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# New method of GPS orbit determination from GCPS network for the purpose of DOP calculations

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**Abstract** The accuracy of GPS measurement satisfies the requirements of some applications, but many applications require an improvement of GPS measurement accuracy. For precise positioning by GPS, it is necessary to perform GPS mission planning. The GPS mission planning is a pre-survey task in which the values of Dilution Of Precision (DOP) should be predicted for the observation points, this task should determine the best observation periods which meet the project requirements. The main purpose of this work is to study a rather simple but still fairly accurate algorithm to determine the artificial satellite orbits for the purpose of DOP calculation. The orbit determination algorithm proposed in this paper is implemented by using several reference stations and calculated the orbits by new algorithm; inverse GPS. Inverse GPS means that reference stations are considered as satellites and satellite as receiver. This new algorithm used to calculate the satellite orbit which is mainly used to calculate the DOP. A comparison is done between the estimated PDOP by using satellite coordinates from new method and from the SP3 (Standard Product # 3) file.



## 1. Introduction

The computation of satellite positions is a fundamental task in all GPS positioning software. The data needed for this computation can come in the form of a broadcast or a precise ephemeris. The broadcast ephemeris is available from the GPS, as a set of parameters is sent to the user via the navigation message. The parameters are updated by the control center quite frequently, approximately every 2 h, and the accuracy of the computed coordinates is about 3 ms. There are several types of precise ephemerides produced by various agencies in the world based on data from permanent GPS sites. Their accuracy ranges from about 0.2 m for predicted to about 0.05 m for a final, post-processed orbit. Precise ephemerides are often distributed in SP3 format, where the coordinates and satellite clock errors for all GPS satellites are usually listed at 15-min intervals [1]. Several networks have been established for orbit determination. Some networks are of regional or even continental size such as Australian GPS orbit determination network [2]. Precise orbits are available from agencies such as the International GPS Service (IGS) on the World Wide Web. The IGS provide three levels of accuracy: predicted, rapid and final. The predicted orbits are available in real time and are accurate to approximately 25 cm. The rapid and final orbits are available with a latency of 17 h and 13 days respectively, and are accurate to 5 cm for the rapid orbits and better

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than 5 cm for the final orbits [3]. The official orbit determination for GPS satellites is based on observations at the five monitor stations of the control segment. The broadcast ephemerides for Block I satellites with cesium clocks were accurate to about 5 m (assuming three uploads per day). For the Block II satellites, the accuracy is the order of about 1 m [2].

The paper is organized as follows: Section 2 discusses Satellite orbits. While Types of satellites ephemerides used to calculating satellites coordinate are presented in Sections 3. Various methods of calculating satellites coordinate are provided in Section 4. Section 5 introduces a proposed model for calculating satellite coordinates. Performance analysis was conducted of numerical case study in Section 6. Conclusions are given in Section 7.

## 2. Satellite orbits

Orbit determination essentially means the determination of orbital parameters and satellite clock biases. In principle, the problem is inverse to the navigational or surveying goal. The applications of GPS depend substantially on knowing satellite orbits. For single receiver positioning, an orbital error is highly correlated with the positional error. In the case of baselines, relative orbital errors are considered to be approximately equal to relative baseline error. Orbital information is either transmitted by the satellite as part of the broadcast message or can be obtained in form of precise ephemerides (typically some days after the observation) from several sources [2]. The satellite ephemeris information is provided from either the broadcast ephemeris in the satellite message or from a precise ephemeris [3]. The satellites orbit the earth with a speed of 3.9 km per second and have a circulation time of 12 h sidereal time, corresponding to 11 h 58 min earth time. This means that the same satellite reaches a certain position about 4 min earlier each day. The mean distance from the middle of the earth is 26,560 km. With a mean earth radius of 6360 km, the height of the orbits is then about 20,200 km. Orbits in this height are referred to as MEO – Medium Earth Orbit. The satellites are arranged on six planes, each of them containing at least four slots where satellites can be arranged equidistantly. Today, typically more than 24 satellites orbit the earth, improving the availability of the system. The inclination angle of the planes towards the equator is 55°, the planes are rotated in the equatorial plane by 60° against each other. By this arrangement of the orbits it is avoided that too many satellites are to often over the north and south pole. The number and constellation of satellites guarantees that the signals of at least four satellites can be received at any time all over the world. The closer you get to the poles, the lower over the horizon the satellites are located. They can still be received very well, but in no case they are directly above. This may lead to a – typically insignificant – loss of the precision of the position determination. This effect, caused by the geometry of the satellite arrangement, happens from time to time on any spot of the earth surface and can be forecasted [4].

