



Channel modulations encountered in GNSS

BPSK: Binary Phase shift Keying (traditional)

BOC: Binary Offset Carrier (sine and cosine variants)

MBOC: Multiplexed BOC

AltBOC: Alternate BOC

Binary Phase Shift Keying (BPSK)

If $p(t)$ denotes the basic pulse used in the construction of the binary data stream, then the BPSK-modulated PRN sequence can be written as:

$$s(t) = \sum_{n=-\infty}^{+\infty} b_n \sum_{k=1}^{+S_F} c_{k,n} p(t - nT - kT_c), \quad b_n = \begin{cases} +1 & \text{for binary symbol "1"} \\ -1 & \text{for binary symbol "0"} \end{cases}$$

No pulse shaping currently used in GNSS $\Rightarrow p(t) = p_{T_c}(t)$ (rectangular pulse)

Therefore, the modulated signal can be seen as the convolution between a code part $c(t)$ (including navigation data) and a modulation pulse

$$s_{BPSK}(t) = p_{T_c}(t)$$

$$s(t) = p_{T_c}(t) \otimes \sum_{n=-\infty}^{+\infty} b_n \sum_{k=1}^{+S_F} c_{k,n} \delta(t - nT - kT_c) = s_{BPSK}(t) \otimes c(t)$$

BPSK (II)

The expression given in the previous slide is the expression for the baseband signal (in continuous-time form), which will be the basis of our models in what follows

The analog passband signal (at carrier frequency) becomes:

$$m(t) = A_c s(t) \cos(2\pi f_{carrier} t)$$

Notation: **BPSK(n)** means a chip rate of $n \cdot 1.023$ MHz

(Power) Spectral Density (PSD)

PSD is the **Fourier transform of the auto-correlation function** of a signal

Since a BPSK-modulated signal has both discrete-time and continuous components, the PSD is not straightforward to be computed and it has two parts :

$$P_{T_c}(f) = \int_{-\infty}^{+\infty} p_{T_c}(t) e^{-j2\pi ft} dt$$
$$G_s(f) = \frac{1}{T_c} |P_{T_c}(f)|^2 G_b(e^{j2\pi f T_c})$$

= the Fourier transform of the pulse waveform

$$G_b(e^{j2\pi f T_c}) = 1$$

= the (power) spectral density function of the transmitted discrete-time symbol train

For infinite-length ideal codes (i.e., Dirac-shaped auto-correlation), we have $G_b(e^{j2\pi f T_c})$
=> the modulated-signal PSD is given by the square of the pulse shape frequency response:

$$G_s(f) = \frac{1}{T_c} |P_{T_c}(f)|^2$$

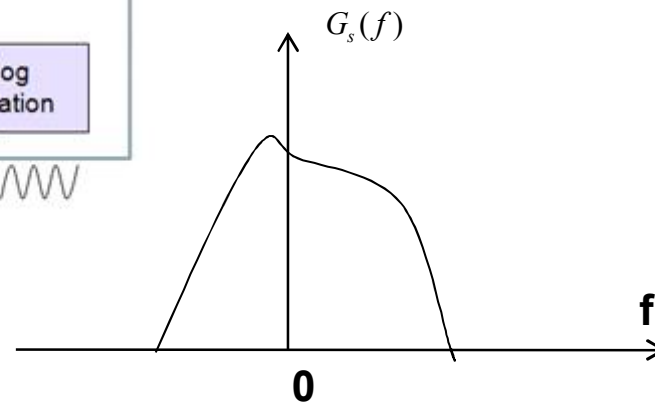
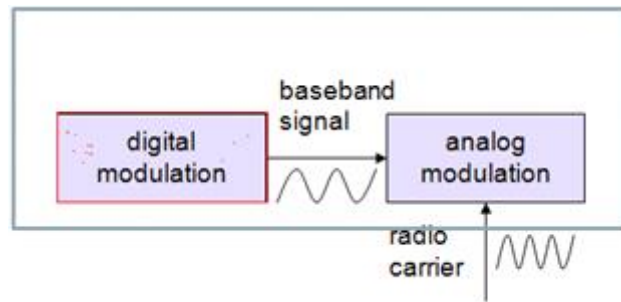
Notes on terminology

Strictly speaking, we deal with **finite-time signals** => **finite energy signals**

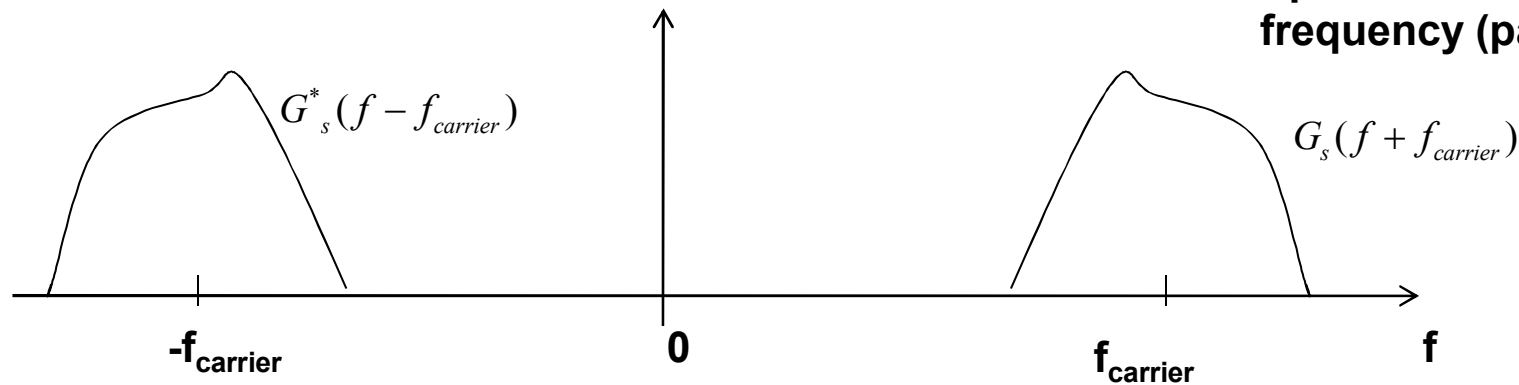
That is why, sometimes the preferred term is 'energy spectral density' (or simply 'spectral density') instead of PSD

However, since the models are equivalent, we'll use the 'power' terminology here

Equivalent baseband representation of modulated signals (why the baseband analysis is enough?)



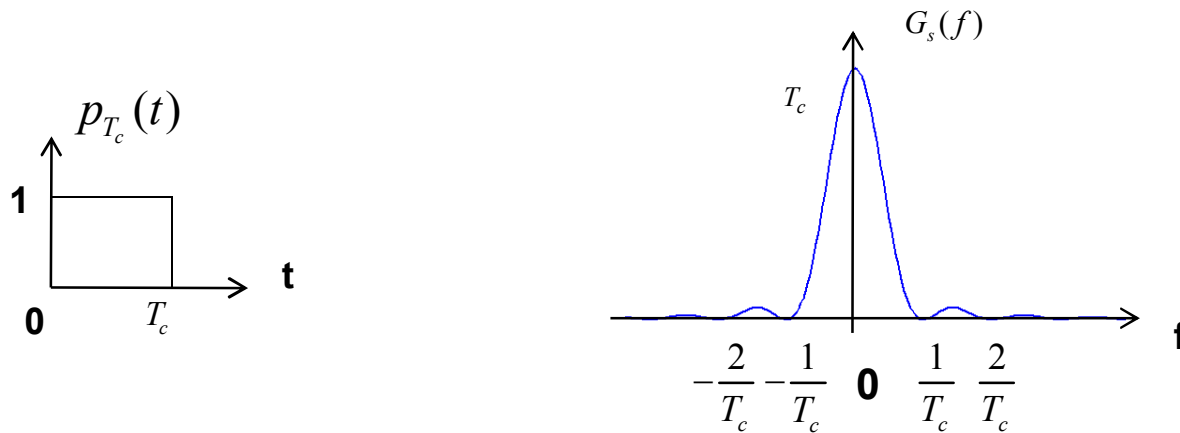
Baseband spectrum



Spectrum at carrier frequency (passband)

PSD of BPSK signals (I)

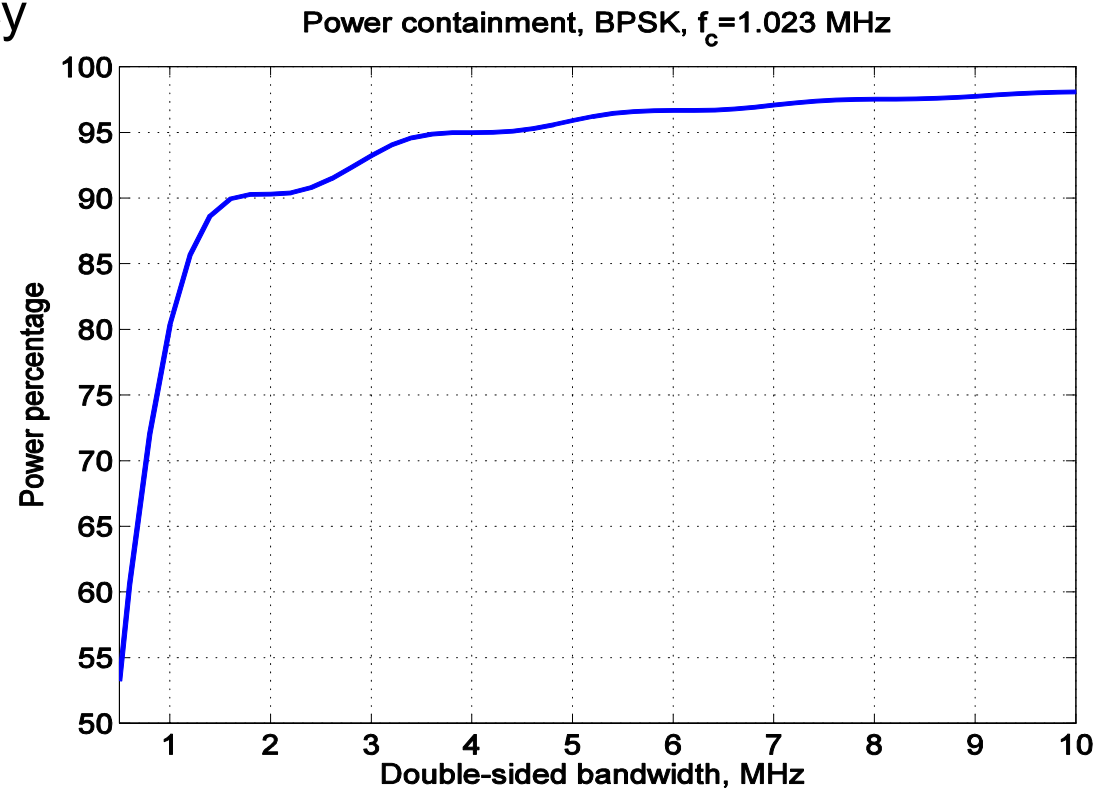
Sinc-shaped PSD, maximum energy at 0 frequency
(baseband representation)



$$G_s(f) = \frac{1}{T_c} \left| P_{T_c}(f) \right|^2 = T_c \operatorname{sinc}^2(\pi f T_c), \quad \operatorname{sinc}(x) \triangleq \frac{\sin(x)}{x}$$

PSD of BPSK signals (II)

Power within main frequency lobe ($2f_c$ bandwidth) is about **90%** of the whole signal power



Binary Offset Carrier (BOC) modulation

- Square sub-carrier modulation, where the PRN code (of chip rate f_c) is multiplied by a rectangular sub-carrier of frequency f_{sc} , which splits the spectrum of the signal.

- Typical notations:

- $\text{BOC}(f_{sc}; f_c)$ or

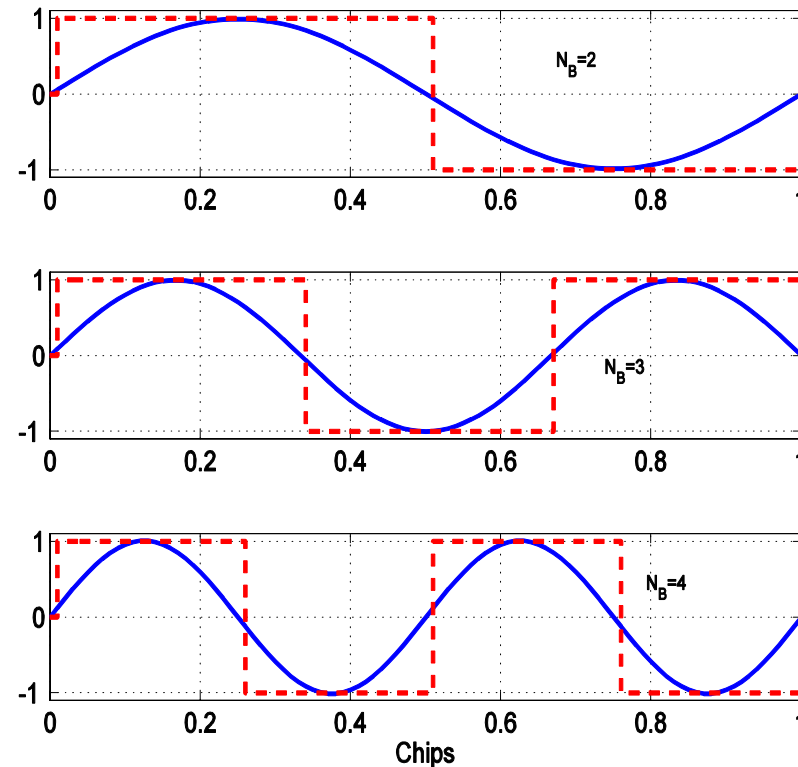
- $\text{BOC}(m; n)$, $m = f_{sc}/1.023 \text{ MHz}$, $n = f_c/1.023 \text{ MHz}$.

