

Training Large Language Models to Reason in a Continuous Latent Space (Coconut)

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<https://arxiv.org/pdf/2412.06769?>

Chain-of-Thought (CoT)

General Idea

Prompt: If there are 3 apples, and you buy 2 more, how many apples do you have in total?

Model Generation Without CoT:

6.



Model Generation With CoT:

Initially, you have 3 apples.

You buy 2 more apples.

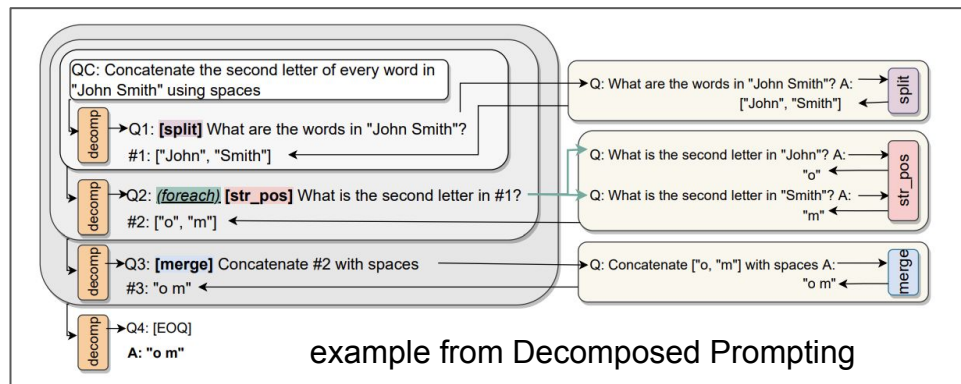
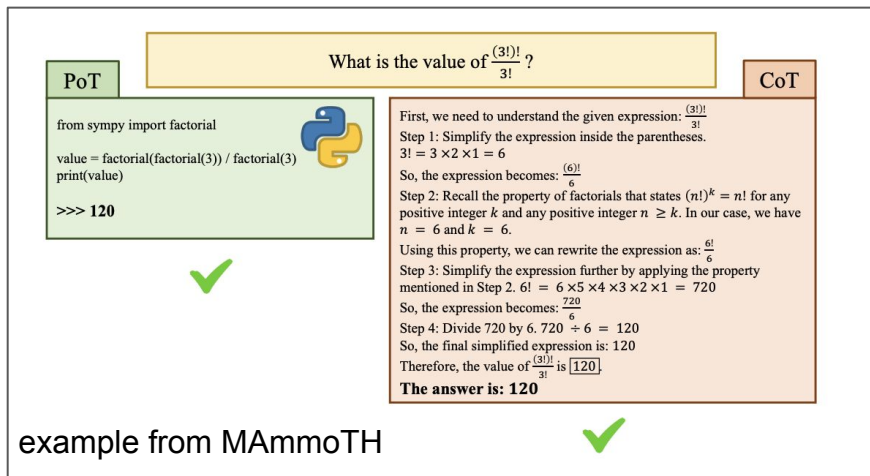
Adding these together, $3+2=5$.

So, the total number of apples is **5**.

Chain-of-Thought (CoT)