### 2.1. Orbit description

Assume two point masses $m_1$ and $m_2$ separated by the distance $r$. Considering for the moment only the attractive force between the masses and applying Newtonian mechanics, the movement of mass $m_2$ relative $m_1$ is defined by the homogeneous differential equation of second order.

$$\ddot{\underline{r}} + \frac{G(m_1 + m_2)}{r^3}\underline{r} = 0 \tag{1}$$

where $\underline{r}$ is the relative position vector with $\|\underline{r}\| = r$, $\ddot{\underline{r}} = \frac{d^2 r}{dt^2}$ is the relative acceleration vector and $G$ is the universal gravitational constant.

The time parameter $t$ being an inertial time. In fact, the inertial time is provided by the GPS system time. In the case of motion of an artificial earth satellite, in a first approximation, both bodies can be considered as point masses and the mass of the satellite can be neglected. The product of $G$ and the earth's mass $M_E$ is denoted as $\mu$ and is known as one defining parameter of the WGS-84 reference system.

$$\mu = GM_E = 3986004.418 * 10^8\ \mathrm{m}^3\ \mathrm{s}^{-2}$$

The orbital Keplerian motion is defined by six orbital parameters. The mathematical procedures of calculating the satellite's orbital parameters and the satellites coordinates will be focuses in Section 3.

## 3. Types of satellites ephemerides used to calculate satellites coordinate

Three sets of data are available to determine position and velocity vectors of the satellites in a terrestrial reference frame at any instant.

The data used to calculate the position vectors of the GPS satellites in Earth Centered Earth Fixed (ECEF) coordinate system are called ephemerides. These ephemerides are divided into three categories according to its accuracy, as shown in Table 1.

### 3.1. Almanac data

The purpose of almanac data is to provide the user with less precise data to facilitate receiver satellite search or for planning tasks such as the computation of visibility charts. The almanac data are updated at least every six days, broadcast as a part of the satellite navigation message and contain orbital information for all satellites, Table 2.

### 3.2. Broadcast ephemerides

The broadcast ephemerides are based on observations at the monitor stations of the GPS control segment. The most recent of these data are used to compute a reference orbit for the satellites. The broadcast ephemerides are part of the satellite message, and also available from a variety of information services such as SOPAC. Essentially, the ephemerides contain records with general information, orbital information, and records with information on the satellite clock, Table 3.

### 3.3. Precise ephemerides

The precise ephemerides consist of satellite positions and velocities at equidistant epochs. Typical spacing of the data is 15 min. The precise ephemerides format consists of a header

**Table 1** Uncertainties of ephemerides [2].

| Ephemerides | Uncertainties |
|---|---|
| Almanac | Some kilometers |
| Broadcast ephemerides | 1 m |
| Precise ephemerides | 0.05–0.20 m |

**Table 2** Almanac data [2].

| Parameter | Explanation |
|---|---|
| ID | Satellite PRN number |
| WEEK | Current GPS week |
| $t_a$ | Reference epoch in sec., current week |
| $\sqrt{a}$ | Square root of semi major axis in $\sqrt{m}$ |
| e | Eccentricity |
| $M_0$ | Mean anomaly at reference epoch |
| $\omega$ | Argument of perigee |
| $i_o$ | Orbital inclination |
| $l_0$ | Longitude of the node at weekly epoch |
| $\Omega$ | Drift of node's right ascension per sec. |
| $a_0$ | Satellite clock offset in seconds |
| $a_1$ | Satellite clock drift |

**Table 3** Broadcast ephemerides [2].

| Parameter | Explanation | |
|---|---|---|
| ID | Satellite PRN number | |
| WEEK | Current GPS week | |
| $t_e$ | Ephemerides reference epoch | |
| $\sqrt{a}$ | Square root of semi major axis in $\sqrt{m}$ | |
| $e$ | Eccentricity | |
| $M_0$ | Mean anomaly at reference epoch | |
| $\omega_o$ | Argument of perigee | |
| $i_o$ | Orbital inclination | |
| $l_0$ | Longitude of node at weekly epoch | |
| $\Delta n$ | Mean motion difference | |
| $i$ | Rate of inclination angle | |
| $\Omega$ | Rate of node's right ascension | |
| Cuc, Cus | Correction coefficients | Argument of perigee |
| Crc, Crs | | Geocentric distance |
| Cic, Cis | | Inclination |
| $t_c$ | Satellite clock reference epoch | |
| $a_0$ | Satellite clock offset | |
| $a_1$ | Satellite clock drift | |
| $a_2$ | Satellite clock frequency drift | |

containing general information (epoch intervals, orbit type, etc.) followed by the data section for successive epochs. These data are repeated for each satellite. The positions are given in kilometers and velocities are given in kilometers per seconds. The position and velocity vectors between the given epochs are obtained by interpolation where the Lagrange interpolation on polynomial base functions is used. The precise ephemerides are available from a variety of information services such as IGS and SOPAC [5].

## 4. Various methods of calculating satellites coordinate

In order to calculate the satellites coordinates using the satellites' ephemerides the satellites orbital parameters should be calculated first. The orbital Keplerian motion is defined by six orbital parameters, as shown in Table 4. Table 5 represents various constants and Table 6 lists anomalies commonly used.