- **BOC modulation order**

- **Sine-BOC time waveform** given by:

$$s_{\text{SinBOC}}(t) = \text{sign} \left(\sin \left(\frac{N_B \pi t}{T_c} \right) \right)$$

Sine BOC with $N_B = 2f_c/f_{sc}$



Notes on Sine BOC

From the point of view of baseband characterization, BOC-modulated signal is fully defined by two parameters, namely N_B and chip frequency f_c

From the point of view of passband signal, also the carrier frequency should be specified.

The sine-BOC modulated signal can be written in a similar form as BPSK-modulated signal:

$$s(t) = s_{SinBOC}(t) \otimes \sum_{n=-\infty}^{+\infty} b_n \sum_{k=1}^{+S_F} c_{k,n} \delta(t - nT - kT_c) = s_{SinBOC}(t) \otimes c(t)$$

Equivalent representation of sine-BOC waveform [LLR06b] as an alternating sequence of +1 and -1

$$s_{SinBOC}(t) = p_{T_B}(t) \otimes \sum_{i=0}^{N_B-1} (-1)^i \delta(t - iT_B), \quad T_B = \frac{T_c}{N_B}$$

Cosine BOC

- **Cosine-BOC time waveform** given by:

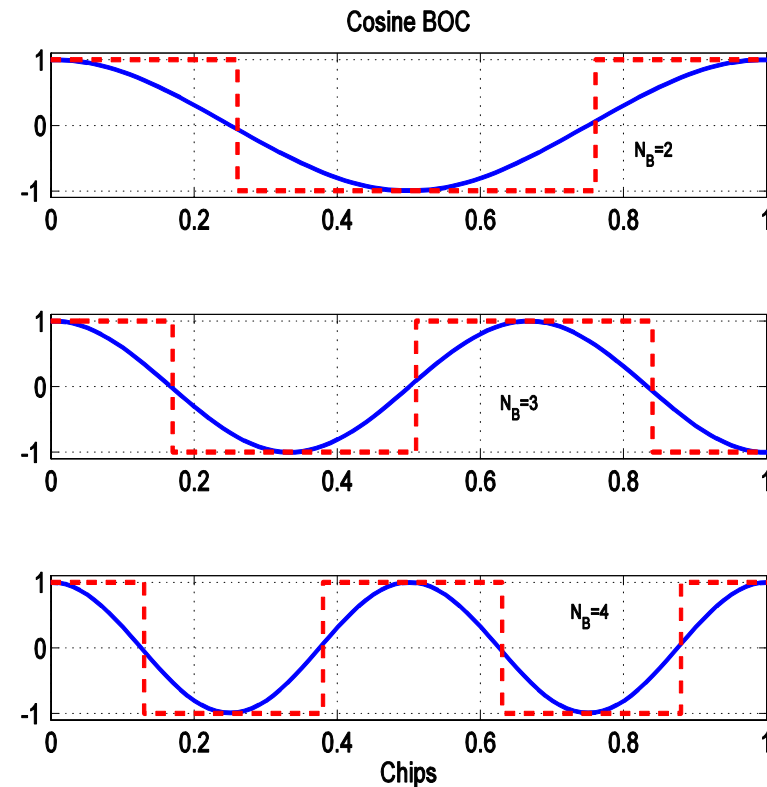
$$s_{CosBOC}(t) = \text{sign} \left(\cos \left(\frac{N_B \pi t}{T_c} \right) \right)$$

- Cosine BOC can be modeled as a double sine BOC modulation: first stage with modulation order N_B , and second stage with modulation order 2

$$s_{CosBOC}(t) = p_{\frac{T_B}{2}}(t) \otimes \sum_{k=0}^1 \sum_{i=0}^{N_B-1} (-1)^{i+k} \delta \left(t - iT_B - k \frac{T_B}{2} \right),$$

$$T_B = \frac{T_c}{N_B}$$

Cosine BOC with $N_B=2f_c/f_{sc}$



Modeling BPSK, sine BOC and cosine BOC

Generic modulated-signal model

$$s(t) = s_{\text{mod}}(t) \otimes \sum_{n=-\infty}^{+\infty} b_n \sum_{k=1}^{+S_F} c_{k,n} \delta(t - nT - kT_c) = s_{\text{mod}}(t) \otimes c(t), \text{mod} = \text{BPSK, SinBOC or CosBOC}$$

Generic modulation waveform (**Double BOC, DBOC** concept):

$$s_{\text{mod}}(t) = p_{T_{B_2}}(t) \otimes \sum_{k=0}^{N_{B_2}-1} \sum_{i=0}^{N_{B_1}-1} (-1)^{i+k} \delta(t - iT_{B_1} - kT_{B_2}), T_{B_1} = \frac{T_c}{N_{B_1}}, T_{B_2} = \frac{T_c}{N_{B_1}N_{B_2}}$$

Modulation factors:

$$N_{B_1} = 1, N_{B_2} = 1$$

BPSK case

$$N_{B_1} = 2 \frac{m}{n}, N_{B_2} = 1$$

Sine BOC(m,n) case

$$N_{B_1} = 2 \frac{m}{n}, N_{B_2} = 2$$

Cosine BOC(m,n) case

PSD of Sine BOC-modulated signals

Obtained by squared Fourier transform of $s_{\text{mod}}(t)$

Random part of the modulated signal ignored (e.g., assumption that we have ideal codes).

$$G_s(f) = \frac{1}{T_c} \left(\frac{\sin\left(\pi f \frac{T_c}{N_B}\right) \sin(\pi f T_c)}{\pi f \cos\left(\pi f \frac{T_c}{N_B}\right)} \right)^2$$

For SinBOC(m,n) with even N_B

$$N_B = 2 \frac{m}{n}$$

$$G_s(f) = \frac{1}{T_c} \left(\frac{\sin\left(\pi f \frac{T_c}{N_B}\right) \cos(\pi f T_c)}{\pi f \cos\left(\pi f \frac{T_c}{N_B}\right)} \right)^2$$

For SinBOC(m,n) with odd N_B

The above expressions are normalized to unit energy over infinite bandwidth

PSD of Cosine BOC-modulated signals

$$G_s(f) = \frac{1}{T_c} \left(\frac{\sin\left(\pi f \frac{T_c}{2N_B}\right) \sin\left(\pi f \frac{T_c}{N_B}\right) \sin(\pi f T_c)}{\pi f \cos\left(\pi f \frac{T_c}{2N_B}\right) \cos\left(\pi f \frac{T_c}{N_B}\right)} \right)^2$$