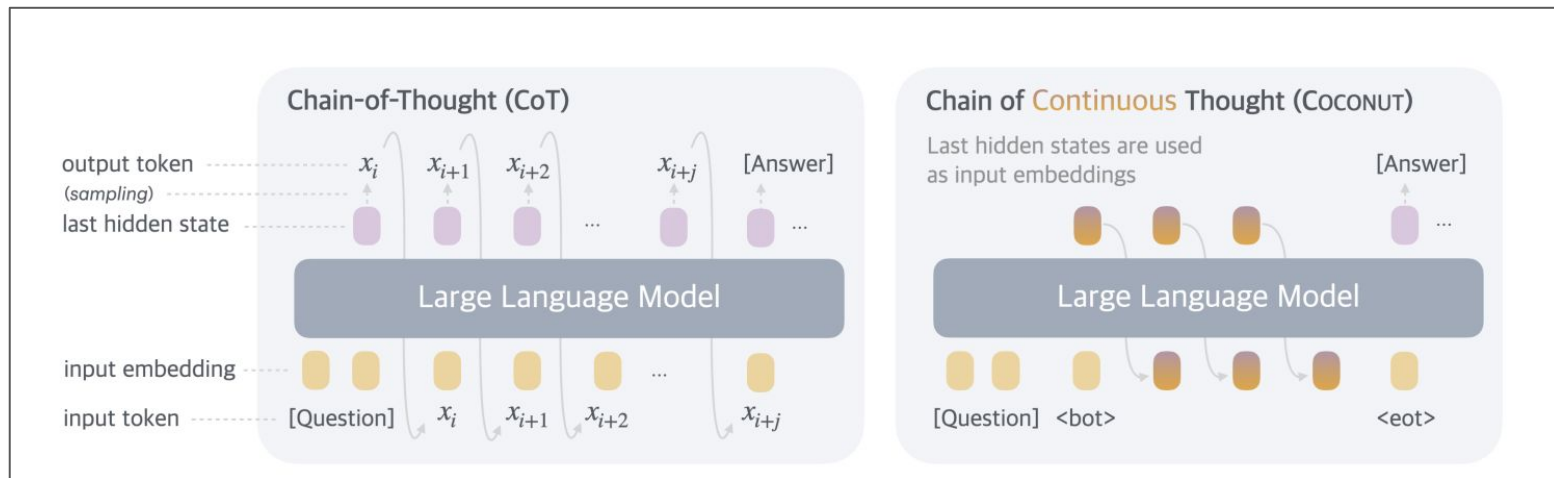
Ways to Implement

- supervised fine tuning (for example [MAMmoTH](#))
- reinforcement learning (for example [Math-Shepherd](#))
- prompting (for example, [Decomposed Prompting](#))



Chain of Continuous Thought (Coconut)

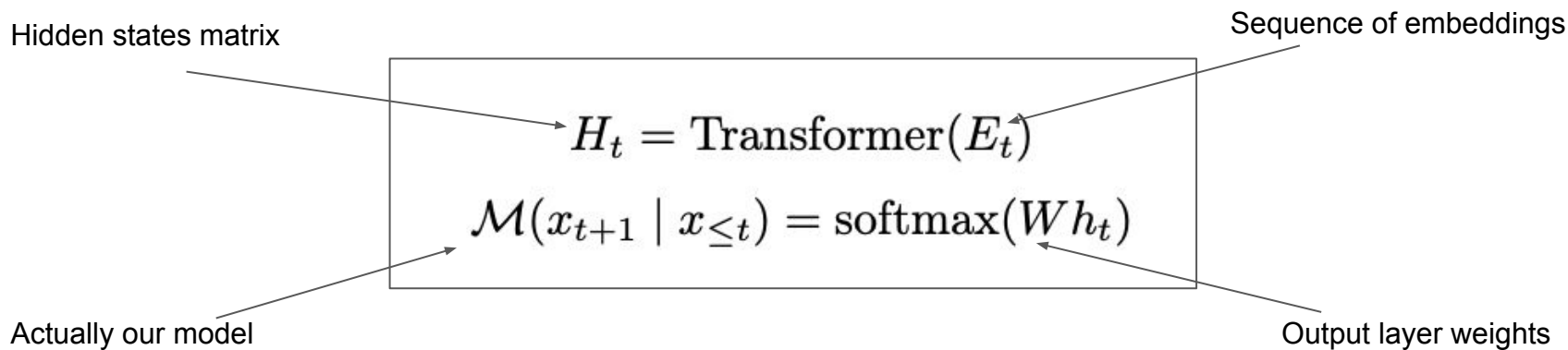
General Idea



If there are 3 apples, and you buy 2 more, how many apples do you have in total?
Initially, you have 3 apples. You buy 2 more apples. Adding these together, $3+2=5$.
So, the total number of apples is 5.

Input tokens, Thoughts tokens (do we need them as tokens?), Answer tokens

Chain of Continuous Thought (Coconut) Formalization



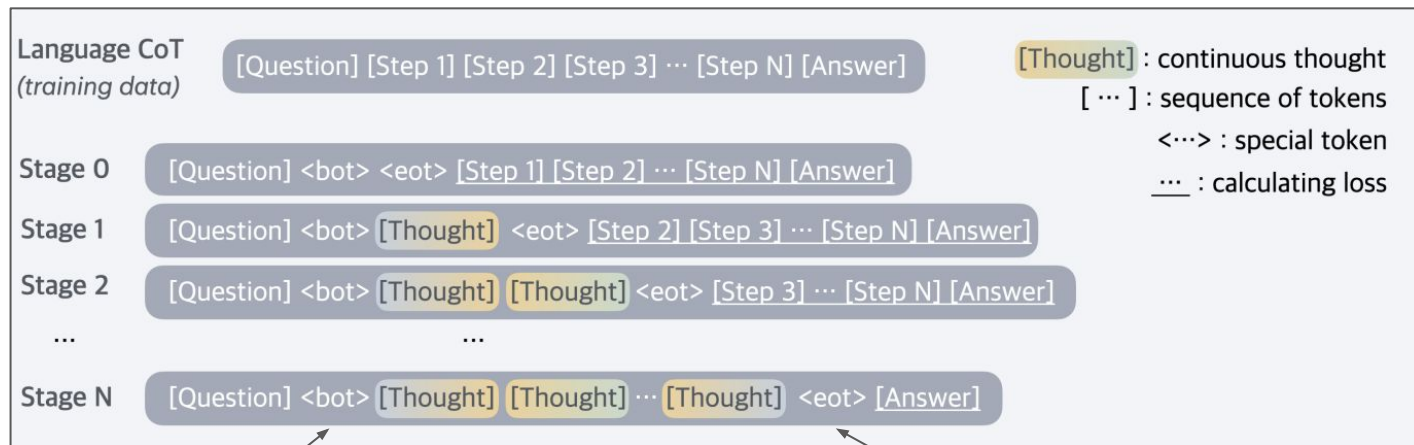
While **input tokens** - we run full pipeline
("language mode")