The three anomalies are related by the formulas.

$$M(t) = n(t - T_o) \tag{2}$$

$$E(t) = M(t) + e \sin E(t) \tag{3}$$

$$\upsilon(t) = 2 \arctan\left[\sqrt{\frac{1+e}{1-e}} \tan\frac{E(t)}{2}\right] \tag{4}$$

This section focuses on the mathematical procedures of calculating the satellite's orbital parameters and the satellites coordinates using the almanac, broadcast and precise ephemerides.

### *4.1. Satellite coordinates, Almanac data*

The almanac data components shown in Table 2 should be kept into consideration. The following steps are for calculating the coordinates of only one GPS satellite in ECEF coordinates system [5].

#### *4.1.1. Calculation steps*

All terms in shaded boxes are transmitted as part of the Broadcast Navigation message; all others are computed or are constants.

*Step 1*: Compute the mean anomaly from $M_o$ and corrected mean motion.

**Table 4** Keplerian orbital parameters [2].

| Parameter | Notation |
|---|---|
| $\Omega$ | Right ascension of ascending node |
| $i_o$ | Inclination of orbital plane |
| $\omega$ | Argument of perigee |
| $a$ | Semi major axis of orbital ellipse |
| $e$ | Numerical eccentricity of ellipse |
| To | Epoch of perigee passage |

**Table 5** Various constants [2].

| | |
|---|---|
| Gravitational constant | $\mu = GM_E = 3.986005\text{E}14\ \text{m}^3/\text{s}^2$ |
| Mean earth rotation rate | $\omega_e = 7.292115147\text{E}{-5}\ \text{rad/s}$ |

**Table 6** Anomalies of the Keplerian orbit [2].

| Notation | Anomaly |
|---|---|
| $M(t)$ | Mean anomaly |
| $E(t)$ | Eccentric anomaly |
| $\upsilon(t)$ | True anomaly |

- Time since reference epoch ($t_k$). Where ($t$) is the time of measurement by the receiver

$$t_k = t - t_{oe} \tag{5}$$

- Corrected mean motion.

$$n_o = \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{P} \tag{6}$$

- Mean anomaly at time of transmission.

$$M_k = M_o + n_o t_k \tag{7}$$

*Step 2*:
- Solve for eccentric anomaly ($E_k$).

Use $E_1 = M_k$ Solve by iteration

$$E_k = M_k + e \sin E_k \tag{8}$$

*Step 3*:
- Compute true anomaly ($\upsilon_k$).

$$\upsilon_k = \arctan\left[\frac{\sqrt{1-e^2}\sin E_k}{\cos E_k - e}\right] \tag{9}$$

*Step 4*:
- Compute argument of latitude ($u_k$).

$$u_k = \omega + \upsilon_k \tag{10}$$

- Orbit radius.

$$r_K = a(1 - e\cos E_k) \tag{11}$$

- Orbit inclination.

$$i_k = i_0 \tag{12}$$

*Step 5*:
- Compute corrected longitude of ascending node. Where $\Delta t$ is transit time

$$\Delta t = a_0 + a_1.t_k \tag{13}$$

$$\Omega_k = l_k = l_o + (\dot{\Omega} - \omega_e)t_k - \omega_e(t_{oe} + \Delta t) \tag{14}$$

*Step 6*:
- Compute satellite coordinates in ECEF frame.

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = [R]\begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

- Where R represents rotation matrices in the X and Z directions.

$$[R] = R_3(-\Omega_k)R_1(-\mathrm{i}_k)R_3(-\mathrm{u}_k) = [\,R_1 \quad R_2 \quad R_3\,] \tag{16}$$

where

$$R_1 = \begin{bmatrix} \cos\Omega_k cos\mathrm{u}_k - \sin\Omega_k \sin\mathrm{u}_k \cos\mathrm{i}_k \\ \sin\Omega_k cos\mathrm{u}_k + \cos\Omega_k \sin\mathrm{u}_k \cos\mathrm{i}_k \\ \sin\mathrm{u}_k \sin\mathrm{i}_k \end{bmatrix}$$

$$R_2 = \begin{bmatrix} -\cos\Omega_k sin\mathrm{u}_k - \sin\Omega_k \cos\mathrm{u}_k \cos\mathrm{i}_k \\ -\sin\Omega_k sin\mathrm{u}_k + \cos\Omega_k \cos\mathrm{u}_k \cos\mathrm{i}_k \\ \cos\mathrm{u}_k \sin\mathrm{i}_k \end{bmatrix}$$

$$R_3 = \begin{bmatrix} \sin\Omega_k \sin\mathrm{i}_k \\ -\cos\Omega_k \sin\mathrm{i}_k \\ \cos\mathrm{i}_k \end{bmatrix}$$

### 4.2. Satellite coordinates, Broadcast ephemerides

The broadcast ephemerides components shown in Table 3 should be kept into consideration. The following steps for calculating the coordinates of only one GPS satellite in ECEF coordinates system [5].