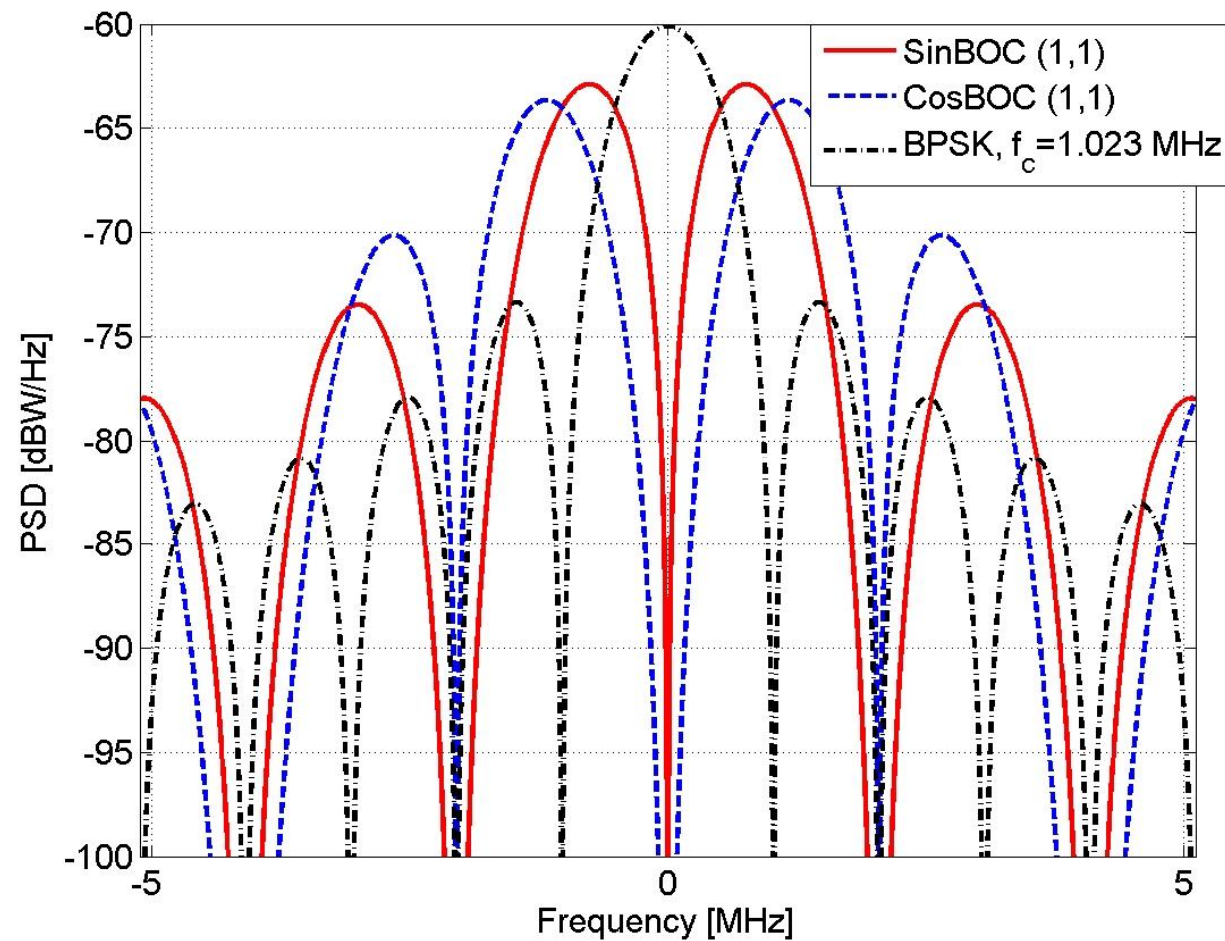
For CosBOC(m,n) with even N_B

$$N_B = 2 \frac{m}{n}$$

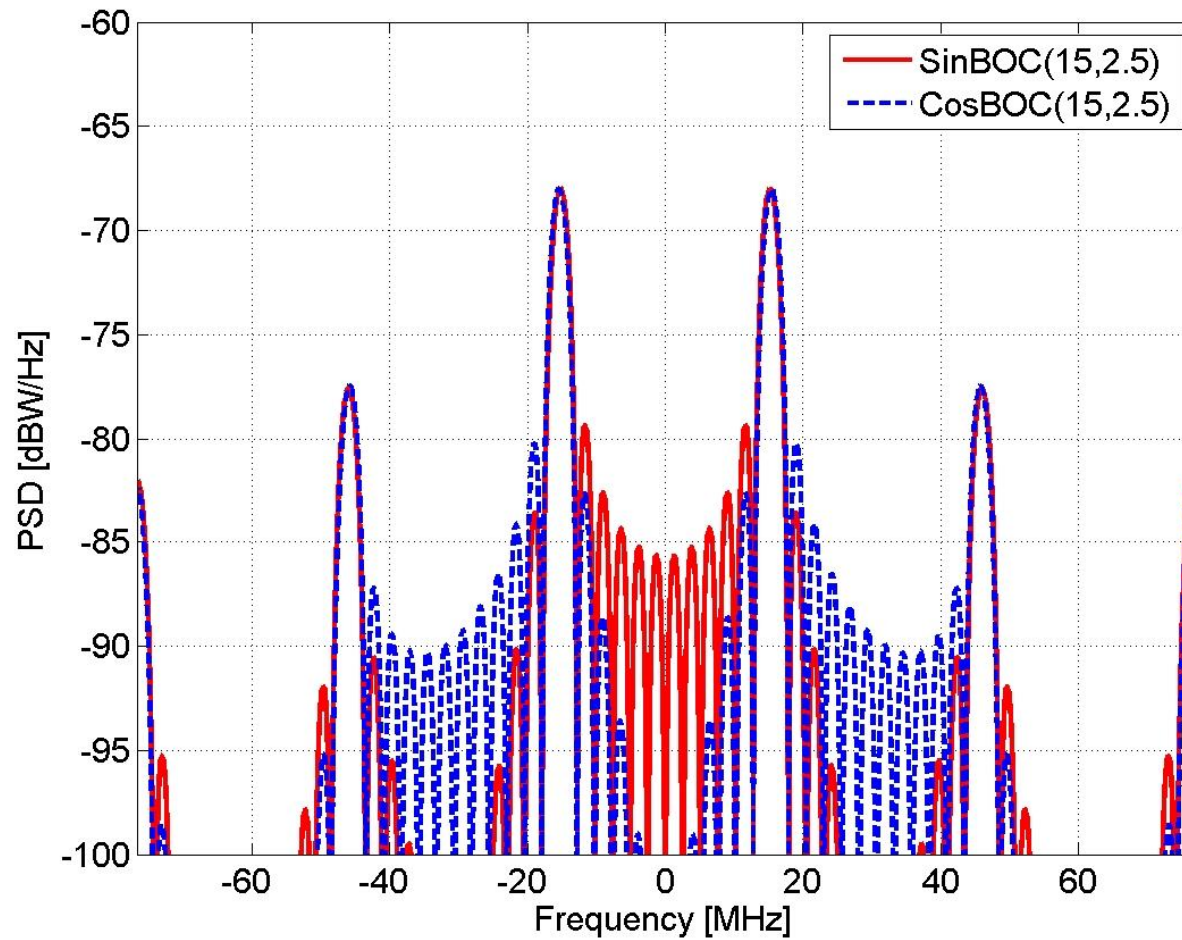
$$G_s(f) = \frac{1}{T_c} \left(\frac{\sin\left(\pi f \frac{T_c}{2N_B}\right) \sin\left(\pi f \frac{T_c}{N_B}\right) \cos(\pi f T_c)}{\pi f \cos\left(\pi f \frac{T_c}{2N_B}\right) \cos\left(\pi f \frac{T_c}{N_B}\right)} \right)^2$$

For CosBOC(m,n) with odd N_B

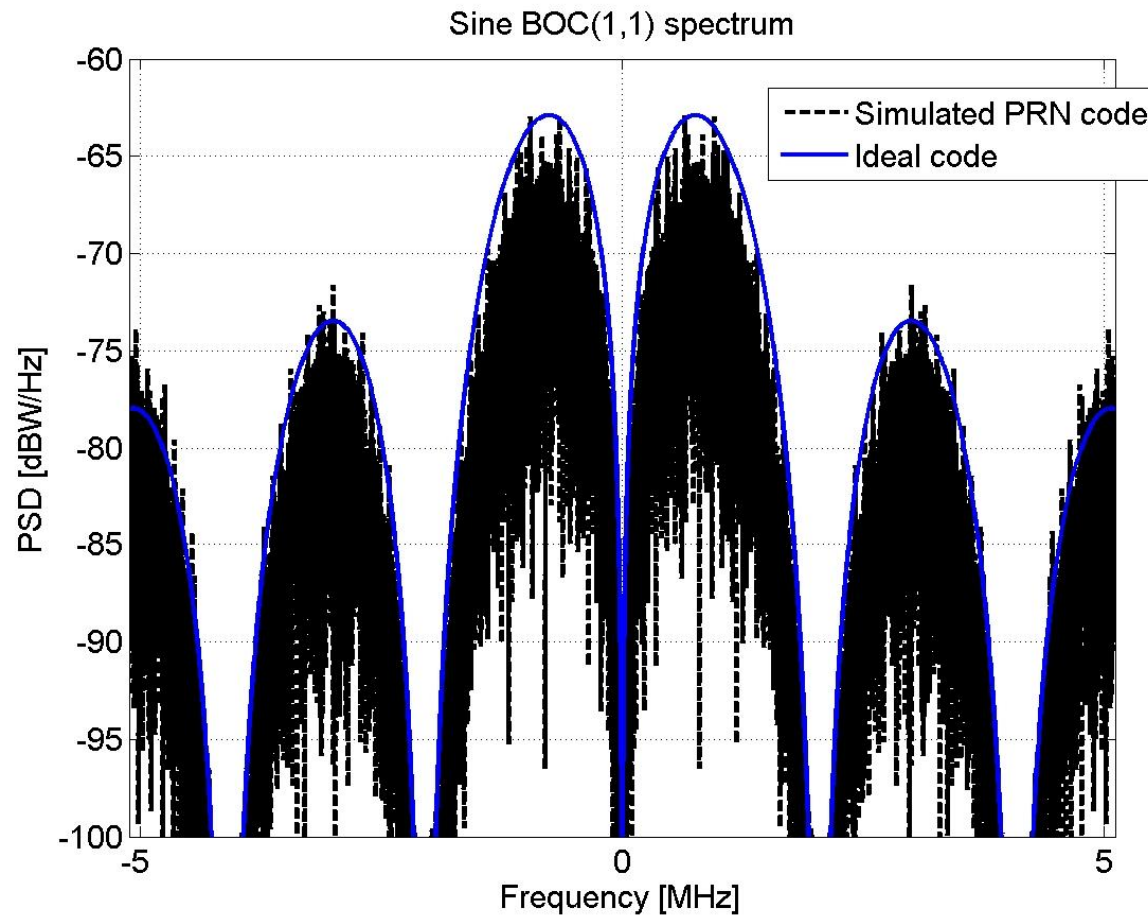
Examples of BOC spectra (I)



Examples of BOC spectra (II)



Effect of non-idealities of the code



Multiplexed BOC (**MBOC**)

- **Proposed (in 2004) for Galileo E1 signals and for modernized GPS L1C signal**
- **Power spectral density of the MBOC signal has to satisfy:**

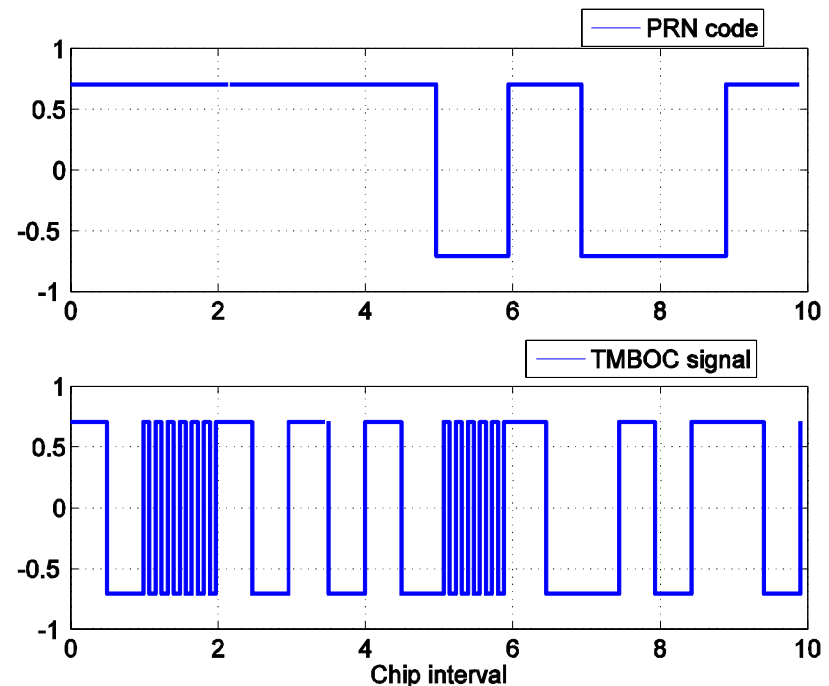
$$G_{MBOC}(f) = \frac{10}{11} G_{SinBOC(1,1)}(f) + \frac{1}{11} G_{SinBOC(6,1)}(f)$$

- **Several implementations possible**

Time-Multiplexed BOC (TMBOC)

TMBOC: time-multiplexed sine BOC(1,1) symbols with sine BOC(6,1) symbols; 2-level waveform.

Example: 10 PRN chips;
chips 2 and 6 are SinBOC(6,1)-modulated



Composite BOC (CBOC)

Weighted combination of SinBOC(1,1) and SinBOC(6,1) code symbols

CBOC(+)

CBOC(-)

CBOC(+/-): odd chips use CBOC(+) and even chips use CBOC(-) (or viceversa)