While **thoughts tokens** - we run only until h_t and use it as embeddings
("latent mode")

Chain of Continuous Thought (Coconut)

Way to implement

Training:



token "beginning of thoughts"

token "end of thoughts"

Inference: same as in casual LLM

Chain of Continuous Thought (Coconut)

Quality Experiments Results

Grade School Math

“Fact check + easy logic”

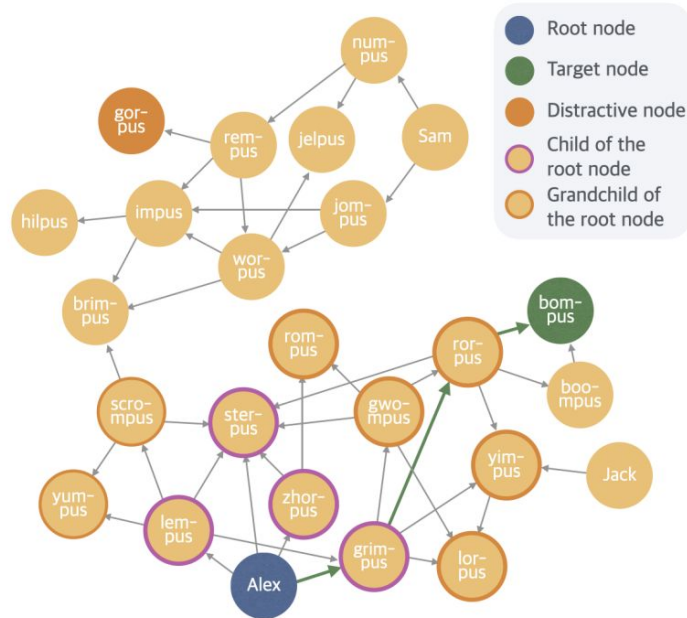
Procedural Question Answering

Method	GSM8k		ProntoQA		ProsQA	
	Acc. (%)	# Tokens	Acc. (%)	# Tokens	Acc. (%)	# Tokens
CoT	42.9 \pm 0.2	25.0	98.8 \pm 0.8	92.5	77.5 \pm 1.9	49.4
No-CoT	16.5 \pm 0.5	2.2	93.8 \pm 0.7	3.0	76.7 \pm 1.0	8.2
iCoT	30.0*	2.2	99.8 \pm 0.3	3.0	98.2 \pm 0.3	8.2
Pause Token	16.4 \pm 1.8	2.2	77.7 \pm 21.0	3.0	75.9 \pm 0.7	8.2
COCONUT (Ours)	34.1 \pm 1.5	8.2	99.8 \pm 0.2	9.0	97.0 \pm 0.3	14.2
- <i>w/o curriculum</i>	14.4 \pm 0.8	8.2	52.4 \pm 0.4	9.0	76.1 \pm 0.2	14.2
- <i>w/o thought</i>	21.6 \pm 0.5	2.3	99.9 \pm 0.1	3.0	95.5 \pm 1.1	8.2
- <i>pause as thought</i>	24.1 \pm 0.7	2.2	100.0 \pm 0.1	3.0	96.6 \pm 0.8	8.2

*a detailed description of the compared models in the original article

Chain of Continuous Thought (Coconut)

Reasoning Experiment Setup



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumps is a yumpus.
Question: **Is Alex a gorpus or bompus?**

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompus.
Alex is a bompus

COCONUT (k=1)

<bot> [Thought] <eot>
Every lempus is a scrompus.
Every scrompus is a brimpus.
Alex is a brimpus ❌

(Wrong Target)