#### 4.2.1. Calculation steps

All terms in shaded boxes are transmitted as part of the Broadcast Navigation message; all others are computed or are constants.

*Step 1*: Compute the mean anomaly from $M_o$ and corrected mean motion.
- Repeat Eqs. (5) and (6) in Step 1, Section 4.1 then:

$$n = n_o + \Delta n \tag{17}$$

- Mean anomaly at time of transmission.

$$M_k = M_o + n t_k \tag{18}$$

*Step 2*:
- The same calculations as Step 2, Section 4.1.

*Step 3*:
- The same calculations as Step 3, Section 4.1.

*Step 4*:
- Compute argument of latitude ($u_k$).

$$u_k = \omega + \upsilon_k \tag{19}$$

*Step 5*:
- Compute corrections:
- Argument of latitude correction.

$$\delta u_k = C_{uc}\cos(2u_k) + C_{us}\sin(2u_k) \tag{20}$$

- Radius correction.

$$\delta r_k = C_{rc}\cos(2u_k) + C_{rs}\sin(2u_k) \tag{21}$$

- Inclination correction.

$$\delta i_k = C_{ic}\cos(2u_k) + C_{is}\sin(2u_k) \tag{22}$$

*Step 6*:
- Compute corrected values:
- Corrected argument of latitude.

$$U_K = u_k + \delta u_k \tag{23}$$

- Corrected radius.

$$r_K = a(1 - e\cos E_k) + \delta r_k \tag{24}$$

- Corrected inclination.

$$i_k = i_0 + it_k + \delta i_k \tag{25}$$

*Step 7*:

- Compute corrected longitude of ascending node. Where $\Delta t$ is transit time

$$\Delta t = a_0 + a_1.t_k + a_2.t_k^2 \tag{26}$$

$$\Omega_k = l_k = l_o + (\dot{\Omega} - \omega_e)t_k - \omega_e(t_{oe} + \Delta t) \tag{27}$$

*Step 8*:

- The same calculations as Step 6, Section 4.1.

### 4.3. Satellite coordinates, precise ephemerides

The precise ephemerides consist of satellite positions at equidistant epochs. Typical spacing of data is 15 min. Therefore there is no need for calculating the orbital parameters, because of the nature of the data obtained from the precise ephemerides file. The precise ephemerides consist of satellite positions and velocities at equidistant epochs. Typical spacing of the data is 15 min. The precise ephemerides format consists of a header containing general information (epoch intervals, orbit type, etc.) followed by the data section for successive epochs. These data are repeated for each satellite. The positions are given in kilometers and velocities are given in kilometers per seconds [2].

## 5. Proposed model for calculating satellite coordinates

The satellites' position is unknown and the distance is computed by measuring the TOA (Time Of Arrival) obtaining pseudoranges. But, these pseudoranges are polluted. The GPS code pseudorange, and carrier phase observables are formulated as:

$$\begin{aligned} \rho &= R + c.\Delta t - d_{sat} + d_{iono} + d_{tropo} + d_{rel} + d_{ins} \\ \lambda\Phi &= R + c.\Delta t - d_{sat} + \lambda N - d_{iono} + d_{tropo} + d_{rel} + d_{ins} \end{aligned} \tag{28}$$

where $R$ represents geometric range, $c.\Delta t$ is the unknown distance caused by the offset of the receiver clock from system time, is the advance of the satellite clock with respect to system time (GPST), $d_{iono}$ represents ionospheric delay, $d_{tropo}$ is the tropospheric delay, $d_{rel}$ is the relativistic delay, and $d_{ins}$ is instrumental delay [6]. It should be noted that the consideration of carrier phase measurements in this study is unusual. Carrier phase is normally used only in precise orbit determination.

Supposing that the pseudoranges were cleaned up from $d_{sat}, d_{iono}, d_{tropo}, d_{rel}, d_{ins}$, the pseudorange measurements become:

$$\rho = R + c.\Delta t \tag{29}$$

The equation can be rewritten

$$\rho = \sqrt{(x - x^S)^2 + (y - y^S)^2 + (z - z^S)^2} + c.\Delta t \tag{30}$$

where $x^S$, $y^S$, $z^S$, $\Delta t$ are unknown and $x$, $y$, $z$ are known. Therefore, it seems obvious that to solve the problem, at least four Ground Control Points (GCPS) network are needed to find satellite position and receiver time offset. Assuming the receiver time offset $\Delta t$ is equal for all GCPS network, the observation equations of the satellite number 1 can be written as

