Basic identities of matrix/vector ops	j.,	Vector norms (beyond euclidean)	The (column) rank of AJ is number of linearly	notice all-but-one minor matrix determinants go to	$-\mathbf{q}_1, \dots, \mathbf{q}_n$ are still eigenvectors of $\underline{\mathbf{A}} = \mathbf{A} = \mathbf{Q} \mathbf{D} \mathbf{Q}^T$	always exists If $n \le m$ [then work with $A^T A \in \mathbb{R}^{n \times n}$]:	$A \approx \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T$
$\frac{(A+B)^T = A^T + B^T}{(AB)^{-1} = B^{-1}A^{-1}} \frac{(AB)^T = B^TA^T}{(A^{-1})^T = (A^T)^{-1}}$	*Notice: $Q_j c_j = \sum_{i=1}^{r} (\mathbf{q}_i \cdot \mathbf{a}_{j+1}) \mathbf{q}_i = \sum_{i=1}^{r} \text{proj}_{\mathbf{q}_i} (\mathbf{a}_{j+1})$, so	-vector norms are such that: x = 0 ⇔ x = 0 , \lambda x = \lambda x , x + y ≤ x + y	independent columns, i.e. rk(A) I.e. its the number of pivots in row-echelon-form	Representing EROs/ECOs as transfor-	(spectral decomposition) -A = QDQ ^T can be interpreted as scaling in direction of	•Obtain eigenvalues $\sigma_1^2 \ge \cdots \ge \sigma_n^2 \ge 0$ of $A^T A$	Cholesky Decomposition
	rewrite as	$\ell_p \mid \text{norms: } \ \mathbf{x}\ _p = (\sum_{i=1}^n \mathbf{x}_i ^p)^{1/p}$	-I.e. its the dimension of the column-space	mation matrices	its eigenvectors:	•Obtain orthonormal eigenvectors $\mathbf{v}_1, \dots, \mathbf{v}_n \in \mathbb{R}^n$ of	Consider positive (semi-)definite $A \in \mathbb{R}^{n \times n}$ Cholesky Decomposition is $A = LL^T$ where L is
For $\underline{A \in \mathbb{R}^{m \times n}}$ $\underline{A_{ij}}$ is the \underline{i} th ROW then \underline{j} th COLUMN	$\mathbf{u}_{j+1} = \mathbf{a}_{j+1} - \sum_{i=1}^{j} (\mathbf{q}_{i} \cdot \mathbf{a}_{j+1}) \mathbf{q}_{i} = \mathbf{a}_{j+1} - \sum_{i=1}^{j} \operatorname{proj}_{\mathbf{q}_{i}} (\mathbf{a}_{j+1})$	$-p = 1 \mid \ \mathbf{x}\ _{1} = \sum_{i=1}^{n} \left \mathbf{x}_{i} \right $	rk(A) = dim(C(A)) -I.e. its the dimension of the image-space	For $A \in \mathbb{R}^{m \times n}$ suppose a sequence of: •EROs transform $A \rightsquigarrow_{EROS} A' \implies$ there is matrix R j.s.t.	Perform a succession of reflections/planar rotations to change coordinate-system	A ^T A (apply normalization e.g. Gram-Schmidt !!!! to	lower-triangular
$(A^T)_{ij} = A_{ji} (AB)_{ij} = A_{i\star} \cdot B_{\star j} = \sum_{k} A_{ik} B_{kj}$	i=1 $i=1•Let a_1,, a_n \in \mathbb{R}^m \mid (m \ge n) be linearly independent,$	$-\underline{p=2} \ddagger \frac{1}{\ \mathbf{x}\ _2} = \sqrt{\sum_{i=1}^n \mathbf{x}_i^2} = \sqrt{\mathbf{x} \cdot \mathbf{x}}$	$rk(A) = dim(im(f_A)) of linear map f_A(x) = Ax $	RA = A'	2.Apply scaling by λ_i to each dimension \mathbf{q}_i 3.Undo those reflections/planar rotations	eigenspaces E_{σ_i} • $V = [v_1 v_n] \in \mathbb{R}^{n \times n}$ is orthogonal so $V^T = V^{-1}$	•For positive semi-definite => always exists, but non-unique
$(Ax)_i = A_{i\star} \cdot \overline{x} = \sum_i A_{ij} x_i x^T y = y^T x = x \cdot y = \sum_i x_i y_i $	i.e. basis of n rdim subspace Un = span{a1,,an}	$-p = \infty_{\Gamma} \ \mathbf{x}\ _{\infty} = \lim_{p \to \infty} \ \mathbf{x}\ _{p} = \max_{1 \le i \le n} \mathbf{x}_{i} $	•The (row) rank of A is number of linearly independent	•ECOs transform A → ECOs A' => there is matrix C s.t. AC = A'	Extension to C ⁿ	$r = rk(A) = no. of strictly + ve \sigma_i$	•For positive-definite => always uniquely exists s.t.
$\mathbf{x}^T \mathbf{A} \mathbf{x} = \sum_i \sum_j A_{ij} \mathbf{x}_i \mathbf{x}_j \mathbf{x} \mathbf{e}_{\mathbf{k}}^T = [0 \dots \mathbf{x} \dots 0] $	-We apply Gram-Schmidt to build ONB $(\mathbf{q}_1,, \mathbf{q}_n) \in \mathbb{R}^m \text{for } U_n \subset \mathbb{R}^m $	•Any two norms in \mathbb{R}^n are equivalent, meaning there	•The row/column ranks are always the same, hence	•Both transform A → EROs•ECOs A' => there are	•Standard inner product: $(x,y) = x^{\dagger} y = \sum_{i} \overline{x_{i}} y_{i}$	•Let $\mathbf{u}_i = \frac{1}{\sigma_i} A \mathbf{v}_i$ then $\mathbf{u}_1, \dots, \mathbf{u}_r \in \mathbb{R}^m$ are orthonormal	diagonals of <u>L</u>]are positive
$e_{k}x^{T} = [0^{T};; x^{T};; 0^{T}]$	$-j=1 \Rightarrow u_1 = a_1$ and $q_1 = \hat{u}_1$, i.e. start of iteration	exist $r>0$; $s>0$ such that: $\forall x \in \mathbb{R}^n$, $r x _a \le x _b \le s x _a$	$rk(A) = dim(C(A)) = dim(R(A)) = dim(C(A^T)) = rk(A^T)$ •Ajis full-rank iff $rk(A) = min(m, n)$, i.e. its as linearly	matrices R, C s.t. RAC = A'	-Conjugate-symmetric: $\langle x, y \rangle = \overline{\langle y, x \rangle}$	(therefore linearly independent)	Finding a Cholesky Decomposition:
Scalar-multiplication + addition distributes over:	$-j=2$ $\Rightarrow u_2 = a_2 - (q_1 \cdot a_2)q_1$ and $q_2 = \hat{u}_2$ etc -Linear independence guarantees that $a_{j+1} \notin U_j$	$\ \mathbf{x}\ _{\infty} \leq \ \mathbf{x}\ _{2} \leq \ \mathbf{x}\ _{1}$	independent as possible	FORWARD: to compute these transformation	•Standard (induced) norm: $ x = \sqrt{\langle x, y \rangle} = \sqrt{x^{\dagger} y}$	-The orthogonal compliment of span $\{u_1,, u_r\}$ =>	Compute <u>LLT</u> and solve <u>A = LLT</u> by matching terms For square roots always pick positive
ocolumn-blocks =>	-For exams: compute $\mathbf{u}_{j+1} = \mathbf{a}_{j+1} - Q_j \mathbf{c}_j$	-Equivalence of ℓ_1 , ℓ_2 and $\ell_{\infty} = \ \mathbf{x}\ _2 \le \sqrt{n} \ \mathbf{x}\ _{\infty}$	Two matrices $\mathbf{A}, \tilde{\mathbf{A}} \in \mathbb{R}^{m \times n}$ are equivalent if there exist	matrices:	•We can diagonalise real matrices in CJwhich lets us diagonalise more matrices than before	$\frac{\operatorname{span}\{\mathbf{u}_1, \dots, \mathbf{u}_r\}^{\perp} = \operatorname{span}\{\mathbf{u}_{r+1}, \dots, \mathbf{u}_m\}}{*\operatorname{Solve for unit-vector }\mathbf{u}_{r+1} \mid \operatorname{s.t. it is orthogonal to}}$	•If there is exact solution then positive-definite
$\lambda A + B = \lambda [A_1 A_C] + [B_1 B_C] = [\lambda A_1 + B_1 \lambda A_C + B_C]$ prow-blocks =>	1. Gather $Q_j = [\mathbf{q}_1 \dots \mathbf{q}_j] \in \mathbb{R}^{m \times j}$	INDICATE IN INTIME INTIME IN INTIME INTIME IN INTIME IN INTIME IN INTIME I	two invertible matrices $P \in \mathbb{R}^{m \times m}$ and $Q \in \mathbb{R}^{n \times n}$	•Start with [I _m A I _n]] i.e. A Jand identity matrices •For every ERO on A J do the same to LHS (i.e. I _m)	Least Square Method	u ₁ ,,u _r	•If there are free variables at the end, then positive semi-definite
$\lambda A + B = \lambda [A_1;; A_r] + [B_1;; B_r] = [\lambda A_1 + B_1;; \lambda A_r + B_r]$	2. Compute $c_j = [\mathbf{q}_1 \cdot \mathbf{a}_{j+1}, \dots, \mathbf{q}_j \cdot \mathbf{a}_{j+1}]^T \in \mathbb{R}^j$	properties:	such that $\mathbf{A} = \mathbf{P} \tilde{\mathbf{A}} \mathbf{Q}^{-1}$ Two matrices $\mathbf{A}, \tilde{\mathbf{A}} \in \mathbb{R}^{n \times n}$ are similar if there exists an	•For every ECO on \underline{A} do the same to RHS (i.e. $\overline{I_n}$) •Once done, you should get $[I_m \mid A \mid I_n] \rightsquigarrow [R \mid \overline{A} \mid C]$	If we are solving $Ax = b$ and $b \notin C(A)$, i.e. no solution,	*Then solve for unit-vector $\underline{\mathbf{u}_{r+2}}$ js.t. it is orthogonal to $\underline{\mathbf{u}_1,,\mathbf{u}_{r+1}}$	–i.e. the decomposition is a solution-set
Matrix-multiplication distributes over: •column-blocks $\Rightarrow AB = A[B_1 B_D] = [AB_1 AB_D]$	3. Compute $Q_j c_j \in \mathbb{R}^m$ and subtract from a_{j+1}	-Translation invariance: $d(x+w,y+w)=d(x,y)$ -Scaling: $d(\lambda x, \lambda y)= \lambda d(x,y) $	invertible matrix $\mathbf{P} \in \mathbb{R}^{n \times n}$ such that $\mathbf{A} = \mathbf{P} \mathbf{A} \mathbf{P}^{-1}$	with $RAC = A'$	then Least Square Method is: •Finding xjwhich minimizes Ax-b 2	*And so on	parameterized on free variables
prow-blocks \Rightarrow AB = $[A_1;; A_D]B = [A_1B;; A_DB]$	Properties: dot-product & norm	Matrix norms	•Similar matrices are equivalent, with Q = P A is diagonalisable iff A is similar to some diagonal	If the sequences of EROs and ECOs were $R_1,, R_{\lambda}$ and	•Recall for $A \in \mathbb{R}^{m \times n}$ we have unique decomposition	$-U = [\mathbf{u}_1 \mid \dots \mid \mathbf{u}_m] \in \mathbb{R}^{m \times m} \text{ is orthogonal so } \underline{U}^T = \underline{U}^{-1}]$ • $S = \operatorname{diag}_{m \times n} (\sigma_1, \dots, \sigma_n) \mid \text{AND DONE}!!$	-e.g. 1 1 1 = LL^T where $L = \begin{bmatrix} 1 & 0 & 0 \\ & & & \end{bmatrix}$, $c \in [0, 1]$
outer-product sum =>	$x^T y = y^T x = x \cdot y = \sum_i x_i y_i \left[x \cdot y = a b \cos x \hat{y} \right]$	-Matrix norms are such that: A = 0 ⇔ A = 0 , \lambda = \lambda A + B	matrix D	C ₁ ,,C _µ respectively	for any $b \in \mathbb{R}^m$ $b = b_i + b_k$	If $m < n$ then let $B = A^T$	1 1 2 1 c √1-c ²
$AB = [A_1 A_p][B_1;; B_p] = \sum_{i=1}^{p} A_i B_i$	$x \cdot y = y \cdot x x \cdot (y + z) = x \cdot y + x \cdot z \alpha x \cdot y = \alpha (x \cdot y)$	-Matrices F ^{m×n} are a vector space so matrix norms	Properties of determinants	$\bullet R = R_{\lambda} \cdots R_{1}$ and $C = C_{1} \cdots C_{\mu}$ so	-where $\mathbf{b}_i \in C(A)$ and $\mathbf{b}_R \in \ker(A^T)$ $\ \mathbf{A}\mathbf{x} - \mathbf{b}\ _2$ is minimized $\iff \ \mathbf{A}\mathbf{x} - \mathbf{b}_i\ _2 = 0 \iff \mathbf{A}\mathbf{x} = \mathbf{b}_i$	•apply above method to \underline{B}] \Rightarrow $\underline{B} = A^T = USV^T$ • $A = B^T = VS^TU^T$	If <u>A = LLT</u> you can use <u>forward/backward substitution</u> to solve equations
oe.g. for $A = [a_1 a_n] B = [b_1;; b_n] \Rightarrow AB = \sum_i a_i b_i$ Projection: definition & properties	$\frac{x \cdot x = x ^2 = 0 \iff x = 0}{\text{for } \underline{x \neq 0}, \text{ we have } x \cdot y = x \cdot z \implies x \cdot (y - z) = 0}$	are vector norms, all results apply •Sub-multiplicative matrix norm (assumed by default)	*Consider $\underline{A} \in \mathbb{R}^{n \times n}$, then $A_{ij}' \in \mathbb{R}^{(n-1) \times (n-1)}$ the	$\frac{(R_{\lambda} \cdots R_{1})A(C_{1} \cdots C_{\mu}) = A'}{1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 +$		Tricks: Computing orthonormal	•For $Ax = b$] \Longrightarrow let $y = L^T x$
•A projection $\pi: V \to V$ is a endomorphism such that	$ x \cdot y \le x y $ (Cauchy-Schwartz inequality)	is also such that AB ≤ A B	(i,j) minor matrix of A obtained by deleting i th row and j th column from A	$R_{\lambda}^{-1} = R_{1}^{-1} \cdots R_{\lambda}^{-1}$ and $C_{\mu}^{-1} = C_{\mu}^{-1} \cdots C_{1}^{-1}$, where	A ^T Ax=A ^T b is the normal equation which gives solution to least square problem:	vector-set extensions	•Solve Ly = b] by forward substitution to find y
<u>поп</u> = п _J i.e. it leaves its image unchanged (its idempotent)	$ u+v ^2 + u-v ^2 = 2 u ^2 + 2 v ^2$ (parallelogram law)	•Common matrix norms, for some $\underline{\mathbf{A}} \in \mathbb{R}^{m \times n}$ } $-\ \mathbf{A}\ _1 = \max_j \ \mathbf{A}_{\star j}\ _1$	Then we define determinant of \underline{A} i.e. $\underline{det(A)} = A $ as	R_i^{-1}, C_j^{-1} are inverse EROs/ECOs respectively	$\ A\mathbf{x} - \mathbf{b}\ _2$ is minimized $\iff A\mathbf{x} = \mathbf{b}_i \iff A^T A\mathbf{x} = A^T \mathbf{b}$	You have orthonormal vectors $\mathbf{u}_1,, \mathbf{u}_r \in \mathbb{R}^m$ \Rightarrow need to extend to orthonormal vectors $\mathbf{u}_1,, \mathbf{u}_m \in \mathbb{R}^m$	•Solve $L^T x = y$ by backward substitution to find \underline{x}
•A square matrix P such that $P^2 = P$ is called a	$\frac{\ u+v\ \le \ u\ + \ v\ \text{(triangle inequality)}}{u \perp v \iff \ u+v\ ^2 = \ u\ ^2 + \ v\ ^2 \text{(pythagorean)}}$	$-\ \mathbf{A}\ _2 = \sigma_1(\mathbf{A}) \text{ [i.e. largest singular value of } \mathbf{A}$	$-\det(A) = \sum_{i=1}^{n} (-1)^{i+k} A_{ik} \det(A_{ik}')$, i.e. expansion along	BACKWARD: once $R_1,, R_{\lambda}$ and $C_1,, C_{\mu}$ for which	Linear Regression	Special case => two 3D vectors => use cross-product =>	For <u>n = 3</u>]=> L = l ₂₁ l ₂₂ 0
projection matrix —It is called an orthogonal projection matrix if	theorem)	(sauare-root of largest eigenvalue of A ^T A or AA ^T	k=1 i th row *(for any i)	RAC = A' are known, starting with $[I_m \mid A \mid I_n]$	•Let $y = f(t) = \sum_{j=1}^{n} s_j f_j(t)$ be a mathematical model, where f : lare pasis functions and s : lare parameters	<u>a×b⊥a,b</u>	[[131 132 133]]
$P^2 = P = P^{\dagger}$ (conjugate-transpose)	$\ \mathbf{c}\ ^2 = \ a\ ^2 + \ b\ ^2 - 2\ a\ \ b\ \cos b\hat{a}$ (law of cosines)	$-\ \mathbf{A}\ _{\infty} = \max_{i} \ \mathbf{A}_{i\star}\ _{1}$ note that $\ \mathbf{A}\ _{1} = \ \mathbf{A}^{T}\ _{\infty}$	$-\det(A) = \sum_{i=1}^{n} (-1)^{k+j} A_{kj} \det(A_{kj}')$ i.e. expansion along	•For $\underline{i} = 1 \rightarrow \lambda$ perform $R_{\underline{i}}$ on \underline{A} , perform $R_{\lambda-\underline{i}+1}^{-1}$ on LHS	where f_j are basis functions and s_j are parameters •Let (t_i, y_i) , $1 \le i \le m, m \gg n$ be a set of observations,	Extension via standard basis $I_m = [e_1 e_m]$ using	$LL^T = \begin{bmatrix} l_1^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{11}l_{21} & l_{21}^2 * l_{22}^2 & l_{21}l_{31} * l_{22}l_{32} \end{bmatrix}$
-Eigenvalues of a projection matrix must be 0 or 1 •Because π: V → V is a linear map , its image space	Transformation matrix & linear maps For linear map $f: \mathbb{R}^n \to \mathbb{R}^m$, ordered bases	-Frobenius norm: $\ \mathbf{A}\ _F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \mathbf{A}_{ij} ^2}$	R=1	(i.e. I _m) •For i = 1 → µ perform C: on Al perform C ⁻¹ on	and $t, y \in \mathbb{R}^m$ are vectors representing those	(tweaked) GS: -Choose candidate vector: just work through	$\begin{bmatrix} l_{11}l_{31} & l_{21}l_{31} + l_{22}l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$
$U = im(\pi)$ and null space $W = ker(\pi)$ are subspaces of V	$(\mathbf{b}_1,, \mathbf{b}_n) \in \mathbb{R}^n$ and $(\mathbf{c}_1,, \mathbf{c}_m) \in \mathbb{R}^m$	V i=1 j=1	j th column (for any j) •When det(A) = 0 we call A a singular matrix	•For $\underline{j=1 \rightarrow \mu}$ perform $\underline{C_j}$ on $\underline{A_j}$ perform $\underline{C_{\mu-j+1}^{-1}}$ on RHS (i.e. I_n)	observations $-f_j(t) = [f_j(t_1),, f_j(t_m)]^T$ is transformed vector	e1 em isequentially starting from e1 i=> denote	Forward/backward substitution
$-\pi_J$ is the identity operator on U -The linear map $\pi^* = I_V - \pi$ is also a projection with	• $A = \mathbf{F}_{CB} \in \mathbb{R}^{m \times n}$ is the transformation-matrix of \underline{f} w.r.t to bases \underline{B} and \underline{C}	•A matrix norm $\ \cdot\ $ on $\mathbb{R}^{m\times n}$ is consistent with the vector norms $\ \cdot\ _a$ on \mathbb{R}^n and $\ \cdot\ _b$ on \mathbb{R}^m if	Common determinants	•You should get [I _m A I _n] → [R ⁻¹ A' C ⁻¹] with	$-A = [f_1(\mathbf{t}) \dots f_n(\mathbf{t})] \in \mathbb{R}^{m \times n}$ is a matrix of columns	the current candidate e_k •Orthogonalize: Starting from $j = r$ going to $j = m$ with	•Forward substitution: for lower-triangular
$W = \operatorname{im}(\pi^*) = \ker(\pi) \operatorname{and} U = \ker(\pi^*) = \operatorname{im}(\pi)$ i.e. they	$f(\mathbf{b}_j) = \sum_{i=1}^m A_{ij} c_i$ \rightarrow each \mathbf{b}_j basis gets mapped to a	-for all $\underline{A} \in \mathbb{R}^{m \times n}$ and $\underline{x} \in \mathbb{R}^n$ \Rightarrow $\ Ax\ _b \le \ A\ \ x\ _a$	-For <u>n = 1</u> , det(A) = A ₁₁ -For <u>n = 2</u> , det(A) = A ₁₁ A ₂₂ -A ₁₂ A ₂₁	A=R ⁻¹ A'C ⁻¹	$-\mathbf{z} = [s_1, \dots, s_n]^T$ is vector of parameters	each iteration ⇒ with current orthonormal vectors	L= 1 %.
swapped *∏is a projection along W Jonto U J	linear combination of $\sum_i a_i c_i$ bases	-If $a = b$, $\ \cdot\ $ is compatible with $\ \cdot\ _a$ -Frobenius norm is consistent with ℓ_2 norm \Rightarrow	-det(I _n) = 1	You can mix-and-match the forward/backward modes	•Then we get equation Az = y => minimizing Az - y 2 is the solution to Linear Regression	u ₁ ,,u _j -Compute	$\frac{\lfloor \ell_{n,1} & \dots & \ell_{n,n} \rfloor}{-\text{For } \underline{L} \mathbf{x} = b \rfloor, \text{ just solve}} \text{ the first row}$
π [] is a projection along U onto W	•If f ⁻¹ exists (i.e. its bijective and m = n) then	Av ₂ ≤ A _F v ₂	•Multi-linearity in columns/rows: if $A = [a_1 a_i a_n] = [a_1 \lambda x_i + \mu y_i a_n]$ [then	•i.e. inverse operations in inverse order for one, and	-So applying LSM to Az = y is precisely what Linear	$\mathbf{w}_{i+1} = \mathbf{e}_k - \sum_{i=1}^{j} (\mathbf{e}_k \cdot \mathbf{u}_i) \mathbf{u}_i = \mathbf{e}_k - \sum_{i=1}^{j} (\mathbf{u}_i)_k \mathbf{u}_i$	$\ell_{1,1} x_1 = b_1 \implies x_1 = \frac{b_1}{\ell_{1,1}}$ and substitute down
π is the identity operator on <u>W</u> -V can be decomposed as V = U⊕W meaning every	$\frac{(\mathbf{F}_{CB})^{-1} = \mathbf{F}^{-1}_{BC}}{\text{transformation-matrix of } f^{-1}}$ is the	•For a vector norm $\ \cdot\ $ on \mathbb{R}^n , the subordinate	$\det(A) = \lambda \det\left(\left[a_1 \mid \dots \mid x_i \mid \dots \mid a_n \mid\right)\right)$	operations in normal order for the other •e.g. you can do $[I_m \mid A \mid I_n] \rightsquigarrow [R^{-1} \mid A' \mid C]$ to get	Regression is -We can use normal equations for this =>	=e _k -U _j c _j	Then solve the second row
vector $\underline{x \in V}$ Jcan be uniquely written as $\underline{x = u + w}$		matrix norm $\ \cdot \ $ on $\mathbb{R}^{m \times n}$ is $\ \mathbf{A} \ = \max \{ \ \mathbf{A} \mathbf{x} \ : \mathbf{x} \in \mathbb{R}^n, \ \mathbf{x} \ = 1 \} $	+ µ det ([a ₁ y _j a _n])	$AC = R^{-1}A'$ => useful for LU factorization	$\ Az - y\ _2$ is minimized $\iff A^T Az = A^T y$	-Where $U_j = [\mathbf{u}_1 \mid \dots \mid \mathbf{u}_j] \mid \text{and } \mathbf{c}_j = [(\mathbf{u}_1)_k, \dots, (\mathbf{u}_j)_k]^T$	$\ell_{2,1} x_1 + \ell_{2,2} x_2 = b_2 \implies x_2 = \frac{b_2 - \ell_{2,1} x_1}{\ell_{2,2}}$ and
$\star \underline{u} \in \underline{U} \text{ [and } \underline{u} = \pi(x) \text{]}$ $\star \underline{w} \in \underline{W} \text{ [and } \underline{w} = x - \pi(x) = (I_V - \pi)(x) = \pi^*(x) \text{]}$	The transformation matrix of the identity map is called change-in-basis matrix	$= \max \left\{ \frac{\ \mathbf{A}\mathbf{x}\ }{\ \mathbf{x}\ } : \mathbf{x} \in \mathbb{R}^n, \mathbf{x} \neq 0 \right\}$	-And the exact same linearity property for rows -Immediately leads to: $ A = A^T \lambda A = \lambda^n A \lambda A $	Eigen-values/vectors	•Solution to normal equations unique iff AJis full-rank, i.e. it has linearly-independent columns	-NOTE: $\mathbf{e}_k \cdot \mathbf{u}_i = (\mathbf{u}_i)_k$ i.e. k th component of \mathbf{u}_i	substitute down
•An orthogonal projection further satisfies <u>U⊥W</u>	•The identity matrix Im represents id pm w.r.t. the	$= \max\{\ \mathbf{A}\mathbf{x}\ : \mathbf{x} \in \mathbb{R}^n, \ \mathbf{x}\ \le 1\}$	$ AB = BA = A B (for any B \in \mathbb{R}^{n \times n})$	•Consider $A \in \mathbb{R}^{n \times n}$ non-zero $\mathbf{x} \in \mathbb{C}^n$ is an eigenvector with eigenvalue $\lambda \in \mathbb{C}$ for A if $A\mathbf{x} = \lambda \mathbf{x}$	Positive (semi-)definite matrices	$ - f w_{j+1} = 0 then e_k \in span\{u_1,, u_j\} \Rightarrow discard $ $ w_{j+1} choose next candidate e_{k+1} try this step $	and so on until all x _i pare solved
i.e. the image and kernel of $\underline{\pi}$ jare orthogonal subspaces	standard basis $E_m = \langle e_1,, e_m \rangle \Rightarrow \overline{i.e. I_m} = I_{EE}$ •If $B = \langle b_1,, b_m \rangle$ [is a basis of \mathbb{R}^m], then	•Vector norms are compatible with their subordinate	 Alternating: if any two columns of Alare equal (or any two rows of Alare equal), then A = 0 (its singular) 	$-\text{If } \underline{Ax = \lambda x} \text{ [then } \underline{A(kx) = \lambda(kx)] for } \underline{k \neq 0} \text{ [i.e. } \underline{kx} \text{] is also an}$	Consider symmetric $\underline{A} \in \mathbb{R}^{n \times n}$, i.e. $\underline{A} = A^T$	again	•Backward substitution: for upper-triangular [u _{1,1} u _{1,n}]
-infact they are each other's orthogonal compliments , i.e. $U^{\perp} = W$, $W^{\perp} = U (because finite-dimensional)$		matrix norms •For $p = 1, 2, \infty$ matrix norm $\ \cdot\ _p$ is subordinate to	-Immediately from this (and multi-linearity) => if	eigenvector -A has at most n distinct eigenvalues	AJis positive-definite iff x ^T Ax>0 for all x ≠0] •AJis positive-definite iff all its eigenvalues are strictly	•Normalize: $\mathbf{w}_{j+1} \neq 0$ so compute unit vector	U =
vectorspaces)	to $E_{IBE} = (I_{EB})^{-1}$, so $\Rightarrow F_{CB} = I_{CE}F_{EE}I_{EB}$	the vector norm $\ \cdot\ _p$ (and thus compatible with)	columns (or rows) are linearly-dependent (some are linear combinations of others) then A = 0	•The set of all eigenvectors associated with eigenvalue	positive	$\mathbf{u}_{j+1} = \hat{\mathbf{w}}_{j+1}$ •Repeat: keep repeating the above steps, now with	$ \begin{array}{c c} 0 & u_{n,n} \\ \hline -For \underline{Ux = b} \text{ just } \textbf{solve} \text{ the last row} \end{array} $
-so we have $\pi(x) \cdot y = \pi(x) \cdot \pi(y) = x \cdot \pi(y)$ -or equivalently, $\pi(x) \cdot (y - \pi(y)) = (x - \pi(x)) \cdot \pi(y) = 0$		Properties of matrices	-Stated in other terms \Rightarrow rk(A) < n \iff A = 0 <=>	λ is called eigenspace E_{λ} of A $= E_{\lambda} = \ker(A - \lambda I)$	•AJis positive-definite => all its diagonals are strictly positive	new orthonormal vectors $\mathbf{u}_1,, \mathbf{u}_{j+1}$	$u_{n,n}x_n = b_n \implies x_n = \frac{b_n}{u_{n,n}}$ and substitute up
	Dot-product uniquely determines a vector w.r.t. to basis	Consider $\underline{A \in \mathbb{R}^{m \times n}}$ If $Ax = x$ for all x then $A = I$	$RREF(A) \neq I_n \iff A = 0$ (reduced row-echelon-form) $\Rightarrow C(A) \neq R^n \iff A = 0$ (column-space)	−The geometric multiplicity of λ is	•AJis positive-definite => $\max(A_{ij}, A_{jj}) > A_{ij} $	SVD Application: Principal Compo-	Then solve the second-to-last row
•By Cauchy–Schwarz inequality we have $\ \pi(x)\ \le \ x\ \ $ •The orthogonal projection onto the line containing	•If $a_i = x \cdot b_i$; $x = \sum_i a_i b_i$, we call \underline{a} the	For square AI, the trace of AI is the sum if its diagonals,	-For more equivalence to the above, see invertible	$\frac{\dim(E_{\lambda}) = \dim(\ker(A - \lambda I))}{\text{The spectrum } Sp(A) = \{\lambda_1, \dots, \lambda_n\} \text{ of } \underline{A} \text{ is the set of all }$	i.e. strictly larger coefficient on the diagonals •A is positive-definite => all upper-left submatrices are	nent Analysis (PCA)	$u_{n-1,n-1} \times_{n-1} + u_{n-1,n} \times_n = b_{n-1}$
vector \underline{u}_{j} is $\underline{proj}_{u} = \hat{u}\hat{u}^{T}$, i.e. $\underline{proj}_{u}(v) = \frac{u \cdot v}{u \cdot u}u$; $\hat{u} = \frac{u}{\ u\ }$	coordinate-vector of x w.r.t. to B Rank-nullity theorem:	i.e. tr(A)	matrix theorem •Interaction with EROs/ECOs:	eigenvalues of A •The characteristic polynomial of A is	also positive-definite	Assume $\underline{A}_{uncentered} \in \mathbb{R}^{m \times n}$ represent \underline{m}_{j} samples of \underline{n}_{j} -dimensional data (with $\underline{m} \ge n$)	$\Rightarrow x_{n-1} = \frac{b_{n-1} - u_{n-1,n-1} x_{n-1}}{u_{n-1,n}}$ and substitute up
-A special case of $\pi(x) \cdot (y - \pi(y)) = 0$ is $u \cdot (v - \text{proj}_{u} v) = 0$	$\dim(\operatorname{im}(f)) + \dim(\ker(f)) = \operatorname{rk}(A) + \dim(\ker(A)) = n$	\underline{A} Jis symmetric iff $\underline{A} = \underline{A}^T$, \underline{A} Jis Hermitian, iff $\underline{A} = \underline{A}^{\dagger}$, i.e.	-Swapping rows/columns flips the sign	P(λ) = $ A - \lambda I = \sum_{i=0}^{n} a_i \lambda^i$	Sylvester's criterion: Alis positive-definite iff all upper-left submatrices have strictly positive	Data centering: subtract mean of each column from that column's elements	and so on until all x; are solved
since $\operatorname{proj}_{U}(u) = u$ •If $U \subseteq \mathbb{R}^{n}$ is a k -dimensional subspace with	f is injective/monomorphism iff $ker(f) = \{0\}$ iff A is full-rank	its equal to its conjugate-transpose •AA ^T and A ^T A are symmetric (and positive	-Scaling a row/column by λ≠0]will scale the determinant by λ](by multi-linearity)	$-\underline{a_0} = A \int_{A=0}^{A} \underline{a_{n-1}} = (-1)^{n-1} \operatorname{tr}(A) \int_{A=0}^{A} \underline{a_n} = (-1)^n \int_{A=0}^{A} \underline{a_n} = (-1)$	determinant	•Let the resulting matrix be $\underline{A \in \mathbb{R}^{m \times n}}$, who's columns	Thin QR Decomposition w/ Gram-
orthonormal basis (ONB) $(\mathbf{u}_1, \dots, \mathbf{u}_R) \in \mathbb{R}^m$	Orthogonality concepts	semi-definite)	*Remember to scale by λ^{-1} to maintain equality, i.e.	- <u>λ∈C</u> is eigenvalue of <u>A</u> iff <u>λ</u> is a root of <u>P(λ)</u> -The algebraic multiplicity of <u>λ</u> is the number of	AJis positive semi-definite iff $x^T Ax \ge 0$ for all x_J	have mean zero PCA is done on centered data-matrices like A	Schmidt (GS) Consider full-rank $A = [a_1 a_n] \in \mathbb{R}^{m \times n} (m \ge n)$, i.e.