$$S_{CBOC(+)}(t) = w_1 S_{SinBOC(1,1)}(t) + w_2 S_{SinBOC(6,1)}(t)$$

$$S_{CBOC(-)}(t) = w_1 S_{SinBOC(1,1)}(t) - w_2 S_{SinBOC(6,1)}(t)$$

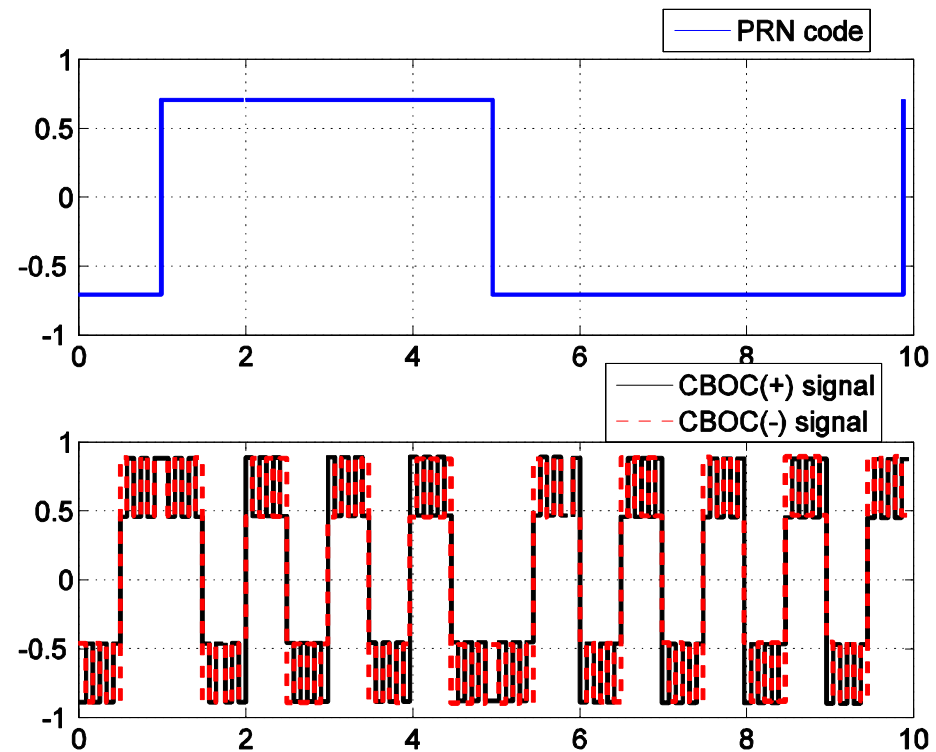
CBOC

Example: typical values

$$w_1 = \sqrt{\frac{10}{11}}, w_2 = \sqrt{\frac{1}{11}}$$

Note that

$$w_1^2 + w_2^2 = 1$$



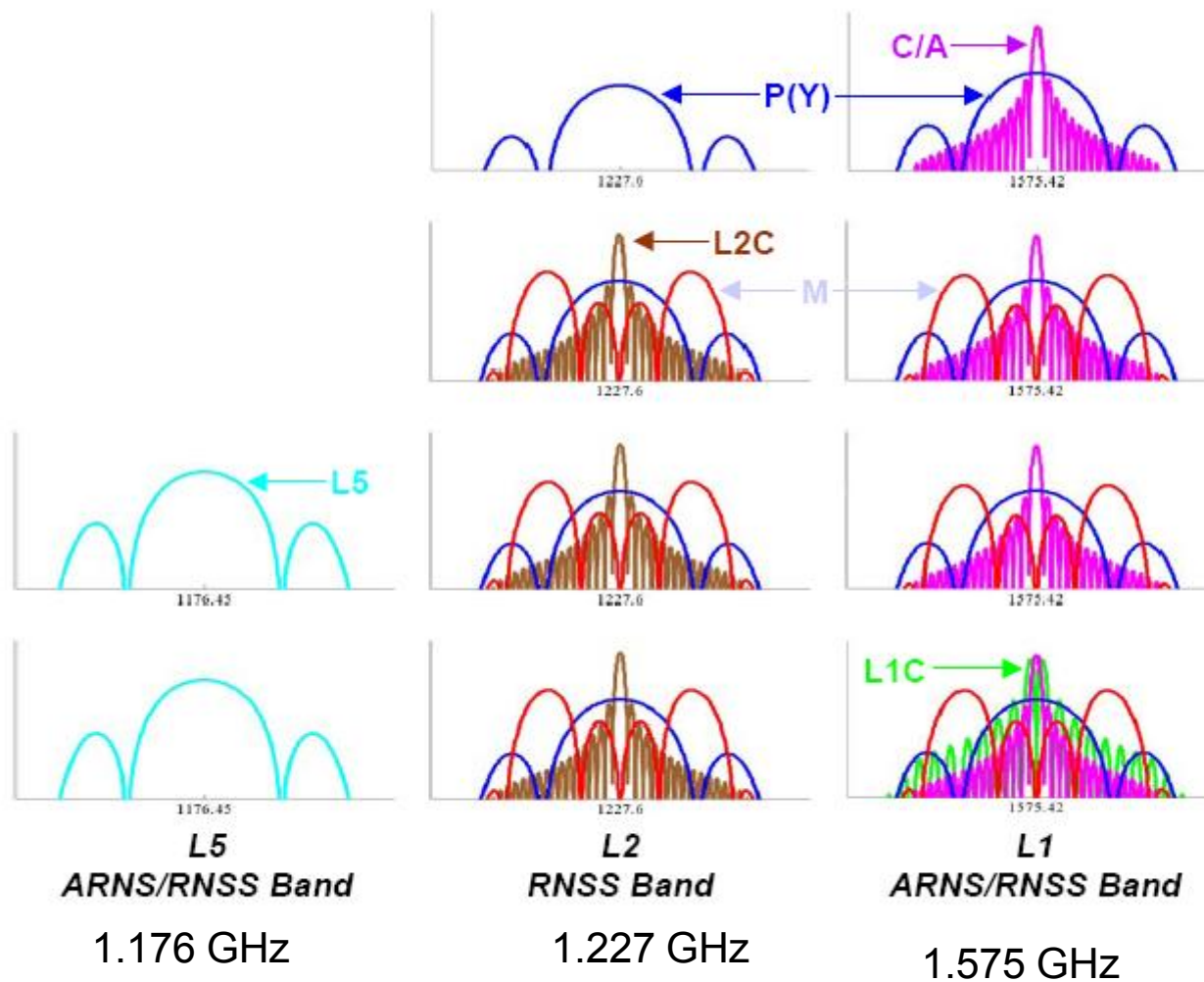
Modulation waveform impact

The **modulation waveform** (= spreading+channel modulation) establishes the shape of the spectrum:

- ❑ Bandwidth and out-of-band spectra
- ❑ Frequencies where power is concentrated
- ❑ Degree of radio frequency interference to receivers in other signals
- ❑ Susceptibility to interference

Recall an example: for rectangular pulse shape and BPSK modulation, the spectrum is sinc^2 -shaped

GPS Signal Spectra



Current GPS
*Dual Frequency w/
Semi-codeless P(Y)*

Block IIR-M
Launch 2005
*Dual Frequency
L1 C/A & L2C*

Block IIF
Launch 2007
*Three Frequency
L1 C/A, L2C, & L5*

Block III
Launch 2013
*L1C, L2C, L5,
& L1 C/A Code*

GPS Signals

❑ Civil signals

- C/A (Coarse/Acquisition) code on L1 band (since the beginning; 1980s)
- L2C on L2 band (since 2005)
- L5 on L5 band (to come, planned for Safety of Life operations)
- L1C on L1 band (to come)

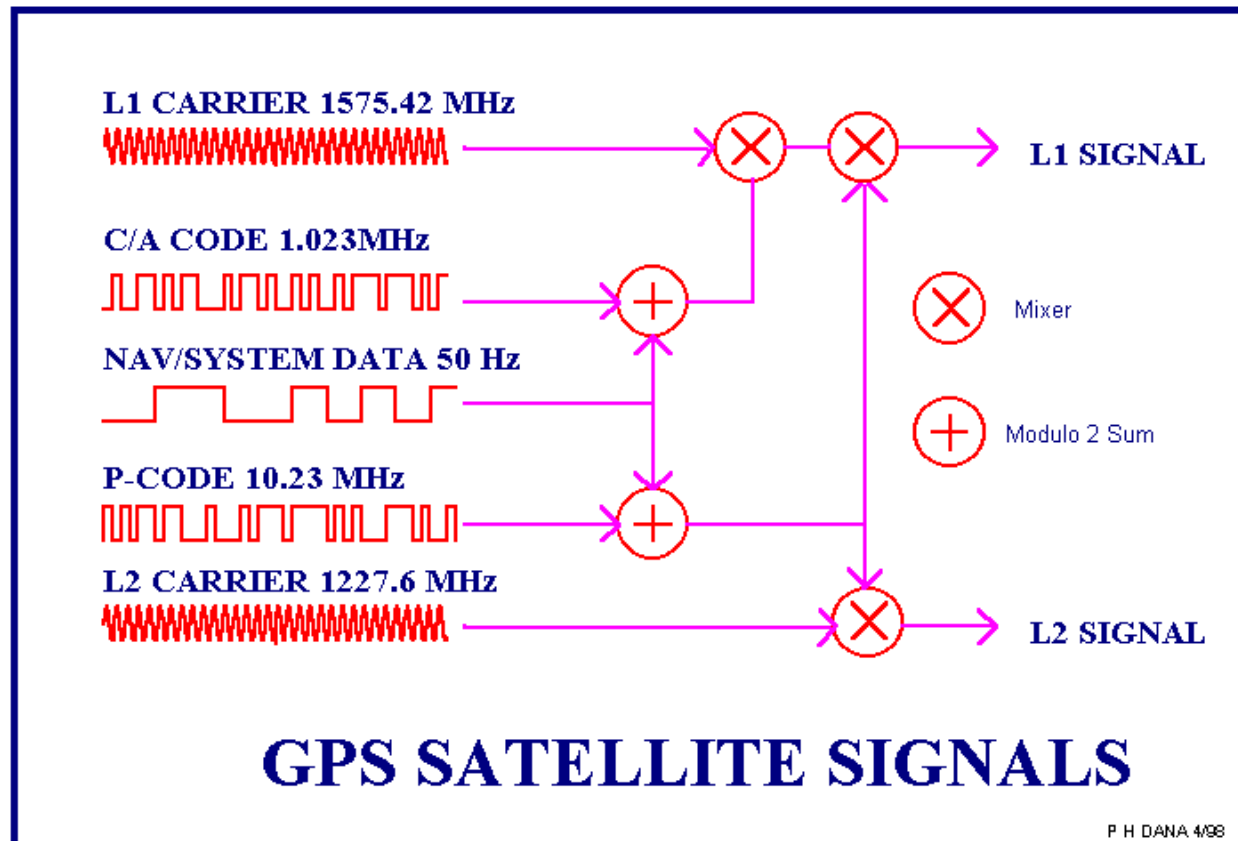
❑ Restricted/military/encrypted signals:

- P(Y) (Precise) code on L1 and L2 bands
- M-code on L1 and L2 bands

Note: GPS L5 signal starts to be transmitted on April 10, 2009.

GPS Basic Signals on L1/L2

C/A code on L1 and P-code (or P(Y)-code) on L1 and L2 bands:



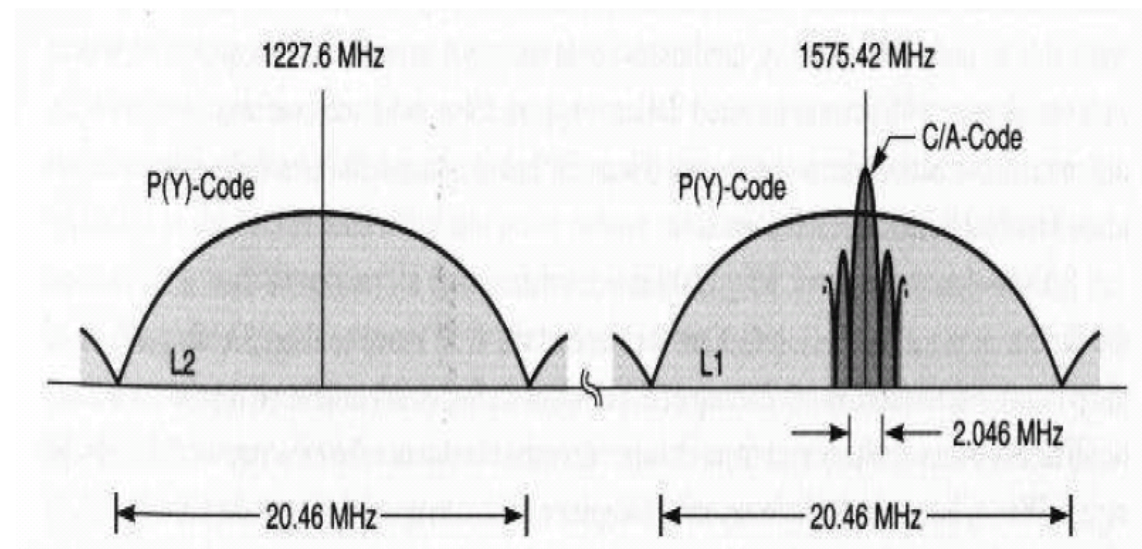
GPS navigation data

- ❑ The GPS Navigation Message consists of time-tagged data bits marking the time of transmission of each subframe at the time they are transmitted by the SV
 - ❑ Data frames (1500 bits) are sent every 30 seconds
 - ❑ 5 subframes (300 bit) over 6 seconds
 - ❑ 3 subframes contain orbital and clock data. SV Clock corrections are sent in subframe 1 and precise SV orbital data sets (ephemeris data parameters) for the transmitting SV are sent in subframes 2 and 3
 - ❑ Subframes 4 and 5 are used to transmit different pages of system data
 - ❑ An entire set of 25 frames (125 subframes) makes up the complete Navigation Message that is sent over a 12.5 minute period
-

GPS Basic Signal spectra

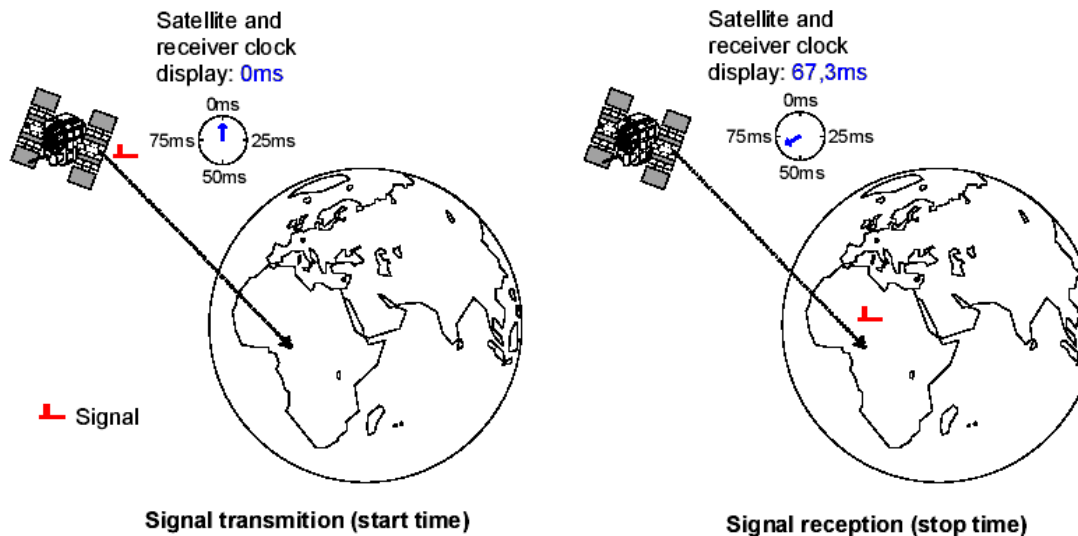
C/A code: 1.023 MHz chip rate, 300 m wavelength, 32 different sequences assigned to GPS satellites.