$$\begin{aligned} \rho_1^{S1} &= \sqrt{(x_1 - x_1^S)^2 + (y_1 - y_1^S)^2 + (z_1 - z_1^S)^2} + c.\Delta t \\ \rho_2^{S1} &= \sqrt{(x_2 - x_1^S)^2 + (y_2 - y_1^S)^2 + (z_2 - z_1^S)^2} + c.\Delta t \\ \rho_3^{S1} &= \sqrt{(x_3 - x_1^S)^2 + (y_3 - y_1^S)^2 + (z_3 - z_1^S)^2} + c.\Delta t \\ \rho_4^{S1} &= \sqrt{(x_4 - x_1^S)^2 + (y_4 - y_1^S)^2 + (z_4 - z_1^S)^2} + c.\Delta t \\ &. \\ \rho_n^{S1} &= \sqrt{(x_n - x_1^S)^2 + (y_n - y_1^S)^2 + (z_n - z_1^S)^2} + c.\Delta t \end{aligned} \tag{31}$$

where $\rho_1^{S1}$ is the pseudoranges from GCP #1 to satellite # 1 and $\rho_2^{S1}$ is the pseudoranges from GCP #2 to satellite # 1 and so on; $x_1, y_1, z_1$ Are the coordinates of GCP #1; $x_2, y_2, z_2$ Are the coordinates of GCP #2 and so on and $x_1^{S1}, y_1^{S1}, z_1^{S1}$ Are the coordinates of satellite #1.

Similarly we can written the observation equation for satellite # 2 as follows:

$$\begin{aligned} \rho_1^{S2} &= \sqrt{(x_1 - x_2^S)^2 + (y_1 - y_2^S)^2 + (z_1 - z_2^S)^2} + c.\Delta t \\ \rho_2^{S2} &= \sqrt{(x_2 - x_2^S)^2 + (y_2 - y_2^S)^2 + (z_2 - z_2^S)^2} + c.\Delta t \\ \rho_3^{S2} &= \sqrt{(x_3 - x_2^S)^2 + (y_3 - y_2^S)^2 + (z_3 - z_2^S)^2} + c.\Delta t \\ \rho_4^{S2} &= \sqrt{(x_4 - x_2^S)^2 + (y_4 - y_2^S)^2 + (z_4 - z_2^S)^2} + c.\Delta t \\ &. \\ \rho_n^{S2} &= \sqrt{(x_n - x_2^S)^2 + (y_n - y_2^S)^2 + (z_n - z_2^S)^2} + c.\Delta t \end{aligned} \tag{32}$$

And so on for other visible satellite from GCPS.

These nonlinear equations can be solved by least square without using iterative techniques as follows:

The set of Eq. (31) of the satellite # 1 can be rewritten as

$$\begin{aligned} (\rho_1^{S1})^2 - R_{S1}^2 + c^2.\Delta t^2 &= R_1^2 + 2\rho_1^{S1}.c.\Delta t - 2X_1.X_1^S - 2Y_1.Y_1^S - 2Z_1.Z_1^S \\ (\rho_2^{S1})^2 - R_{S1}^2 + c^2.\Delta t^2 &= R_2^2 + 2\rho_2^{S1}.c.\Delta t - 2X_2.X_1^S - 2Y_2.Y_1^S - 2Z_2.Z_1^S \\ (\rho_3^{S1})^2 - R_{S1}^2 + c^2.\Delta t^2 &= R_3^2 + 2\rho_3^{S1}.c.\Delta t - 2X_3.X_1^S - 2Y_3.Y_1^S - 2Z_3.Z_1^S \\ (\rho_4^{S1})^2 - R_{S1}^2 + c^2.\Delta t^2 &= R_4^2 + 2\rho_4^{S1}.c.\Delta t - 2X_4.X_1^S - 2Y_4.Y_1^S - 2Z_4.Z_1^S \\ &. \\ (\rho_n^{S1})^2 - R_{S1}^2 + c^2.\Delta t^2 &= R_n^2 + 2\rho_n^{S1}.c.\Delta t - 2X_n.X_1^S - 2Y_n.Y_1^S - 2Z_n.Z_1^S \end{aligned} \tag{33}$$

where $R_{S1}^2$ is the $((X_1^S)^2 + (Y_1^S)^2 + (Z_1^S)^2)$; $R_1^2$ is the $((X_1)^2 + (Y_1)^2 + (Z_1)^2)$ and $R_2^2$ is the $((X_2)^2 + (Y_2)^2 + (Z_2)^2)$ and so on.

Alternatively, it is suggested in the present work to take differences between each two consecutive observation equations to get rid of second order terms. In this it is no need to any linearization process or iteration. Thus a new derived set of $(n - 1)$ equations can be written as follows:

The four variables ($X_1^{S1}$, $Y_1^{S1}$, $Z_1^{S1}$ and $\Delta t$) can now be solved according to the rules of Least Squares.