CoT

Alex is a lempus.
Every lempus is a scrompus.
Every scrompus is a yumpus.
Every yumpus is a rempus.
Every rempus is a gorpus.
Alex is a gorpus ❌

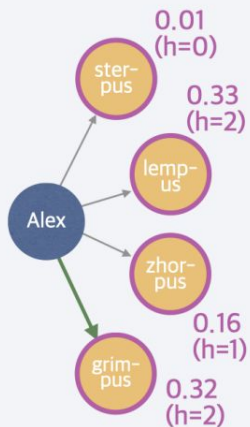
(Hallucination)

COCONUT (k=2)

<bot> [Thought] [Thought] <eot>
Every rorpus is a bompus.
Alex is a bompus ✅

(Correct Path)

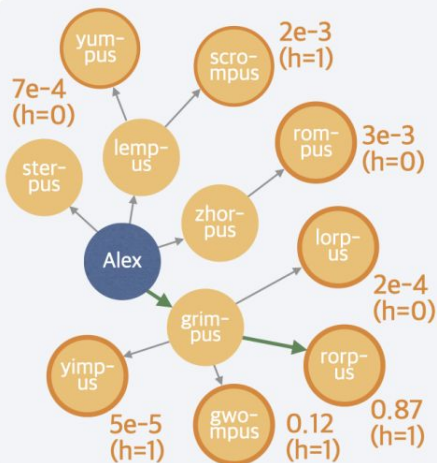
Chain of Continuous Thought (Coconut) Reasoning Experiment Setup



COCONUT (k=1)

<bot> [Thought] <eot>
Every lempus ...

$$\begin{aligned} p(\text{"lempus"}) \\ &= p(\text{"le"})p(\text{"mp"})p(\text{"us"}) \\ &= 0.33 \end{aligned}$$



COCONUT (k=2)

<bot> [Thought] [Thought] <eot>
Every rorpus ...

$$\begin{aligned} p(\text{"rorpus"}) \\ &= p(\text{"ro"})p(\text{"rp"})p(\text{"us"}) \\ &= 0.87 \end{aligned}$$

Chain of Continuous Thought (Coconut)

Reasoning Experiments

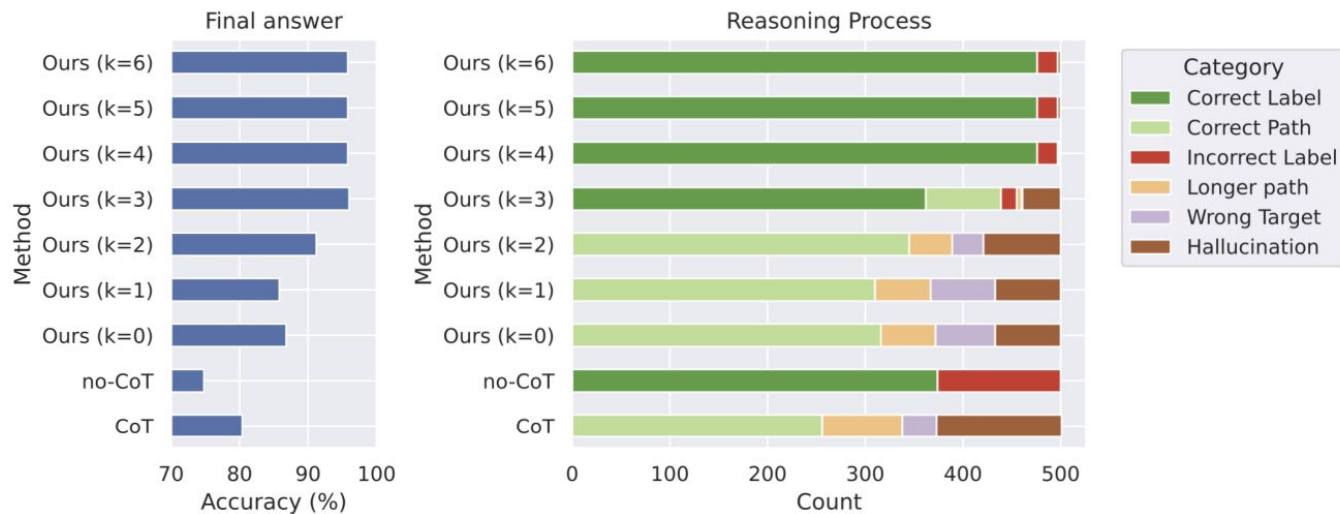


Figure 5 The accuracy of final answer (left) and reasoning process (right) of multiple variants of COCONUT and baselines on ProsQA.

*a detailed description of metrics in the original article

That's all :)