-Let $\mathbf{U} = [\mathbf{u}_1 \mid \mid \mathbf{u}_k] \in \mathbb{R}^{m \times k}$ matrix	• <u>u ⊥ v ⇔ u · v = 0</u> } i.e. <u>u</u> jand <u>v</u> jare orthogonal • <u>u</u> jand <u>v</u> jare orthonormal iff u ⊥ v, u = 1 = v	•For real matrices, Hermitian/symmetric are equivalent conditions	$\det(A) = \lambda^{-1} \det([a_1 \lambda a_i a_n])$ -Invariant under addition of rows/columns	times it is repeated as root of $P(\lambda)$	•AJ is positive semi-definite iff all its eigenvalues are non-negative	*SVD exists i.e. A = USV ^T and r = rk(A)	$\mathbf{a}_1, \dots, \mathbf{a}_n \in \mathbb{R}^m$ are linearly independent
-Orthogonal projection onto U is $\pi_U = \mathbf{U}\mathbf{U}^T$	$\bullet A \in \mathbb{R}^{n \times n}$ is orthogonal iff $A^{-1} = A^T$	•Every eigenvalue λ_i of Hermitian matrices is real	•Link to invertable matrices => A ⁻¹ = A ⁻¹ which	-1]≤ geometric multiplicity of \(\lambda\) ≤ algebraic multiplicity of \(\lambda\)	•AJ is positive semi-definite => all its diagonals are	•Let $A = [\mathbf{r}_1;; \mathbf{r}_m]$ be rows $\mathbf{r}_1,, \mathbf{r}_m \in \mathbb{R}^n$ \Rightarrow each row corresponds to a sample	•Apply $\underline{GS} \underline{q_1,, q_n} \leftarrow GS(a_1,, a_n)$ to build ONB $(\underline{q_1,, q_n}) \in \mathbb{R}^m for C(A) $
-Can be rewritten as $\pi_U(v) = \sum_i (\mathbf{u}_i \cdot v) \mathbf{u}_i$	-Columns of $A = [a_1 a_n]$ are orthonormal basis (ONB) $C = \langle a_1,, a_n \rangle \in \mathbb{R}^n$ so $A = I_{EC}$ is	-geometric multiplicity of λ_i = geometric multiplicity of λ_i	means A is invertible $\iff A \neq 0$, i.e. singular matrices are not invertible	•Let $\lambda_1,, \lambda_n \in \mathbb{C}$ be (potentially non-distinct)	non-negative •Ajis positive semi-definite => $\max(A_{ii}, A_{ji}) \ge A_{ji} $	•Let $A = [c_1 c_n]$ be columns $c_1,, c_n \in \mathbb{R}^m$ \Rightarrow each	•For exams: more efficient to compute as
$-\operatorname{If}\left(\underbrace{\mathbf{u}_{1},,\mathbf{u}_{k}}\right)$ is not orthonormal , then "normalizing	change-in-basis matrix	-eigenvectors x_1, x_2 associated to distinct eigenvalues λ_1, λ_2 are orthogonal , i.e. $x_1 \perp x_2$	•For block-matrices:	eigenvalues of \underline{A} with $\underline{x_1,, x_n \in \mathbb{C}^n}$ their eigenvectors	i.e. no coefficient larger than on the diagonals	column corresponds to one dimension of the data Let X ₁ ,,X _n be random variables where each X _i	$\frac{\mathbf{u}_{j+1} = \mathbf{a}_{j+1} - Q_j \mathbf{c}_j}{\mathbf{u}_{j+1}}$
factor" $(\underline{\mathbf{U}^T \mathbf{U}})^{-1}$ is added $\Rightarrow \pi_U = \mathbf{U}(\underline{\mathbf{U}^T \mathbf{U}})^{-1}\underline{\mathbf{U}^T}$ *For line subspaces $U = \text{span}\{u\}$, we have	-Orthogonal transformations preserve lengths/angles/distances \Rightarrow $ Ax _2 = x _2$, $AxAy = xy$		$-\det\begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = \det(A) \det(B) = \det\begin{pmatrix} A & 0 \\ C & D \end{pmatrix}$	$-\operatorname{tr}(A) = \sum_{i} \lambda_{i}$ and $\operatorname{det}(A) = \prod_{i} \lambda_{ij}$	•AJ is positive semi-definite => all upper-left submatrices are also positive semi-definite	corresponds to column c; •i.e. each X; corresponds to i th component of data	1. Gather $Q_j = [\mathbf{q}_1 \mid \dots \mid \mathbf{q}_j] \in \mathbb{R}^{m \times j}$ all-at-once
$(\mathbf{U}^T \mathbf{U})^{-1} = (u^T u)^{-1} = 1/(u \cdot u) = 1/ u $	*Therefore can be seen as a succession of reflections	AJis triangular iff all entries above (lower-triangular) or below (upper-triangular) the main diagonal are zero	$-\frac{\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A) \det(D - CA^{-1}B)}{\det(A) \det(D - CA^{-1}B)} \text{ if } \underline{A} \text{ jor } \underline{D} \text{ jare}$	-AJis diagonalisable iff there exist a basis of R ⁿ consisting of x₁,,x _n	•Alis positive semi-definite => it has a Cholesky	•i.e. each x_i corresponds to i the component of data •i.e. random vector $X = [X_1,, X_n]^T$ models the data	2. Compute $\mathbf{c}_j = [\mathbf{q}_1 \cdot \mathbf{a}_{j+1}, \dots, \mathbf{q}_j \cdot \mathbf{a}_{j+1}]^T \in \mathbb{R}^j$ all-at-once
Gram-Schmidt (GS) to gen. ONB from	and planar rotations -det(A) = 1 or det(A) = -1, and all eigenvalues of Alare	•Determinant $\Rightarrow A = \prod_i a_{ii}$, i.e. the product of	$= \det(D) \det(A - BD^{-1}C)$ if Ajor Djare	-AJis diagonalisable iff r _i = g _i where	Decomposition	r ₁ ,,r _m	3. Compute $Q_j c_j \in \mathbb{R}^m$ and subtract from a_{j+1}
lin. ind. vectors Gram-Schmidt is iterative projection => we use	s.t. \(\lambda\right) = 1	diagonal elements	invertible, respectively	r_i = geometric multiplicity of λ_i and g_i = geometric multiplicity of λ_i	For any $M \in \mathbb{R}^{m \times n}$, MM^T and M^TM are symmetric and	•Co-variance matrix of \underline{X} is $Cov(A) = \frac{1}{m-1} A^T A =>$	all-at-once
current j dim subspace, to get next (j+1) dim	• <u>A</u> ∈ $\mathbb{R}^{m \times n}$ is semi-orthogonal iff $A^T A = I$ or $AA^T = I$ −If $n > m$ then all m prows are orthonormal vectors	A Jis diagonal iff $A_{ij} = 0, i \neq j$ i.e. if all off-diagonal	•Sylvester's determinant theorem: det (I _m +AB) = det (I _n +BA)	-Eigenvalues of A^k are $\lambda_1,, \lambda_n$	positive semi-definite Singular Value Decomposition (SVD) &	$(A^T A)_{ij} = (A^T A)_{ji} = \text{Cov}(X_i, X_j)$	•Can now rewrite $\underline{\mathbf{a}_j = \sum_{i=1}^{J} (\mathbf{q}_i \cdot \mathbf{a}_j) \mathbf{q}_i = Q_j \mathbf{c}_j}$
subspace -Assume orthonormal basis (ONB) $(\mathbf{q}_1,, \mathbf{q}_i) \in \mathbb{R}^m$	-If m > n then all n columns are orthonormal vectors	entries are zero •Written as	•Matrix determinant lemma:	•Let P = [x ₁ x _n] , then	Singular Values	v ₁ ,, v _r (columns of <u>V</u>)) are principal axes of <u>A</u>]	Choose $\mathbf{Q} = Q_n = [\mathbf{q}_1 \dots \mathbf{q}_n] \in \mathbb{R}^{m \times n}$, notice its
for j -dim subspace $U_j \subset \mathbb{R}^m$	• $U \perp V \subset \mathbb{R}^n \iff \underline{\mathbf{u}} \cdot \underline{\mathbf{v}} = 0$ for all $\underline{\mathbf{u}} \in U, \underline{\mathbf{v}} \in V$, i.e. they are orthogonal subspaces	$\operatorname{diag}_{m \times n}(\mathbf{a}) = \operatorname{diag}_{m \times n}(a_1, \dots, a_p), p = \min(m, n)$ where	$-\det(\mathbf{A} + \mathbf{u}\mathbf{v}^T) = (1 + \mathbf{v}^T \mathbf{A}^{-1}\mathbf{u})\det(\mathbf{A})$	$AP = [\lambda_1 \mathbf{x}_1 \dots \lambda_n \mathbf{x}_n] = [\mathbf{x}_1 \dots \mathbf{x}_n] \operatorname{diag}(\lambda_1, \dots, \lambda_n) = PD$ $\Rightarrow \text{if } P^{-1} \text{ exists then}$	Singular Value Decomposition of $A \in \mathbb{R}^{m \times n}$ is any decomposition of the form $A = USV^{T}$, where	Let $\underline{\mathbf{w} \in \mathbb{R}^n}$ be some unit-vector \Rightarrow let $\alpha_j = \mathbf{r}_j \cdot \mathbf{w}$ be the	semi-orthogonal since $Q^TQ = I_n$
*Let $Q_j = [\mathbf{q}_1 \mid \mid \mathbf{q}_j] \in \mathbb{R}^{m \times j}$ be the matrix	•Orthogonal compliment of $U \subset \mathbb{R}^n$ is the subspace	$\mathbf{a} = [a_1, \dots, a_p]^T \in \mathbb{R}^p$ diagonal entries of $\underline{\mathbf{A}}$	$-\det(\mathbf{A} + \mathbf{U}\mathbf{V}^T) = \det(\mathbf{I}_m + \mathbf{V}^T\mathbf{A}^{-1}\mathbf{U})\det(\mathbf{A})$	-A=PDP-1 i.e. Ajis diagonalisable	•Orthogonal $U = [\mathbf{u}_1 \mid \mid \mathbf{u}_m] \in \mathbb{R}^{m \times m}$ and	projection/coordinate of sample r	•Notice \Rightarrow $\mathbf{a}_j = Q_j \mathbf{c}_j = \mathbf{Q}[\mathbf{q}_1 \cdot \mathbf{a}_j,, \mathbf{q}_j \cdot \mathbf{a}_j, 0,, 0]^T = \mathbf{Q}\mathbf{r}_j$
$*P_j = Q_j Q_j^T$ is orthogonal projection onto U_j	$U^{\perp} = \{x \in \mathbb{R}^{n} \mid \forall y \in \mathbb{R}^{n} : x \perp y\}$ $= \{x \in \mathbb{R}^{n} \mid \forall y \in \mathbb{R}^{n} : x \le x + y \}$	•For $x \in \mathbb{R}^n$ $Ax = \operatorname{diag}_{m \times n}(a_1,, a_p)[x_1 x_n]^T$ (if	$\det \left(\mathbf{A} + \mathbf{U} \mathbf{W} \mathbf{V}^{T}\right) = \det \left(\mathbf{W}^{-1} + \mathbf{V}^{T} \mathbf{A}^{-1} \mathbf{U}\right) \det(\mathbf{W}) \det(\mathbf{A})$	$-P = I_{EB}$ is change-in-basis matrix for basis $B = (\mathbf{x}_1,, \mathbf{x}_n)$ of eigenvectors	$V = [\mathbf{v}_1 \mid \mid \mathbf{v}_n] \in \mathbb{R}^{n \times n}$	•Variance (Bessel's correction) of $\alpha_1,, \alpha_m$ is	Let $R = [r_1 r_n] \in \mathbb{R}^{n \times n}$ =>
$*P_{\perp j} = I_m - Q_j Q_j^T$ is orthogonal projection onto	$-\mathbb{R}^n = U \oplus U^{\perp}$ and $(U^{\perp})^{\perp} = U$	=[a ₁ x ₁ a _p x _p 00]' ∈ R'''	Tricks for computing determinant	-If A = F _{EE} is transformation-matrix of linear map f	$\frac{-S = \operatorname{diag}_{m \times n}(\sigma_1, \dots, \sigma_p)}{\sigma_1 \ge \dots \ge \sigma_p \ge 0}$ where $p = \min(m, n)$ and	$Var_{\mathbf{W}} = \frac{1}{m-1} \sum_{j} \alpha_{j}^{2} = \frac{1}{m-1} \mathbf{w}^{T} \left(\sum_{j} \mathbf{r}_{j}^{T} \mathbf{r}_{j} \right) \mathbf{w}$	$A = QR = Q$ $\begin{bmatrix} q_1' a_1 & \dots & q_1' a_n \\ & \ddots & \vdots \end{bmatrix}$ notice its
$\left(U_{j}\right)^{\perp}$ (orthogonal compliment)	$-\frac{U \perp V \iff U^{\perp} = V}{-Y \subseteq X} \text{ and } \text{vice-versa}$ $-\frac{V \subseteq X}{-Y \subseteq X} \Rightarrow X^{\perp} \subseteq Y^{\perp} \text{ and } X \cap X^{\perp} = \{0\}$	$p = m \text{ those tail-zeros don't exist)}$ $diag_{m \times n}(\mathbf{a}) * diag_{m \times n}(\mathbf{b}) = diag_{m \times n}(\mathbf{a} * \mathbf{b})$	•If block-triangular matrix then apply	then $\mathbf{F}_{EE} = \mathbf{I}_{EB} \mathbf{F}_{BB} \mathbf{I}_{BE}$ • Spectral theorem: if A is Hermitian then P^{-1} exists:	• $\sigma_1,, \sigma_p$ are singular values of A _J .	$= \frac{1}{m-1} \mathbf{w}^T A^T A \mathbf{w}$ • First (principal) axis defined =>	$\begin{bmatrix} 0 & \mathbf{q}_n^T \mathbf{a}_n \end{bmatrix}$
-Uniquely decompose next U _j ∌ a _{j+1} = v _{j+1} + u _{j+1}	-Any x ∈ R ⁿ can be uniquely decomposed into	•Consider diag $_{n \times k}(c_1,, c_q), q = \min(n, k)$ then	$\det \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = \det(A) \det(B)$	-If \mathbf{x}_i , \mathbf{x}_i associated to different eigenvalues then	-(Positive) singular values are (positive) square-roots of eigenvalues of AAT or ATA	$w(1) = \arg \max_{\ \mathbf{w}\ = 1} \mathbf{w}^T A^T A \mathbf{w}$	upper-triangular
$*v_{j+1} = P_j(a_{j+1}) \in U_j$ => discard it!!	$x = x_i + x_k$, where $x_i \in U$ and $x_k \in U^{\perp}$ •For matrix $A \in \mathbb{R}^{m \times n}$ and for row-space R(A),	$\operatorname{diag}_{m \times n}(a_1, \dots, a_p)\operatorname{diag}_{n \times k}(c_1, \dots, c_q)$	•If close to triangular matrix apply EROs/ECOs to get it there, then its just product of diagonals	x _i ±x _j	of eigenvalues of AA^T or A^TA -i.e. $\sigma_1^2,, \sigma_p^2$ are eigenvalues of AA^T or A^TA	= $\arg \max_{\ \mathbf{w}\ =1} (m-1) \operatorname{Var}_{\mathbf{w}} = \mathbf{v}_1$	Full QR Decomposition •Consider full-rank $A = [a_1 a_n] \in \mathbb{R}^{m \times n} (\underline{m \ge n}),$
$*\mathbf{u}_{j+1} = P_{\perp j}(\mathbf{a}_{j+1}) \in (U_j)^{\perp}$ => we're after this!!	•For matrix A ∈ R''' A'' and for row-space R(A), column-space C(A) and null space ker(A)	= diag $_{m \times k}(a_1c_1,, a_rc_r, 0,, 0)$ = diag(s) -Where $r = \min(p, q) = \min(m, n, k)$, and	•If Cholesky/LU/QR is possible and cheap then do it,	-If associated to same eigenvalue λ] then eigenspace E_{λ}] has spanning-set $\{x_{\lambda_i},\}$	- A ₂ = o ₁ (link to matrix norms	•i.e. w ₍₁₎ the direction that maximizes variance Var _w	i.e. $\mathbf{a}_1, \dots, \mathbf{a}_n \in \mathbb{R}^m$ are linearly independent
-Let $\mathbf{q}_{j+1} = \hat{\mathbf{u}}_{j+1} \implies$ we have next ONB $\langle \mathbf{q}_1,, \mathbf{q}_{j+1} \rangle$	$-R(A)^{\perp} = ker(A)$ and $C(A)^{\perp} = ker(A^{T})$	$s \in \mathbb{R}^S$, $s = \min(m, k)$	then apply AB = A B •If all else fails, try to find row/column with MOST zeros	*X1Xn Jare linearly independent => apply	Let r = rk(A), then number of strictly positive singular	i.e. maximizes variance of projections on line $Rw_{(1)}$ $\sigma_1 \mathbf{u}_1, \dots, \sigma_r \mathbf{u}_r (columns of \underline{US}) \text{ are principal}$	-Apply QR decomposition to obtain: -ONB $(\mathbf{q}_1,, \mathbf{q}_n) \in \mathbb{R}^m \text{for C(A)} $
for $U_{j+1} \Rightarrow$ start next iteration	-Any $\mathbf{b} \in \mathbb{R}^m$ can be uniquely decomposed into $\mathbf{b} = \mathbf{b}_i + \mathbf{b}_k$ where $\mathbf{b}_i \in C(A)$ and $\mathbf{b}_k \in \ker(A^T)$	•Inverse of square-diagonals => diag $(a_1,, a_n)^{-1}$ = diag $(a_1^{-1},, a_n^{-1})$, i.e. diagonals	-Perform minimal EROs/ECOs to get that row/column	Gram-Schmidt \mathbf{q}_{λ_i} , $\leftarrow \mathbf{x}_{\lambda_i}$,	values is r_1 •i.e. $\sigma_1 \ge \cdots \ge \sigma_r > 0$ and $\sigma_{r+1} = \cdots = \sigma_p = 0$	components/scores of A	-ONB $(\mathbf{q}_1,, \mathbf{q}_n) \in \mathbb{R}^m$ for $C(A)$ -Semi-orthogonal $Q_1 = [\mathbf{q}_1 \mathbf{q}_n] \in \mathbb{R}^{m \times n}$ and
			to be all-but-one zeros	*Then $\{q_{\lambda_i},\}$ is orthonormal basis (ONB) of E_{λ}		•Recall: $A = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T$ with $\sigma_1 \ge \cdots \ge \sigma_r > 0$, so that	
$*\mathbf{u}_{j+1} = (\mathbf{I}_m - Q_j Q_j^T) \mathbf{a}_{j+1} = \mathbf{a}_{j+1} - Q_j \mathbf{c}_j$ where	$*\mathbf{b} = \mathbf{b}_i * \mathbf{b}_k$, where $\mathbf{b}_i \in R(A)$ and $\mathbf{b}_k \in \ker(A)$	cannot be zero (division by zero undefined)	*Don't forget to keep track of sign-flipping &		$ A = \sum_{i=1}^{n} O_i \mathbf{u}_i \mathbf{v}_i $		upper-triangular $R_1 \in \mathbb{R}^{n \times n}$, where $A = Q_1 R_1$
$\mathbf{v}_{1,+1} = (\mathbf{I}_m - Q_j Q_j^T) \mathbf{a}_{j+1} = \mathbf{a}_{j+1} - Q_j c_j$ where $\mathbf{c}_j = [\mathbf{q}_1 \cdot \mathbf{a}_{j+1}, \dots, \mathbf{q}_j \cdot \mathbf{a}_{j+1}]^T$	$\star \mathbf{b} = \mathbf{b}_i \star \mathbf{b}_k$, where $\mathbf{b}_i \in R(A)$ and $\mathbf{b}_k \in \ker(A)$	-Determinant of square-diagonals => $ \operatorname{diag}(a_1,,a_n) = \prod_i a_i (since they are technically)$	*Don't forget to keep track of sign-flipping & scaling-factors -Do Laplace expansion along that row/column =>	$-Q = \langle \mathbf{q}_1, \dots, \mathbf{q}_n \rangle \text{ is an ONB of } \underline{\mathbb{R}^n} \Longrightarrow Q = [\mathbf{q}_1 \mid \dots \mid \mathbf{q}_n] \text{ is}$ orthogonal matrix i.e. $Q^{-1} = Q^T$	$ \frac{\bullet A = \sum_{i=1}^{I} \sigma_{i} \mathbf{u}_{i} \mathbf{v}_{i}^{I}}{\mathbf{SVD} \text{ is } similar \text{ to spectral decomposition, except it}} $	relates principal axes and principal components • Data compression: If $\sigma_1 \gg \sigma_2$ then compress AJby	upper-triangular $R_1 \in \mathbb{R}^{m \times m}$ where $A = Q_1 R_1$ •Compute basis extension to obtain remaining $q_{n+1}, \dots, q_m \in \mathbb{R}^m$ where (q_1, \dots, q_m) is ONB for \mathbb{R}^m

-Notice $(\mathbf{q}_{n+1}, \dots, \mathbf{q}_m)$ is ONB for $C(A)^{\perp} = \ker(A^T)$	Let $\mathbf{u}_{k}^{(j)} = \left(\prod_{i=1}^{j} P_{\perp \mathbf{q}_{i}}\right) \mathbf{a}_{k}$, i.e. \mathbf{a}_{k} without its	$ \mathbf{H}(f) = \nabla^2 f = \mathbf{J}(\nabla f)^T$ is Hessian $\Rightarrow \mathbf{H}(f)_{ij} = \frac{\partial^2 f}{\partial \mathbf{x}_i \partial \mathbf{x}_i}$	$\{\Phi(f_1,,f_n): f_1 \in O(g_1),,f_n \in O(g_n)\}$	$\bullet L_{ij}(\lambda)^{-1} = L_{ij}(-\lambda)$ both <u>triangular matrices</u>	$\mathbf{x}_{i}^{(k+1)} = \frac{1}{A_{ij}} \left(\mathbf{b}_{i} - \sum_{j \neq i} A_{ij} \mathbf{x}_{j}^{(k)} \right) \Rightarrow \mathbf{x}_{i}^{(k+1)}$ only needs	$(A-\sigma I)$ has $\frac{\lambda_2 - \sigma}{\lambda_1 - \sigma}$	
-Let $Q_2 = [\mathbf{q}_{n+1} \dots \mathbf{q}_m] \in \mathbb{R}^{m \times (m-n)}$ let	components along q ₁ ,,q _j	Interpret $F: \mathbb{R}^n \to \mathbb{R}^m$ as m functions $F_i: \mathbb{R}^n \to \mathbb{R}$	-e.g. $\epsilon^{O(1)} = \{\epsilon^{f(\epsilon)} : f \in O(1)\}$ •General case:	LU factorization w/ Gaussian elimina-	$ \mathbf{b}_i $; $\mathbf{x}^{(k)}$; $\mathbf{A}_{i\star}$ => row-wise parallelization	-Eigenvector guess => estimated eigenvalue	
$Q = [Q_1 Q_2] \in \mathbb{R}^{m \times m}$ let $R = [R_1; 0_{m-n}] \in \mathbb{R}^{m \times n}$ •Then full QR decomposition is	-Notice: $\mathbf{u}_j = \mathbf{u}_i^{(j-1)}$ thus $\mathbf{q}_j = \hat{\mathbf{u}}_j = \mathbf{u}_i^{(j-1)} / r_{jj}$ where	(one per output-component)	$\Phi_1(O(f_1),,O(f_m)) = \Phi_2(O(g_1),,O(g_n))$ means	Recall: you can represent EROs and ECOs as	Gauss-Seidel (G-S) Method:	•Inverse (power-)iteration: perform power iteration on $(A-\sigma I)^{-1}$ to get $\lambda_{1,\sigma}$ closest to $\underline{\sigma}_J$	
$A = QR = [Q_1 Q_2] \begin{bmatrix} R_1 \\ 0_{m-n} \end{bmatrix} = Q_1 R_1$	$r_{jj} = \left\ \mathbf{u}_{j}^{(j-1)} \right\ $	• $\underline{\mathbf{J}}(F) = \left[\nabla^T F_1;; \nabla^T F_m \right] $ is $Jacobian \Rightarrow \underline{\mathbf{J}}(F)_{ij} = \frac{\partial F_i}{\partial x_j}$	$\frac{\boldsymbol{\Phi}_{1}(O(f_{1}),,O(f_{m})) \subseteq \boldsymbol{\Phi}_{2}(O(g_{1}),,O(g_{n}))}{-\text{e.g. } \epsilon^{O(1)} = O\left(k^{\epsilon}\right) \mid \text{means} \left\{\epsilon^{f(\epsilon)} : f \in O(1)\right\} \subseteq O\left(k^{\epsilon}\right) \mid$	transformation matrices R, C respectively	$G = D + L; R = U \mid \Rightarrow M = -(D + L)^{-1} U; c = (D + L)^{-1} b$	$-(A-\sigma)^{-1}$ has eigenvalues $(\lambda-\sigma)^{-1}$ so power iteration	
$-Q \text{is orthogonal, i.e. } Q^{-1} = Q^{T} \text{ so its a basis}$	-Iterative step:	Conditioning	not necessarily true $e^{-e.g.} e^{-e.g.} e^{$	<u>LU</u> factorization => finds <u>A = LU</u> where <u>L</u> , <u>U</u> are lower/upper triangular respectively	$ \frac{\mathbf{x}_{i}^{(k+1)} = \frac{1}{A_{ij}}}{\mathbf{b}_{i}} \left[\mathbf{b}_{i} - \sum_{j=1}^{i-1} A_{ij} \mathbf{x}_{j}^{(k+1)} - \sum_{j=i+1}^{n} A_{ij} \mathbf{x}_{j}^{(k)} \right] $	will yield largest $(\lambda_{1,\sigma} - \sigma)^{-1}$	
transformation	$\mathbf{u}_{k}^{(j)} = \left(P_{\perp \mathbf{q}_{j}}\right)\mathbf{u}_{k}^{(j-1)} = \mathbf{u}_{k}^{(j-1)} - \left(\mathbf{q}_{j} \cdot \mathbf{u}_{k}^{(j-1)}\right)\mathbf{q}_{j}$	A problem is some $f: X \to Y$ where X, Y are normed vector-spaces	• Special case: $f = \Phi(O(g_1),, O(g_n))$ means	Naive Gaussian Elimination performs	•Computing $\mathbf{x}_{i}^{(k+1)}$ needs \mathbf{b}_{i} ; $\mathbf{x}^{(k)}$; $A_{i\star}$ and $\mathbf{x}_{i}^{(k+1)}$ for	-i.e. will yield smallest $\lambda_{1,\sigma}$ - σ , i.e. will yield $\lambda_{1,\sigma}$	
$-\operatorname{proj}_{C(A)} = Q_1 Q_1^T$, $\operatorname{proj}_{C(A)} \perp = Q_2 Q_2^T$ are	-i.e. each iteration j of MGS computes P ₁ q _j (and	 A problem instance is f with fixed input x ∈ X . 	$f \in \Phi(O(g_1), \dots, O(g_n))$ $-e.g. (\varepsilon + 1)^2 = \varepsilon^2 + O(\varepsilon)$ means	$[I_m \mid A \mid I_n] \rightsquigarrow [R^{-1} \mid U \mid I_n]$ to get $AI_n = R^{-1} U$ using	j < i ⇒ lower storage requirements	closest to o	
orthogonal projections onto $C(A)$ $C(A)$ $C(A)$ = $C(A)$	projections under it) in one go	shortened to just "problem" (with $\underline{x \in X}$ implied) • $\underline{\delta x}$ is small perturbation of \underline{x} $\underline{s} = \delta f = f(x + \delta x) - f(x)$	$\epsilon \mapsto (\epsilon+1)^2 \in \{\epsilon^2 + f(\epsilon) : f \in O(\epsilon)\}$ not necessarily true	only row addition		$-\ \mathbf{b}^{(k)} - a_k \mathbf{x}_{1,\sigma}\ = O\left(\left\ \frac{\lambda_{1,\sigma} - \sigma}{\lambda_{2,\sigma} - \sigma}\right\ ^{\kappa}\right) \text{ where } \mathbf{x}_{1,\sigma}$	
-Notice: $QQ^T = \mathbf{I}_m = Q_1 Q_1^T + Q_2 Q_2^T$	-At start of iteration $j \in 1n$ we have ONB	A problem (instance) is:	Let f. = O(a.) f= = O(a.) land let h = O he a constant	$\cdot R^{-1}$, i.e. inverse EROs in reversed order, is lower-triangular so $L = R^{-1}$	Successive over-relaxation (SOR): $G = \omega^{-1} D + L; R = (1 - \omega^{-1}) D + U =>$	corresponds to $\lambda_{1,\sigma}$ and $\lambda_{2,\sigma}$ is 2nd-closest to σ	
 Generalizable to A∈C^{m×n} by changing transpose to conjugate-transpose 	$ \underline{\mathbf{q}_1, \dots, \mathbf{q}_{j-1} \in \mathbb{R}^m} $ and residual $\underline{\mathbf{u}}_j^{(j-1)}, \dots, \underline{\mathbf{u}}_n^{(j-1)} \in \mathbb{R}^m$	•Well-conditioned if <u>all</u> small $\underline{\delta x}$ lead to small $\underline{\delta f}$ i.e. if $\underline{\kappa}$ is small (e.g. $\underline{1}$ j $\underline{10}$ j $\underline{10^2}$ j	Let $f_1 = O(g_1)$, $f_2 = O(g_2)$ and let $k \neq 0$ be a constant • $f_1 f_2 = O(g_1 g_2)$ $f \cdot O(g) = O(fg)$ $O(k \cdot g) = O(g)$	Algorithm 1 Gaussian elimination	$M = -(\omega^{-1}D + L)^{-1}((1 - \omega^{-1})D + U); c = -(\omega^{-1}D + L)^{-1}b$	- Efficiently compute eigenvectors for known	
Lines and hyperplanes in $E^n(=\mathbb{R}^n)$	-Compute $r_{jj} = \left\ \mathbf{u}_{j}^{(j-1)} \right\ \Rightarrow \mathbf{q}_{j} = \overline{\mathbf{u}_{j}^{(j-1)} / r_{jj}}$	•Ill-conditioned if some small δx lead to large δf i.e.	$f_1 + f_2 = O(\max(g_1 , g_2))$ => if $g_1 = g = g_2$ then $f_1 + f_2 = O(g)$	1: $U = A, L = I$ 2: for $k = 1$ to $m - 1$ do	$\frac{\omega}{\mathbf{x}_{i}^{(k+1)}} = \frac{\omega}{A_{ij}} \left(\mathbf{b}_{i} - \sum_{j=1}^{i-1} A_{ij} \mathbf{x}_{j}^{(k+1)} - \sum_{j=i+1}^{n} A_{ij} \mathbf{x}_{j}^{(k)} \right) $ for	eigenvalues on — Eigenvalue guess => estimated eigenvector	
Consider standard Euclidean space E ⁿ (=R ⁿ)	-For each $k \in (j+1)n$ compute $r_{jk} = \mathbf{q}_j \cdot \mathbf{u}_k^{(j-1)} = >$	if <u>K</u> j is large (e.g. <u>10⁶</u>) <u>10¹⁶</u>)	Floating-point numbers	3: for $j = k + 1$ to m do	+(1-ω)x _i ^(k)	-![[Pasted image 20250420131643.png 300]] -Can reduce matrix inversion $O(m^3)$ to $O(m^2)$ by	
•with standard basis $(e_1,, e_n) \in \mathbb{R}^n$ •with standard origin $0 \in \mathbb{R}^n$	$\mathbf{u}_{k}^{(j)} = \mathbf{u}_{k}^{(j-1)} - r_{jk} \mathbf{q}_{j}$	Absolute condition number cond(x) = $\hat{\kappa}(x) = \hat{\kappa} \text{ of } f \text{ at } \underline{x}$;	Consider base/radix β≥2 (typically 2) and precision	4: $\ell_{j,k} = u_{j,k}/u_{k,k}$ 5: $u_{j,k;m} = u_{j,k;m} - \ell_{j,k}u_{k,k;m}$	relaxation factor <u>\omega > 1</u>	pre-factorization	