It should be noted that these equations are not independent because they are functions of the original ones and the new variances and covariances must be derived. Proper treatment

$$\begin{aligned}
&(\rho_1^{S1})^2-(\rho_2^{S1})^2-R_1^2+R_2^2=2c.(\rho_1^{S1}-.\rho_2^{S1}).\Delta t-2(X_1-X_2)X_1^{S1}-2(Y_1.-Y_2)Y_1^{S1}-2(Z_1-.Z_2)Z_1^{S1}\\
&(\rho_2^{S1})^2-(\rho_3^{S1})^2-R_2^2+R_3^2=2c.(\rho_2^{S1}-.\rho_3^{S1}).\Delta t-2(X_2-X_3)X_1^{S1}-2(Y_2.-Y_3)Y_1^{S1}-2(Z_2-.Z_3)Z_1^{S1}\\
&(\rho_3^{S1})^2-(\rho_4^{S1})^2-R_3^2+R_4^2=2c.(\rho_3^{S1}-.\rho_4^{S1}).\Delta t-2(X_3-X_4)X_1^{S1}-2(Y_3.-Y_4)Y_1^{S1}-2(Z_3-.Z_4)Z_1^{S1}\\
&(\rho_4^{S1})^2-(\rho_5^{S1})^2-R_4^2+R_5^2=2c.(\rho_4^{S1}-.\rho_5^{S1}).\Delta t-2(X_4-X_5)X_1^{S1}-2(Y_4.-Y_5)Y_1^{S1}-2(Z_4-.Z_5)Z_1^{S1}\\
&.\\
&(\rho_{n-1}^{S1})^2-(\rho_n^{S1})^2-R_{n-1}^2+R_n^2=2c.(\rho_{n-1}^{S1}-.\rho_n^{S1}).\Delta t-2(X_{n-1}-X_n)X_1^{S1}-2(Y_{n-n}.-Y_n)Y_1^{S1}-2(Z_{n-1}-.Z_n)Z_1^{S1}
\end{aligned} \tag{34}$$

of these equations should, therefore, involve the use of a derived weighting system. In the following sections, regular and proper weighting solutions will be introduced:

### *5.1. Case of equal weights*

Observation equations can be expressed in the matrix form as follows:

$$V = A.X + L \tag{35}$$

where:

$V$ = Vector of residuals

$$A=\begin{bmatrix}2c.(\rho_1^{S1}-.\rho_2^{S1}) & -2(X_1-X_2) & -2(Y_1.-Y_2) & -2(Z_1-.Z_2)\\ 2c.(\rho_2^{S1}-.\rho_3^{S1}) & -2(X_2-X_3) & -2(Y_2.-Y_3) & -2(Z_2-.Z_3)\\ 2c.(\rho_3^{S1}-.\rho_{44}^{S1}) & -2(X_3-X_4) & -2(Y_3.-Y_4) & -2(Z_3-.Z_4)\\ 2c.(\rho_4^{S1}-.\rho_5^{S1}) & -2(X_4-X_5) & -2(Y_4.-Y_5) & -2(Z_4-.Z_5)\\ . & . & . & .\\ 2c.(\rho_{n-1}^{S1}-.\rho_n^{S1}) & -2(X_{n-1}-X_n) & -2(Y_{n-1}-Y_n) & -2(Z_{n-1}-Z_n)\end{bmatrix}$$

$$X=\begin{bmatrix}\Delta t\\ X_1^S\\ Y_1^S\\ Z_1^S\end{bmatrix}$$

Are unknown, which will be solved using the least square techniques, and

$$L=\begin{bmatrix}(\rho_1^{S1})^2-(\rho_2^{S1})^2-R_1^2+R_2^2\\ \rho_2^{S1})^2-(\rho_3^{S1})^2-R_2^2+R_3^2\\ (\rho_3^{S1})^2-(\rho_4^{S1})^2-R_3^2+R_4^2\\ (\rho_4^{S1})^2-(\rho_5^{S1})^2-R_4^2+R_5^2\\ .\\ (\rho_{n-1}^{S1})^2-(\rho_n^{S1})^2-R_{n-1}^2+R_n^2\end{bmatrix}$$

The normal equations become

$$(A^T.A).X = A^T.L$$

or

$$N.X = A^T.L \tag{36}$$

The least square will solve these equations by minimizing the sum of the squares of the residuals, and the solution will be in the form:

$$X = N^{-1}.A^T.L \tag{37}$$

### *5.2. Case of unequal weights*

Observation equations can be expressed in the matrix form as follows:

$$WV = WA.X + WL \tag{38}$$

where:

$V$ = Vector of residuals

$$A=\begin{bmatrix}2c.(\rho_1^{S1}-.\rho_2^{S1}) & -2(X_1-X_2) & -2(Y_1.-Y_2) & -2(Z_1-.Z_2)\\ 2c.(\rho_2^{S1}-.\rho_3^{S1}) & -2(X_2-X_3) & -2(Y_2.-Y_3) & -2(Z_2-.Z_3)\\ 2c.(\rho_3^{S1}-.\rho_{44}^{S1}) & -2(X_3-X_4) & -2(Y_3.-Y_4) & -2(Z_3-.Z_4)\\ 2c.(\rho_4^{S1}-.\rho_5^{S1}) & -2(X_4-X_5) & -2(Y_4.-Y_5) & -2(Z_4-.Z_5)\\ . & . & . & .\\ 2c.(\rho_{n-1}^{S1}-.\rho_n^{S1}) & -2(X_{n-1}-X_n) & -2(Y_{n-1}-Y_n) & -2(Z_{n-1}-Z_n)\end{bmatrix}$$

$$X=\begin{bmatrix}\Delta t\\ X_1^S\\ Y_1^S\\ Z_1^S\end{bmatrix}$$

Are unknown, which will be solved using the least square techniques, and

$$L=\begin{bmatrix}(\rho_1^{S1})^2-(\rho_2^{S1})^2-R_1^2+R_2^2\\ (\rho_2^{S1})^2-(\rho_3^{S1})^2-R_2^2+R_3^2\\ (\rho_3^{S1})^2-(\rho_4^{S1})^2-R_3^2+R_4^2\\ (\rho_4^{S1})^2-(\rho_5^{S1})^2-R_4^2+R_5^2\\ .\\ (\rho_{n-1}^{S1})^2-(\rho_n^{S1})^2-R_{n-1}^2+R_n^2\end{bmatrix}$$

The covariance matrix in this model can be written as

$$\sum\nolimits_{FF} = A\sum A^T \tag{39}$$

where $\sum_{FF}$ is the covariance matrix of the model and $\sum$ is the covariance matrix of the observations.

$$\sum=\begin{bmatrix}\sigma_{\Delta t}^2 & \sigma_{\Delta tx} & \sigma_{\Delta ty} & \sigma_{\Delta tz}\\ \sigma_{x\Delta t} & \sigma_x^2 & \sigma_{xy} & \sigma_{xz}\\ \sigma_{y\Delta t} & \sigma_{yx} & \sigma_y^2 & \sigma_{yz}\\ \sigma_{z\Delta t} & \sigma_{zx} & \sigma_{zy} & \sigma_z^2\end{bmatrix} \tag{40}$$

Assuming equal weights of the observations therefore the covariance matrix of the observations $\sum$ equal

$$\sum = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{41}$$

The weight matrix $W$ is

$$W = \sigma_0^2 \sum_{FF}^{-1} \tag{42}$$

where $\sigma_0^2$ is the reference variance.

$$\sigma_0^2 = \frac{L^T.\sum_{FF}.L}{r} \tag{43}$$

$r$ = number of equations – number of unknowns.

The normal equations become

$$(A^T.W.A).X = A^T W.L$$

or

$$N.X = A^T W.L \tag{44}$$

The least square will solve these equations by minimizing the sum of the squares of the residuals, and the solution will be in the form:

$$X = N^{-1}.A^T W.L \tag{45}$$

## 6. Case study

Performance analysis was conducted using data from regional GPS reference networks. The data from Oregon Real-time Network GPS reference network (ORGN) has been used to evaluate the new method of estimation the satellite positions for purpose of calculating Dilution Of Precision (DOP). The used stations of (ORGN) network (Fig. 1) consist of 12 continuously operating GPS. Oregon Real-time GPS Network, including network status, products and services, support information, and contacts is located at website www.theorgn.net [8].

By using the proposed model for calculating satellite coordinates in Section 5. The DOP value was calculated twice the first one by using satellite coordinates from precise ephemerides from SP3 file and the second one from satellite coordinates calculated by proposed method in Section 5. The eleven GCPS network was used to calculate the satellite coordinates at each epoch for about 2 h (120 min). Fig. 2 shows the difference between satellite coordinates for the two sources (SP3 file and proposed method) for the satellite # 11, the figure shows that the approximate coinciding in coordinates between two methods.