P(Y) code: 10.23 MHz chip rate, repeats every week



Exercise: How long it takes for a GPS signal to reach a receiver? (I)

- ❑ Satellites broadcast signals in the L-band of the microwave spectrum (between microwave and radio wavelengths: 1 cm to 1 m wavelengths). These wavelengths can pass through some obstacles (e.g., forests), but may be blocked by tree trunks or tall buildings if the signal comes in at an angle low on the horizon.
- ❑ The speed of each signal emitted by a satellite is 3×10^8 m/s (speed of light).
- ❑ The satellite is about 20200 km away \Rightarrow it takes about 1/14 of a second for the signal to leave the satellite and reach a position on Earth's surface located directly below the satellite (i.e., 67.3 ms). Signal needs further $3.33 \mu\text{s}$ for each km (distance = travel time * speed of light)

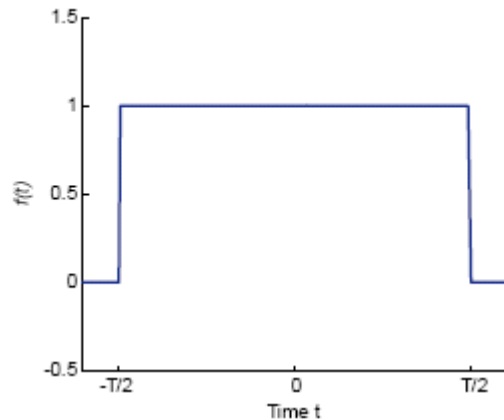


GPS signals characteristics

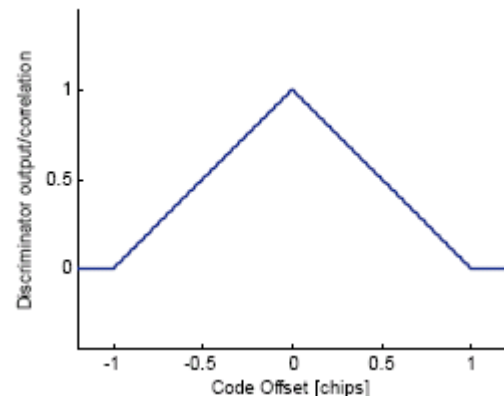
- ❑ Main focus: civil signals. Modulation types:
 - Binary Phase Shift Keying (BPSK) modulation for C/A signal
 - BPSK for L2C
 - BPSK for L5
 - Multiplexed Binary Offset Carrier Modulation (MBOC) for L1C (to be discussed later)
 - ❑ Code lengths:
 - 1023 chips for C/A
 - 2 codes: one with 10230 chip length and another one with 767250 chip length for L2C
 - 2 codes of 10230 chip length on L5
 - 10230 chip length on L1C
 - ❑ Pilot (dataless) channels:
 - No pilot code for C/A (navigation data present)
 - Time-multiplexed pilot signal on L2C
 - Quadrature-phase pilot signal on L5
 - Power multiplexed pilot signal on L1C (current proposal: 75% signal power on pilot channel; 25% signal power on data channel)
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BPSK-modulated sequences - Auto-correlation function (I)

- Convolution between two rectangular pulses is a triangular pulse



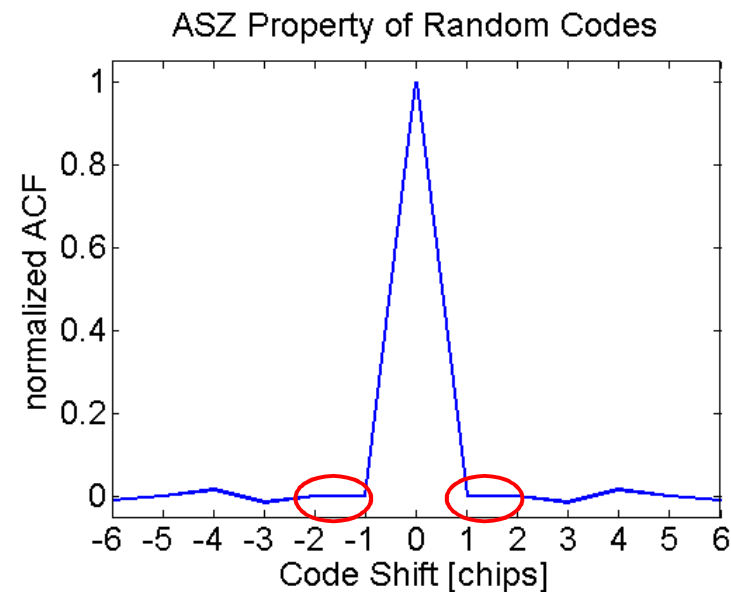
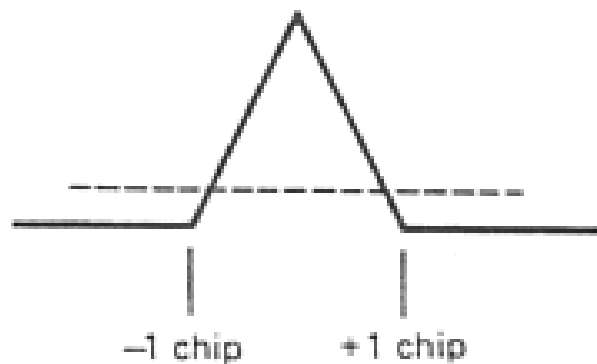
$$f(t) = \begin{cases} 1, & |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$



$$r_f(\tau) = \begin{cases} T \left(1 - \frac{|\tau|}{T} \right), & \text{for } |\tau| \leq T \\ 0, & \text{otherwise} \end{cases}$$

BPSK-modulated sequences - Auto-correlation function(II)

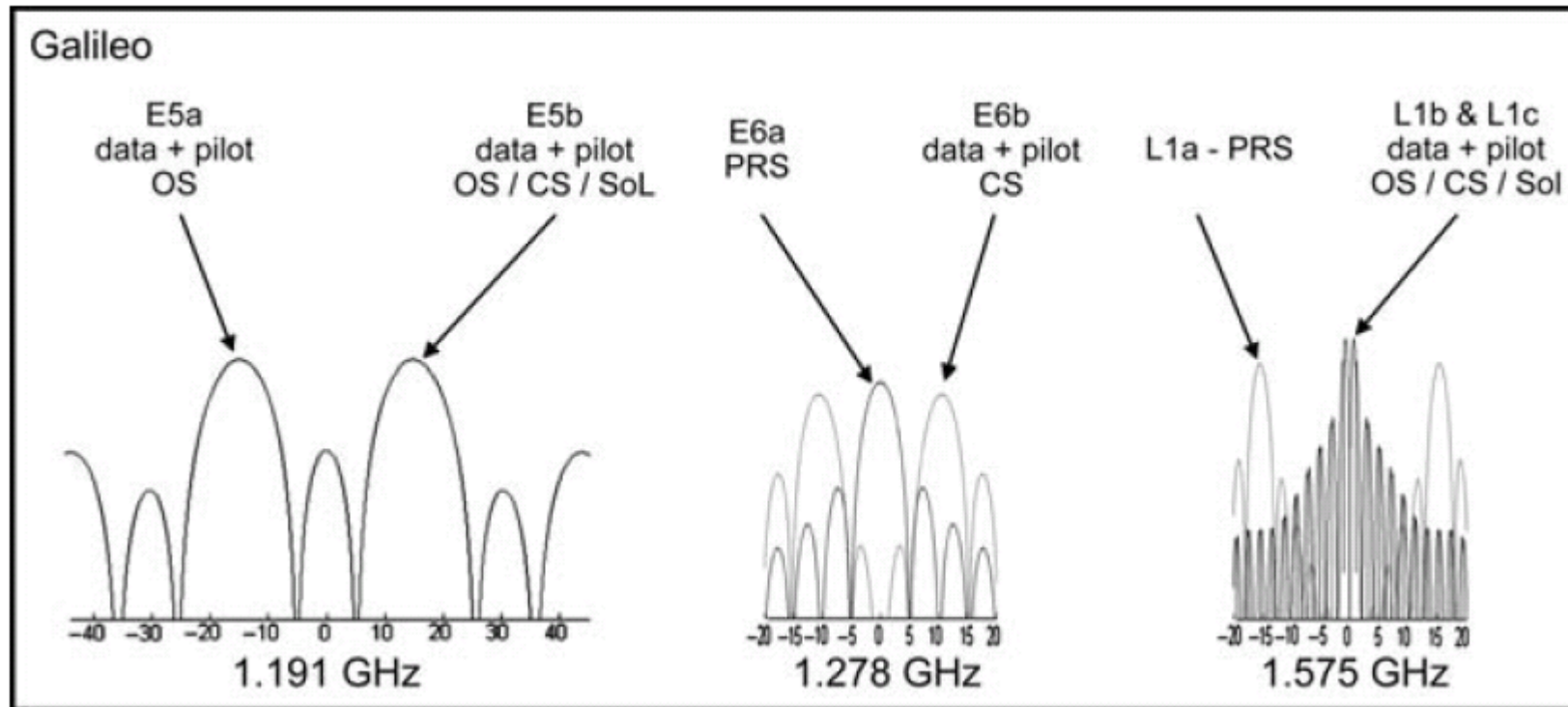
- If the PRN codes would be with infinite lengths and i.i.d symbols, the global ACF will be a triangle
- Due to non-ideal correlation properties of the code, there are always some non-zero cross-correlation and auto-correlation values.



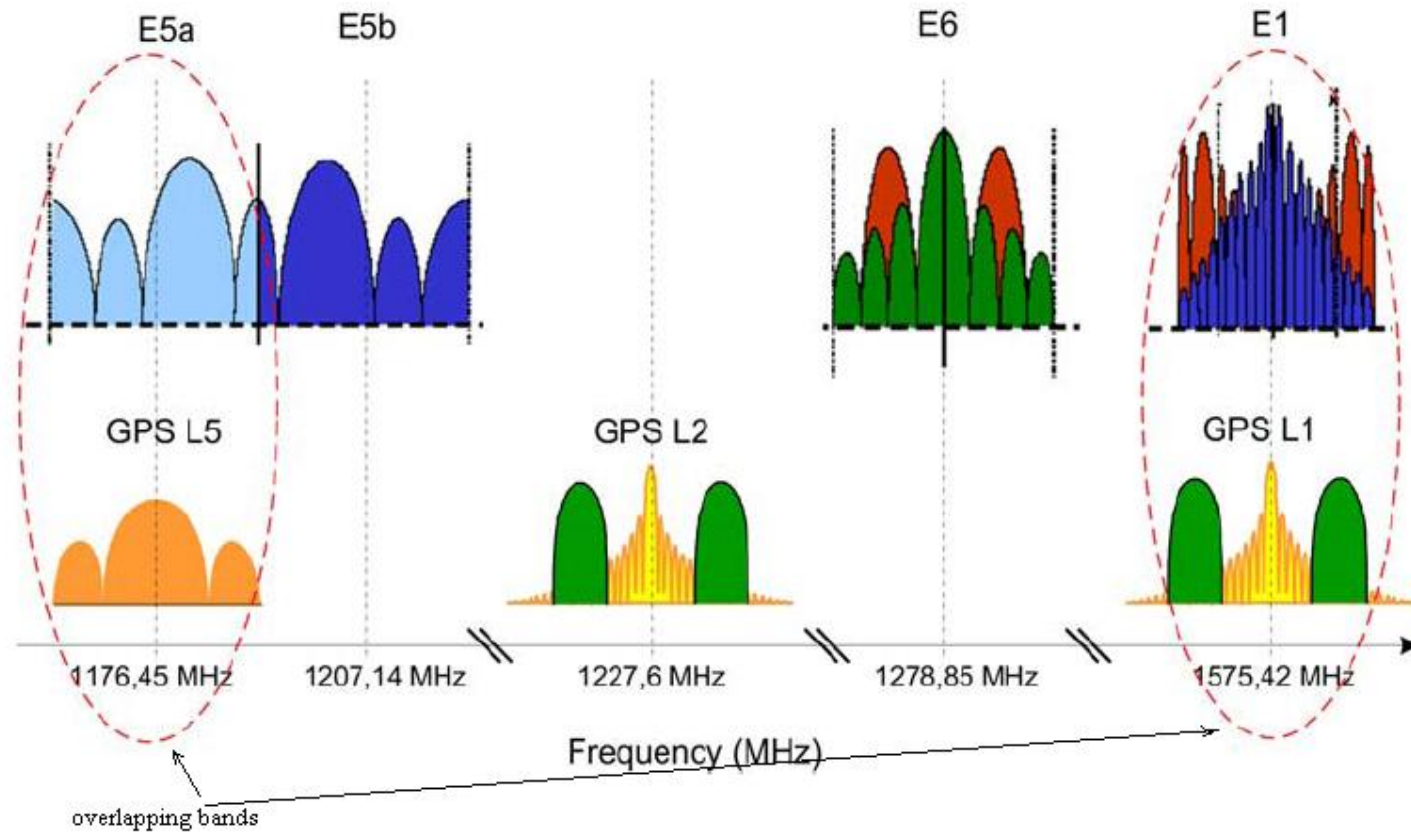
ASZ= autocorrelation sidelobe zero

Galileo Signals - spectra

Source: Nel Salama book on 'Global positioning- Technologies and applications'



Galileo/GPS – spectra comparison



Note: E1 band is sometimes called also L1 in Galileo. Also the denomination of E1-L1-E2 is sometimes used (obsolete)

Galileo carrier frequencies and receiver bandwidths

- Source: Galileo Signal In Space Interface Control Document (SIS-ICD), Status 2008

- Some Galileo frequencies are overlapping with GPS bands (in E5/L5 and L1 bands)