	-Next ONB $\langle \mathbf{q}_1,, \mathbf{q}_j \rangle$ and next residual $\mathbf{u}_{j+1}^{(i)},, \mathbf{u}_n^{(i)}$	$\hat{\kappa} = \lim_{\delta \to 0} \sup_{\ \delta x\ \le \delta} \frac{\ \delta f\ }{\ \delta x\ }$	t≥1](24]or 53]for IEEE single/double precisions) Floating-point numbers are discrete subset	6: end for 7: end for	If AJ is strictly row diagonally dominant then	Nonlinear Systems of Equations: Itera-	
A line $L = \mathbb{R} \mathbf{n} + \mathbf{c}$ is characterized by direction $\mathbf{n} \in \mathbb{R}^n$ $(\mathbf{n} \neq 0)$ and offset from origin $\mathbf{c} \in L$	-NOTE: for $j=1$]=> $\mathbf{q}_1,, \mathbf{q}_{j-1} = \emptyset$, i.e. none yet	\Rightarrow for most problems simplified to $\hat{\kappa}$ = sup _{δx} $\frac{\ \delta f\ }{\ \delta x\ }$	$\mathbf{F} = \left\{ (-1)^{S} \left(m/\beta^{t} \right) \beta^{e} \mid 1 \le m \le \beta^{t}, \ s \in \mathbb{B}, m, e \in \mathbb{Z} \right\}$		Jacobi/Gauss-Seidel methods converge; AJis strictly	tive Techniques •[[tutorial 6#Multivariate Calculus [Recall]] that $\nabla f(x)$ [is	
•It is customary that:	•By end of iteration j = n , we have ONB	•If <u>Jacobian</u> $J_f(x)$ exists then $\hat{k} = \ J_f(x)\ $, where	•s_iis sign-bit, m/\(\text{\beta}^t\) is mantissa, e_iis exponent (8)-bit	•The pivot element is simply <u>diagonal entry</u> $u_{kk}^{(R-1)}$	row diagonally dominant if $A_{ii} > \sum_{j \neq i} A_{ij}$ If A_{ji} positive-definite then G-S and SOR ($\omega \in (0, 2)$)	direction of max. rate-of-change ∇f(x)	
-n is a unit vector, i.e. $\ \mathbf{n}\ = \ \hat{\mathbf{n}}\ = 1$ -c $\in L$ is closest point to origin, i.e. $\mathbf{c} \perp \mathbf{n}$	$\langle \mathbf{q}_1, \dots, \mathbf{q}_n \rangle \in \mathbb{R}^m$	matrix norm - induced by norms on X and Y	for single, 11 bit for double) • Equivalently, can restrict to $\beta^{t-1} \le m \le \beta^t - 1$ for unique	fails if $u_{Rk}^{(k-1)} \approx 0$	converge	•Search for stationary point by gradient descent : $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha \nabla f(\mathbf{x}^{(k)})$ for step length α	
 If c ≠ \(\lambda \n \right => L \) not vector-subspace of \(\mathbb{R}^n\right \) 	$-A = [a_1 a_n] = [q_1 q_n]$ $r_{11} r_{1n}$ $r_{1n} r_{1n}$ $r_{1n} r_{1n}$ $r_{1n} r_{1n}$	Relative condition number $\underline{\kappa}(x) = \underline{\kappa} \int \underline{f} dx \underline{x} is$ $\kappa = \lim_{\delta \to 0} \sup_{\ \delta x\ \le \delta} \left(\frac{\ \delta f\ }{\ f(x)\ } / \frac{\ \delta x\ }{\ x\ } \right)$	mjand ej •F⊂R Jis idealized (ignores over/underflow), so is	• $\tilde{L}\tilde{U} = A + \delta A$] $\frac{\ \delta A\ }{\ L\ \cdot \ U\ } = O(\varepsilon_{mach})$ only backwards	Metric spaces & limits	•A is positive-definite solving Ax = b and	
 i.e. 0 ∉ L i.e. L doesn't go through the origin L is affine-subspace of Rⁿ 	0 r _{nn}]	*K=um ₆ →0 ^{Sup} 6x ≤6 \(f(x)	countably infinite and self-similar (i.e. F=βF)	stable if $ L \cdot U \approx A $ •Work required: $\sim \frac{2}{3} m^3 flops \sim O(m^3) $	Metrics obey these axioms $d(x, x) = 0 x \neq y \implies d(x, y) > 0 d(x, y) = d(y, x) $	$\min_{\mathbf{X}} f(\mathbf{X}) = \frac{1}{2} \mathbf{X}^T A \mathbf{X} - \mathbf{X}^T \mathbf{b}$ are equivalent	
•If $c = \lambda n$, i.e. $L = Rn$] => L] is vector-subspace of R^n	corresponds to thin QR decomposition -Where $\underline{A} \in \mathbb{R}^{m \times n}$ is full-rank, $\underline{Q} \in \mathbb{R}^{m \times n}$ is	$\kappa = \sup_{\delta x} \left(\frac{\ \delta f\ }{\ f(x)\ } / \frac{\ \delta x\ }{\ x\ } \right)$	•For all $x \in \mathbb{R}$ there exists $f(x) \in F$ s.t. $ x-f(x) \le \epsilon_{mach} x $	•Solving $\underline{Ax = LUx}$ is $\sim \frac{2}{3}$ m ³ flops (back substitution is	$d(x,z) \le d(x,y) * d(y,z)$	-Get iterative methods $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha^{(k)} \mathbf{p}^{(k)}$ for step	
 i.e. 0∈LJ, i.e. LJgoes through the origin LJhas dim(L) = 1 and orthonormal basis (ONB) { n̂ } 	semi-orthogonal, and $R \in \mathbb{R}^{n \times n}$ is upper-triangular	Je(x)	$- \frac{ x - t(x) \le \epsilon_{mach} x }{- \text{Equivalently } f(x) = x(1 + \delta), \delta \le \epsilon_{mach}}$	0(m ²)	For metric spaces, mix-and-match these	length $\underline{a}^{(k)}$ and directions $\underline{p}^{(k)}$ •Conjugate gradient (CG) method: if $\underline{A} \in \mathbb{R}^{n \times n}$ also	
	Classical vs. Modified Gram-Schmidt These algorithms both compute thin	•If Jacobian $J_f(x)$ exists then $K = \frac{ f(x) / x }{ f(x) / x }$	Machine epsilon $\epsilon_{\text{machine}} = \epsilon_{\text{mach}} = \frac{1}{2} \beta^{1-t}$	•NOTE: Householder triangularisation requires $\sim \frac{4}{3} m^3$	$\frac{\inf_{x \to +\infty} f(x) = +\infty}{\inf_{x \to +\infty} f(x) = +\infty} \iff \forall r \in \mathbb{R}, \exists N \in \mathbb{N}, \forall x > N : f(x) > r$	symmetric then $(\mathbf{u}, \mathbf{v})_A = \mathbf{u}^T A \mathbf{v}$ is an inner-product	
A hyperplane $P = (\mathbb{R}\mathbf{n})^{\perp} + \mathbf{c} = \{x + \mathbf{c} \mid x \in \mathbb{R}^n, x \perp \mathbf{n}\} \mathbf{c} = \{x \in \mathbb{R}^n \mid x \cdot \mathbf{n} = \mathbf{c} \cdot \mathbf{n}\} \mathbf{c} = \{x \in \mathbb{R}^n \mid x \cdot \mathbf{n} = \mathbf{c} \cdot \mathbf{n}\} \mathbf{c} = \{x \in \mathbb{R}^n \mid x \cdot \mathbf{n} = \mathbf{c} \cdot \mathbf{n}\} \mathbf{c} = \{x \in \mathbb{R}^n \mid x \in \mathbb{R}^n $	•These algorithms both compute thin thin QR decomposition	•More important than k for numerical analysis	is maximum relative gap between FPs Half the gap between 1 and next largest FP	Partial pivoting computes PA = LU where P is a		-GC chooses p(k) that are conjugate w.r.t. A₁	
characterized by normal $\underline{\mathbf{n}} \in \mathbb{R}^n$ $(\underline{\mathbf{n}} \neq 0)$ and offset from	Modified Gram-Schmidt 1: for i = 1 to n do	Matrix condition number $Cond(A) = \kappa(A) = A A^{-1} $ => comes up so often that has its own name	$\star 2^{-24} \approx 5.96 \times 10^{-8}$ and $2^{-53} \approx 10^{-16}$ for single/double	permutation matrix => PPT = I, i.e. its orthogonal	$\bullet \lim_{\chi \to p} f(\chi) = L \iff \begin{array}{l} \forall \varepsilon > 0, \exists \delta > 0, \forall x \in X : \\ 0 < d_\chi(x,p) < \delta \implies d_Y(f(x),L) < \varepsilon \end{array}$	i.e. $(\mathbf{p}^{(i)}, \mathbf{p}^{(j)})_A = 0$ for $i \neq j$	
origin <u>c ∈ P</u>] •It represents an (n - 1) dimensional slice of the	Classical Gram-Schmidt 2: $u_j = a_j$	•A∈C ^{m×m} is well-conditioned if κ(A) is small,		•For each column j finds largest entry and row-swaps to make it new pivot => P_i	•Cauchy sequences, i.e. $\forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall m, n \ge N : d(a_m, a_n) < \varepsilon$, converge in	-And chooses $\underline{a}^{(k)}$ s.t. residuals $\mathbf{r}^{(k)} = -\nabla f(\mathbf{x}^{(k)}) = \mathbf{b} - \mathbf{A}\mathbf{x}^{(k)}$ are orthogonal	
n-dimensional space	2: $u_i = a_i$ 4: for $i = 1$ to a do	$\frac{\text{ill-conditioned}}{\kappa(\mathbf{A}) = \kappa(\mathbf{A}^{-1}) \kappa(\mathbf{A}) = \kappa(\gamma \mathbf{A}) \ \cdot \ = \ \cdot \ _2 \implies \kappa(\mathbf{A}) = \frac{\sigma_1}{\sigma_m}$	FP arithmetic: let *, (*) Jbe <u>real</u> and <u>floating</u> counterparts of arithmetic operation	•Then performs <u>normal elimination</u> on that column =>	complete spaces You can manipulate matrix limits much like in real	$*k=0 \Rightarrow p^{(0)} = -\nabla f(x^{(0)}) = r^{(0)}$	
-It is customary that: -njis a unit vector, i.e. n = n̂ = 1	3: for $i = 1$ to $j - 1$ do 5: $r_{jj} = u_j _2$ 4: $r_{ij} = q_i^* a_j$ 6: $q_j = u_j / r_{jj}$		•For x, y ∈ F we have	<u>'j</u>	You can manipulate <u>matrix limits</u> much like in real analysis , e.g. $\lim_{n\to\infty} (A^n B + C) = (\lim_{n\to\infty} A^n) B + C$	$*_{\underline{k \ge 1}} \stackrel{=}{\underset{\longrightarrow}{}} \mathbf{p}^{(k)} = \mathbf{r}^{(k)} - \sum_{i < k} \frac{\langle \mathbf{p}^{(i)}, \mathbf{r}^{(k)} \rangle_{A}}{\langle \mathbf{p}^{(i)}, \mathbf{p}^{(i)} \rangle_{A}} \mathbf{p}^{(i)}$	
$-c \in P$ is closest point to origin, i.e. $c = \lambda n$	5: $u_j = u_j - r_{ij}q_i$ 7: for $k = j + 1$ to n do 6: end for 8: $r_{jk} = q_j^*u_k$	For $\underline{\mathbf{A} \in \mathbb{C}^{m \times n}}$ the problem $f_{\underline{\mathbf{A}}}(x) = \underline{\mathbf{A}}x$ has	$x \circledast y = fl(x * y) = (x * y)(1 * \varepsilon), \delta \le \varepsilon_{mach}$ -Holds for any arithmetic operation $\circledast = \bullet, \bullet, \bullet, \bullet$	•Result is $L_{m-1}P_{m-1}L_2P_2L_1P_1A=U$ where $L_{m-1}P_{m-1}L_2P_2L_1P_1=L'_{m-1}L'_1P_{m-1}P_1$		$= \sum_{i < k} (\mathbf{p}^{(i)}, \mathbf{p}^{(i)})_{A}$	
-With those $\Rightarrow P = \{x \in \mathbb{R}^n \mid x \cdot \mathbf{n} = \lambda\}$ •If $\mathbf{c} \cdot \mathbf{n} \neq 0 \Rightarrow P$ not vector-subspace of \mathbb{R}^n	7: $r_{ij} = u_j _2$ 9: $u_k = \dot{u_k} - r_{jk}q_j$ 8: $q_j = u_j/r_{jj}$ 10: end for	$\kappa = \ \mathbf{A}\ \frac{\ \mathbf{x}\ }{\ \mathbf{A}\mathbf{x}\ } \Longrightarrow \text{if } \frac{\mathbf{A}^{-1}}{\ \mathbf{A}\mathbf{x}\ } \text{ exists then } \frac{\kappa \leq \text{Cond}(\mathbf{A})}{\ \mathbf{A}\mathbf{x}\ }$	Complex floats implemented pairs of real floats, so above applies to complex ops as-well	*Setting $L = (L'_{m-1} L'_1)^{-1} P = P_{m-1} P_1 gives$	Turn metric limit $\lim_{n\to\infty} x_n = L$ into real limit $\lim_{n\to\infty} d(x_n, L) = 0$ to leverage real analysis	$\star \alpha^{(R)} = \operatorname{argmin}_{\Omega} f(\mathbf{x}^{(R)} + \alpha^{(R)} \mathbf{p}^{(R)}) = \frac{\mathbf{p}^{(R)} \cdot \mathbf{r}^{(R)}}{\langle \mathbf{p}^{(R)}, \mathbf{p}^{(R)} \rangle_{A}}$	
-i.e. 0 ∉ P J, i.e. P J doesn't go through the origin	9: end for 11: end for	•If $\underline{\mathbf{A}} \times = \underline{b}$, problem of finding \underline{x} given \underline{b} is just $f_{\underline{\mathbf{A}}^{-1}}(b) = \underline{\mathbf{A}}^{-1}\underline{b} = \times = \ \underline{\mathbf{A}}^{-1}\ \frac{\ \underline{b}\ }{\ \underline{x}\ } \le Cond(\underline{\mathbf{A}})$	-Caveat: $\epsilon_{mach} = \frac{1}{2} \beta^{1-t}$ must be scaled by factors	PA=LU	 Bounded monotone sequences converge in R 	$(p^{(\kappa)}, p^{(\kappa)})_A$ -Without rounding errors, CG converges in $\leq n$	
-PJis affine-subspace of \mathbb{R}^n •If $\mathbf{c} \cdot \mathbf{n} = 0$, i.e. $P = (\mathbb{R}\mathbf{n})^{\perp} = PJ$ is vector-subspace of	•Computes at j th step: •Classical GS ⇒ j th column of Q J and the j th column	For $b \in \mathbb{C}^m$ the problem $f_b(A) = A^{-1}b$ (i.e. finding x jin	on the order of 2 ^{3/2} , 2 ^{5/2} for \otimes , \otimes respectively	Algorithm 2 Gaussian elimination with partial pivoting 1: $U = A, L = I, P = I$	• <u>Sandwich theorem</u> for limits in <u>R</u> J=> pick easy upper/lower bounds	iterations	
R ⁿ	of RI	$Ax = b$ has $\kappa = A A^{-1} = Cond(A)$	(* * * * *)	2: for $k = 1$ to $m - 1$ do 3: $i = \operatorname{argmax} u_{i,k} $	$-\overline{\lim}_{n\to\infty} r^n = 0 \iff r < 1$ and	*Similar to to [[tutorial 1#Gram-Schmidt method to generate orthonormal basis from any linearly	
-i.e. 0∈PJ, i.e. PJgoes through the origin	-Modified GS ⇒ j th column of Q and the j th row of		$(x_1 + \dots + x_n) + \sum_{i=1}^n x_i \left(\sum_{j=i}^n \delta_j \right)^{i-1} \delta_j \leq \epsilon_{\text{mach}}$	4: $U_{k,k:m} \leftrightarrow U_{i,k:m}$	$\lim_{n\to\infty} \sum_{i=0}^{n} ar^{i} = \frac{a}{1-r} \iff r < 1$	independent vectors Gram-Schmidt]] (different	
		Stability		5. 0k,k:m \ 7. 0i,k:m	11-10 - 1-7		
- <u>P_J</u> has <u>dim(P) = n - 1</u>	Both have flop (floating-point operation) count of	Stability Given a problem $f: X \to Y$, an algorithm for f is	$(x_1 \otimes \cdots \otimes x_n) \approx (x_1 \times \cdots \times x_n)(1+\epsilon), \epsilon \le 1.06(n-1)\epsilon_{\text{mach}}$	5: $\ell_{k,1:k-1} \leftrightarrow \ell_{i,1:k-1}$ 6: $p_{k,i} \leftrightarrow p_{i,i}$	Eigenvalue Problems: Iterative Tech-	inner-product) $\star(p^{(0)},,p^{(n-1)})$ and $(\mathbf{r}^{(0)},,\mathbf{r}^{(n-1)})$ are bases for	
Notice $L = Rn$ and $P = (Rn)^{\perp}$ are	Both have flop (floating-point operation) count of $o(2mn^2)$ NOTE: Householder method has $2(mn^2 - n^3/3)$ [flop	Given a problem $f: X \to Y$, an algorithm for f is $\tilde{f}: X \to Y$	• $(x_1 \otimes \cdots \otimes x_n) \times (x_1 \times \cdots \times x_n)(1 + \epsilon), \epsilon \le 1.06(n-1)\epsilon_{mach}$ • $f((\sum x_i y_i) = \sum x_i y_i(1 + \epsilon_i))$ where	5: $\ell_{k,1:k-1} \leftrightarrow \ell_{\ell,1:k-1}$ 6: $p_{k,i} \leftrightarrow p_{\ell,i}$ 7: for $j = k + 1$ to m do 8: $\ell_{i,k} = u_{i,k}/u_{k,k}$	-	$\begin{array}{c} \textit{inner-product)} \\ *(p^{(0)}, \dots, p^{(n-1)}) \middle] \text{and} \underbrace{(\mathbf{r}^{(0)}, \dots, \mathbf{r}^{(n-1)})}_{} \middle] \text{are bases for} \\ \underline{\mathbb{R}^n} \middle] \end{array}$	
Notice $\underline{L} = \mathbf{Rn} $ and $P = (\mathbf{Rn})^{\perp}$ are orthogonal compliments, so: •proj _l = $\hat{\mathbf{n}}\hat{\mathbf{n}}^T$ is orthogonal projection onto $\underline{L}_{\parallel}(along P)$	$O(2mn^2)$ -NOTE: Householder method has $2(mn^2 - n^3/3)$ flop count, but better numerical properties	Given a problem $\underline{f}: X \to Y$], an algorithm for \underline{f} is $\underline{f}: X \to Y$] input $\underline{x} \in X$ is first rounded to $fl(x)$ i.e. $\underline{f}(x) = \overline{f}(fl(x))$]. Absolute error $\Rightarrow \ \overline{f}(x) - f(x)\ $	$\begin{array}{l} \bullet(x_1 \otimes \cdots \otimes x_n) \otimes (x_1 \times \cdots \times x_n)(1 + \varepsilon), & \varepsilon \leq 1.06(n-1)\varepsilon_{\text{mach}} \\ \bullet f \mathbb{I}\left(\sum x_i y_i\right) = \sum x_i y_i (1 + \varepsilon_i) \text{ where} \\ \hline 1 + \varepsilon_i = (1 + \delta_i) \times (1 + \eta_i) \cdots (1 + \eta_n) \\ \hline -1 + \varepsilon_i \approx 1 + \delta_i + (\eta_i + \cdots + \eta_n) \end{bmatrix} \text{ and } \underbrace{ \begin{vmatrix} \delta_j \\ 1 \end{vmatrix}, \eta_i \leq \varepsilon_{\text{mach}} \end{vmatrix}}_{}$	5. $\ell_{k,1:k-1} \leftrightarrow \ell_{j,1:k-1}$ 6. $\rho_{k,i} \leftrightarrow \rho_{j,i}$ 7. $for j = k + 1$ to m do 8. $\ell_{j,k} = u_{j,k}/u_{k,k}$ 9. $u_{j,k:m} = u_{j,k:m} - \ell_{j,k}u_{k,k:m}$ 10. end for	Eigenvalue Problems: Iterative Techniques -If Aljis [[tutorial 1#Properties of matrices diagonalizable]] then [[tutorial	$\begin{array}{l} \underset{(p,0)}{\text{inner-product}} \\ *(\underline{p^{(0)}}_{\dots, p}(n^{-1})) \text{ and } \underbrace{(\underline{r^{(0)}}_{\dots, r}(n^{-1}))}_{\text{lare bases for }} \\ \text{QR Algorithm to find Schur decomposi-} \end{array}$	
Notice $\underline{\iota} = \operatorname{Rn} \operatorname{jand} P = (\operatorname{Rn})^{\perp} \operatorname{Jare}$ orthogonal compliments, so: *proj_ = $\hat{\mathbf{n}}\hat{\mathbf{n}}^T$ is orthogonal projection onto $\underline{\iota} (\operatorname{along} P) $ *proj_ = $\operatorname{id}_{\mathbf{p}} n - \operatorname{proj}_{\mathbf{i}} = \underline{1}_{n} - \hat{\mathbf{n}}\hat{\mathbf{n}}^T$ is orthogonal	O($2mn^2$) NOTE: Householder method has $2(mn^2 - n^3/3)$ flop count, but better numerical properties Recall: $Q^{\dagger}Q = I_n$ \Longrightarrow check for loss of orthogonality	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \parallel is \ \hat{f}: X \to Y \ $ -input $\underline{x} \in X$ is first rounded to $fl(x) \rfloor$ i.e. $\hat{f}(x) = \hat{f}(fl(x)) \rfloor$ -absolute error $\Rightarrow \ \hat{f}(x) - f(x) \ \ $ -relative error $\Rightarrow \ \hat{f}(x) - f(x) \ \ $	$\begin{split} & \cdot \langle x_1 \otimes \cdots \otimes x_n \rangle (x_1 \times \cdots \times x_n) \langle 1 + c_i \rangle \leqslant 1.06 \langle n - 1 \rangle \epsilon_{\text{mach}} \\ & \cdot f \{ [\sum_x i_y y_j] = \sum_x j_y i_j (1 + c_j) \} \text{where} \\ & \cdot f \leqslant_x \langle 1 \cdot \delta_y \rangle_x \langle 1 \cdot \eta_j \rangle \cdots \langle 1 \cdot \eta_n \rangle \\ & - 1 \cdot \epsilon_j = 1 \cdot \delta_y \langle 1 \cdot \eta_j \rangle \cdots \langle 1 \cdot \eta_n \rangle \\ & - 1 \cdot \epsilon_j = 1 \cdot \delta_y \langle \eta_j \rangle \cdots \langle \eta_n \rangle \\ & - \\ & - \end{aligned}$	5: $\ell_{k,1k-1} \leftrightarrow \ell_{l,1k-1}$ 6: $p_{k,c} \leftrightarrow p_{l}$ 7: for $j = k+1$ to m do 8: $\ell_{j,k} = u_{j,k} \ell_{k,k}$ 9: $u_{j,k} \ell_{k,k} = u_{j,k} \ell_{k,k}$ 10: end for 11: end for	Eigenvalue Problems: Iterative Techniques	$\begin{array}{l} \underset{(p(0),\dots,p^{(n-1)})}{\underbrace{h(p(0),\dots,p^{(n-1)})}} \text{ and } \underbrace{h(p(0),\dots,p^{(n-1)})} \text{ are bases for } \\ \frac{g^n}{QR} \text{ Algorithm to find Schur decomposition } A = QUQ^{\dagger} \end{array}$	
Notice $\underline{L} = \operatorname{Rn} \operatorname{and} P = (\operatorname{Rn})^{\perp} \int_{\operatorname{are}} \operatorname{orthogonal compliments}$, so: $\text{"proij.} = \widehat{\operatorname{nh}}\widehat{I}^{\top} \operatorname{is orthogonal projection onto } \underline{L}[\operatorname{along} P]$ $\text{"projp.} = \widehat{\operatorname{id}}_{\operatorname{Rn}} - \operatorname{proj}_{\operatorname{in}} = \widehat{\operatorname{In}}_{\operatorname{nh}}\widehat{I}^{\top} \operatorname{is orthogonal}$ $\operatorname{projection onto} P[\operatorname{"along} \underline{L}]$	$\begin{array}{l} O(2mn^2) \\ -\text{NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \text{ flop} \\ \text{count, but better numerical properties} \\ -\text{Recall: } Q^{\dagger}Q \circ I_n \\ \end{array}$ -> check for loss of orthogonality with $\ I_n - Q^{\dagger}Q\ = \log S$	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] in f [input $\underline{x} \in X$] is first rounded to f [(x)], i.e. $\hat{f}(x) = \hat{f}(f$ [(x))] *Absolute error $\Rightarrow \ \hat{f}(x) - f(x)\ \ _{2}$ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ _{2}$ [input f] *Input f] *Input f [input f] *Input f] *	$(x_1 = -\infty x_n) (x_1 = -\infty x_n) (1 + \varepsilon_i) \in S.106(n-1) \in mach$ $(T_i(\sum_x y_i)_y) = S_y y_i (1 + \varepsilon_j) Mere$ $(T_i \in T_i(\sum_x y_i)_y) = S_y y_i (1 + \varepsilon_j) Mach$ $(T_i \in T_i(\sum_x y_i)_y) = S_y (T_i(\sum_x y_i)_y) \log \frac{1}{ S_j } (1 + y_i) \log \frac{1}{ S_j } ($	5. $\ell_{k,1:k-1} \leftrightarrow \ell_{j,1:k-1}$ 6. $\rho_{k,i} \leftrightarrow \rho_{j,i}$ 7. $for j = k + 1$ to m do 8. $\ell_{j,k} = u_{j,k}/u_{k,k}$ 9. $u_{j,k:m} = u_{j,k:m} - \ell_{j,k}u_{k,k:m}$ 10. end for	Eigenvalue Problems: Iterative Techniques 'if_A]is [[uttorial 1#Properties of matrices[diagonalizable]] then [[uttorial 1#Eigen-values/vectors[eigen-decomposition]] A=X/XX^-1] -Dominant \(\)_1, \(\)_1 are such that \(\)_1 [is strictly largest	$\begin{array}{l} \underset{R^n}{\operatorname{inne-product}} \\ * & * \langle p(0), \dots, p^{(n-1)} \rangle \big \operatorname{and} \langle r^{(0)}, \dots, r^{(n-1)} \rangle \big \operatorname{are bases for} \\ \overset{R^n}{\mathbb{R}^n} \\ = QR \ Algorithm \ to \ find \ Schur \ decomposition \ A = QUQ^{\dagger} \\ - \operatorname{Any} A \in \mathbb{C}^{m \cdot m} \big \operatorname{has \ Schur \ decomposition \ } A = QUQ^{\dagger} \\ - \operatorname{Qis \ unitary, i.e.} \ Q^{\dagger} = q^{-1} \ \big \ \operatorname{and \ upper-triangular \ } U \\ \end{array}$	
Notice $\underline{L} = \mathbf{R} \mathbf{n}$ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: *proj_ = $\hat{\mathbf{n}} \hat{\mathbf{n}}^T$ is orthogonal projection onto \underline{L} (along \underline{P}) *proj_ = $\mathbf{id}_{\mathbf{p}} \mathbf{n}$ -proj_ = $\mathbf{I}_{\mathbf{n}} - \hat{\mathbf{n}} \hat{\mathbf{n}}^T$ is orthogonal	$\begin{array}{l} O(2mn^2) \\ - \text{NOTE: Householder method has } 2\left(mn^2 - n^3/3\right) \text{ flop} \\ \text{count, but better numerical properties} \\ \text{Recall: } Q^{\dagger} Q \circ I_n \Big \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \ I_n - Q^{\dagger} Q\ = \text{loss} \Big \\ - \text{Classical GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \Big \end{array}$	Given a problem $f: X \to Y \rfloor$ an algorithm for \underline{f} [is $f: X \to Y \rceil$ = input $\underline{x} \in Y$ is first rounded to $f!(x) \rfloor$ i.e. $\underline{\hat{f}}(x) = \underline{\hat{f}}(f!(x)) \rfloor$ -Absolute error $\Rightarrow \ \underline{f}(x) - f(x)\ \ _{\underline{f}(x)}$ relative error $\Rightarrow \ \underline{f}(x) - f(x)\ \ _{\underline{f}(x)}$ = $\frac{\ \underline{f}(x) - f(x)\ }{\ f(x)\ } = 0$ (ϵ_{mach})	$\begin{split} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_i) \in \pm 1.05(n - 1) \in \text{mach} \\ & = 1([\sum_i x_j y_i)_i \in x_j) \text{mach} -1 + \varepsilon_j = (1 + \varepsilon_j)_i = (1 + n_j) - (1 + n_n) \text{and} \delta_j \mid_i \mid_{n_j} \mid_i \leq \varepsilon_{\text{mach}} \\ & = 1 + \varepsilon_j = (1 + \varepsilon_j)_i = (1 + n_j) - (1 + n_n) \text{and} \delta_j \mid_i \mid_{n_j} \mid_i \leq \varepsilon_{\text{mach}} \\ & = 1 + \varepsilon_j = (1 + \varepsilon_j)_i = (1 + n_j) - (1 + n_n) \text{and} \delta_j \mid_i \mid_{n_j} \mid_i \leq \varepsilon_{\text{mach}} \\ & = 1 + (1 + \varepsilon_j)_i = (1 + \varepsilon_j)_i =$	5: $\ell_{k,1k-1} \leftrightarrow \ell_{l,1k-1}$ 6: $p_{k,c} \leftrightarrow p_{l,c}$ 7: for $j = k+1$ to m do 8: $\ell_{j,k} = u_{j,k}/u_{k,k}$ 9: $u_{j,k,m} = u_{j,k,m} - \ell_{j,k}u_{k,k,m}$ 10: end for 10: the for 10: Work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{jj} \le 1$ so $\ L\ = O(1)$	Eigenvalue Problems: Iterative Techniques If AJis [[tutorial 1#Properties of matrices[diagonalizable]] then [[tutorial 1#Eigen-values/vectors]eigen-decomposition]] Δ=ΧΛΧ ⁻¹ -Dominant λ ₁ : χ ₁ are such that λ ₁	$\begin{array}{l} & \text{inner-product)} \\ *(\underline{p}^{(0)}, \dots, \underline{p}^{(n-1)}) \text{ and } (\underline{r}^{(0)}, \dots, \underline{r}^{(n-1)}) \text{ are bases for } \\ \underline{R}^{p}] \\ & \mathbf{QR} \ \mathbf{Algorithm} \ \mathbf{to} \ \mathbf{find} \ \mathbf{Schur} \ \mathbf{decomposition} \ A = QUQ^{\dagger} \\ * Any \underline{A} \in (\underline{m} \cdot \underline{m}] \ \text{ has Schur} \ \mathbf{decomposition} \ A = QUQ^{\dagger} \\ -Q \text{ jis unitary, i.e. } \underline{Q}^{\dagger} = \underline{Q}^{-1} \text{ and upper-triangular } \underline{U} \\ -\text{Diagonal of } \underline{U} \text{ jcontains eigenvalues of } \underline{A} \text{ j} \end{array}$	