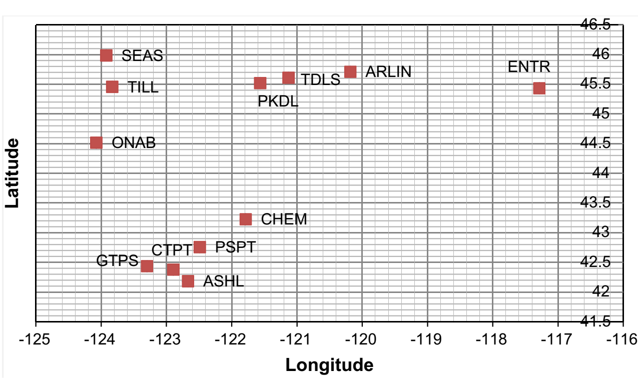


**Figure 1** Oregon Real-time Network (ORGN) reference stations.

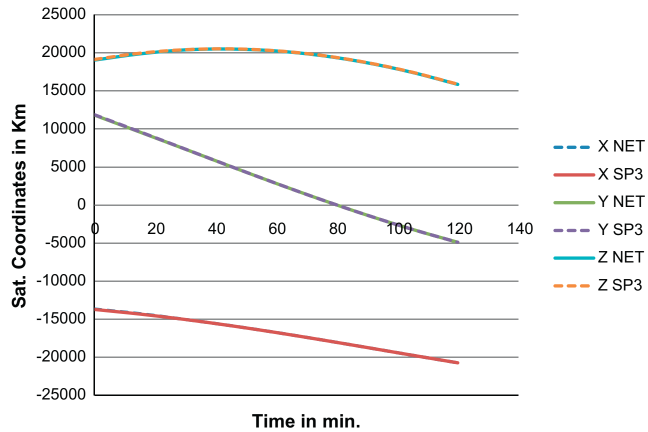


**Figure 2** Satellite #11 coordinates difference between coordinates from SP3 and proposed method.

**Table 7** Error in coordinates for satellite # 11 between two methods.

| Unknown | Form proposed model | From SP3 file | Error in coordinates |
|---|---|---|---|
| $X_{11}^S$ (km) | −15041.119 | −15043.8 | 2.651235 |
| $Y_{11}^S$ (km) | 7344.7896 | 7343.78 | 1.009573 |
| $Z_{11}^S$ (km) | 20393.288 | 20397.18 | −3.89642 |
| $\Delta t$ (s) | 8.76117E−09 | | |

For example; the difference between two methods for satellite # 11 at epoch 30 min as shown in Table 7.

Once the satellite coordinates were obtained the PDOP at GCP (TDLS) can be calculated by using the known coordinates of GCP (TDLS) and the satellite coordinates from two sources (SP3 file and proposed method).

The coordinates of GCP (TDLS) are

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| TDLS | −2,243,186 | −3,856,771 | 4,542,745 |

The Position Dilution Of Precision (PDOP) can be defined from the following equations:

$$PDOP = \sqrt{q_{xx} + q_{yy} + q_{zz}} \tag{46}$$

$$Q_x = \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{zy} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{bmatrix} \tag{47}$$

$$Q_x = (A^T A)^{-1} \tag{48}$$

**Table 8** Actual and estimated values of PDOP for each epoch.

| Epoch (min) | PDOP value from network | PDOP value from SP3 | Error in PDOP% |
|---|---|---|---|
| 0 | 2.09930 | 2.10075 | 0.0691 |
| 15 | 1.91738 | 1.91826 | 0.0458 |
| 30 | 1.77838 | 1.77794 | 0.0246 |
| 45 | 2.83493 | 2.83626 | 0.0470 |
| 60 | 4.02251 | 4.02725 | 1.1764 |
| 75 | 4.32880 | 4.33609 | 0.1681 |
| 90 | 3.87006 | 3.87456 | 0.1162 |
| 105 | 2.82919 | 2.82975 | 0.0195 |
| 120 | 2.78831 | 2.78916 | 0.0307 |

$$A = \begin{bmatrix} \frac{(x_1^S - x)}{\rho_1} & \frac{(y_1^S - y)}{\rho_1} & \frac{(z_1^S - z)}{\rho_1} & c \\ \frac{(x_2^S - x)}{\rho_2} & \frac{(y_2^S - y)}{\rho_2} & \frac{(z_2^S - z)}{\rho_2} & c \\ \frac{(x_3^S - x)}{\rho_3} & \frac{(y_3^S - y)}{\rho_3} & \frac{(z_3^S - z)}{\rho_3} & c \\ \frac{(x_4^S - x)}{\rho_4} & \frac{(y_4^S - y)}{\rho_4} & \frac{(z_4^S - z)}{\rho_4} & c \end{bmatrix} \tag{49}$$

If the model derived can be compared to its directly estimated from the SP3 file, the comparison results are provided in Table 8. Table 8 shows the estimated values from two sources and the difference between them for each epoch, where the differences indicated an agreement between the model-derived estimates and the direct estimation from SP3 file at the level of max. 1.1764%.

The difference error estimated in percentage is calculated using the following equation:

$$DE = \frac{PDOP_{from\ SP3} - PDOP_{from\ network}}{PDOP_{from\ SP3}} .100$$

## 7. Conclusion

Based on the experimental results obtained so far, the following conclusions can be drawn:

- The initial fundamental goal of this research is the development of new method for calculations of satellite positions from GPS GCPS network.
- A comparison is done between the estimated PDOP from new method and from the SP3 file.
- The differences between estimated PDOP from two methods indicated an agreement between the model-derived estimates and the direct estimated from SP3 file at the level of max. 1.1764%.
- The experimental results indicate that there is an approximate coinciding in satellite coordinates between two methods.
- Orbit determination techniques have been discussed based on GPS ephemerides.

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