

1. The likelihood function,  $L(\theta) = \prod_{i=1}^n f(x_i | \theta)$  is the product of the n pmf/pdf viewed as a function of a parameter  $\theta$ .
2. Take the natural log of the likelihood to get the log-likelihood function  $l(\hat{\theta}) = \log(L(\theta))$  and collect terms involving  $\theta$ .
3. Find the value of  $\theta$  for which log-likelihood is maximised. This is typically done by finding  $\hat{\theta}$  that solves  $l'(\hat{\theta}) = \frac{d}{d\theta} \log(L(\hat{\theta})) = 0$ .
4. If the estimate  $\hat{\theta}$  obtained in step 3 corresponds to a maximum  $\frac{d^2}{d\theta^2} l(\hat{\theta}) < 0$ , then  $\hat{\theta}$  is confirmed as the maximum likelihood estimator (MLE) of  $\theta$ .

Remarks:

- In large sample sizes, the MLE progressively becomes unbiased, efficient and consistent. This can be proved under mild technical assumptions.
- In small sample sizes there is no such guarantee and the quality of a MLE can vary. Bayesian parameter estimation is an area of statistics that deals with this problem.
- MLE generalizes to multi-parameter distributions. Yet this requires multivariate calculus and the maximization may give more than one answer if  $l$  has several peaks (local maxima).