- All signals share the same spectrum via CDMA multiple access technique (same as in GPS)

- Wider receiver bandwidths than for GPS receivers (typically, BOC modulation uses more spectrum than BPSK modulation)

Signal	Carrier Frequency (MHz)
E1	1575.420
E6	1278.750
E5	1191.795
E5a	1176.450
E5b	1207.140

Signal	Receiver Reference Bandwidth (MHz)
E1	24.552
E6	40.920
E5	51.150
E5a	20.460
E5b	20.460

Galileo Signals characteristics

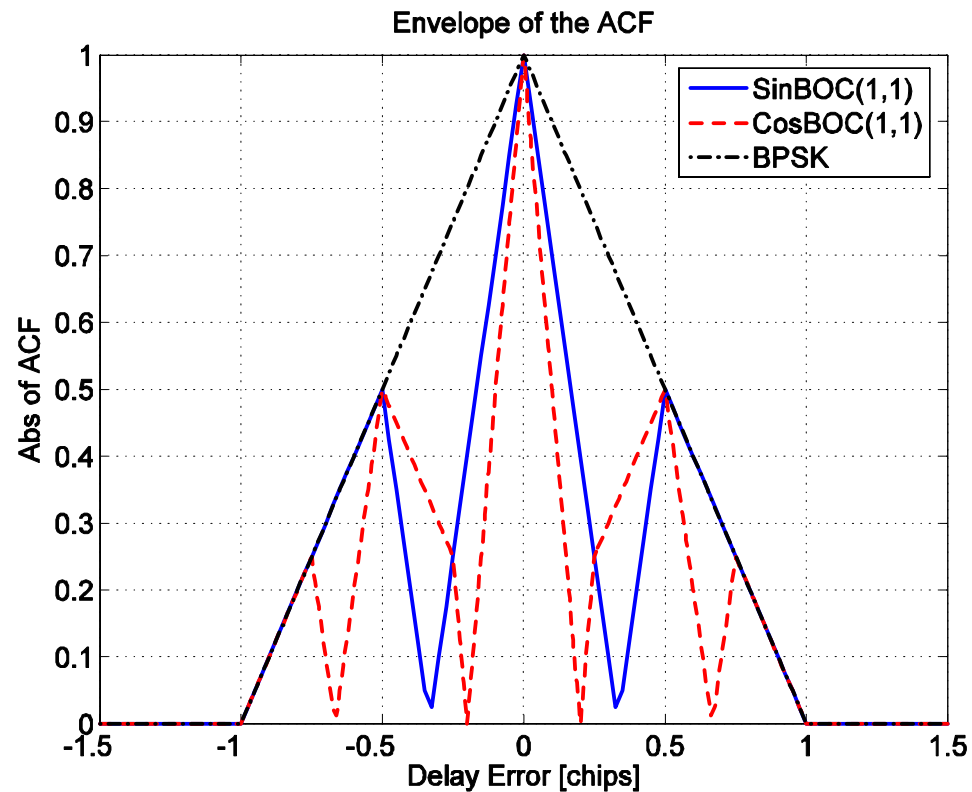
- ❑ Modulation types:
 - MBOC modulation for L1/E1 signals
 - Alternate BOC (AltBOC) modulation for E5
 - BPSK for E6
 - ❑ Code lengths:
 - 4092 chips for E1 signals (but the spreading factor is still 1023)
 - 2 PRN codes of 10230 chip length on E5 (I/Q multiplexed)
 - 2 PRN codes of 5115 chip length on E6
 - ❑ Pilot (dataless) channels: present for all Galileo signals
 - Power-multiplexed pilots of E1
 - Quadrature-phase multiplexed pilots on E5
 - Code multiplexed pilots on E6
-

Galileo: main differences with GPS at physical layer

- ❑ New modulation types: variants of **Binary-Offset-Carrier (BOC)** modulation, which provide better spectral separation with GPS signals (see next slides)
 - ❑ Different code lengths for some signals (e.g., C/A code in GPS is 1023 chip length; OS signals in Galileo are 4092 chip length); longer spreading codes.
 - ❑ Higher data symbol rates compared to GPS (e.g., C/A code in GPS has 50 sps data rates; in Galileo rates between 50 and 1000 sps are specified)
 - ❑ Presence of data-less signals (pilot signals) – this is also valid in modernized GPS signals
 - ❑ Block Interleaving (bit scattering) - to make the long data losses manageable.
-

BOC modulation Auto-Correlation Function (ACF)

Compared with BPSK, the main lobe of the ACF envelope is narrower, but there are more lobes and some deep fades (=ambiguities) within 2 chip interval) => challenges in the acquisition and tracking.



Spectral properties used as performance criteria [Betz &.al] (I)

- **Spectral Separation Coefficient** (SSC) should be as low as possible:

$$SSC = \int_{BW} P_{desired}(f) P_{interference}(f) df,$$

where $P_{desired}(f)$ is the PSD of the BOC-modulated signal (e.g., Galileo) and $P_{interference}(f)$ is the PSD of the existing GPS signals (e.g., C/A code, P(Y) code and M-code).

- **Maximum Value of the spectrum** (MVS): m_{MVS} should be as small as possible \Rightarrow less interference.

$$m_{MVS} \triangleq \max_{f \in B_T} P_s(f), \quad s = \text{SinBOC or CosBOC}.$$

Spectral properties used as performance criteria (II)

- **The RMS bandwidth of the signal:** the smaller the better

$$\beta_{RMS} = \int_{BW} f^2 P_{desired}(f) df$$

- **Power containment factor ε** is the percentage of the signal power contained within a certain bandwidth B_T ; the higher ε is, the better the demodulation process will be.

$$\varepsilon \triangleq \int_{-B_T/2}^{B_T/2} P_s(f) df, \quad s = \text{SinBOC or CosBOC}.$$

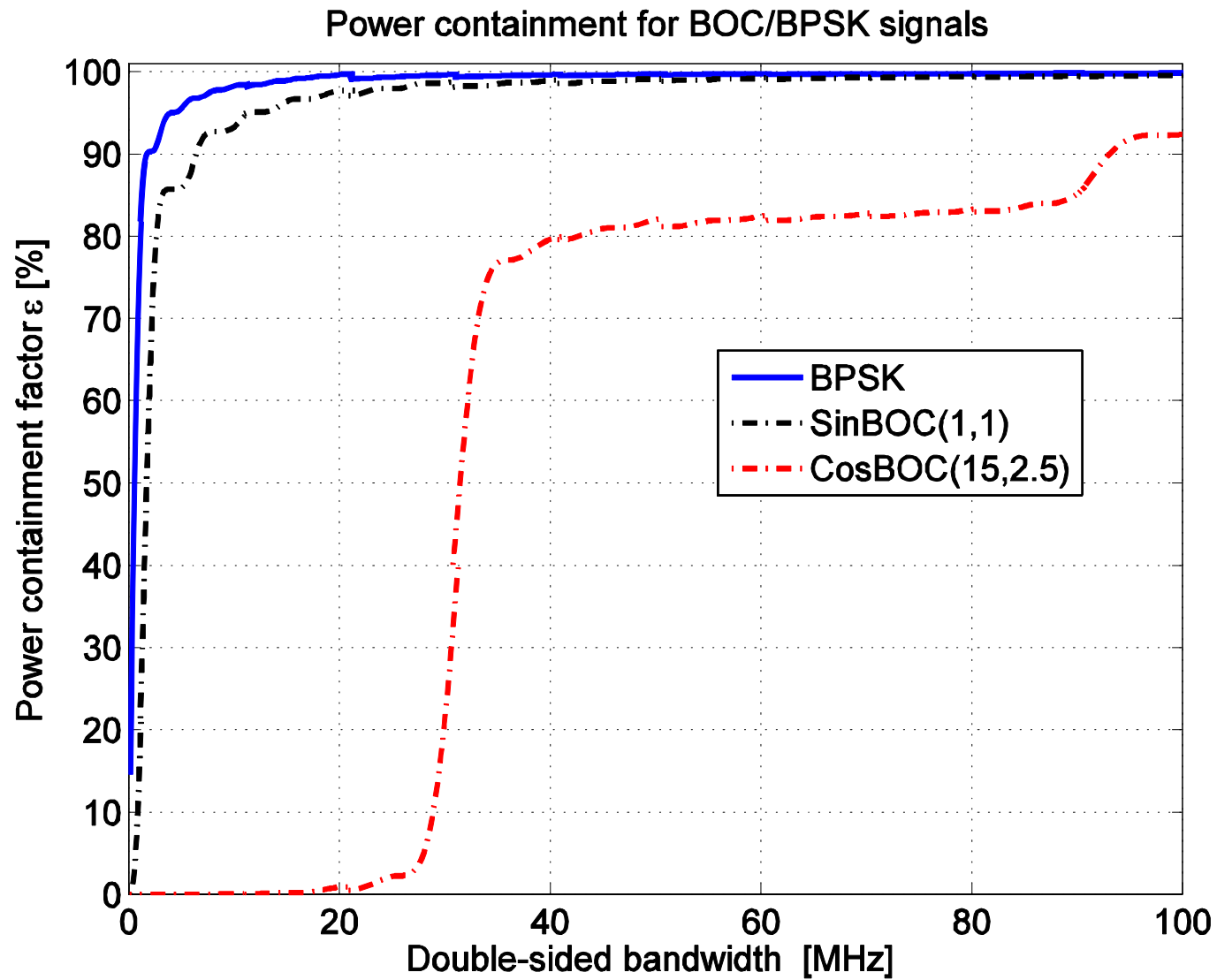
- **Time $(\Delta t)_{res}$ and distance $(\Delta d)_{res}$ resolution factors:** the time-resolution factor is given by half of the width of the main lobe of the absolute value of ACF; the distance-resolution factor is its value in meters. E.g., for SinBOC(1, 1): $(\Delta t)_{res} = 0.3438$ chips; $(\Delta d)_{res} \triangleq c(\Delta t)_{res}/f_c$ depends on chip rate.
-

Numerical example

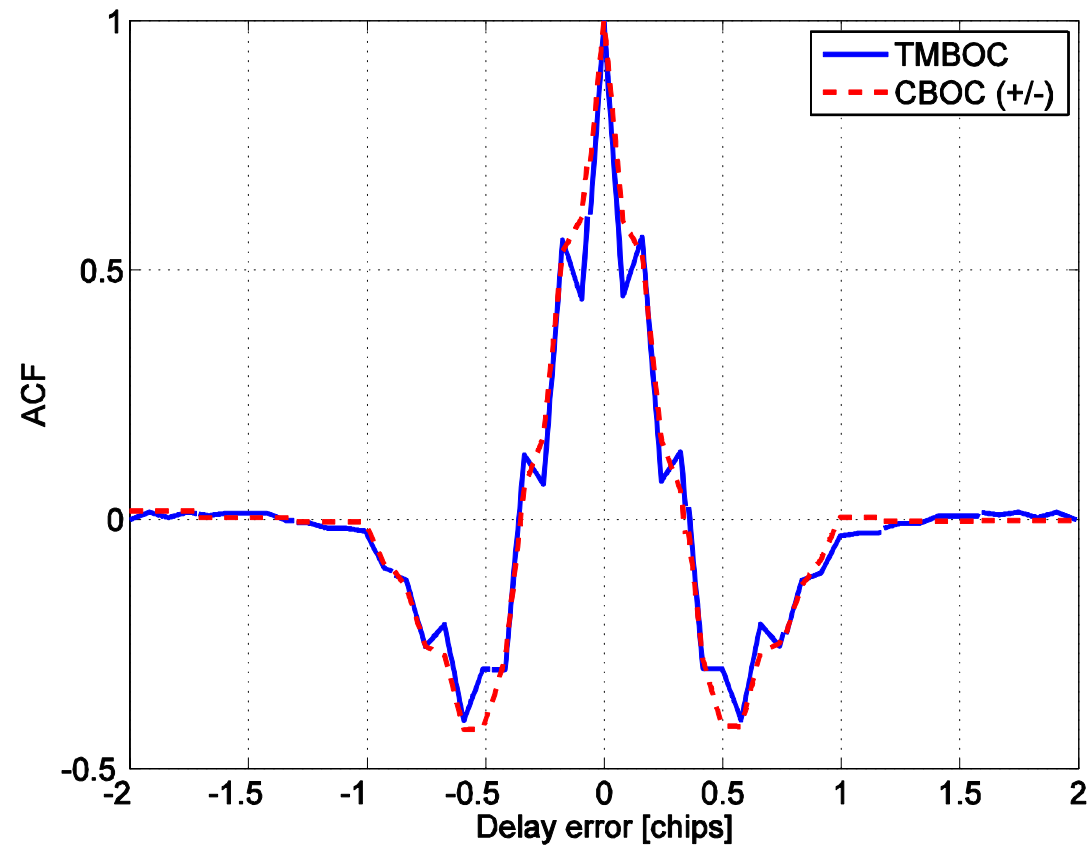
Criterion \ BOC type		SinBOC (1,1)
$\kappa_{SSC,C/A_code}$ [dBW-Hz]	at $B_T = 4$ MHz	-66.99
	at $B_T = 8$ MHz	-67.43
κ_{SSC,M_code} [dBW-Hz]	at $B_T = 4$ MHz	-68.06
	at $B_T = 8$ MHz	-72.44
$\kappa_{SSC,P(Y)_code}$ [dBW-Hz]	at $B_T = 4$ MHz	-65.99
	at $B_T = 8$ MHz	-68.52
$\kappa_{SSC,itself}$ [dBW-Hz]	at $B_T = 4$ MHz	-63.55
	at $B_T = 8$ MHz	-64.19
m_{MVS} [dBW-Hz]	$B_T = 4$ MHz and $B_T = 8$ MHz	-62.89
β_{RMS} [MHz]	at $B_T = 4$ MHz	0.86
	at $B_T = 8$ MHz	1.17
Power containment factor ε [%]	at $B_T = 4$ MHz	85.57%
	at $B_T = 8$ MHz	92.51%
$(\Delta d)_{res}$ [m]		100.80

Exercise: using the formulas from slides 18 and 21, verify the values of this table (in Matlab). Note: BPSK modulation corresponds to $N_{BOC}=1$.