Notice $\underline{L} = \mathbf{R} \mathbf{n}$ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: $\mathbf{P} \mathbf{r} \mathbf{o}_{\underline{l}} = \hat{\mathbf{n}} \hat{\mathbf{n}}^{T} \ \mathbf{s} \text{ orthogonal projection } \mathbf{onto} \underline{L} \ (\mathbf{along} P_{\underline{l}} \ \mathbf{s})^{-1} \mathbf{p} \mathbf{r} \ _{2} + \ \mathbf{n}_{\underline{l}} \ \mathbf{n}$	$\begin{array}{l} O(2mn^2) \\ - \text{NOTE: Householder method has } 2\left(mn^2 - n^3/3\right) \text{flop} \\ \text{count, but better numerical properties} \\ \text{Recalt: } Q^{\dagger}Q = I_n \longrightarrow \text{check for loss of orthogonality} \\ \text{with } \prod_{n} - Q^{\dagger}Q = \text{loss} \\ - \text{Classical G} S \Longrightarrow \prod_{n} - Q^{\dagger}Q \ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ - \text{Modified GS} \Longrightarrow \prod_{n} - Q^{\dagger}Q \ = \text{Cond}(A)\epsilon_{\text{mach}} \end{array}$	Given a problem $f: X \to Y$] an algorithm for f] is $\hat{f}: X \to Y$] an algorithm for f] is $\hat{f}: X \to Y$] in the final $\hat{f}: X \to Y$. The following final $\hat{f}: X \to Y$ is first rounded to $f!(x)$, i.e. $\hat{f}(x) = \hat{f}(f!(x))$] and the error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ for all $\ f(x)\ \ $ is accurate if $\ Y \times EX\ $ $\ \hat{f}(x) - f(x)\ \ $ or $\ f(x)\ $ or $\ f(x)\ $ is stable if $\ Y \times EX\ $ $\ X \to Y\ $ and $\ X \to Y\ $.	$\begin{split} & \cdot \langle \mathbf{x}_1 \otimes \cdots \otimes \mathbf{x}_n \rangle (\mathbf{x}_1 \times \cdots \times \mathbf{x}_n) \langle \mathbf{t}_1 \cdot \langle \mathbf{c}_1 \cdot \mathbf{c}_1 \cdot \mathbf{c}_1 \rangle - \mathbf{c}_{mach} \\ & \cdot \langle \mathbf{t}_1 \cdot \langle \mathbf{x}_1 \cdot \mathbf{y}_1 \rangle + \langle \mathbf{t}_1 \cdot \mathbf{t}_n \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle + (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle + (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle + (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_n) \\ & \cdot \langle \mathbf{t}_1 \cdot \mathbf{t}_1 \rangle - (\mathbf{t}_1 \cdot \mathbf{t}_$	5: $\ell_{k,1:k-1} \leftrightarrow \ell_{l,1:k-1}$ 6: $p_{k,c} \leftrightarrow p_{l,c}$ 7: for $j = k+1$ to m do 8: $\ell_{j,k} = u_{j,k} \cdot m_{k,k}$ 9: $u_{j,k,m} = u_{j,k,m} - \ell_{j,k} \cdot u_{k,k,m}$ 10: end for 10: end for 10: Work required: $-\frac{2}{3} m^3 \text{flops} - O(m^3) $ results in $L_{jj} \le 1$ 50: $ L = O(1) $ 1: Stability depends on growth-factor $p = \frac{\max_{l,j} u_{l,j} }{\max_{l,j} u_{l,j} }$	Eigenvalue Problems: Iterative Techniques if Ajis [[tutorial 1#Properties of matrices]diagonalizable]] then [[tutorial 1#Eigen-values/vectors]eigen-decomposition]] $A = X N X^{-1}$ -Dominant $\lambda_1 : \mathbf{x}_1$ are such that $\ \lambda_1\ $ is strictly largest for which $\underline{A \mathbf{x}} = \lambda N X$ -Rayleigh quotient for Hermitian $\underline{A} = \underline{A}^{\dagger}$ is	$\begin{array}{l} \underset{R^n}{\operatorname{inne-product}} \\ * & * \langle p(0), \dots, p^{(n-1)} \rangle \big \operatorname{and} \langle r^{(0)}, \dots, r^{(n-1)} \rangle \big \operatorname{are bases for} \\ \overset{R^n}{\mathbb{R}^n} \\ = QR \ Algorithm \ to \ find \ Schur \ decomposition \ A = QUQ^{\dagger} \\ - \operatorname{Any} A \in \mathbb{C}^{m \cdot m} \big \operatorname{has \ Schur \ decomposition \ } A = QUQ^{\dagger} \\ - \operatorname{Qis \ unitary, i.e.} \ Q^{\dagger} = q^{-1} \ \big \ \operatorname{and \ upper-triangular \ } U \\ \end{array}$	
Notice $\underline{L} = \operatorname{Rn} \operatorname{and} P = (\operatorname{Rn})^{\perp} \operatorname{are}$ orthogonal compliments, so: $ \operatorname{"prol}_{\underline{l}} = \widehat{\operatorname{nil}}^{\underline{l}} \operatorname{[s]} \operatorname{orthogonal} \operatorname{projection} \operatorname{onto} \underline{L} \operatorname{[clong} P \underline{J}] $ $ \operatorname{"proj}_{\underline{l}} = \widehat{\operatorname{nin}}^{\underline{l}} \operatorname{nin}^{\underline{l}} \operatorname{[s]} \operatorname{orthogonal} \operatorname{projection} \operatorname{onto} \underline{P} \operatorname{[clong} \underline{L}] $ $ \operatorname{"L} = \operatorname{im} (\operatorname{proj}_{\underline{l}}) + \operatorname{ker} (\operatorname{proj}_{\underline{l}}) \operatorname{and} $ $ P = \operatorname{ker} (\operatorname{proj}_{\underline{l}}) - \operatorname{im} (\operatorname{proj}_{\underline{l}}) \operatorname{le}. \operatorname{all} \operatorname{vectors} \underline{v} \in \mathbb{R}^{n} \operatorname{uniquely} $ $ \operatorname{decomposed into} \underline{v} = \underline{v}_{\underline{l}} \cdot \underline{v}_{\underline{l}} $ $ \operatorname{uniquely} \operatorname{decomposed into} \underline{v} = \underline{v}_{\underline{l}} \cdot \underline{v}_{\underline{l}} $	$\begin{array}{l} \underline{O(2mn^2)} \\ -\text{NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \end{bmatrix} \text{flop} \\ \text{count, but better numerical properties} \\ \text{Recall: } \underline{Q^{\dagger}Q = I_n} \Big \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \ \underline{I_n - Q^{\dagger}Q}\ = \text{loss} \Big \\ -\text{Classical GS} \Rightarrow \ \underline{I_n - Q^{\dagger}Q}\ \approx \text{Cond}(A)^2 \epsilon_{\text{mach}} \Big \\ -\text{Modified GS} \Rightarrow \ \underline{I_n - Q^{\dagger}Q}\ \approx \text{Cond}(A)\epsilon_{\text{mach}} \Big \\ -\text{NOTE: Householder method has } \ \underline{I_n - Q^{\dagger}Q}\ \approx \epsilon_{\text{mach}} \Big \end{array}$		$\begin{array}{ll} (x_j = -\infty x_j) & (x_j = -\infty x_j) & (1 + \ell_i) \in \mathbb{S} \setminus OS(n-1) \in mach \\ \hline \cdot ([\xi, x_j])_i \in \mathbb{S}_y \setminus (1 + \epsilon_j) \mid where \\ \hline \cdot e_i = (1 + \delta_j) \cdot x (1 + \eta_j) \cdots (1 + \eta_m) \mid and \mid \delta_j \mid_i \mid \eta_j \mid i \leq \epsilon_{mach} \\ \hline -1 + \epsilon_j = 1 + \delta_j \cdot (\eta_j = -m \eta_m) \mid \\ -1 + \epsilon_j = 1 + \delta_j \cdot (\eta_j = -m \eta_m) \mid \\ \hline \cdot ([f(x^j) y - x^j] y \mid y \in [\chi_j x_j] \mid \epsilon_j \mid j \mid \\ \hline -Assuming \mid_{mach} \leq 0.1 \mid 2 \cdot y \mid f(x_j - y) \mid x_j \in [\chi_j x_j] \mid y \mid y \mid x_j \in [\chi_j x_j] \mid y \mid y \mid x_j \in [\chi_j x_j] \mid y \mid $	5: $f_{k,1,k-1} \mapsto f_{k,1,k-1}$ 6: $p_{k_i} \circ p_i$ 7: for $j = k+1$ to m do 8: $f_{j,k} = g_{j,k} m = g_{j,k} m \cdot f_{j,k} g_{k,k,m}$ 10: $g_{j,k} = g_{j,k} m - f_{j,k} g_{k,k,m}$ 10: $g_{j,k} = g_{j,k} m - f_{j,k} g_{k,k,m}$ 10: $g_{j,k} = g_{j,k} m - g_{j,k} g_{k,k,m}$ 10: $g_{j,k} = g_{j,k} m - g_{j,k} g_{k,k,m}$ 11: $g_{j,k} = g_{j,k} g_{j,k}$ 12: $g_{j,k} = g_{j,k} g_{j,k}$ 13: $g_{j,k} = g_{j,k} g_{j,k}$ 14: $g_{j,k} = g_{j,k} g_{j,k}$ 15: $g_{j,k} = g_{j,k} g_{j,k}$ 16: $g_{j,k} = g_{j,k} g_{j,k}$ 17: $g_{j,k} = g_{j,k} g_{j,k}$ 18: $g_{j,k} = g_{j,k} g_{j,k}$ 19: $g_{j,k} = g_{j,k} g_{j,k}$ 19: $g_{j,k} = g_{j,k}$ 10: $g_{j,k} = g_{j,k}$ 11: $g_{j,k} = g_{j,k}$ 11: $g_{j,k} = g_{j,k}$ 12: $g_{j,k} = g_{j,k}$ 13: $g_{j,k} = g_{j,k}$ 14: $g_{j,k} = g_{j,k}$ 15: $g_{j,k} = g_{j,k}$ 16: $g_{j,k} = g_{j,k}$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial 1#Properties of matrices]diagonalizable]] then [[tutorial 1#Eigen-values/vectors]eigen-decomposition]] $A = \lambda \Delta \lambda \lambda^{-1}$ - Dominant $\lambda_1 : x_1$ are such that $ \lambda_1 $ is strictly largest for which $\lambda_1 = \lambda \lambda \lambda^{-1}$ - Rayleigh quotient for Hermitian $\underline{A} = \underline{A}^{\dagger}$ is $R_A(x) = \frac{x^{\dagger}}{x^{\dagger}} \underline{x}$	$\begin{array}{l} & inner-product) \\ & \times \{p(0), \dots, p^{(n-1)}\} \text{ and } \{x^{(Q)}, \dots, x^{(n-1)}\} \text{ are bases for } \\ & \mathbb{R}^n \end{bmatrix} \\ & \times \{p(0), \dots, p^{(n-1)}\} \text{ and } \mathbf{E} \text{ for } \mathbf{E}$	
Notice $\underline{L} = \operatorname{Rn} \operatorname{jand} P = (\operatorname{Rn})^{\perp} \operatorname{jare}$ orthogonal compliments, so: $\operatorname{proj}_{\underline{L}} = \widehat{\operatorname{nh}}^T \operatorname{is} \operatorname{orthogonal} \operatorname{projection} \operatorname{onto} \underline{L} (\operatorname{along} P)^{\underline{J}} $ $\operatorname{proj}_{\underline{L}} = \widehat{\operatorname{nh}}^T \operatorname{is} \operatorname{orthogonal} \operatorname{projection} \operatorname{onto} \underline{L} (\operatorname{along} P)^{\underline{J}} $ $\operatorname{projection} \operatorname{onto} \underline{P} (\operatorname{along} \underline{L})^{\underline{J}} \operatorname{is} \operatorname{orthogonal} \operatorname{projection} \operatorname{onto} \underline{P} (\operatorname{along} \underline{L})^{\underline{J}} \operatorname{is} \operatorname{orthogonal} \operatorname{projection} \operatorname{projection} \underline{L} = \operatorname{im} (\operatorname{proj}_{\underline{L}}) = \operatorname{ker} (\operatorname{proje}_{\underline{J}}) \operatorname{projection} \underline{L} = \operatorname{im} (\operatorname{projection}_{\underline{J}}) = \operatorname{ker} (\operatorname{projection}_$	$\begin{array}{l} O(2mn^2) \\ -\text{NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \end{bmatrix} \text{flop} \\ \text{count, but better numerical properties} \\ -\text{Recall: } Q^{\dagger} Q = \mathbb{I}_n \end{bmatrix} = \text{check for loss of orthogonality} \\ \text{with } \ \mathbb{I}_n - Q^{\dagger} Q\ = \text{loss} \end{bmatrix} \\ -\text{Classical GS} \Rightarrow \ \mathbb{I}_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ -\text{Modified GS} \Rightarrow \ \mathbb{I}_n - Q^{\dagger} Q\ = \text{Cond}(A) \in_{\text{mach}} \\ -\text{NOTE: Householder method has } \ \mathbb{I}_n - Q^{\dagger} Q\ = \epsilon_{\text{mach}} \\ \\ \hline \text{Multivariate Calculus} \\ \hline \text{Consider } f : \mathbb{R}^n - \mathbb{R} \} \end{array}$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variable $f: X \to X \rfloor$ in the variable $f: X \to X \rfloor$ in the variable $f: X \to X \rfloor$ is accurate if $f: X \to X \rfloor$ in the variable $f: X \to X \to X \rfloor$ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X $ in the variable $f: X \to X \to X \to X $ in the variable $f: X \to X \to X \to X \to X $ in the variable $f: X \to X $	$\begin{array}{l} (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_i) \in \pm 1.06(n-1) \in \mathrm{mach} \\ -1([\sum_x y_i]) \in \sum_x y_i (1 + \varepsilon_j) Mere \\ 1 + (\varepsilon_i = (1 + \delta_j) \cdot (1 + \eta_i) - (1 + \eta_n) mach \delta_j , \eta_i \le \varepsilon_{mach} \\ -1 + \varepsilon_i = 1 + \delta_j \cdot (\eta_i) = -\eta_n \delta_i \delta_j , \eta_i \le \varepsilon_{mach} \\ -1 + (\varepsilon_i = 1 + \delta_j \cdot (\eta_i) = -\eta_n \delta_i) = \delta_i \delta_$	5: $\ell_{k,1k-1} \mapsto \ell_{k,1k-1}$ 6: $p_{k,i} \circ p_{k}$ 7: for $j = k+1$ to m do 8: $\ell_{j,k} = m_{j,k} m_{k}$ 9: $m_{j,k,m} = m_{j,k} m_{k} \in \ell_{j,k} m_{k,k,m}$ 9: $m_{j,k,m} = m_{j,k,m} = \ell_{j,k} m_{k,k,m}$ 11: end for 11: end for 12: $m_{j,k} = m_{j,k} m_{k,k,m}$ 13: $m_{j,k} = m_{j,k} m_{k,k,m}$ 14: $m_{j,k} = m_{j,k} m_{j,k} = m_{j,k} m_{j,k}$ 15: $m_{j,k} = m_{j,k} m_{j,k} = m_{j,k} m_{j,k}$ 15: $m_{j,k} = m_{j,k} m_{j,k} = m_{j,k} m_{j,k}$ 16: $m_{j,k} = m_{j,k} m_{j,k} = m_{j,k} m_{j,k}$ 16: $m_{j,k} = m_{j,k} m_{j,k} = m_{j,k}$ 16: $m_{j,k} = m_{j,k} = m_{j,k} m_{j,k}$ 16: $m_{j,k} = m_{j,k} = m_{j,k} m_{j,k}$ 17: $m_{j,k} = m_{j,k} = m_{j,k} m_{j,k}$ 18: $m_{j,k} = m_{j,k} = m_{j,k} m_{j,k}$ 18: $m_{j,k} = m_{j,k} = m_{j,k} m_{j,k}$	Eigenvalue Problems: Iterative Techniques if Ajis [[tutorial $\pm Properties$ of matrices[diagonalizable]] then [[tutorial $\pm Eigen-values]$ when $(\pm Varian) = Varian = Varian) = Varian = Vari$	$\begin{array}{l} & inner-product) \\ & \times_{\mathbb{R}^{D}} (m, p^{(n-1)}) \text{and } (\underline{r}^{(0)}, \dots, \underline{r}^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{D} & \text{ and } \mathbf{r}^{(n)} \text{ and } \mathbf{r}^{(n)} \text{ are bases for } \\ & \mathbb{R}^{D} & \text{ and } \mathbf{r}^{(n)} \text{ and } \mathbf{r}^{(n)} \text{ and } \mathbf{r}^{(n)} \\ & -Q \text{ sunitary, i.e. } \underline{q}^{t} = \underline{q}^{-1} \text{ and upper-triangular } \underline{U} \\ & -\mathrm{Diagonal } of \underline{U} \text{ contains eigenvalues } of \underline{A}] \\ & \text{ il [Pasted image 2020542013566, png] 2001]} \\ & \text{ For } \underline{A} \in \mathbb{R}^{m \times m} \text{ sach iteration } \underline{A}^{(k)} = \underline{Q}^{(k)} \underline{R}^{(k)} \text{ produces } \\ & \text{ orthogonal } \underline{Q}^{(k)} = \underline{Q}^{(k)} \\ & \text{ 5.5} \\ \end{array}$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{p} = (\mathbf{R} \mathbf{n})^{\perp} \mathbf{n} \mathbf{r}$ orthogonal compliments, so: $\mathbf{r} = \mathbf{r} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} n$	$\begin{array}{l} \underline{O(2mn^2)} \\ -\text{NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \Big \text{flop} \\ \text{count, but better numerical properties} \\ -\text{Recall: } \underline{Q^{\dagger}} Q = I_n \Big \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \underbrace{\ I_n - Q^{\dagger}} Q\ = \log s \Big \\ -\text{classical GS} \Rightarrow \underbrace{\ I_n - Q^{\dagger}} Q\ = \text{cond}(a)^2 \epsilon_{\text{mach}} \Big \\ -\text{Modified GS} \Rightarrow \underbrace{\ I_n - Q^{\dagger}} Q\ = \text{cond}(a) \epsilon_{\text{mach}} \Big \\ -\text{NOTE: Householder method has } \underbrace{\ I_n - Q^{\dagger}} Q\ = \epsilon_{\text{mach}} \Big \\ \text{Multivariate Calculus} \end{array}$	Given a problem $f: X \to Y$ an algorithm for f is $\hat{f}: X \to Y$ in algorithm for f is $\hat{f}: X \to Y$ in first rounded to f if X in \hat{f} is accurate if \hat{f} in \hat{f} in \hat{f} in \hat{f} is stable if \hat{f} in \hat{f}	$\begin{array}{ll} (x_j = -\infty x_j) & (x_j = -\infty x_j) & (1 + \ell_i) \in \mathbb{S} \setminus OS(n-1) \in mach \\ \hline \cdot ([\xi, x_j])_i \in \mathbb{S}_y \setminus (1 + \epsilon_j) \mid where \\ \hline \cdot e_i = (1 + \delta_j) \cdot x (1 + \eta_j) \cdots (1 + \eta_m) \mid and \mid \delta_j \mid_i \mid \eta_j \mid i \leq \epsilon_{mach} \\ \hline -1 + \epsilon_j = 1 + \delta_j \cdot (\eta_j = -m \eta_m) \mid \\ -1 + \epsilon_j = 1 + \delta_j \cdot (\eta_j = -m \eta_m) \mid \\ \hline \cdot ([f(x^j) y - x^j] y \mid y \in [\chi_j x_j] \mid \epsilon_j \mid j \mid \\ \hline -Assuming \mid_{mach} \leq 0.1 \mid 2 \cdot y \mid f(x_j - y) \mid x_j \in [\chi_j x_j] \mid y \mid y \mid x_j \in [\chi_j x_j] \mid y \mid y \mid x_j \in [\chi_j x_j] \mid y \mid $	5: $\ell_{k,1:k-1} \leftrightarrow \ell_{l,1:k-1}$ 6: $p_{k,c} \leftrightarrow p_{l,c}$ 7: for $j = k+1$ to m do 8: $\ell_{j,k} = \nu_{j,k}/m_{k,k}$ 9: $u_{j,k,m} = u_{j,k,m} - \ell_{j,k}u_{k,k,m}$ 10: end for 11: end for 12: Mork required: $-\frac{2}{3}m^3$ flops $-O\left(m^3\right)$; results in $L_{ij} \le 1$] 5: $SIL[=O(1)]$ • stability depends on growth-factor $\rho = \frac{\max_{i,j} u_{i,j} }{\max_{i,j} u_{i,j} }$ \Rightarrow for partial pivotting $\rho \le 2^{m-1}$ • $\ U\ = O(\rho\ A\) \Rightarrow \widehat{L}U = \widehat{P}A + \delta A\}$ $\frac{\ \delta A\ }{\ A\ } = O\left(\rho \in machine\right)$ \Rightarrow only backwards stable if $\rho = O(1)$	Eigenvalue Problems: Iterative Techniques $\frac{1}{H_1^2 \mathbf{s} } [\mathbf{t} \mathbf{t}\mathbf{t}\mathbf{t}\mathbf{v}\mathbf{r}] = \mathbf{t}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{s}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}\mathbf{r}r$	$\begin{array}{l} & inner-product) \\ *(\underline{p}^{(0)}, \dots, \underline{p}^{(n-1)}) \big] \text{and } (\underline{r}^{(0)}, \dots, \underline{r}^{(n-1)}) \big] \text{are bases for } \\ & \underline{R}^{p} \big] \\ & \mathbf{QR} \ \mathbf{Algorithm} \ \mathbf{to find Schur decomposition} \ A = QUQ^{\dagger} \\ & - Any \underline{A} \in \mathbb{C}^{m \times m} \big] \text{has Schur decomposition} \ A = \underline{Q} \underline{U} q^{\dagger} \big] \\ & - Diagonal of \ \underline{U} \ \text{contains eigenvalues} \ of \ A \big] \\ *![[Pasted image 20250420135506, png] \ \text{Son} \ A \big[R^{m \times m} \big] \ \text{each iteration} \ \underline{A}^{(k)} = \underline{Q}^{(k)} R^{(k)} \big] \text{produces} \\ & \text{orthogonal} \ \underline{Q}^{(k)} \overline{I} = \underline{Q}^{(k)} \overline{I} \\ & - So \\ \underline{A}^{(k+1)} = \underline{R}^{(k)} \underline{Q}^{(k)} = \underline{Q}^{(k)} \overline{I}_{Q}^{(k)} \big] \\ & \text{similar to } \underline{A}^{(k)} \big] \\ & \text{similar to } \underline{A}^{(k)} \big] \end{aligned}$	
Notice $\underline{L} = \operatorname{Rn} \operatorname{and} P = (\operatorname{Rn})^{\perp}$ are orthogonal compliments, so: *proj_* = $\widehat{\operatorname{nn}}^T$ is orthogonal projection onto $\underline{L} (\operatorname{along} P)^{\parallel}$ *proj_* = $\operatorname{id}_{R}^n - \operatorname{proj} = \mathbb{I}_n - \widehat{\operatorname{nn}}^T$ is orthogonal projection onto $\underline{P} (\operatorname{along} \underline{L})^{\parallel} = \operatorname{lim}(\operatorname{proj}_{L}) = \ker(\operatorname{proj}_{L}) = \ker(\operatorname{proj}_{L}) = \ker(\operatorname{proj}_{L}) = \ker(\operatorname{proj}_{L}) = \ker(\operatorname{proj}_{L}) = \operatorname{im}(\operatorname{proj}_{L}) = \operatorname{lim}(\operatorname{proj}_{L}) = \operatorname{lim}(pro$	O(2mn²)	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] -input $x \in X$ [is first rounded to $f(x)$] i.e. $\hat{f}(x) = \hat{f}(f(x))$] -Absolute error $\Rightarrow \frac{\ \hat{f}(x) - f(x)\ }{\ f(x)\ }$ relative error $\Rightarrow \frac{\ \hat{f}(x) - f(x)\ }{\ f(x)\ } = 0$ (ϵ_{mach}) \hat{f} [is saccurate if $Yx \in X$] $\hat{J} \in X$ [s. $\hat{J} \in X$] $\hat{J} \in X$ [s. nearly the right answer to nearly the right question -outer-product is stable $\hat{J} \in X$ [s. $\hat{J} \in X$] and $\hat{J} \in X$ [s. $\hat{J} \in X$] $\hat{J} \in X$ [s. $\hat{J} \in X$] and $\hat{J} \in X$ [s. $\hat{J} \in X$] $\hat{J} \in X$ [s. $\hat{J} \in X$ [s. $\hat{J} \in X$] $\hat{J} \in X$ [s. \hat	$\begin{array}{l} (\exists_i = -\infty x_n) \in (x_1 = -\infty x_n) \cap \{+\emptyset_i \in \mathbb{E} : 1.05(n-1) \in \text{mach} \} \\ f([\sum_i x_j)_i) = \sum_i y_i (1+\varepsilon_j) \ \text{where} \\ \vdots = \{-1 \in \mathbb{F}_j\} \times \{-1 \in \mathbb{F}_j\} \text{ where} \\ \vdots = \{-1 \in \mathbb{F}_j \times \{-1 \in \mathbb{F}_j\} \cap \{-1 \in \mathbb{F}_j\} \text{ where} \} \\ -1 \in \mathbb{F}_i = 1 + \mathbb{G}_j \times \{0_1 = -\infty n_n\} \\ -1 \in \mathbb{F}_i = 1 + \mathbb{G}_j \times \{0_1 = -\infty n_n\} \\ -1 \in \mathbb{F}_i = 1 + \mathbb{G}_j \times \{0_1 = -\infty n_n\} \\ -1 \in \mathbb{F}_i = 1 + \mathbb{G}_j \times \{0_1 = -\infty n_n\} \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb{F}_i = 1 + \mathbb{F}_i \times \{-1 \in \mathbb{F}_i\} \\ +1 \in \mathbb$	5. $(k_{\perp}k_{\perp}) + (\ell_{\perp}k_{\perp}) = 0$ 5. $(p_{\perp} + p_{\perp}) = p_{\perp}k_{\perp}p_{\perp}k_{\perp}$ 7. If or $j = k + 1$ to m do 8. $(j_{\perp}k = m_{\perp}k_{\perp}) = k_{\perp}k_{\perp}$ 9. $w_{\parallel}k_{\perp}m = w_{\parallel}k_{\perp}m - \ell_{\parallel}k_{\perp}k_{\perp}m}$ 10. end for 11: end for 11: end for 12: $m = m_{\perp}k_{\perp}m - \ell_{\parallel}k_{\perp}m + \ell_{\parallel}k_{\perp}m}$ 13: $m = m_{\perp}k_{\parallel}m - \ell_{\parallel}k_{\perp}m + \ell_{\parallel}k_{\perp}m}$ 15: $m = m_{\perp}k_{\parallel}m - \ell_{\parallel}k_{\parallel}m + \ell$	Eigenvalue Problems: Iterative Techniques If AJIs [Itutorial 1#Properties of matrices]diagonalizable]] then [Itutorial 1#Eigen-values/vectors]eigen-decomposition]] $A=X\Lambda X^{-1}$ -Dominant $\Lambda_1: X_1$ are such that $ \lambda_1 $ [is strictly largest for which $X \in X \setminus X$] -Rayleigh quotient for Hermitian $A=A^{\frac{1}{2}}$ is $R_A(x) = \frac{x^{\frac{1}{2}}Ax}{x^{\frac{1}{2}}}$ *Eigenvectors are stationary points of R_Δ] * $R_A(x)$ is closest to being like eigenvalue of \underline{x}_1 . i.e. $R_A(x)$ argmin $Ax - xX_2$] * $R_A(x) - xX_1$ * $R_A(x) - xX_2$ * $R_A(x) - xX_1$ * $R_A(x) - xX_2$ * $R_A(x) -$	$\begin{array}{l} & inner-product) \\ & \times (p(0), \dots, p^{(n-1)}) \text{and } (\underline{x}^{(0)}, \dots, \underline{x}^{(n-1)}) \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } (\underline{x}^{(0)}, \dots, \underline{x}^{(n-1)}) \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ and } \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are bases for } \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are bases } are bases $	
Notice $\underline{L} = \mathbf{R} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{p} = (\mathbf{R} \mathbf{n})^{\perp} \mathbf{n} \mathbf{r}$ orthogonal compliments, so: $\mathbf{r} = \mathbf{r} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} n$	O(2mm²)	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] in the rounded to $f(x)$, i.e. $\hat{f}(x) = \hat{f}(f(x))$] relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ for $\ \hat{f}(x)\ = 0$ ($\ \hat{f}(x)\ = 0$) ($\ f$	$\begin{split} & \{x_1 = -\infty x_n\} (x_1 = -x_n x_n) (1 + \varepsilon_i), \varepsilon \le 1.06(n - 1) \varepsilon_{mach} \\ & = \{1(\sum x_i y_i) = \sum x_j y_i (1 + \varepsilon_j) \ where \\ & = 1 + \varepsilon_i = \{1 + \varepsilon_j\} \cdot \{1 + \eta_i\} - [1 + \eta_n] \ and \ \delta_j \ _1 \ \eta_i \ \le \varepsilon_{mach} \\ & = 1 + \varepsilon_i = 1 + \varepsilon_j \cdot \{\eta_i\} \cdots \eta_n \} \\ & = -1 + \varepsilon_i = 1 + \varepsilon_j \cdot \{\eta_i\} \cdots \eta_n \} \\ & = -1 + \varepsilon_i = 1 + \varepsilon_j \cdot \{\eta_i\} \cdots \eta_n \} \\ & = -1 + \varepsilon_i = 1 + \varepsilon_j \cdot \{\eta_i\} \cdots \eta_n \} \\ & = -1 + \varepsilon_i = 1 + \varepsilon_i + \varepsilon_i$	5. $\ell_{k,1k-1} \mapsto \ell_{i,1k-1}$ 6. $p_{k,i} \circ p_{i}$ 7. for $j = k+1$ to m do 8. $\ell_{j,k} = u_{j,k} \circ m_{k,k}$ 9. $u_{j,k,m} = u_{j,k,m} - \ell_{j,k} \circ u_{k,k,m}$ 9. $u_{j,k,m} = u_{j,k,m} - \ell_{j,k} \circ u_{k,k,m}$ 11. end for 11. end	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] if Ajis Ajis Ajis Ajis Ajis Ajis Ajis Ajis	$\begin{array}{l} \underset{(p,q)}{\operatorname{linner-product}} \\ \times \underset{(p,q)}{\operatorname{(p')}} \\ \times \underset{(p,q)}{\operatorname{(p')}} \\ = 0 \end{array} \begin{array}{l} \operatorname{linner-product} \\ \times \underset{(p,q)}{\operatorname{(p')}} $	