Example: power containment versus bandwidth

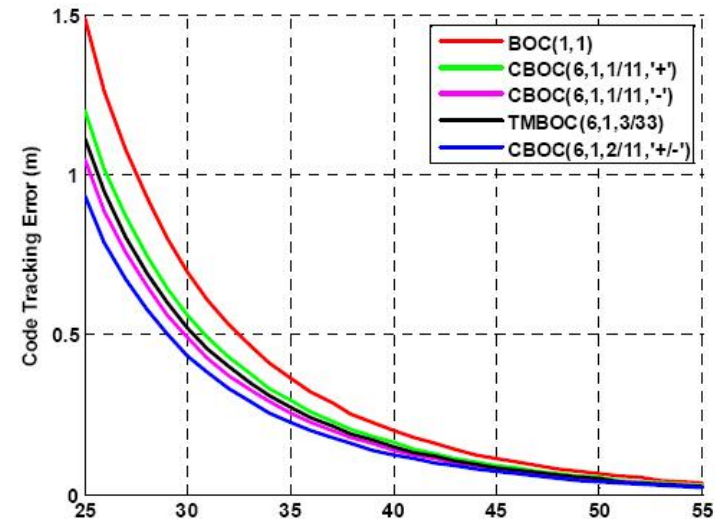


MBOC modulation Auto-Correlation Function (ACF)



MBOC advantages/disadvantages

- + Better spectral separation with GPS C/A codes
- + Better tracking properties compared with SinBOC(1,1) (because of the SinBOC(6,1) component; typically, higher BOC modulation order => better tracking properties)
- More complex
- Slightly more difficult acquisition



Source: "CBOC – AN IMPLEMENTATION OF MBOC", Jose-Angel Avila-Rodriguez, Stefan Wallner, Guenter W. Hein, Emilie Rebeyrol, Olivier Julien & al., 2006, ION proceedings

Reasons for different codes and frequencies

Codes are used for:

- Satellite identification
- Correlation => need for good auto/cross-correlation properties
- Immunity from interference domains: narrowband interference rejection is proportional with the spreading factor (or code processing gain)
- Encryption (optional) => long codes needed in this case

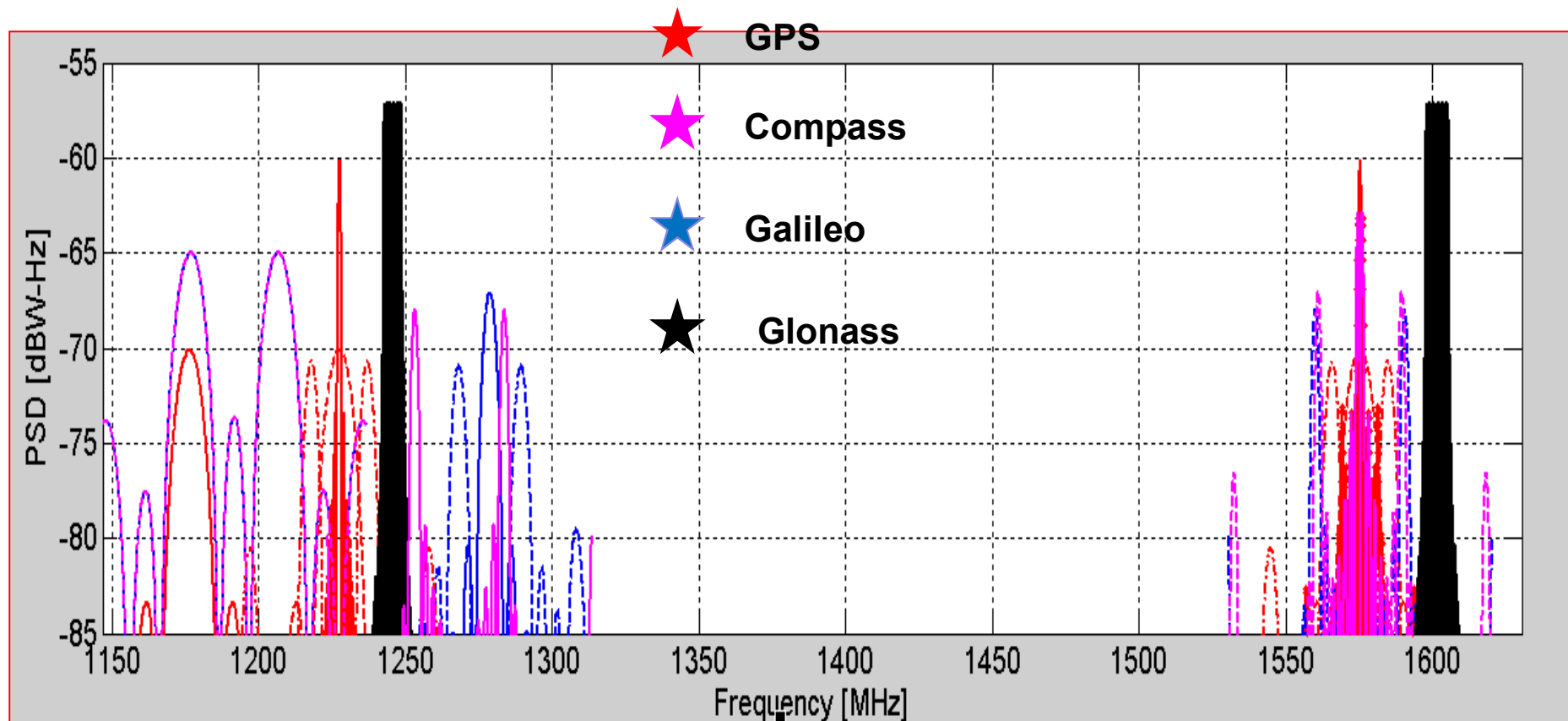
Frequency allocation:

- Standardization bodies
 - Dual frequencies might help with ionospheric corrections
 - Different services may use different frequencies
 - Increased system reliability when multiple channels are available
-

System comparison (source: Hein & al., InsideGNSS)

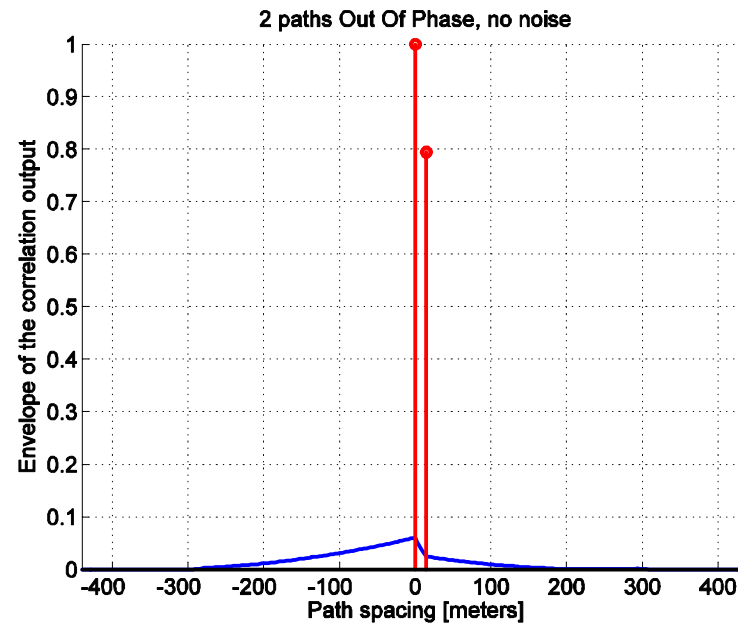
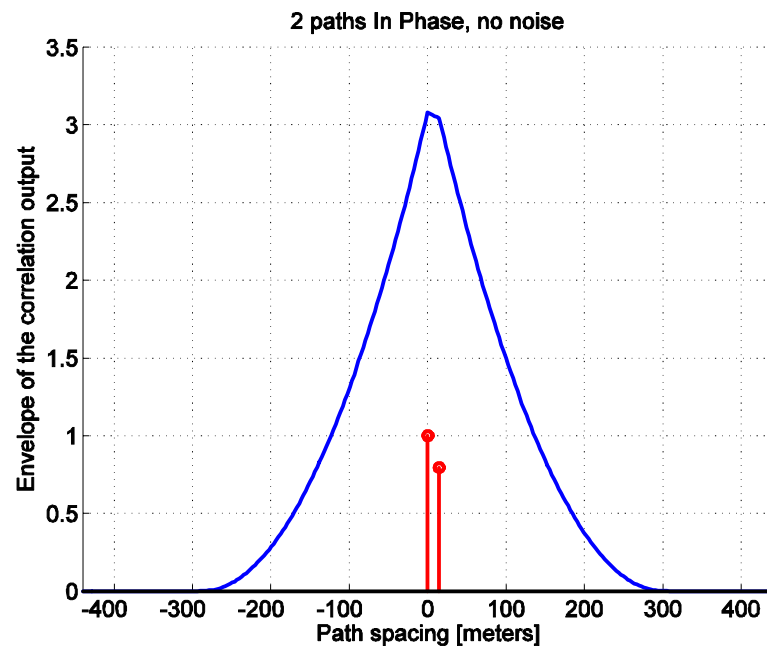
	GPS	GLONASS	GALILEO	QZSS
Number of Satellites	21+3 nominal 28 (27 Dec 2005)	21+3 nominal 13 (27 Dec. 2005)	27+3 nominal	3 IGSO
Number of Orbital Planes	6 (Trend to 3)	3	3	3
Satellite Life Time	GPS IIR: 10 yrs	GLONASS: 3yrs GLONASS-M: 7yrs GLONASS-K: 10-12 yrs	> 12 yrs	12 yrs
Satellite Mass	GPS IIR ca. 2000 kg	GLONASS and GLONASS-M: 1415 kg GLONASS-K: 850 kg	ca. 700 kg	?
Signal Access Scheme	CDMA	FDMA	CDMA	CDMA
Number of Frequencies	3 L1, L2, L5(=E5a)	One per two antipodal satellites	4 L1, E6, E5a(=L5), E5b	4 L1, L2, E6 (experim.), E5a(=L5)
Number of Codes	One per service and satellite	One per service and frequency (band)	One per service and satellite	One per service and satellite
Orbit Altitude	ca. 20,200 km above earth	ca. 19,100 km above earth	ca. 23,200 km above earth	ca. 36,000 km above earth
Intersatellite Links	Yes	GLONASS: No GLONASS-M, -K: Yes	No	No
Inclination	55°	64,8°	56°	45°
D – Dual Use C – Civilian	D	D	C (D PRS)	C
Commercial Service	No	No	Yes	Yes
Integrity Transmission	No (GPS III – Yes)	No (GLONASS-K – Yes)	Yes	Yes
Funding	Public	Public	Public/Private ?	Public

GNSS CDMA signals together



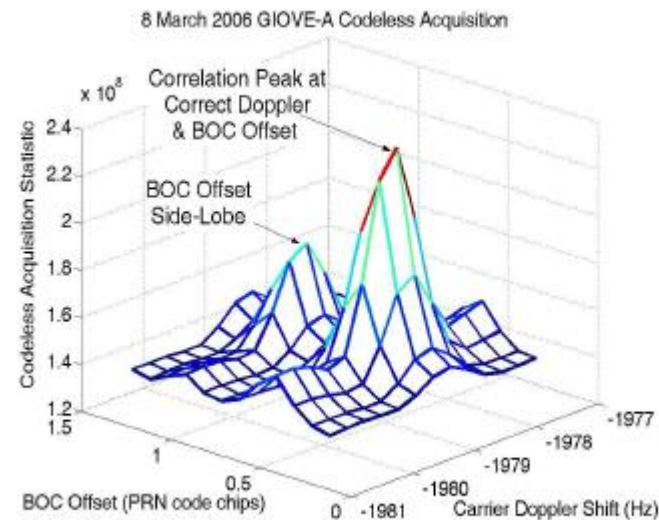
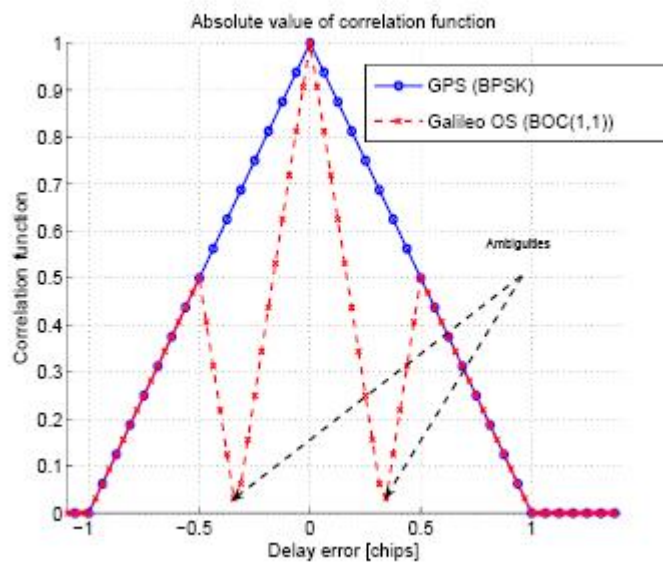
GNSS challenges: multipaths

- **Multipath propagation.** Below: an example of 2 paths adding constructively (left) or destructively (right); paths are 14.6 m apart and the second one is 1 dB smaller than the first one. BPSK modulation



Galileo specific challenges in acquisition

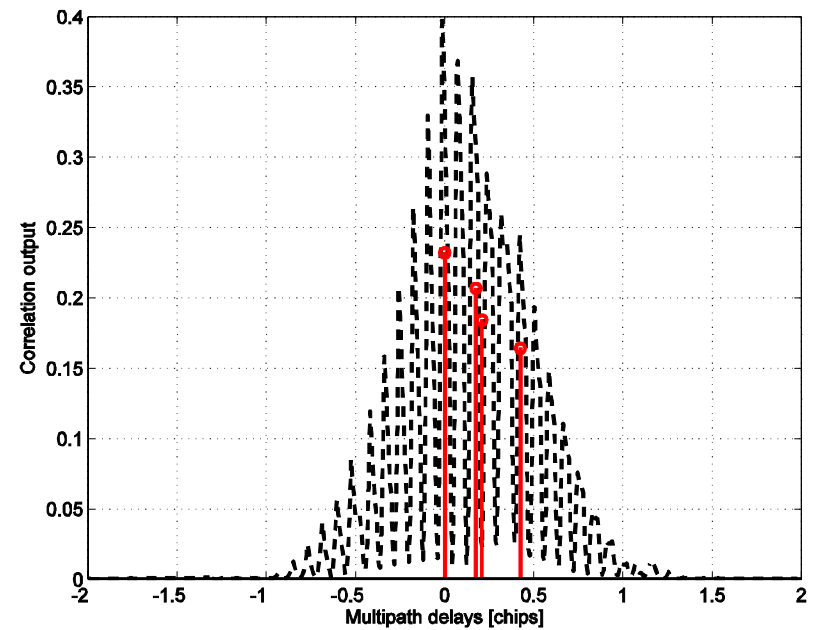
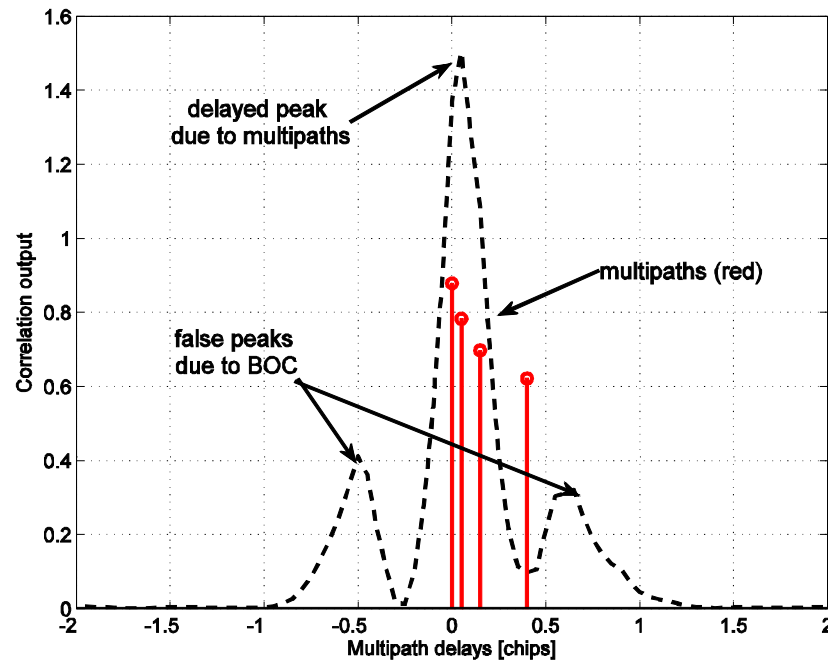
Ambiguities in the correlation function (due to BOC-modulation) =>
smaller search steps needed in the acquisition stage and methods to deal with the false lock points in the tracking stage



Galileo specific challenges in tracking

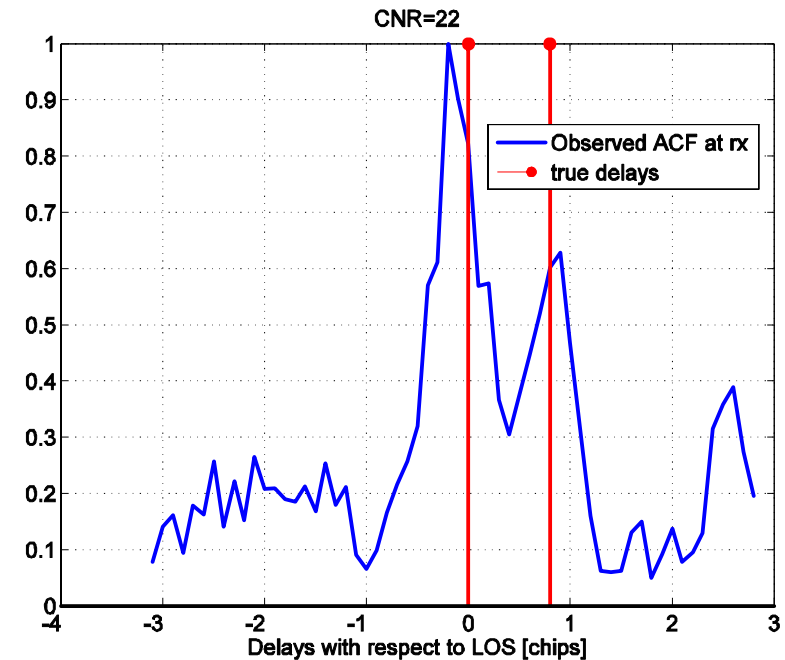
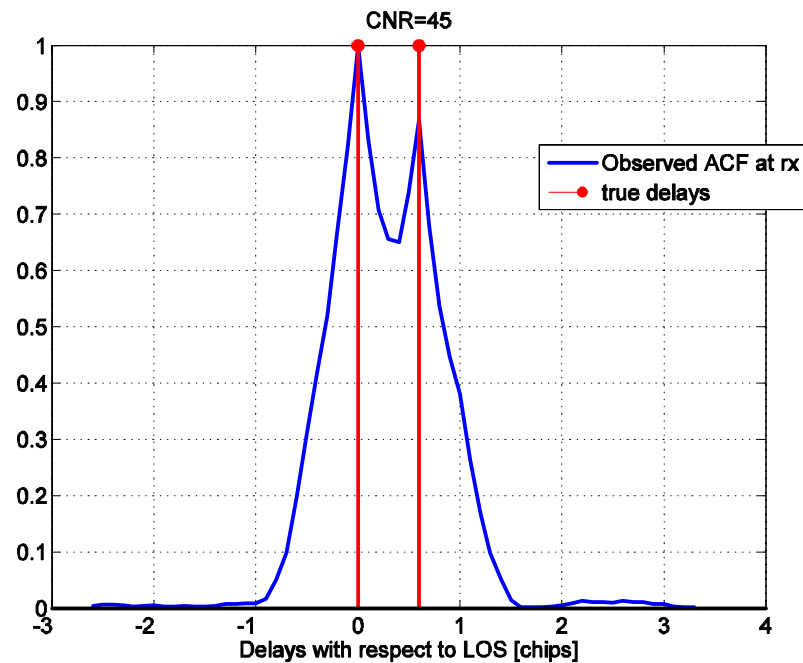
Additional false-lock peaks due to the BOC/MBOC modulation sidelobes.

Left $N_{\text{BOC}}=2$; right $N_{\text{BOC}}=12$



GNSS challenges: noise effect

High noise (low CNR): left plot shows the autocorrelation function (ACF) in good CNR (45 dB-Hz); right plot shows the ACF at low CNR (22 dB-Hz). BPSK modulation.



Carrier To Noise Ratio (CNR) definition

$$CNR[dB/Hz] = 10\log_{10}\frac{E_b}{N_0} + 10\log_{10}(B_w) = 10\log_{10}\frac{E_b}{N_0} + 30, \quad (1)$$

where $B_w = 1$ kHz bandwidth. Sometimes, CNR is expressed in dBm, and the relationship with bit-energy-to-noise ratio becomes:

$$CNR[dBm] = 20\log_{10}\left(\frac{E_b}{N_0}\right) + 20\log_{10}(B_w) + 10\log_{10}(kT_0). \quad (2)$$

where, k is Boltzmann constant ($k = 1.3806503 \times 10^{-23}$ Joule/Kelvin) and T_0 is the room temperature in Kelvin ($T \approx 300$ K). That is,

$$CNR[dBm] \approx CNR[dB/Hz] - 174 \text{ dBm/Hz}. \quad (3)$$

In this lecture we use the CNR given in dB-Hz.

$\frac{E_b}{N_0}$ is the bit-energy to noise ratio, as used before in DS-SS systems (including WCDMA)

GNSS challenges: interference effects

The extremely low power of GNSS received signal makes it more sensitive to various sources of interference. Interference is typically divided into:

- ❑ **Intra-system interference**: between signals of the same system (e.g., between various signals sharing E1 band in Galileo or between different frequency bands, such as E5a and E5b)
- ❑ **Inter-system interference**: between various GNSS systems, e.g., between Galileo and GPS. It is reduced between BOC/MBOC-modulated signals and BPSK-modulated signals (see the Spectral Separation Coefficients)

Another classification:

- ❑ **Wideband interference**: over the whole bandwidth of interest (e.g., interference from other satellites)
 - ❑ **Narrowband interference**: over a smaller bandwidth than the whole signal bandwidth (e.g., intentional jamming, unintentional UHF/VHF television signal interference)
-

Summary & core content

- ✓ 3 main frequency bands per system (GPS/Galileo)
- ✓ Several signals transmitted in each band (more to come in the future)
- ✓ Basic GPS signal is C/A code
- ✓ Basic Galileo signals are the L1C/L1B signals for Open Services (OS)
- ✓ Different modulation in Galileo compared to GPS (BOC/MBOC)
- ✓ New challenges in Galileo compared to GPS

Core content:

- What is the task of the spreading modulation? What are the pilot channels? What properties do you need for PRN codes used in GNSS?
 - Basic principle of BOC and CBOC modulations (signal spectra, autocorrelation function, comparison with BPSK)
 - Main differences between Galileo and GPS (including the new challenges in Galileo)
 - Relationship between CNR and SNR
-

Further references

- ❑ J. Betz, "Galileo, GPS and Other GNSS signals with receiver processing and technology, NavtechGPS courses
 - ❑ G. Hein, J.A. Avila-Rodriguez and S. Wallner, "The Galileo code and others, InsideGNSS journal, Sep 2006
"http://www.insidegnss.com/auto/0906%20WP%20Galileo.pdf
 - ❑ Jose-Angel Avila-Rodriguez, Stefan Wallner, Guenter W. Hein, Emilie Rebeyrol, Olivier Julien & al, "CBOC – AN IMPLEMENTATION OF MBOC", ION proceedings, 2006.
 - ❑ M. Petovello and G. Lachapelle, "Mathematical Models and GNSS Interference", InsideGNSS journal, Mar/Apr 2008,
http://www.insidegnss.com/auto/igm_022-027.pdf
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