Notice $\underline{L} = Rn \rfloor$ and $P = (Rn)^{\perp} \rfloor$ are orthogonal compliments, so: $ \underbrace{Proi_{j} = \hat{\mathbf{n}} \hat{\mathbf{n}}^{\top}}_{\text{los}} \text{ is orthogonal projection onto } \underline{L}[along P] $ $\underbrace{Proi_{j} = \hat{\mathbf{n}} \hat{\mathbf{n}}^{\top}}_{\text{los}} \text{ is orthogonal projection onto } \underline{P}] \text{ is orthogonal } \underline{P} = \ker\{proi_{L}\} \text{ is } \ker\{proi_{D}\}] \text{ and } \underline{P} = \ker\{proi_{L}\} \text{ is } \min\{proi_{D}\}] \text{ and } \underline{P} = \ker\{proi_{L}\} \text{ is } \min\{proi_{D}\}]$ $\underbrace{R^{n} = \operatorname{Rn} \cap \{\mathbf{R}\} \prod_{i}^{\perp} i. \text{ c. all vectors } \underline{v} \in R^{n} \text{ uniquely decomposed into } \underline{v} = \underline{v}_{L} \text{ v. } \underline{v} = \underline{P} \text{ uniquely onto } \underline{v} = \underline{v}_{L} \text{ is } \underline{v} = \underline{v}_{L} \text{ is } \underline{v} = \underline{v}_{L} \text{ in } \underline{v} = \underline{v} \text{ is } \underline{v} = \underline{v} \text{ in } \underline{v} \text{ in } \underline{v} = \underline{v} \text{ in } \underline{v} = \underline{v} \text{ in } \underline{v} \text{ in } \underline{v} = \underline{v} \text$	$\begin{array}{l} \frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{Rop} \\ \text{count, but better numerical properties} \\ \text{-Recall: } \frac{Q^{\frac{1}{2}} \otimes 1_n}{2} \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\frac{1}{2}} Q\ = \text{coss}}{\ I_n - Q^{\frac{1}{2}} Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ -\text{Modified Gs} \Rightarrow \frac{\ I_n - Q^{\frac{1}{2}} Q\ = \text{cond}(A)^2 \in_{\text{mach}}}{\ I_n - Q^{\frac{1}{2}} Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ -\text{NOTE: Householder method has } \frac{\ I_n - Q^{\frac{1}{2}} Q\ = \text{cond}(A)^2 \in_{\text{mach}}}{\ Multivariate Calculus} \\ \text{Consider } f: \mathbb{R}^n - \mathbb{R} \\ \text{When clear write } \frac{1}{2} \text{th component of input as } \frac{1}{2} \text{instead of } \frac{1}{2} \text{supposed } \frac{1}{2} \text{conditions} \\ \text{-level curve } \text{w.t. t. } \text{t. } \text{c.} \in \mathbb{R} \text{ jis all points s.t. } f(x) = c \\ \text{-Projecting level curves} \text{ onto } \mathbb{R}^n \text{ jives } f \text{ js} \text{ contour-map} \\ \end{array}$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the x-y-like first rounded to $f!(x) \rfloor$ i.e. $f(x) = f(f!(x)) \rfloor$ absolute error $\Rightarrow \ f(x) - f(x)\ $ relative error $\Rightarrow \ f(x) - f(x)\ $ $\ f(x)\ = 0$ is accurate if $Y \le X \rfloor \ f(x) - f(x)\ = 0$ (ϵ_{mach}) $\ f(x) - f(x)\ = 0$ (ϵ_{mach}) and $\ f(x) - f(x)\ = 0$ (ϵ_{mach}) and $\ f(x) - f(x)\ = 0$ (ϵ_{mach}) and $\ f(x) - f(x)\ = 0$ (ϵ_{mach}) i.e. nearly the right answer to nearly the right question outer-product is stable $\ f(x) - f(x)\ = 0$ (ϵ_{mach}) i.e. exactly the right answer to nearly the right question, a subset of stability is exactly the right answer to nearly the right question, a subset of stability	$\begin{split} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_1 x_n\} (1 + \varepsilon_i) \in \pm 1.05(n-1) \in \text{mach} \} \\ & = \{1(\sum_i x_i y_i) \geq \sum_j y_i (1 + \varepsilon_j) \text{where} \\ & = \{i + \delta_j\} \times \{1 + \eta_j\} \cdots (1 + \eta_n) \} \text{ and } \ \delta_j\ _1 \ \eta_j\ \leq \varepsilon_{\text{mach}} \\ & = 1 + \varepsilon_j + \{1 + \delta_j\} \times \{1 + \eta_j\} \cdots (1 + \eta_n) \} \text{ and } \ \delta_j\ _1 \ \eta_j\ \leq \varepsilon_{\text{mach}} \\ & = 1 + \{i + \delta_j\} \times \{1 + \eta_j\} \cdots (1 + \eta_n) \} \\ & = 1 + \{i + \delta_j\} \times \{1 + \eta_j\} \cdots \{1 + \eta_n\} \\ & = 1 + \{i + \delta_j\} \times \{i + \eta_n\} \times \{i + \eta_n\} \cdots \{i + \eta_n\} \} \\ & = 1 + \{i + \delta_j\} \times \{i + \eta_n\} \times \{i + \eta_n\} \cdots \{i + \eta_n\} \} \\ & = 1 + \{i + \delta_j\} \times \{i + \eta_n\} \times \{i + \eta_n\} \cdots \{i + \eta_n\} \} \\ & = 1 + \{i + \delta_j\} \times \{i + \eta_n\} \times$	5. $(k_{\perp}k_{\perp}) + (\ell_{\perp}k_{\perp}) = 0$ 5. $(p_{\perp} + p_{\perp}) = p_{\perp}k_{\perp}k_{\perp}$ 7. If or $j = k + 1$ to m do 8. $(j_{\perp} = v_{\parallel}k) (p_{\perp}k_{\perp}) = 0$ 9. $v_{\parallel}k_{\parallel}m = v_{\parallel}k_{\parallel}m - \ell_{\parallel}k_{\parallel}k_{\perp}m}$ 10. end for 11. end for 12. Work required: $-\frac{2}{3}m^3$ flops $-O\left(m^3\right)$ results in $L_{jj} \le 1$ 13. Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 13. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 15. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 16. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 17. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 18. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depends on growth-factor $p = \max_{i,j} u_{i,j} $ 19. *Stability depen	Eigenvalue Problems: Iterative Techniques If AJIs [[tutorial 1#Properties of matrices]diagonalizable]] then [[tutorial 1#Eigen-values/vectors]eigen-decomposition]] $A=XNX^{-1}$ -Dominant $A_1: X_1$ are such that $ \lambda_1 $ [is strictly largest for which $Ax=\lambda X$] -Rayleigh quotient for Hermitian $A=A^{\frac{1}{2}}$ is $R_A(x) \equiv \frac{x^{\frac{1}{2}}Ax}{x^{\frac{1}{2}}}$ *Eigenvectors are stationary points of R_Δ] *Eigenvectors are stationary points of R_Δ] ** $R_A(x)$ is closest to being like eigenvalue of \underline{x} , i.e. $R_A(x)$ argmin $\ Ax-\alpha x\ _2$ ** $R_A(x) = x^{\frac{1}{2}}Ax$ **Eigenvector are stationary points of R_Δ] ** $R_A(x) = x^{\frac{1}{2}}Ax$ **Eigenvector are stationary points of R_Δ] ** $R_A(x) = x^{\frac{1}{2}}Ax$ **Eigenvector are stationary points of R_Δ] ***Power iteration: define sequence b(R+1) = $Ab(R)$ ** [Ab(R)]	$\begin{array}{l} & inner-product) \\ & \times_{\mathbb{R}^{D}}(\dots, p^{(n-1)}) \big \text{and } (\underline{\mathbf{r}}^{(0)}, \dots, \underline{\mathbf{r}}^{(n-1)}) \big \text{ are bases for } \\ & \mathbb{R}^{D} \big \\ $	
Notice $\underline{L} = Rn_j$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $\underline{Prol}_{\underline{L}} = \widehat{\mathbf{n}}\widehat{\mathbf{n}}^{T}$ is orthogonal projection onto $\underline{L}_{\underline{J}}(along P_{\underline{J}})$ $\underline{Prol}_{\underline{J}} = \widehat{\mathbf{n}}\widehat{\mathbf{n}}^{T}$ is orthogonal projection onto $\underline{P}_{\underline{J}}(along P_{\underline{J}})$ $\underline{Prol}_{\underline{J}} = \widehat{\mathbf{n}}\widehat{\mathbf{n}}^{T}$ is orthogonal projection onto $\underline{P}_{\underline{J}}(along P_{\underline{J}})$ $\underline{L} = \underline{\mathbf{i}}\widehat{\mathbf{n}}(proj_{\underline{J}}) = ker(proj_{\underline{J}})$ and $\underline{P} = ker(proj_{\underline{J}}) = ker(proj_{\underline{J}})$ $\underline{R}^{n} = \underline{\mathbf{R}}\widehat{\mathbf{n}} = (\underline{\mathbf{R}}\widehat{\mathbf{n}})^{\perp}$ j. i.e. all vectors $\underline{\mathbf{v}} \in R^{n}$ uniquely decomposed into $\underline{\mathbf{v}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$ uniquely decomposed into $\underline{\mathbf{v}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$. Two points $\underline{\mathbf{v}} \in R^{n}$ is $\underline{\mathbf{v}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$. Two points $\underline{\mathbf{v}} \in R^{n}$ if: 1)The translation $\underline{x}\widehat{\mathbf{v}} = \underline{\mathbf{v}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$ is $\underline{\mathbf{p}}$ and $\underline{\mathbf{p}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$. Suppose $\underline{P}_{\underline{\mathbf{u}}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$. Suppose $\underline{P}_{\underline{\mathbf{u}}} = \underline{\mathbf{v}}$ is goes through the origin with unit normal $\underline{\mathbf{u}} = \underline{\mathbf{v}} = \underline{\mathbf{v}}$. Suppose $\underline{P}_{\underline{\mathbf{u}}} = \underline{\mathbf{v}}$ is goes through the origin with unit normal $\underline{\mathbf{u}} = \underline{\mathbf{v}}$.	O(2ma²)] -NOTE: Householder method has $2(mn^2 - n^3/3)$] flop count, but better numerical properties -Recall: $Q^{\dagger}Q = \mathbf{I}_n$ \Rightarrow check for loss of orthogonality with $\ \mathbf{I}_n - Q^{\dagger}Q\ = \cos s$ -Classica GS $\Rightarrow \ \mathbf{I}_n - Q^{\dagger}Q\ = \cos s$ -Modified GS $\Rightarrow \ \mathbf{I}_n - Q^{\dagger}Q\ = \operatorname{Cond}(a)^2 \in_{\operatorname{mach}}$ -NOTE: Householder method has $\ \mathbf{I}_n - Q^{\dagger}Q\ = \epsilon_{\operatorname{mach}}$ Multivariate Calculus Consider $f : \mathbb{R}^n \to \mathbb{R}$ -When clear write: j -th component of input as j -instead of s_j -level curve w.r.t. $t_0 \in \mathbb{R}$ [is all points s.t. $f(x) = c$ -Projecting level curves onto \mathbb{R}^n gives f [S contour-map	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] is first rounded to $f([x)]$, i.e. $\hat{f}(x) = \hat{f}(f([x))]$. Absolute error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ [if $\ \hat{f}(x)\ - \ f(x)\ \ $] if $\ \hat{f}(x)\ - \ f(x)\ \ $ [if $\ \hat{f}(x)\ - \ f(x)\ \ $] is accurate if $\ Y \times \mathbb{E}X\ = \ \hat{f}(x) - f(x)\ \ $ $\ f(x)\ = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ - \ f(x)\ \ $) and $\ \frac{\ X - X\ \ }{\ X\ } = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ $) and $\ \frac{\ X - X\ \ }{\ X - X\ } = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ - \ f(x)\ \ $) is backwards stable if $\ Y \times \mathbb{E}X\ = X$ and $\ X - X\ = X$ an	$\begin{split} & \cdot \{x_1 = -\infty x_n\} \in \{x_1 = -x_2 x_n\} \cap \{+c\}, c \leq 1.06(n-1) \in \text{mach} \\ & \cdot \{(\sum_i x_j)_i > \sum_j x_j \cap \{-c_j\} \text{where} \\ & \cdot \{c_j = (1+\delta_j) \times (1+\eta_j) - (1+\eta_n) \} \text{ and } \ \delta_j\ , \ \eta_j\ \leq \epsilon_{\text{mach}} \\ & -1 + \{c_j = 1+\delta_j \times (\eta_j) + \cdots (\eta_n) \} \\ & -1 + \{c_j = 1+\delta_j \times (\eta_j) + \cdots (\eta_n) \} \\ & -1 + \{c_j = 1+\delta_j \times (\eta_j) + \cdots (\eta_n) \} \\ & -1 + \{(i,j) + i,j + i,$	50 $\{k_{\perp},k_{-1} + \ell_{\perp},k_{\perp}\}$ 50 $\{k_{\perp},k_{-1} + \ell_{\perp},k_{\perp}\}$ 71 for $j = k+1$ to m do 81 $\{\ell_{j,k} = \ell_{j,k},\ell_{k,k,m}\}$ 92 $\{m_{j,k,m} = \ell_{j,k,m} - \ell_{j,k},\ell_{k,k,m}\}$ 93 $\{m_{j,k,m} = \ell_{j,k,m} - \ell_{j,k},\ell_{k,k,m}\}$ 94 of $\{m_{j,k,m} = \ell_{j,k,m} - \ell_{j,k},\ell_{k,k,m}\}$ 95 $\ L\ = O(1)$ 1-Stability depends on growth-factor $\rho = \frac{\max_{i,j} \mu_{i,j} }{\max_{i,j} \alpha_{i,j} }$ 95 of partial pivoting $\rho \le 2^{m-1}$ 1- $\ U\ = O(\rho\ A\) \Rightarrow \widehat{L}U = \widehat{P}A + \delta A\}$ 1- $\ A\ = O(\rho - \ A\) \Rightarrow \widehat{L}U = \widehat{P}A + \delta A$ 1- $\ A\ = O(\rho + \ A\)$ 1-Stability depends stable if $\rho = O(1)$ 1-Stability depends on growth-factor $\rho = \frac{\max_{i,j} \mu_{i,j} }{\max_{i,j} \alpha_{i,j} }$ 1- $\ U\ = O(\rho\ A\) \Rightarrow \widehat{L}U = \widehat{P}A + \delta A$ 1- $\ A\ = O(\rho + \ A\)$ 1-Stability depends on growth-factor $\rho = 0$ 1- $\ A\ = O(\rho + \ A\)$ 1-Stability depends on growth-factor $\rho = 0$ 1- $\ A\ = O(\rho + \ A\)$ 1-Stability depends on growth-factor $\rho = 0$ 1-Stability depends on growth-factor	Eigenvalue Problems: Iterative Techniques If AJIs [[Itutorial 1#Properties of matrices]diagonalizable]] then [[Itutorial 1#Eigen-values/vectors]eigen-decomposition]] A=X/X^-1] -Dominant λ_1 : λ_1 : are such that $ \lambda_1 $ is strictly largest for which $\underline{\lambda} \underline{x} \cdot \underline{\lambda} \underline{x}$ -Rayleigh quotient for Hermitian $\underline{A} = A^{\frac{1}{2}}$ is $R_A(x) = \frac{x^{\frac{1}{2}}Ax}{x^{\frac{1}{2}}x}$ *Eigenvectors are stationary points of R_A * $R_A(x)$ is closest to being like eigenvalue of \underline{x} i.e. $R_A(x)$ -argmin $ Ax-x x _2$ * $R_A(x)$ - $R_A(y)$ - $O(x-y ^2)$] as $\underline{x} \rightarrow y$ where y is eigenvector -Power iteration: define sequence $b^{(k+1)}$ = $\frac{Ab^{(k)}}{ Ab^{(k)} }$ with initial $b^{(0)}$ s.t. $ b^{(0)} = 1$	$\begin{array}{l} & inner-product) \\ *(\underline{p}(0), \dots, \underline{p}^{(n-1)}) \big \text{and } \underline{t}^{(Q)}, \dots, \underline{t}^{(n-1)}) \big \text{ are bases for } \\ \underbrace{R^n} \big \\ \text{QR Algorithm to find Schur decomposition } A = QUQ^{\dagger} \big \\ -Q_1 \big \text{sunitary, i.e. } \underline{Q}^t = \underline{Q}^{-1} \big \text{ and upper-triangular } \underline{U} \big \\ -D_1 \big \text{sunitary, i.e. } \underline{Q}^t = \underline{Q}^{-1} \big \text{ and upper-triangular } \underline{U} \big \\ -D_2 \big \text{sunitary, i.e. } \underline{Q}^t = \underline{Q}^{-1} \big \text{ and upper-triangular } \underline{U} \big \\ -D_3 \big \text{ and produces or thogonal } \underline{Q}^t \big \text{ (In } Q$	
Notice $\underline{L} = Rn \rfloor$ and $P = (Rn)^{\perp} \rfloor$ are orthogonal compliments, so: $ \underbrace{proj_{k} = \widehat{\mathbf{n}}\widehat{\mathbf{n}}^{T}}_{\mathbf{l}} \text{ is orthogonal projection onto } \underline{L}(along P)^{t} $ $\underbrace{proj_{k} = \widehat{\mathbf{n}}\widehat{\mathbf{n}}^{T}}_{\mathbf{l}} \text{ is orthogonal projection onto } \underline{P}^{t} \text{ is orthogonal } \underline{P}^{t} i$	O(2ma²)] -NOTE: Householder method has $2(mn^2 - n^3/3)$] flop count, but better numerical properties -Recall: $Q^{\dagger}Q = \mathbf{I}_n$ \Rightarrow check for loss of orthogonality with $\ \mathbf{I}_n - Q^{\dagger}Q\ = \cos s$ -Classica GS $\Rightarrow \ \mathbf{I}_n - Q^{\dagger}Q\ = \cos s$ -Modified GS $\Rightarrow \ \mathbf{I}_n - Q^{\dagger}Q\ = \operatorname{Cond}(a)^2 \in_{\operatorname{mach}}$ -NOTE: Householder method has $\ \mathbf{I}_n - Q^{\dagger}Q\ = \epsilon_{\operatorname{mach}}$ Multivariate Calculus Consider $f : \mathbb{R}^n \to \mathbb{R}$ -When clear write: j -th component of input as j -instead of s_j -level curve w.r.t. $t_0 \in \mathbb{R}$ [is all points s.t. $f(x) = c$ -Projecting level curves onto \mathbb{R}^n gives f [S contour-map	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] is first rounded to $f([x)]$, i.e. $\hat{f}(x) = \hat{f}(f([x))]$. Absolute error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ $ [if $\ \hat{f}(x)\ - \ f(x)\ \ $] if $\ \hat{f}(x)\ - \ f(x)\ \ $ [if $\ \hat{f}(x)\ - \ f(x)\ \ $] is accurate if $\ Y \times \mathbb{E}X\ = \ \hat{f}(x) - f(x)\ \ $ $\ f(x)\ = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ - \ f(x)\ \ $) and $\ \frac{\ X - X\ \ }{\ X\ } = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ $) and $\ \frac{\ X - X\ \ }{\ X - X\ } = 0$ ($\mathbb{E}\{x\} - \ f(x)\ \ - \ f(x)\ \ $) is backwards stable if $\ Y \times \mathbb{E}X\ = X$ and $\ X - X\ = X$ an	$ \begin{aligned} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_n x_n\} \cap \{+c\}, \in \pm 1, 0 \leq t_n\} - \{\epsilon = n \} \\ & \{x_1 = x_n\} = \{x_n = x_n\} \cap \{+c\}, \{-c\}, $	5. $(k_{\perp})_{k-1} + \ell_{\perp}(j_{k-1})$ 5. $(k_{\perp})_{k-1} + \ell_{\perp}(j_{k-1})$ 7. for $j = k+1$ to m do 8. $(j_{j_{k}} = \mu_{j_{k}}) \ell_{j_{k}}$ 9. $u_{j_{k},m} = u_{j_{k},m-\ell} \ell_{j_{k}} u_{k,k,m}$ 10. end for 11. end for 11. end for 12. Sability depends on growth-factor $\rho = \frac{\max_{i,j} u_{i,j} }{\max_{i,j} u_{i,j} }$ 12. Stability depends on growth-factor $\rho = \frac{\max_{i,j} u_{i,j} }{\max_{i,j} u_{i,j} }$ 13. For partial pivoting $\rho \le 2^{m-1}$ 14. $\ U\ = O(\rho\ A\) \Rightarrow \widehat{L}U = \widehat{L}U$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Beproperties of matrices[diagonalizable]] then [[tutorial \bot] Eigenvalues/vectors[eigen-decomposition]] $A = X A X^{-1}$] -Dominant $\lambda_1 : x_1$ are such that $ \lambda_1 $ jis strictly largest for which $\lambda_1 = \lambda_1 = \lambda$	$\begin{array}{l} & \operatorname{inner-product}) \\ & \times (p(0), \dots, p^{(n-1)}) \operatorname{and} \left(\chi^{(0)}, \dots, \chi^{(n-1)} \right) \operatorname{are} \operatorname{bases} \operatorname{for} \\ & \times (p(0), \dots, p^{(n-1)}) \operatorname{and} \left(\chi^{(0)}, \dots, \chi^{(n-1)} \right) \operatorname{are} \operatorname{bases} \operatorname{for} \\ & \times (p(0), \dots, p^{(n-1)}) \operatorname{and} \operatorname{corposition} A = QUQ^{\dagger} \\ & - Q \operatorname{Is} \operatorname{unitary}, \operatorname{i.e.} Q^{\dagger} = Q^{-1} \operatorname{and} \operatorname{upper-triangular} U \\ & - \operatorname{Diagonal} \operatorname{of} U \operatorname{cortains} \operatorname{eigenvalues} \operatorname{of} A \operatorname{distance} \operatorname{distance} \operatorname{produces} \operatorname{orthogonal} Q^{(k)} = Q^{(k)} \operatorname{distance} \operatorname{distance} \operatorname{orthogonal} Q^{(k)} = Q^{(k)} \operatorname{distance} \operatorname{distance} \operatorname{orthogonal} Q^{(k)} = Q^{(k)} \operatorname{distance} \operatorname{orthogonal} orthogona$	
Notice $\underline{L} = Rn_j$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $-prol_{\perp} = \hat{\mathbf{n}}_{n} - \hat{\mathbf{n}}_{n}^{T}$ is orthogonal projection onto $\underline{L}_{\parallel}(along P_{\parallel})$ $-prol_{\perp} = \hat{\mathbf{n}}_{n} - \hat{\mathbf{n}}_{n}^{T}$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel})$ $-prol_{\parallel} = \hat{\mathbf{n}}_{n} - \hat{\mathbf{n}}_{n}^{T}$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel})$ $-1 = \lim_{n \to \infty} (prol_{\parallel}) + \lim_{n \to \infty} (pr$	$\begin{array}{l} \frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{Rop} \\ \text{count, but better numerical properties} \\ \text{-Recall: } \frac{Q^{\frac{1}{2}}Q \cdot I_n}{2} \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \cos\ }{\ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \cos\ } \\ -\text{Classical GS} \Rightarrow \ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \cos\ \\ -\text{Modified GS} \Rightarrow \ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \cos\ \\ +\text{Mot Subseholder method has } \frac{\ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \sin\ }{\ I_n - Q^{\frac{1}{2}}Q\ \cdot \ \sin\ } \\ \text{Consider } f \colon \mathbb{R}^n \to \mathbb{R}^1 \\ \text{When clear write } j \cdot \text{th component of input as } j \sinstead of x_j \\ \text{-level curve w.t.t. } to \subseteq \mathbb{R} \text{is all points s.t. } f(x) \cdot c \cdot \ \sin\ \\ \text{-Projecting level curves noto } \mathbb{R}^n \text{gives } f \text{S} \\ \text{contour-map} \\ \\ \frac{n_k}{n} \cdot \text{th order partial derivative w.t.t.} i_{j_1} \int f \cdot \int_{j_1}^{j_1} \text{s.} \\ \frac{n_k}{n_n} \cdot \frac{n_{j_1}}{n_{j_1}} = n_{j_1}^n - n_{j_1}^n f = f_{j_1}^{(n_1, \dots, n_k)} \\ \frac{n_k}{n_n} \cdot \frac{n_{j_1}}{n_n} = n_{j_1}^n - n_{j_1}^n f = f_{j_1}^{(n_1, \dots, n_k)} \\ \frac{n_k}{n_n} \cdot \frac{n_{j_1}}{n_n} = n_{j_1}^n - n_{j_1}^n f = f_{j_1}^{(n_1, \dots, n_k)} \\ \\ \end{array}$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variable $f: X \to X \to X \rfloor$ in the variable $f: X \to X \to X \to X \rfloor$ in the variable $f: X \to X $	$ \begin{array}{ll} (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_1, \varepsilon \leq 1, 0 \times (n-1) \in \max) \\ -(1(\sum_i x_i_i)_i) = \sum_i y_i (1 + \varepsilon_i_i) (1 + \eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i = 1 + \varepsilon_i + (\eta_i) + (\eta_{in}) - (1 + \eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i = 1 + \varepsilon_i + (\eta_i) + (\eta_{in}) + (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i) = (1 + \gamma_i) \leq \varepsilon_i (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_i \leq \varepsilon_i \right\} \\ -(1 + \gamma_i) = (1 + \gamma_i) \leq \varepsilon_i (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_i \leq \varepsilon_i \right\} \\ -(1 + \gamma_i) = (1 + \varepsilon_i) =$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 6. $p_{\perp} \circ p_{\perp}$ 7. for $j = k+1$ to m do 8. $(j_{\perp} = u_{\parallel}k)(p_{\perp}k_{\perp})$ 9. $u_{\parallel}k_{\parallel}m = u_{\parallel}k_{\parallel}m - \ell_{\parallel}ku_{\parallel}k_{\perp}m}$ 10. end for 11. end for 12. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 13. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 14. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 15. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 15. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 16. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 17. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 18. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 19. ability depends on $p \ge 1$ 19. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 21. ability depends on $p \ge 1$ 22. ability depends on $p \ge 1$ 23. ability depends on $p \ge 1$ 24. ability depends on $p \ge 1$ 25. ability depends on $p \ge 1$ 26. ability depends on $p \ge 1$ 27. ability depends on $p \ge 1$ 28. ability depends on $p \ge 1$ 29. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 210. ability depends on $p \ge 1$ 210. ability	Eigenvalue Problems: Iterative Techniques If AJIs [Itutorial 1#Properties of matrices]diagonalizable]] then [[Itutorial 1#Eigen-values/vector]eigen-decomposition]] A=X/N^T] -Dominant λ_1 ; x_1 are such that $ \lambda_1 $ is strictly largest for which $\Delta x = \lambda x$. -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $R_A(x) = \frac{x^{\dagger}}{Ax}$ *Eigenvectors are stationary points of R_A ** $R_A(x)$] is closest to being like eigenvalue of \underline{x}_1 i.e. $R_A(x) = a_{\min}[Ax = \alpha x]$ ** $R_A(x)$ - $R_A(y) = O(x - y ^2)$] as $\underline{x} \to y_1$ where y_1 is eigenvector -Power iteration: define sequence $b(R+1) = \frac{Ab(R)}{ Ab(R) }$ with initial $b(0)$ s.t. $ b(0) = 1$ -Assume dominant λ_1 : λ_1 exist for Δ_1 and that A_1 : λ_1 and A_2 : Under above assumptions.	$\begin{array}{l} & inner-product) \\ **(\underline{R}^{0})_{,p}(n^{-1})_{j} \text{ and } (\underline{t}^{(Q)},,\underline{t}^{(n-1)})_{j} \text{ are bases for } \\ & \underline{R}^{n}_{j} \\ & \underline{R}^{n}_{j} \text{ and } (\underline{t}^{(Q)},,\underline{t}^{(n-1)})_{j} \text{ are bases for } \\ & \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j} \text{ and } \underline{R}^{n}_{j} \\ \text{ and } \underline{R}^{n}_{j$	
Notice $\underline{L} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $-prol_{\underline{L}} = \widehat{\mathbf{n}} \widehat{\mathbf{n}}^{\top}$ is orthogonal projection onto $\underline{L}_{\underline{J}}(along P_{\underline{J}})$ $-prol_{\underline{J}} = \widehat{\mathbf{n}} \widehat{\mathbf{n}}^{\top}$ is orthogonal projection onto $\underline{P}_{\underline{J}}(along P_{\underline{J}})$ $-prol_{\underline{J}} = \widehat{\mathbf{n}} \widehat{\mathbf{n}}^{\top}$ is orthogonal projection onto $\underline{P}_{\underline{J}}(along P_{\underline{J}})$ $-1 = \lim_{t \to \infty} (prol_{\underline{J}}) = \lim_{t \to \infty$	$\begin{array}{l} O(2mn^2) \\ - \text{NOTE: Householder method has } 2\left(mn^2 - n^3 / 3\right)] \text{flop} \\ \text{count, but better numerical properties} \\ \text{-Recall: } Q^{\dagger} Q = I_n \Big \Longrightarrow \text{check for loss of orthogonality} \\ \text{with } \ I_n - Q^{\dagger} Q\ = \log s \Big \\ - \text{classical GS} \Longrightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ - \text{Modified GS} \Longrightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ - \text{MOTE: Householder method has } \ I_n - Q^{\dagger} Q\ = \epsilon_{\text{mach}} \\ \text{Multivariate Calculus} \\ \text{Consider } f : \mathbb{R}^n \to \mathbb{R} \\ \text{-When clear write } J \text{-th component of input as } j] \text{instead of } x_j \\ \text{-Level curve w.r.t. } to \subseteq \mathbb{R} \text{ jis all points s.t. } f(x) = c \\ \text{-Projecting } \text{evel curves onto } \mathbb{R}^n \text{ gives } f \text{ js} \\ \text{contour-map} \\ n_k \text{ th order partial derivative w.r.t. } i_1 \text{ jof } f \text{-i} \text{ or } n_1 \text{ jth order partial derivative w.r.t. } \frac{n_1}{n_1} \text{ for } f \text{-i} \text{ or } n_2 \text{ just } f \text{-i} \\ \frac{n_1}{n_2} \frac{n_1}{n_1} = \delta_{j_1}^n - \delta_{j_1}^n = f_{j_1}^n - j_n - j_n - j_n \text{-j} \\ 1 \text{-i} \frac{n_1}{n_2} \frac{n_1}{n_1} = \delta_{j_1}^n - \delta_{j_1}^n = f_{j_1}^n - j_n - j_n \text{-j} \\ 1 \text{-i} \text{ is an } M \text{ broder partial derivative where } N = \sum_k n_k \text{ near } f \text{-i} \\ \text{-i} \end{cases}$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ is accurate if $f: X \to X \rfloor$ in the variance of $f: X \to X \rfloor$ is accurate if $f: X \to X \rfloor$ in the variance of $f: X \to Y \rfloor$ in the variance of $f: X \to Y \rfloor$ is accurate if $f: X \to X \rfloor$ in the variance of $f: X \to Y \rfloor$ is accurate if $f: X \to X \rfloor$ in the variance of $f: X \to Y \rfloor$ in	$ \begin{array}{ll} (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_1, \varepsilon \leq 1, 0 \times (n-1) \in \max) \\ -(1(\sum_i x_i_i)_i) = \sum_i y_i (1 + \varepsilon_i_i) (1 + \eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i = 1 + \varepsilon_i + (\eta_i) + (\eta_{in}) - (1 + \eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i = 1 + \varepsilon_i + (\eta_i) + (\eta_{in}) + (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_{\max} \right\} \\ -(1 + \varepsilon_i) = (1 + \gamma_i) \leq \varepsilon_i (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_i \leq \varepsilon_i \right\} \\ -(1 + \gamma_i) = (1 + \gamma_i) \leq \varepsilon_i (\eta_{in}) = \min \left\{ j_i, \eta_{ii} \leq \varepsilon_i \leq \varepsilon_i \right\} \\ -(1 + \gamma_i) = (1 + \varepsilon_i) =$	5. $f_{k,1,k-1} \mapsto f_{k,1,k-1}$ 5. $f_{k,1,k-1} \mapsto f_{k,1,k-1}$ 7. If or $j = k+1$ to m do 8. $f_{j,k} = m_{j,k}m_{k} = g_{j,k}m_{k,k}m_{k}$ 9. $m_{j,k,m} = m_{j,k,m} = f_{j,k}m_{k,k,m}$ 11. $m_{j,k} = m_{j,k,m} = f_{j,k}m_{k,k,m}$ 12. $m_{j,k} = m_{j,k,m} = g_{j,k}m_{k,k}m_{k}$ 13. $m_{j,k} = m_{j,k}m_{k} = g_{j,k}m_{k,k}m_{k}$ 14. $m_{j,k} = m_{j,k} = g_{j,k}m_{k} = g_{j,k}m_$	Eigenvalue Problems: Iterative Techniques If AJIs [Itutorial 1#Properties of matrices]diagonalizable]] then [[Itutorial 1#Eigen-values/vector]eigen-decomposition]] A=X/N^T] -Dominant λ_1 ; x_1 are such that $ \lambda_1 $ is strictly largest for which $\Delta x = \lambda x$. -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $R_A(x) = \frac{x^{\dagger}}{Ax}$ *Eigenvectors are stationary points of R_A ** $R_A(x)$] is closest to being like eigenvalue of \underline{x}_1 i.e. $R_A(x) = a_{\min}[Ax = \alpha x]$ ** $R_A(x)$ - $R_A(y) = O(x - y ^2)$] as $\underline{x} \to y_1$ where y_1 is eigenvector -Power iteration: define sequence $b(R+1) = \frac{Ab(R)}{ Ab(R) }$ with initial $b(0)$ s.t. $ b(0) = 1$ -Assume dominant λ_1 : λ_1 exist for Δ_1 and that A_1 : λ_1 and A_2 : Under above assumptions.	$\begin{array}{l} & inner-product) \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } (\chi^{(Q)}, \dots, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \end{bmatrix} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } (\chi^{(Q)}, \dots, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \end{bmatrix} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) are producted$	
Notice $\underline{L} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $\underline{Prol}_{\underline{L}} = \widehat{n}\widehat{n}^T$ is orthogonal projection onto $\underline{L}_{\underline{L}}[along P_{\underline{L}}]$ $\underline{Prol}_{\underline{L}} = \widehat{n}\widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\underline{L}}[along P_{\underline{L}}]$ $\underline{Prol}_{\underline{L}} = \underline{n}\widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\underline{L}}[along P_{\underline{L}}]$ $\underline{L} = \underline{n}\widehat{n}(Proj_{\underline{L}}) + \ker(Proj_{\underline{L}})$ and $\underline{P} = \ker(Proj_{\underline{L}}) + \ker(Proj_{\underline{L}})$ is orthogonal projection onto $\underline{P}_{\underline{L}}[along P_{\underline{L}}]$ $\underline{R}^n = \underline{R}\underline{n} \in R(\underline{n})^{\perp}$ j. i.e. all vectors $\underline{v} \in R^n$ uniquely decomposed into $\underline{v} = \underline{v}_{\underline{L}} + \underline{v}_{\underline{L}}$ Householder Maps: reflections $\underline{P} = (Rn)^{\perp} + \underline{c}$ if: 1) The translation $\underline{x}_{\underline{V}} = \underline{y} = \underline{v}_{\underline{L}}$ is parallel to normal $\underline{n}_{\underline{L}}$ i.e. $\underline{m} = \underline{n} = \underline{c} = \underline{n}$ \underline{S} upon the \underline{n} is goes through the origin with unit normal \underline{n} is \underline{R} is reflection w.r.t. hyperplane $\underline{P}_{\underline{L}}$ $\underline{P}_{\underline{L}}$ Householder matrix $\underline{H}_{\underline{U}} = \underline{I}_{\underline{n}} - 2\underline{u}\underline{u}^T$ is reflection w.r.t. hyperplane $\underline{P}_{\underline{U}}$ $\underline{P}_{\underline{L}}$ recall: let $\underline{L}_{\underline{L}} = \underline{R}\underline{u}$ $\underline{P}_{\underline{L}} = \underline{u}\underline{u}^T$ and $\underline{P}_{\underline{L}} = \underline{u}\underline{u}^T$ is $\underline{P}_{\underline{L}} = \underline{u}\underline{u}^T$ and $\underline{P}_{\underline{L}} = \underline{u}\underline{u}^T$	$\begin{array}{l} O(2mn^2) \\ - \text{NOTE: Householder method has } 2\left(mn^2 - n^3/3\right)] \text{flop} \\ \text{count, but better numerical properties} \\ \text{-Recall: } Q^{\dagger} Q = I_n \Big \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \ I_n - Q^{\dagger} Q\ = \log s \Big \\ - \text{classical GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ - \text{Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A) \epsilon_{\text{mach}} \\ - \text{NOTE: Householder method has } \ I_n - Q^{\dagger} Q\ = \epsilon_{\text{mach}} \\ \text{Multivariate Calculus} \\ \text{Consider } f: \mathbb{R}^n \to \mathbb{R} \} \\ \text{-When clear write } [\text{-th component of input as } j] \text{instead of } s_j \\ \text{-Level curve w.r.t. to } c \in \mathbb{R} \text{ jis all points s.t. } f(x) = c \\ \text{-Projecting level curves onto } \mathbb{R}^n \text{ gives } f s \\ \text{-contour-map} \\ \\ n_k \text{-th order partial derivative w.r.t. } i_k \text{-} of \dots, of } n_1 \text{-} \text{th} \\ \text{-} \frac{\partial h^{k-m-n} f_1}{\partial s_k} = a_{j_k}^n - a_{j_1}^n f = f_{j_1-j_k}^n \\ \frac{\partial h^{k-m-n} f_1}{\partial s_k} = a_{j_k}^n - a_{j_1}^n f = f_{j_1-j_k}^n \\ \frac{\partial h^{k-m-n} f_1}{\partial s_k} = a_{j_k}^n - a_{j_1}^n f = f_{j_1-j_k}^n \\ \frac{\partial f}{\partial s_k} = \frac{\partial f}{\partial s_k} f \\ \\ \text{-} \mathcal{I} \text{-} \text{-} \text{-} \text{-} \text{-} \text{-} \text{-} -$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variable $f: X \to X \to X \rfloor$ in the variable $f: X \to X \to X \to X \rfloor$ in the variable $f: X \to X $	$ (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_1, \varepsilon \leq 1.05(n-1) \in \text{mach}) $ $ (x_1 \subseteq x_1 y_1) = x_1 y_1 (1 + \varepsilon_1) \text{ where} $ $ (1 \in x_1 = 1 + \varepsilon_1 + (\varepsilon_1 + \varepsilon_1) - (1 + \varepsilon_n) \text{ and } \delta_1 _1 _1 _1 \leq \varepsilon_{\text{mach}} $ $ (1 + \varepsilon_1 = 1 + \varepsilon_1 + (\varepsilon_1 + \varepsilon_1) - (1 + \varepsilon_n) \text{ and } \delta_1 _1 _1 _1 \leq \varepsilon_{\text{mach}} $ $ (1 + \varepsilon_1 = 1 + \varepsilon_1 + (\varepsilon_1 + \varepsilon_1) + (\varepsilon_1 + \varepsilon_1) \text{ and } \delta_1 _1 _1 _1 \text{ and } _1 _1 _1 \text{ and } _1 _1 _1 _1 _1 _1 _1 _1 _1 _1$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 6. $p_{\perp} \circ p_{\perp}$ 7. for $j = k+1$ to m do 8. $(j_{\perp} = u_{\parallel}k)(p_{\perp}k_{\perp})$ 9. $u_{\parallel}k_{\parallel}m = u_{\parallel}k_{\parallel}m - \ell_{\parallel}ku_{\parallel}k_{\perp}m}$ 10. end for 11. end for 12. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 13. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 14. work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{ij} \le 1$ 15. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 15. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 16. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 17. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 18. ability depends on growth-factor $p = \max_{i,j} u_{i,j} u_{i,j} $ 19. ability depends on $p \ge 1$ 19. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 21. ability depends on $p \ge 1$ 22. ability depends on $p \ge 1$ 23. ability depends on $p \ge 1$ 24. ability depends on $p \ge 1$ 25. ability depends on $p \ge 1$ 26. ability depends on $p \ge 1$ 27. ability depends on $p \ge 1$ 28. ability depends on $p \ge 1$ 29. ability depends on $p \ge 1$ 20. ability depends on $p \ge 1$ 210. ability depends on $p \ge 1$ 210. ability	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Eigenvalues (vectors] eigen-decomposition]] $A = X A X^{-1}$] -Dominant λ_1 ; λ_1 are such that $ \lambda_1 $ jis strictly largest for which $ \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_3 \times \lambda_4 \times \lambda_4 \times \lambda_4 \times \lambda_3 \times \lambda_4 \times$	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} $ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: • $\mathbf{P} \mathbf{n} \mathbf{i}_{\perp} = \mathbf{n} \mathbf{n} \mathbf{i}^{\top}$ is orthogonal projection onto $\underline{L} (\mathbf{along} \ P)^{\parallel} \mathbf{j} $ • $\mathbf{P} \mathbf{n} \mathbf{j}_{\perp} = \mathbf{n} \mathbf{n}^{\top} \mathbf{j}^{\top}$ is orthogonal projection onto $\underline{P} \mathbf{j} \mathbf{n} \mathbf{j}$ • $\mathbf{L} = \mathbf{i} \mathbf{m} [\mathbf{p} \mathbf{n} \mathbf{j}_{\perp}] \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{j}^{\top}$ is orthogonal projection onto $\underline{P} \mathbf{j} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{j}^{\top}$ is orthogonal projection onto $\underline{P} \mathbf{j} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n}^{\top} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n}^{\top} $	$\begin{array}{l} \frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{flop}}{\text{count, but better numerical properties}} \\ \text{-Recall: } \frac{Q^{\dagger} Q = I_n}{2} \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\dagger} Q\ = \log s}{\ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \\ \text{-Multivariate Calculus} \\ \text{Consider } f: \mathbb{R}^n \to \mathbb{R} \\ \text{-When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ \text{-When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ \text{-Level curve w.r.t. to } c \in \mathbb{R} \text{ jis all points s.t. } f(\mathbf{x}) = c \\ \text{-Projecting level curves onto } \mathbb{R}^n \text{ gives } f \\ \text{s.c.} \\ \text{-Injection } s_j \text{-the order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative where } N = \sum_k n_k \\ \frac{n_k}{N_k} - \frac{n_1}{N_k} = a_{j_k}^n - a_{j_1}^n = f_{j_1}^n - i_n \\ \frac{n_k}{N_k} - \frac{n_k}{N_k} \text{-dos}_{j_1}^n \text{j.s.} \text{j.s.} \\ \frac{n_j}{N_k} + \text{order partial derivative where } N = \sum_k n_k \\ \frac{n_j}{N_k} - \frac{n_j}{N_k} \text{j.s.} \text{j.s.} \\ \frac{n_j}{N_k} - \frac{n_j}{N_k} \text{j.s.} \\ \frac{n_j}{N$	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$] in algorithm for f [is $\hat{f}: X \to Y$] in the control of $f(x)$] i.e. $\hat{f}(x) = \hat{f}(f(x))$] relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ _{F(x)}$ relative error $\Rightarrow \ \hat{f}(x) - f(x)\ \ _{F(x)}$ $\ \hat{f}(x)\ _{F(x)}$ $\ \hat{f}(x$	$ \begin{aligned} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_i) \in \pm 1.06(n-1) \in \mathrm{mach} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \varepsilon_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_iy_i (1 + \varepsilon_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_iy_i (1 + \varepsilon_i) \mathrm{Mere} \\ & \cdot \{(1 + \delta_i) \times (1 + \eta_i) - (1 + \eta_n)\} \mathrm{and} \ \ \delta_j \ _1 \ \eta_j \ \le \varepsilon_{\mathrm{mach}} \\ & \cdot \{(1 \le i - \delta_i) \times (1 + \eta_i) - (1 + \eta_n)\} \mathrm{and} \ \ \delta_j \ _1 \ \eta_j \ \le \varepsilon_{\mathrm{mach}} \\ & \cdot \{(1 \le i - \delta_i) \times (1 + \eta_i) + (1 + \eta_i)\} \mathrm{and} \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ _1 \ \ \delta_j \ \ \delta_j$	50 $f_{k,1,k-1} \mapsto f_{k,1,k-1}$ 50 $f_{k,1,k-1} \mapsto f_{k,1,k-1}$ 71 for $j = k+1$ to m do 61 $f_{j,k} = u_{j,k} m_{k,k}$ 72 $u_{j,k} = u_{j,k} m_{k-1}$ 73 $u_{j,k} = u_{j,k} m_{k-1}$ 74 whork required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{jj} \le 1$ 75 $u_{j,k} = u_{j,k} m_{k-1}$ 75 $u_{j,k} = u_{j,k} m_{k-1}$ 76 results in $L_{jj} \le 1$ 77 $u_{j,k} = u_{j,k}$ 78 $u_{j,k} = u_{j,k}$ 79 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 71 $u_{j,k} = u_{j,k}$ 72 $u_{j,k} = u_{j,k}$ 73 $u_{j,k} = u_{j,k}$ 74 $u_{j,k} = u_{j,k}$ 75 $u_{j,k} = u_{j,k}$ 76 $u_{j,k} = u_{j,k}$ 77 $u_{j,k} = u_{j,k}$ 78 $u_{j,k} = u_{j,k}$ 79 $u_{j,k} = u_{j,k}$ 79 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 71 $u_{j,k} = u_{j,k}$ 72 $u_{j,k} = u_{j,k}$ 73 $u_{j,k} = u_{j,k}$ 74 $u_{j,k} = u_{j,k}$ 75 $u_{j,k} = u_{j,k}$ 76 $u_{j,k} = u_{j,k}$ 77 $u_{j,k} = u_{j,k}$ 78 $u_{j,k} = u_{j,k}$ 79 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 70 $u_{j,k} = u_{j,k}$ 71 $u_{j,k} = u_{j,k}$ 72 $u_{j,k} = u_{j,k}$ 73 $u_{j,k} = u_{j,k}$ 74 $u_{j,k} = u_{j,k}$ 75 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_2$ * $R_A(x) = a \operatorname{constant} Ax = \alpha x _2$ *with initial $\underline{b}(0)$ s.t. $\underline{b}(0)$ $\underline{b}(0)$ $\underline{b}(0)$ and that $\underline{b}(0)$ \underline{x}_1	$\begin{array}{l} & inner-product) \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } (\chi^{(Q)}, \dots, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \end{bmatrix} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } (\chi^{(Q)}, \dots, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \end{bmatrix} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ and } \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) \text{ are producted} \\ & \times (p(0), \dots, p^{(n-1)}) are producted$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} $ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: **Prol_i = $\mathbf{n} \mathbf{n}^{\top}$ is orthogonal projection onto $\underline{L} (along \ P)^{\parallel}$ **Prol_i = $\mathbf{n}^{\top} \mathbf{n}^{\top}$ is orthogonal projection onto $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is reflection such that $\underline{P} \underline{P} $	$\begin{array}{l} \frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{flop}}{\text{count, but better numerical properties}} \\ \text{-Recall: } \frac{Q^{\dagger} Q = I_n}{2} \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\dagger} Q\ = \log s}{\ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \\ \text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \\ \text{-Multivariate Calculus} \\ \text{Consider } f: \mathbb{R}^n \to \mathbb{R} \\ \text{-When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ \text{-When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ \text{-Level curve w.r.t. to } c \in \mathbb{R} \text{ jis all points s.t. } f(\mathbf{x}) = c \\ \text{-Projecting level curves onto } \mathbb{R}^n \text{ gives } f \\ \text{s.c.} \\ \text{-Injection } s_j \text{-the order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative w.r.t.} i_n \text{of } \dots, \text{of } n_1 \text{ jth order partial derivative where } N = \sum_k n_k \\ \frac{n_k}{N_k} - \frac{n_1}{N_k} = a_{j_k}^n - a_{j_1}^n = f_{j_1}^n - i_n \\ \frac{n_k}{N_k} - \frac{n_k}{N_k} \text{-dos}_{j_1}^n \text{j.s.} \text{j.s.} \\ \frac{n_j}{N_k} + \text{order partial derivative where } N = \sum_k n_k \\ \frac{n_j}{N_k} - \frac{n_j}{N_k} \text{j.s.} \text{j.s.} \\ \frac{n_j}{N_k} - \frac{n_j}{N_k} \text{j.s.} \\ \frac{n_j}{N$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variable $f: X \to X \rfloor$ in the variabl	$ \begin{aligned} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_n x_n\} \cap \{+c\}, \in \pm 1.06(n-1) \in \text{mach} \} \\ & = \{1(\sum x_i y_i) \ge x_i y_i \in +\epsilon_j\} \text{where} \\ & = \{i = \{i + \delta_j\} \times \{1 + n_j\} - (1 + n_n)\} \text{and } \delta_j \mid_i \mid_{n \mid i} \mid \le \epsilon_{\text{mach}} \} \\ & = \{i = \{i + \delta_j\} \times \{1 + n_j\} - (1 + n_n)\} \text{and } \delta_j \mid_i \mid_{n \mid i} \mid \le \epsilon_{\text{mach}} \} \\ & = \{i = \{i + \delta_j\} \times \{1 + n_j\} - (1 + n_n)\} \text{and } \delta_j \mid_i \mid_{n \mid i} \mid \le \epsilon_{\text{mach}} \} \\ & = \{i \in \{i + \delta_j\} \times \{1 + n_j\} - (1 + n_n)\} \text{where } x_i \mid_j \mid_j \mid_{n \mid i} \mid_{n \mid i$	50 $\{k_{\perp},k_{-1} + \ell_{\perp},k_{\perp} - 1\}$ 50 $\{p_{\perp}, e^{-1}\}_{\perp} = 0$ 71 for $j = k + 1$ to m do 81 $\{l_{j,k} = u_{j,k}, l_{m,k,m}\}$ 92 $u_{j,k,m} = u_{j,k,m} - \ell_{j,k}u_{k,k,m}$ 93 $u_{j,k,m} = u_{j,k,m} - \ell_{j,k}u_{k,k,m}$ 101 end for 111 end for 112 end for 113 so $\ L\ = O(1)$ 113 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 114 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 115 for partial pivoting $\rho \le 2^{m-1}$ 116 $\ u_{i,j}\ = 0$ 117 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 118 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 119 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 119 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 121 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 122 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 123 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 124 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 125 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 125 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 125 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 126 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 127 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 128 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 129 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 120 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 121 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 122 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 123 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 124 Sability depends on growth-factor $\rho = \max_{i,j} u_{i,j} $ 125 Sability depends on growth-f	Eigenvalue Problems: Iterative Techniques if Ajis [[tutorial \bot #Properties of matrices]diagonalizable]] then [[tutorial \bot #Eigenvalues] with Ajis and that \bot #Eigenvalues/vectors]eigen-decomposition]] \bot = \bot **Dominant \bot **, \bot ** are such that \bot **, \bot strictly largest for which \bot **\tilde{\lambda} \subseteq \frac{1}{3} \text{ is strictly largest for which }\frac{1}{3} \text{ is } \text{ strictly largest for which }\frac{1}{3} \text{ is } \text{ is } \text{ strictly largest for which }\frac{1}{3} \text{ is } \text{ strictly largest for which }\frac{1}{3} \text{ is } \text{ strictly largest for which }\frac{1}{3} \text{ is } \text{ strictly largest for which }\frac{1}{3} \text{ is } \text{ strictly largest for }\frac{1}{3} strictly lar	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} $ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: **Prol_i = $\mathbf{n} \mathbf{n}^{\top}$ is orthogonal projection onto $\underline{L} (along \ P)^{\parallel}$ **Prol_i = $\mathbf{n}^{\top} \mathbf{n}^{\top}$ is orthogonal projection onto $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is orthogonal $\underline{P} = \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ in $\underline{P} \mathbf{n}^{\top}$ is reflection w.r.t. hyperplane $\underline{P} \underline{P} $	$\begin{aligned} & \frac{o(2m\alpha^2)}{-NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \mathbf{Rop} \\ & \text{count, but better numerical properties} \\ & \cdot \mathbf{Recall:} \ Q^{\frac{1}{4}} Q \cdot \mathbf{I}_n \Rightarrow \text{check for loss of orthogonality} \\ & \text{with } \frac{\ \mathbf{I}_n - Q^{\frac{1}{4}} Q\ = \cos \ \mathbf$	Given a problem $f: X \to Y$] an algorithm for f [is $\hat{f}: X \to Y$]. Finally a set $\hat{f}: X \to Y$] is first rounded to $f(x)$, i.e., $\hat{f}(x) = \hat{f}(f(x))$. Absolute error $\Rightarrow \ \hat{f}(x) - f(x)\ $ Felative error $\Rightarrow \ \hat{f}(x) - f(x)\ $ $\ \hat{f}(x)\ = 0$ ($\ \hat{f}(x)\ - \ \hat{f}(x)\ $). $\ \hat{f}\ = \ \hat{f}\ = $	$ \begin{aligned} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \epsilon_i), \in \pm 1.06(n-1) \in \text{mach} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le x_i) \text$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 5. (p_{\perp}, e^{+}) 6. (p_{\perp}, e^{+}) 7. for $j = k + 1$ to m do 8. $(j_{\perp} = m_{\perp}) k_{\parallel} k_{\perp}$ 9. $w_{j,k,m} = w_{j,k,m} - \ell_{j,k} w_{k,k,m}$ 10. end for 11. end for 12. Work required: $-\frac{2}{3}m^{3}$ flops $-O\left(m^{3}\right)$ results in $L_{\parallel j} \leq 1$ 13. $L_{\parallel l} = O(1)$ 13. *Stability depends on growth-factor $p = \frac{\max_{i,j} w_{i,j} }{\max_{i,j} w_{i,j} }$ 14. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\perp} \otimes A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 15. **only backwards stable if $p = O(1)$] 16. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\perp} \otimes A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 16. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\perp} \otimes A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 17. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 18. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 18. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 18. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 18. **Jull = $O(p_{\parallel}A_{\parallel}) = \sum_{l,l} \frac{p_{l}A_{\parallel}}{\ A_{\parallel}\ } = O\left(p_{l}e_{machine}\right)$ 18. **Jull = $O(p_{\parallel}A_{\parallel}A_{\parallel}) = O\left(p_{\parallel}A_{\parallel}A_{\parallel}A_{\parallel}A_{\parallel}A_{\parallel}A_{\parallel}A_{\parallel}A$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Eigenvalues (vectors] eigen-decomposition]] $A = X A X^{-1}$] -Dominant $\lambda_1 : x_1$ are such that $ \lambda_1 $ [is strictly largest for which $\lambda_1 \in X$] are such that $ \lambda_1 $ [is strictly largest for which $\lambda_2 \in X$] -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $A_1(x) = \frac{x^{\dagger}A_1}{x^{\dagger}X}$ *Eigenvectors are stationary points of A_1 *Eigenvectors are stationary points of A_2 *Eigenvectors are stationary points of A_1 *Eigenvectors are stationary points of A_2 *Ap_(x)] is closest to being like eigenvalue of x_2 i.e. $A_2(x) = x_2$ *Ap_(x) = x_2	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} $ and $P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: **Prol_* = $\mathbf{n} \mathbf{n}^{\perp}$ is orthogonal projection onto \underline{L} [$a \mathbf{l} \mathbf{n} \mathbf{p} \mathbf{v} \mathbf{p}^{\perp}$] are orthogonal compliments, so: **Prol_* = $\mathbf{n}^{\perp} \mathbf{n}^{\perp}$ is orthogonal projection onto \underline{L} [$a \mathbf{l} \mathbf{n} \mathbf{p} \mathbf{v} \mathbf{j} \mathbf{e}^{\perp}$] is orthogonal projection onto \underline{L} [$a \mathbf{l} \mathbf{n} \mathbf{p} \mathbf{v} \mathbf{j} \mathbf{e}^{\perp}$] is orthogonal projection onto \underline{L} [$a \mathbf{l} \mathbf{n} \mathbf{p} \mathbf{v} \mathbf{j} \mathbf{e}^{\perp} \mathbf{e}^{\perp}$] is orthogonal projection onto \underline{L} [$a \mathbf{l} \mathbf{n} \mathbf{v} \mathbf{v} \mathbf{j} \mathbf{e}^{\perp} \mathbf{e}^$	$\begin{array}{l} O(2mn^2) \\ -\text{NOTE:} \ \text{Householder method has } 2\left(mn^2-n^3/3\right) \ \text{flop} \\ \text{count, but better numerical properties} \\ \text{-Recall:} \ Q^{\dagger} \ Q = I_n \ \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \ \ I_n - Q^{\dagger} \ Q\ = \log s \ \\ -\text{-Classical GS} \Rightarrow \ I_n - Q^{\dagger} \ Q\ = \operatorname{Cond}(A)^2 \epsilon_{\text{mach}} \\ -\text{-Modified GS} \Rightarrow \ I_n - Q^{\dagger} \ Q\ = \operatorname{Cond}(A)^2 \epsilon_{\text{mach}} \\ -\text{NOTE:} \ \text{Householder method has } \ \ I_n - Q^{\dagger} \ Q\ = \epsilon_{\text{mach}} \ \\ \text{Multivariate Calculus} \\ \text{Consider } f: \ \mathbb{R}^n \to \mathbb{R} \ \\ \text{-When clear write:} \ \text{Jth component of input as } ij \text{ instead of } x_i \ \\ \text{-Level curve w.r.t. } \ \text{to } c \in \mathbb{R} \ \text{is all points s.t. } f(x) = c \ \\ \text{-Projecting } \ \text{evel curves onto } \ \mathbb{R}^n \ \text{gives } f \mid \text{S} \ \\ \text{-contour-map} \\ n_k \ \text{th order partial derivative w.r.t.} \ i_1 \ \text{of } f \mid_{1} = \delta_1 \ \\ \text{-} \frac{n_k}{2} \ \frac{n_k}{3} \ $	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in the variable $f: X \to X \rfloor$ in the variabl	$ \begin{tabular}{ll} & (x_i) = (x_i) + (x_i)$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 6. $(p_{\perp} + p_{\perp}) = (p_{\perp}k_{\perp}) = (p_{\perp}k_{\perp}) = (p_{\perp}k_{\perp}) = (p_{\perp}k_{\perp}k_{\perp}k_{\perp})$ 9. $(q_{\perp}k_{\perp}m) = (q_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (q_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (q_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (q_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Eigenvalues (vectors] eigen-decomposition]] $A = X A X^{-1}$] -Dominant $\lambda_1 : x_1$ are such that $ \lambda_1 $ [is strictly largest for which $\lambda_1 \in X$] are such that $ \lambda_1 $ [is strictly largest for which $\lambda_2 \in X$] -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $A_1(x) = \frac{x^{\dagger}A_1}{x^{\dagger}X}$ *Eigenvectors are stationary points of A_1 *Eigenvectors are stationary points of A_2 *Eigenvectors are stationary points of A_1 *Eigenvectors are stationary points of A_2 *Ap_(x)] is closest to being like eigenvalue of x_2 i.e. $A_2(x) = x_2$ *Ap_(x) = x_2	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: **prol_* = $\hat{n} \hat{n}^T$ is orthogonal projection onto $\underline{L}_{\parallel}$ (along P_{\parallel}^{p}) proj_* = $\hat{n}_{n}^{-} \hat{n} \hat{n}^T$ is orthogonal projection onto \underline{P} if proj_* = $\hat{n}_{n}^{-} \hat{n} \hat{n}^T$ is orthogonal projection onto \underline{P} if orthogonal projection onto \underline{P} (along \underline{L}) ** **L = $\hat{n} (Proj_*) = \hat{n} (Proj_*)$ and $P = ker (Proj_*) = ker (Proj_*)$ in (Proj_*) **R^* = $Rn \in (Rn)^{\perp}$ i.e. all vectors $\underline{v} \in R^n$ uniquely decomposed into $\underline{v} = \underline{v}_{1} + \underline{v}_{2}$ **Householder Maps: reflections** **Two points $\underline{v}_{2} \in R^n$ is parallel to normal \underline{n}_{1} i.e. $\underline{v}_{2} \in R^n$ in the origin with unit normal $\underline{n}_{2} \in R^n$ is reflection w.r.t. hyperplane $\underline{P}_{\underline{u}}$ (Bos) and $\underline{P}_{\underline{u}}$ is reflection w.r.t. hyperplane $\underline{P}_{\underline{u}}$ is proj_* \underline{u} and $\underline{proj}_{\underline{u}}$ = $\underline{n}_{\underline{u}} = \underline{u}_{\underline{u}}$ and $\underline{proj}_{\underline{u}}$ = $\underline{n}_{\underline{u}} = \underline{u}_{\underline{u}}$ is involutory, orthogonal and symmetric, i.e. $\underline{H}_{\underline{u}} = \underline{H}_{\underline{u}}^{-1} = \underline{H}_{\underline{u}}^{-$	O(2mm²) O(2mm	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is accurate if $Y \times Z Y \rfloor$ if $Y \to Y \rfloor$ in a subset of $Y \to Y \to Y \rfloor$ in a subset of $Y \to Y \to Y \to Y \rfloor$ is accurate if $Y \to Z Y \rfloor$ if $Y \to Y $	$ \begin{aligned} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \epsilon_i), \in \pm 1.06(n-1) \in \text{mach} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le \epsilon_j) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 \le x_i) \text$	50 $\{k_{\pm,k=1} + k_{\pm,k=1} = 0, p_{\pm} = 0, $	Eigenvalue Problems: Iterative Techniques If Jis [[tutorial 1#Properties of matrices]diagonalizable]] then [[tutorial 1#Eigen-values/vectors]eigen-decomposition]] $A=XDX^{-1}$ -Dominant λ_1 ; λ_1 are such that $ \lambda_1 $ [is strictly largest for which $\lambda_1 \times \lambda_2 \times \lambda_3$ -Rayleigh quotient for Hermitian $A=A^{\frac{1}{2}}$ is $R_A(x) = \frac{x^{\frac{1}{2}}Ax}{x^{\frac{1}{2}}}$ *Eigenvectors are stationary points of R_{Δ}] * $R_A(x)$ is closest to being like eigenvalue of \underline{x}_1 i.e. $R_A(x)$ argmin $Ax - \alpha x \ x \ $ * $R_A(x)$ argmin $Ax - \alpha x \ x \ $ * $R_A(x)$ argmin $Ax - \alpha x \ x \ $ with initial $\underline{b}(0)$ s.t. $\underline{b}(0) \ = 1$ -Assume dominant $\lambda_1 : \underline{x}_1 \ x \ $ exist for \underline{A} and that $\underline{b}(0) = 1$ -Under above assumptions, $\underline{\mu}_R = R_A(b(R)) = \frac{b(R)^{\frac{1}{2}} + b(R)}{b(R)^{\frac{1}{2}} + b(R)}$ converges to dominant $\lambda_1 : \underline{x}_1 \ x \ x \ $ - $\lambda_1 = 1$ -	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $-\operatorname{Prol}_{\underline{L}} = \widehat{\operatorname{Ani}}^T$ is orthogonal projection onto $\underline{L}_{\underline{J}}(along P_{\underline{J}}^0)$ $-\operatorname{Prol}_{\underline{J}} = \widehat{\operatorname{Ani}}^T$ is orthogonal projection onto $\underline{L}_{\underline{J}}(along P_{\underline{J}}^0)$ $-\operatorname{Prol}_{\underline{J}} = \widehat{\operatorname{Ani}}^T$ is orthogonal projection onto \underline{P} is orthogonal \underline{P} is $-\operatorname{Prol}_{\underline{J}} = \operatorname{Prol}_{\underline{J}} = \operatorname{Prol}_$	$\begin{aligned} & \frac{O(2mn^2)}{-NOTE:} \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{Rop} \\ & \text{count, but better numerical properties} \\ & \text{-Recall: } Q^{\frac{1}{4}}Q = \mathbf{I}_n \Rightarrow \text{check for loss of orthogonality} \\ & \text{with } \ \mathbf{I}_n - Q^{\frac{1}{4}}Q\ = \text{cons} \ \mathbf{I}_n - Q^{\frac{1}{4}}Q\ = \text{cond}(A)^2 \in_{\text{mach}} \ \\ & -\text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\frac{1}{4}}Q\ = \text{cond}(A)^2 \in_{\text{mach}} \ \\ & -\text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\frac{1}{4}}Q\ = \text{cond}(A)^2 \in_{\text{mach}} \ \\ & -\text{Motivariate Calculus} \\ & \text{Consider } f: \mathbb{R}^n \to \mathbb{R} \\ & \text{when clear write} j \text{-th component of input as } j \text{instead of } S_j \\ & \text{-level curve w.t.t. to } c \in \mathbb{R} \text{is all points s.t. } f(x) = c \\ & \text{-Projecting level curves onto } \mathbb{R}^n \text{gives } f \text{s} \\ & \text{-contour-map} \\ & n_k \text{th order partial derivative w.t.t.} i_j \text{of } \dots, \text{of } n_1 \text{th} \\ & \text{order partial derivative w.t.t.} i_j \text{of} f f \text{s} \\ & \frac{\partial n_k - \cos n_j}{\partial x_j} = \partial_{i_k}^n - \partial_{i_j}^n f = \int_{i_1 - \cos n_j}^{i_1 - \cos n_j} f f \text{s} \\ & \frac{\partial n_k - \cos n_j}{\partial x_j} = \partial_{i_k}^n - \partial_{i_j}^n f f f \text{s} \\ & \frac{\partial n_j}{\partial x_j} f f f f \text{s} \\ & \frac{\partial n_j}{\partial x_j} f f f f f \text{s} \\ & \frac{\partial n_j}{\partial x_j} f f f f f \text{s} \\ & \frac{\partial n_j}{\partial x_j} f f f f f f f f f $	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is accurate if $Y \le X \le Y \le $	$ \begin{aligned} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \varepsilon_i) \in \pm 1.06(n-1) \in \text{mach} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \varepsilon_i) \text{where} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + \varepsilon_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \varepsilon_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i (1 + \varepsilon_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) \text{where} \\ & \cdot \{(1 \le x_i) \ge x_iy_i \le x_iy_i (1 \le x_i) (1$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 5. (p_{\perp}, e^{-k}) 7. If or $j = k+1$ to m do 6. $(j_{\perp} = u_{\parallel}k)(n_{\perp}k_{\perp})$ 9. $u_{\parallel}k_{\parallel}m = u_{\parallel}k_{\parallel}m - \ell_{\parallel}k(k_{\perp}k_{\parallel}m)}$ 10. end for 11: end for 11: end for 12: end for 13: end for 13: end for 14: end for 15: end for 15: end for 16: end for 16: end for 16: end for 17: end for 18: end for 19: end for 19	Eigenvalue Problems: Iterative Techniques If Ajis [Itutorial 1#Properties of matrices]diagonalizable]] then [Itutorial 1#Eigen-values/vectors]eigen-decomposition]] $A=XNX^{-1}$ -Dominant $A_1: X_1$ are such that $ \lambda_1 $ [is strictly largest for which $Ax=\lambda X_2$] -Rayleigh quotient for Hermitian $A=A^{\frac{1}{2}}$ is $R_A(x) = \frac{x^{\frac{1}{2}}Ax^{\frac{1}{2}}}{Ax^{\frac{1}{2}}}$ *Eigenvectors are stationary points of R_A] *Eigenvector are stationary points of R_A] *Eigenvector are stationary points of R_A] *Ag.(X) is closest to being like eigenvalue of \underline{x}_1 . i.e. $R_A(x) = \alpha_1 x_1$ as $\underline{x} \to y$ where y is eigenvector -Power iteration: define sequence $b^{(R+1)} = \frac{Ab^{(R)}}{ Ab^{(R)} }$ with initial $\underline{b}^{(Q)}$ s.t. $ \underline{b}^{(Q)} = 1$] -Assume dominant $\lambda_1 : \underline{x}_1 = x = 1$ for \underline{A}_1 and that propial $\underline{b}^{(Q)}$ $$	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{t} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $Prol_L = \widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{t} [along \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} [along \underline{P} if \underline{P}	$\begin{array}{l} O(2mn^2) \\ -\text{NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \textbf{flop} \\ \text{count, but better numerical properties} \\ -\text{Recall: } Q^{\dagger} Q = I_n \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\dagger} Q\ = \ \text{oss}\ }{\ I_n - Q^{\dagger} Q\ = \ \text{cond}(A)^2 \in_{\text{mach}} \ } \\ -\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \ } \\ -\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \ } \\ -\text{Moltivariate Calculus} \\ \text{Consider } f: \mathbb{R}^n \to \mathbb{R} } \\ \text{When clear write } j\text{-th component of input as } j \text{instead of } s_j } \\ \text{When clear write } j\text{-th component of input as } j \text{instead of } s_j } \\ \text{-level curve w.r.t. to } c \in \mathbb{R} \text{ jis all points s.t. } f(\mathbf{x}) = c } \\ \text{-Projecting level curves onto } \mathbb{R}^n \text{ gives } f \text{s.c.} \\ \text{-orthour-map} \\ \\ \frac{n_k}{N} + \text{order partial derivative w.r.t. } i_j \text{ of } f_{j} \text{s.c.} \\ \text{-orthour-map} \\ \\ \frac{n_k}{N} + \text{-orthour-map} } = a_{jk}^n - a_{j1}^n f = f_{j1}^n - i_n R \\ \\ \frac{n_j}{N} + \text{-orthour-map} } = a_{jk}^n - a_{j1}^n f = f_{j1}^n - i_n R \\ \\ \frac{n_j}{N} + \text{-orthour-map} } \\ \text{-its an } \underline{M} + \text{broder partial derivative where } N - \sum_k n_k \\ \\ \frac{n_j}{N} + \text{-orthour-map} } \\ \text{-its tane, } a_j f f \text{is transpose of } y f } \text{ is } \frac{a_j f}{N} + \text{order partial derivative when } \\ \text{-v} f = (2nf_1, \dots, a_n f)^T } \text{ is transpose of } y f } \text{ is } \frac{a_j f}{N} + \text{order partial derivative } \frac{n_j}{N} + \text{order partial derivative } \\ \text{-v} f f = (2nf_1)^T } \text{ is transpose of } y f } \text{ is } \frac{a_j f}{N} + \text{order partial derivative } \frac{n_j}{N} + \text{order partial derivative } \\ \text{-v} f f = (2nf_1)^T } \text{ is transpose of } y f } \text{ is } \frac{a_j f}{N} + \text{order partial derivative } \frac{n_j f}{N} + \text{order partial derivative } \frac{n_j f}{N} + \text{order partial derivative } \\ \text{-v} f f = (2nf_1)^T } \text{ is transpose } \text{order partial derivative } \text{ when } 0 = \mathbb{R}^n f } \text{ is } \\ \text{-order partial derivative } \frac{n_j f}{N} + \text{order partial derivative } \frac{n_j f}{N} + \text{order partial derivative } \\ -order partial derivat$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rceil$ in algorithm for $f \rfloor$ is $f: X \to Y \rceil$ input $x \in X \rfloor$ is first rounded to $f!(x) \rfloor$ i.e. $f(x) = f(f!(x)) \rfloor$ relative error $\Rightarrow \ f(x) - f(x)\ $ $\ f(x) - f(x)\ $ $\ f(x)\ $ is accurate if $Y \le X \le Y $ $\ f(x) - f(x)\ $ $\ f(x)\ $ is $\ f(x) - f(x)\ $ $\ f(x)\ $	$ \begin{aligned} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_n x_n\} \cap \{+c\}, \in \pm 1.06(n-1) \in \text{mach} } \\ & = \{1(\sum x_i y_i) \ge x_j y_i \cap \{+c\}\} \text{where} \\ & = \{i = \{i + \delta_j\} \times \{1 + n_j\} - (i + n_n)\} \text{and} \delta_j \mid_i \mid_{i \mid i} \le \epsilon_{\text{mach}} \\ & = 1 + \epsilon_j + \{n_j\} - (i + n_n) - (i + n_n)\} \text{and} \delta_j \mid_i \mid_{i \mid i} \le \epsilon_{\text{mach}} \\ & = 1 + \epsilon_j + \{n_j\} - (i + n_n) - (i + n_n)\} \text{and} \delta_j \mid_i \mid_{i \mid i} \le \epsilon_{\text{mach}} \\ & = 1 + \epsilon_j + \{n_j\} - (i + n_n) - (i + n_n)\} \text{and} \delta_j \mid_i \mid_{i \mid i} \le \epsilon_{\text{mach}} \\ & = 1 + \{i \mid i $	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 5. $(p_{\perp}, e_{\parallel}) + (p_{\perp}k_{\perp})$ 7. $(f \circ f) = k + 1 \text{ to } m \text{ do}$ 6. $(p_{\perp}, e_{\parallel}) + (p_{\perp}k_{\perp}) + (p_{\perp}k_{\perp}k_{\perp})$ 9. $(q_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}m))$ 10. $(q_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}m))$ 11. $(g_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m + (p_{\perp}k_{\perp}k_{\perp}m))$ 12. $(g_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}k_{\perp}m)$ 13. $(g_{\perp}k_{\perp}m) = (p_{\perp}k_{\perp}m) = (p_\perpk_{\perp}m) = (p_\perpk_{\perp}m) = (p_\perpk_{\perp}m) = (p_\perpk_{\perp}m) = (p_\perp k_{\perp}m) $	Eigenvalue Problems: Iterative Techniques If A is [Itutorial A is A is a such that A is is A i	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{t} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: $Prol_L = \widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{t} [along \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} if $Prol_D = I_0\widehat{n}\widehat{n}^T$ is orthogonal projection onto \underline{P} [along \underline{P} if \underline{P}	$\frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right)^{\text{Rop}} \text{count, but better numerical properties} \\ \text{Recall: } \frac{Q^{\frac{1}{2}}Q = I_n}{2} \Rightarrow \text{check for loss of orthogonality} \\ \text{with } \frac{\ I_n - Q^{\frac{1}{2}}Q\ = \text{cond}(A)^2 \in_{\text{mach}}}{\ I_n - Q^{\frac{1}{2}}Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ -\text{Modified GS} \Rightarrow \ I_n - Q^{\frac{1}{2}}Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ -\text{Modified GS} \Rightarrow \ I_n - Q^{\frac{1}{2}}Q\ = \text{cond}(A)^2 \in_{\text{mach}}} \\ -\text{MOTE: Householder method has } \ I_n - Q^{\frac{1}{2}}Q\ = \varepsilon_{\text{mach}}} \\ \frac{\text{Multivariate Calculus}}{\text{consider } f : \mathbb{R}^n \to \mathbb{R}} \\ \text{When clear write } \frac{1}{2} \text{th component of input as } \frac{1}{2} \text{instead of } \frac{1}{N} \\ \text{The order partial derivative } \text{w.r.t. } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \text{w.r.t. } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \frac{1}{N} = \frac{1}{N} \text{contour-map}} \\ \frac{n_k}{N} \text{ th order partial derivative } \text{w.r.t. } \frac{1}{N} \text{p. } 0 \text{f.} \frac{1}{N} \text{p. } $	Given a problem $f: X \to Y$] an algorithm for f [is $f: X \to Y$] an algorithm for f [is $f: X \to Y$] in the problem $f: X \to Y$] an algorithm for f [is $f: X \to Y$] input $x \in X$] is first rounded to $f((x))$ i.e. $f(x) = f(f((x)))$ absolute error $\Rightarrow \ f(x) - f(x)\ $ $\ f(x)\ $ is accurate if $Y \times X$] $\ f(x) - f(x)\ $ $\ f(x)\ $ is able if $Y \times X = X$] $\ f(x)\ = 0$ ($\{mach\}$) and $\ \frac{X \to X}{\ X\ } = 0$ ($\{mach\}$) and $\ \frac{X \to X}{\ $	$ \begin{aligned} & (x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + c_i) \in \pm 1.06(n-1) \in \mathrm{mach} \\ & \cdot \{(1 \le x_iy_i) \ge x_iy_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_iy_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_iy_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i)^2 \ge x_i y_i (1 + c_i) \mathrm{Mere} \\ & \cdot \{(1 \le x_i$	5. $(k_{\perp}k_{\perp}) + (k_{\perp}k_{\perp})$ 5. (p_{\perp}, e^{-k}) 7. If or $j = k+1$ to m do 6. $(j_{\perp} = u_{\parallel}k)(n_{\perp}k_{\perp})$ 9. $u_{\parallel}k_{\parallel}m = u_{\parallel}k_{\parallel}m - \ell_{\parallel}k(k_{\perp}k_{\parallel}m)}$ 10. end for 11: end for 11: end for 12: end for 13: end for 13: end for 14: end for 15: end for 15: end for 16: end for 16: end for 16: end for 17: end for 18: end for 19: end for 19	Eigenvalue Problems: Iterative Techniques If A is [Itutorial A is A is a such that A is is A i	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n}_1 \mathbf{n} \mathbf{n} P = (\mathbf{R} \mathbf{n})^{\perp}$ are orthogonal compliments, so: $\mathbf{r} \mathbf{r} \mathbf{n}_1 = \mathbf{n} \mathbf{n}^T$ is orthogonal projection onto $\underline{L}_1(\mathbf{along} \ P_1^0)$ proj _p = $\mathbf{l}_n - \mathbf{n} \mathbf{n}^T$ is orthogonal projection onto $\underline{P}_1(\mathbf{along} \ P_1^0)$ proj _p = $\mathbf{l}_n - \mathbf{n} \mathbf{n}^T$ is orthogonal projection onto $\underline{P}_1(\mathbf{along} \ P_1^0)$ proj _p = $\mathbf{l}_n - \mathbf{n} \mathbf{n}^T$ is orthogonal projection onto $\underline{P}_1(\mathbf{along} \ P_1^0)$ projection onto $\underline{P}_1(\mathbf{along} \ P_1^0)$ projection onto $\underline{P}_1(\mathbf{along} \ P_1^0)$ i.e. all vectors $\underline{\mathbf{v}} \in \mathbf{R}^n$ uniquely decomposed into $\mathbf{v} = \mathbf{v}_1 \cdot \mathbf{v} = \mathbf{v}_1$ uniquely decomposed into $\mathbf{v} = \mathbf{v}_1 \cdot \mathbf{v} = \mathbf{v}_1$ is parallel to normal $\underline{\mathbf{n}}_1$ i.e. alvo points $\underline{\mathbf{x}}_1 = \mathbf{v}_1$ if $\underline{\mathbf{r}}_1 = \mathbf{v}_1$ if $\underline{\mathbf{r}}_1 = \mathbf{v}_1$ if $\underline{\mathbf{r}}_1 = \mathbf{v}_1$ is parallel to normal $\underline{\mathbf{n}}_1$ i.e. $\underline{\mathbf{x}}_2 = \mathbf{n}_1$ i.e. $\underline{\mathbf{x}}_2 = \mathbf{n}_1$ i.e. $\underline{\mathbf{r}}_1 = \mathbf{n}_1$ i.e. orthogonal compliment of line Rq] i.e. $\underline{\mathbf{r}}_1 = \mathbf{n}_1$ i.e. orthogonal complime	$\begin{aligned} &\frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{flop}}{\text{count, but better numerical properties}} \\ &\text{-Recall: } Q^{\dagger}Q = I_n \Rightarrow \text{check for loss of orthogonality} \\ &\text{with } \frac{\ I_n - Q^{\dagger}Q\ = \ \cos\ }{\ I_n - Q^{\dagger}Q\ = \ \cos\ } \\ &-\text{Classical GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Multivariate Calculus} \\ &\text{Consider } f: \mathbb{R}^n \to \mathbb{R} \\ &\text{When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ &\text{When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clear urite } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clear urite } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clurye w.r.t.} \ \text{to } c_i \in \mathbb{R} \text{is all points s.t. } f(\mathbf{x}) = c_i \\ &-\text{Projecting level curves onto } \mathbb{R}^n \text{gives } f \text{so} \\ &-\text{Vice curve w.r.t.} \ \text{to } c_i \in \mathbb{R} \text{is an } j \text{of } \dots, \text{of } n_1 \text{ j th} \\ &-\text{Order partial derivative w.r.t.} \text{jof } f \text{jis:} \\ &-\text{So} \frac{n_1 \text{w. of } n_1 \text{ j th}}{n_1 \text{-w. of } n_2 } = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n \text{mach} \text{jof } f \text{jos} \\ &-\text{Vice } a_{j_1}^n + a_{j_1}^n + a_{j_2}^n + a_{j_$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $\hat{f}: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $\hat{f}: X \to Y \rfloor$ in the variance of $f \ x \ = 1$ and $f \ $	$ \begin{aligned} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_n x_n\} \cap \{+c\}, \in \pm 1.06(n-1) \in \text{mach} } \\ & = \{1(\sum x_i y_i) \ge x_j y_i \cap \{+c\}_i\} \text{ where} \\ & = \{i - \epsilon_i\} \times \{i + \epsilon_j\} \times \{i + \epsilon_n\} - (i + \epsilon_n)_n\} \text{ and } \ \hat{y}_i \ _1 \ _1 \ \le \epsilon_{\text{mach}} \\ & = 1 + \{i - \epsilon_j\} \times \{i + \epsilon_n\} - (i + \epsilon_n)_n\} \text{ and } \ \hat{y}_i \ _1 \ _1 \ \le \epsilon_{\text{mach}} \\ & = 1 + \{i - \epsilon_j\} \times \{i + \epsilon_n\} - (i + \epsilon_n)_n\} \text{ and } \ \hat{y}_i \ _1 \ _1 \ \le \epsilon_{\text{mach}} \ -1 + \epsilon_i\} \times \{i + \epsilon_n\} \times \{i + \epsilon_n\} \times \{i + \epsilon_n\} \text{ of } \ -1 + \epsilon_n$	50 $\{k_1,k_{n-1} \mapsto \ell_1,k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 54 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 55 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 56 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 57 $\{n_j, k_{n-1}\}$ 58 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 50 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 54 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 55 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 56 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 57 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 58 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 50 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 66 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 67 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 68 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 69 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 61 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 62 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 63 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 66 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 67 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 68 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 69 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 61 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 62 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 63 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Effigenvalues/vectors[eigen-decomposition]] \bot =	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{p} = (\mathbf{R} \mathbf{n})^{\perp} \mathbf{n} \mathbf{r} \mathbf{n}$ orthogonal compliments, so: **Prol_* = $\mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n}^{\top} \mathbf{n}^{\top$	$\begin{aligned} & O(2m\alpha^2) \\ &-NOTE: Householder method has 2 \left(mn^2 - n^3 / 3 \right) \text{Rop} \\ & \text{count, but better numerical properties} \end{aligned} & \cdot \text{Recall: } Q^{\dagger} Q \cdot \mathbf{I}_n \rightarrow \text{check for loss of orthogonality} \\ & \text{with } \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{coss} \rightarrow \text{closs of orthogonality} \\ & \text{with } \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{mach} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A) \in_{\text{mach}} \rightarrow $	Given a problem $f: X \to Y \rfloor$ an algorithm for $f s f: X \to Y $ in algorithm for $f s f: X \to Y $ input $x \in X \rfloor$ is first rounded to $f (x) \rfloor$ i.e. $\hat{f}(x) = \hat{f}(f (x)) \rfloor$ relative error $\Rightarrow \ f(x) - f(x) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$ \begin{aligned} & \{x_1 = -\infty x_n\} \in \{x_1 = -x_n x_n\} \cap \{+c\}, \in \pm 1.06(n-1) \in \text{mach} \} \\ & = \{1(\sum_i x_i y_i)_i \ge x_i y_i \cap \{+c\}_i\} \text{ where} \\ & = \{i = \{i + \delta_j\} \times \{i + n_j\} \dots (i + n_m)\} \text{ and } \ \delta_j\ _1 \ n_j\ \le \epsilon_{\text{mach}} \\ & = \{i + \delta_j\} \times \{i + n_j\} \dots (i + n_m)\} \text{ and } \ \delta_j\ _1 \ n_j\ \le \epsilon_{\text{mach}} \\ & = \{i = \{i + \delta_j\} \times \{i + n_j\} \dots (i + n_m)\} \text{ and } \ \delta_j\ _1 \ n_j\ \le \epsilon_{\text{mach}} \\ & = \{i = \{i + \delta_j\} \times \{i + n_j\} \dots (i + n_m)\} \text{ and } \ \delta_j\ _1 \ n_j\ \le \epsilon_{\text{mach}} \\ & = \{i \in \{i + \delta_j\} \times \{i + n_j\} \dots (i + n_m)\} \text{ and } \ \delta_j\ \le \epsilon_{\text{mach}} \ \delta_j\ \le \epsilon_$	50 $\{L_{i,k-1} + \ell_{i,k-1} = 0\}$ 50 $\{P_{i,k} + P_{i,k-1} = 0\}$ 71 for $j = k+1$ to m do 61 $\{\ell_{j,k} = \ell_{j,k} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} = \ell_{j,k,m} = \ell_{j,k,m} \ell_{k,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} \ell_{k,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} \ell_{k,k,m} = \ell_{j,k,m} \ell_{k,k,m} \ell_{k,$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot Properties of matrices]diagonalizable]] then [[tutorial \bot Ajis [Itutorial \bot Ajis [Itutorial \bot Azis \bot	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = \mathbf{R} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{p} = (\mathbf{R} \mathbf{n})^{\perp} \mathbf{n} \mathbf{r} \mathbf{n}$ orthogonal compliments, so: **Prol_* = $\mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n} \mathbf{n}^{\top} \mathbf{n} \mathbf{n}^{\top} \mathbf{n}^{\top$	$\begin{aligned} &\frac{O(2mn^2)}{-NOTE: \text{Householder method has } 2\left(mn^2-n^3/3\right) \text{flop}}{\text{count, but better numerical properties}} \\ &\text{-Recall: } Q^{\dagger}Q = I_n \Rightarrow \text{check for loss of orthogonality} \\ &\text{with } \frac{\ I_n - Q^{\dagger}Q\ = \ \cos\ }{\ I_n - Q^{\dagger}Q\ = \ \cos\ } \\ &-\text{Classical GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Modified GS} \Rightarrow \ I_n - Q^{\dagger}Q\ = \text{Cond}(A)^2 \in_{\text{mach}} \\ &-\text{Multivariate Calculus} \\ &\text{Consider } f: \mathbb{R}^n \to \mathbb{R} \\ &\text{When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ &\text{When clear write } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clear urite } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clear urite } j\text{-th component of input as } j \text{instead of } s_j \\ &-\text{Vice clurye w.r.t.} \ \text{to } c_i \in \mathbb{R} \text{is all points s.t. } f(\mathbf{x}) = c_i \\ &-\text{Projecting level curves onto } \mathbb{R}^n \text{gives } f \text{so} \\ &-\text{Vice curve w.r.t.} \ \text{to } c_i \in \mathbb{R} \text{is an } j \text{of } \dots, \text{of } n_1 \text{ j th} \\ &-\text{Order partial derivative w.r.t.} \text{jof } f \text{jis:} \\ &-\text{So} \frac{n_1 \text{w. of } n_1 \text{ j th}}{n_1 \text{-w. of } n_2 } = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n = a_{j_R}^n - a_{j_1}^n \text{mach} \text{jof } f \text{jos} \\ &-\text{Vice } a_{j_1}^n + a_{j_1}^n + a_{j_2}^n + a_{j_$	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ in algorithm for $f \rfloor$ is $f: X \to Y \rfloor$ input $x \in X \rfloor$ is first rounded to $f((x)) \rfloor$ i.e. $f(x) = f(f((x)) \rfloor$ relative error $\Rightarrow \ f(x) - f(x)\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$ \begin{aligned} & \{(x_1 = -\infty x_n) \in (x_1 = -\infty x_n) \cap \{+\emptyset_i \in \mathbb{E} : 0.06(n-1) \in \text{mach} \} \\ & = \{(1 \le x_i y_i) \in \mathbb{E} : y_i y_i \in +\varepsilon_j \} \text{where} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i \times (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i \times (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_$	50 $\{k_1,k_{n-1} \mapsto \ell_1,k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 54 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 55 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 56 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 57 $\{n_j, k_{n-1}\}$ 58 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 50 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 54 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 55 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 56 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 57 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 58 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 59 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 50 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 51 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 52 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 53 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 66 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 67 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 68 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 69 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 61 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 62 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 63 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 65 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 66 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 67 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 68 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 69 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 60 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 61 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 62 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 63 $\{n_j, k_j \mapsto \ell_j, k_{n-1}\}$ 64 $\{n_j$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Effigenvalues/vectors[eigen-decomposition]] $A = \lambda \Delta \lambda^{-1}$] -Dominant $\lambda_1; x_1$ are such that $ \lambda_1 $ jis strictly largest for which $\lambda x \le \lambda x$] -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $A = \lambda x = \frac{x^{\dagger} A x}{x^{\dagger} x}$ *Eigenvectors are stationary points of R_A] **Eigenvectors are stationary points of R_A] **	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{L} = Rn_1$ and $P = (Rn)^{\perp}$ are orthogonal compliments, so: **Prol_* = $\widehat{n} \widehat{n}^T$ is orthogonal projection onto $\underline{L}_{\parallel}(along P_{\parallel}^0)$ **proj_* = $\widehat{n}_n - \widehat{n} \widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel}^0)$ **proj_* = $\widehat{n}_n - \widehat{n} \widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel}^0)$ **proj_* = $\widehat{n}_n - \widehat{n} \widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel}^0)$ **er (proj_*) = $\widehat{n}_n - \widehat{n} \widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel}^0)$ **er (proj_*) = $\widehat{n}_n - \widehat{n}^T$ is orthogonal projection onto $\underline{P}_{\parallel}(along P_{\parallel}^0)$ **er **er**	$\begin{aligned} & O(2mn^2) \\ & - NOTE: Householder method has 2 \left(mn^2 - n^3 / 3 \right) \text{Rop} \\ & \text{count, but better numerical properties} \\ & \cdot \text{Recall: } Q^{\dagger} Q \cdot \mathbf{I}_n \rightarrow \text{check for loss of orthogonality} \\ & \text{with } \frac{\ \mathbf{I}_n - Q^{\dagger} Q\ = \cos \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{Modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow \text{modified GS} \Rightarrow \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{cond}(A)^2 \in_{\text{mach}} \rightarrow modified $	Given a problem $f: X \to Y \rfloor$ an algorithm for $f \rfloor$ is $f: X \to Y \rceil$ in algorithm for $f \rfloor$ is $f: X \to Y \rceil$ input $x \in X \rfloor$ is first rounded to $f((x) \rfloor$ i.e. $\hat{f}(x) = \hat{f}(f((x)) \rfloor$ relative error $\Rightarrow \ \hat{f}(x) - f(x) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$ \begin{array}{ll} \{(x_1 = -\infty x_n) \in (x_1 = -\infty x_n) (1 + \epsilon), \in \pm 1.06(n-1) \in \mathrm{mach} \} \\ = \{(1 \leq x_1 y_i) \geq \sum_{i \neq j} (1 + \epsilon_j) \mathrm{mach} \} \\ = \{(1 \leq x_1 y_i) \geq \sum_{i \neq j} (1 + \epsilon_j) \mathrm{mach} \} \\ = \{(1 \leq i) \geq (1 + \epsilon_j) \geq (1 + \epsilon_j) \mathrm{mach} \} \\ = \{(1 \leq i) \geq (1 + \epsilon_j) \geq (1 + \epsilon_j) \mathrm{mach} \} \\ = \{(1 \leq i) \geq (1 \leq i) \geq (1 + \epsilon_j) \mathrm{mach} \} \\ = \{(1 \leq i) \geq (1 \leq i) \geq (1 \leq i) \} \\ =$	50 $\{k_{\perp},k_{-1} + \ell_{\perp},k_{\perp} - 1\}$ 50 $\{p_{\perp}, e_{\perp}, e_{\perp}\}$ 71 for $j = k + 1$ to m do 61 $\{\ell_{j,k} = \ell_{j,k}, \ell_{j,k,k,m}\}$ 72 $\{\ell_{j,k} = \ell_{j,k}, \ell_{j,k,k,m}\}$ 73 $\{\ell_{j,k} = \ell_{j,k}, \ell_{j,k,k,m}\}$ 74 work required: $-\frac{2}{3}m^3$ flops $-O(m^3)$ results in $L_{jj} \le 1$ 75 $\ L\ = O(1)$ 75 stability depends on growth-factor $\rho = \frac{\max_{i,j} u_{i,j} }{\max_{i,j} a_{i,j} }$ 75 so $\ L\ = O(1)$ 76 stability depends on growth-factor $\rho = \frac{\max_{i,j} u_{i,j} }{\max_{i,j} a_{i,j} }$ 77 so final pivoting $\rho \le 2^{m-1}$ 78 so $\ L\ = O(\rho(\ A\)) \Rightarrow [\tilde{L}\bar{U} + \tilde{P}A + \delta A]$ 79 $\ A\ = O(\rho \epsilon_{machine})$ 79 so only backwards stable if $\rho = O(1)$ 70 Full pivoting is $PAQ = LU$ finds largest entry in bottom-right submatrix Makes it pivot with row/column swaps before normal elimination 79 very expensive $O(m^3)$ search-ops, partial pivoting only needs $O(m^2)$ 70 Iterative Techniques 70 Systems of Equations 71 Let $A, C \in R^{n,n}$ where C^{-1} exists \Rightarrow splitting 71 $A = O(n^2)$ begin iteration 71 $A = O(n^2)$ iteration 72 $A = O(n^2)$ iteration $A = O(n^2)$ 73 solution to $A = O(n^2)$ 74 limit of $A = O(n^2)$ iteration of $A = O(n^2)$ 75 solution to $A = O(n^2)$ 76 $A = O(n^2)$ 77 $A = O(n^2)$ 78 solution to $A = O(n^2)$ 79 unique fixed point of $A = O(n^2)$ 70 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 72 $A = O(n^2)$ 73 $A = O(n^2)$ 74 $A = O(n^2)$ 75 solution to $A = O(n^2)$ 76 $A = O(n^2)$ 77 $A = O(n^2)$ 78 $A = O(n^2)$ 79 $A = O(n^2)$ 79 $A = O(n^2)$ 70 $A = O(n^2)$ 70 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 72 $A = O(n^2)$ 73 $A = O(n^2)$ 74 $A = O(n^2)$ 75 $A = O(n^2)$ 76 $A = O(n^2)$ 77 $A = O(n^2)$ 78 $A = O(n^2)$ 79 $A = O(n^2)$ 79 $A = O(n^2)$ 70 $A = O(n^2)$ 70 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 72 $A = O(n^2)$ 73 $A = O(n^2)$ 74 $A = O(n^2)$ 75 $A = O(n^2)$ 76 $A = O(n^2)$ 77 $A = O(n^2)$ 78 $A = O(n^2)$ 79 $A = O(n^2)$ 79 $A = O(n^2)$ 70 $A = O(n^2)$ 70 $A = O(n^2)$ 71 $A = O(n^2)$ 71 $A = O(n^2)$ 72 $A = O(n^2)$ 73 $A = O(n^2)$ 74 $A = O(n^2)$ 75 $A = O(n^2)$	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot Properties of matrices]diagonalizable]] then [[tutorial \bot Ajis] in [Itutorial \bot Ajis] in [Itutorial \bot Ajis] in [Itutorial \bot Ajis] is strictly largest for which \bot Azis] are such that \bot Ajis strictly largest for which \bot Azis] are such that \bot Ajis strictly largest for which \bot Azis] -Rayleigh quotient for Hermitian \bot Azis] is \bot Azis \bot A	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	
Notice $\underline{l} = Rn \operatorname{and} P = (Rn)^{\perp} \operatorname{are}$ orthogonal compliments, so: **proi_* = \(\text{ni} \) \(\text{is} \) orthogonal projection onto $\underline{L} (\operatorname{along} P j + n - \hat{n} \hat{n}^T is) = 1 + n - \hat{n} \hat{n}^T is) = 1 + n - \hat{n} \hat{n}^T is orthogonal projection onto \underline{L} (\operatorname{along} P j + n - \hat{n} \hat{n}^T is) = 1 + n - \hat{n} \hat{n}^T is) = 1 + n - \hat{n} \hat{n}^T is orthogonal projection onto \underline{L} (\operatorname{along} L \hat{n}) = 1 + n - \hat{n} \hat{n}^T is) = 1 + n - \hat{n} \hat{n}^T is orthogonal projection onto \underline{L} (\operatorname{along} L \hat{n}) = 1 + n - \hat{n} \hat{n}^T \hat{n}^$	$\begin{aligned} &\frac{O(2mn^2)}{-NOTE: Householder method has } 2\left(mn^2-n^3/3\right) \mathbf{flop} \\ &\text{count, but better numerical properties} \\ &\text{-Recall: } Q^{\dagger} Q = \mathbf{I}_n \Rightarrow \text{-check for loss of orthogonality} \\ &\text{with } \frac{\ \mathbf{I}_n - Q^{\dagger} Q\ = \log \ \mathbf{S} - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ &-\text{Modified GS} = \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ &-\text{Modified GS} = \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ &-\text{Modified GS} = \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ &-\text{Modified GS} = \ \mathbf{I}_n - Q^{\dagger} Q\ = \text{Cond}(A)^2 \epsilon_{\text{mach}} \\ &-\text{Multivariate Calculus} \\ &\text{Consider } f : \mathbb{R}^n \to \mathbb{R} \\ &\text{When clear write } j\text{-th component of input as } j \text{instead of } \kappa_j \\ &-\text{When clear write } j\text{-th component of input as } j \text{instead of } \kappa_j \\ &-\text{Level curve w.t.t. } to \subseteq \mathbb{R} \text{is all points s.t. } f(\mathbf{x}) = c \\ &-\text{Projecting } \text{perce curves onto } \mathbb{R}^n \text{gives } f \text{s} \\ &-\text{Contour-map} n_k \text{-th order partial derivative w.t.t.} j_k \text{-} of j_k \\ &-\text{N}_n f = n_1 f - n_1 f f \\ &-\text{N}_n f - n_1 f f f \\ &-\text{N}_n f - n_1 f f f \\ &-\text{N}_n f f f f f \\ &-\text{N}_n f f f f f \\ &-\text{N}_n f f f f f f f \\ &-\text{N}_n f f f f f f f f \\ &-\text{N}_n f f f f f f f f f $	Given a problem $f: X \to Y$] an algorithm for f [is $f: X \to Y$] in algorithm for f [is $f: X \to Y$] in the content of the problem $f: X \to Y$] in the content of the cont	$ \begin{aligned} & \{(x_1 = -\infty x_n) \in (x_1 = -\infty x_n) \cap \{+\emptyset_i \in \mathbb{E} : 0.06(n-1) \in \text{mach} \} \\ & = \{(1 \le x_i y_i) \in \mathbb{E} : y_i y_i \in +\varepsilon_j \} \text{where} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \eta_j\ \le \epsilon_{\text{mach}} \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) - (1+\eta_n) \} \text{and } \ \delta_j\ _1 \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\delta_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) + (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i = (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i \times (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \times (1+\eta_i) \} \text{and } \ \delta_j\ _1 \\ & = \{-\varepsilon_i \times (1+\varepsilon_j) \times (1+\eta_i) \times (1+\eta_$	5 (k_1.k=1 + \(\ell_{i,k} - 1\) (i.k=1) 5 (p_i = \(\text{p}_i \) (i.k_i = 1) 6 (p_i = \(\text{p}_i \) (i.k_i = 1) 7 (or j = k + 1 to m do 8 (j_i = \(\text{p}_i \) (i.k_i = 1) 9 (i.k_i = \(\text{p}_i \) (i.k_i = 1) 9 (i.k_i = \(\text{p}_i \) (i.k_i = 1) 10 (ord for 11 end for 12 end for 13 end for 14 end for 15 end for 16 end for 16 end for 17 end for 18 end for 19 end for 19 end for 10 end	Eigenvalue Problems: Iterative Techniques If Ajis [[tutorial \bot] Properties of matrices[diagonalizable]] then [[tutorial \bot] Effigenvalues/vectors[eigen-decomposition]] $A = \lambda \Delta \lambda^{-1}$] -Dominant $\lambda_1; x_1$ are such that $ \lambda_1 $ jis strictly largest for which $\lambda x \le \lambda x$] -Rayleigh quotient for Hermitian $A = A^{\dagger}$ is $A = \lambda x = \frac{x^{\dagger} A x}{x^{\dagger} x}$ *Eigenvectors are stationary points of R_A] **Eigenvectors are stationary points of R_A] **	$\begin{array}{l} & inner-product) \\ & \times (\mathbb{P}^{0})_{m-1} \mathbb{P}^{(n-1)}) \text{ and } (\chi^{(0)}_{m-1}, \chi^{(n-1)}) \text{ are bases for } \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \\ & \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \text{ and } \mathbb{R}^{n} \\ & \mathbb{R}^{n$	