Normal distribution $N(\mu, \sigma^2)$: A Normal (or Gaussian) random variable X with range $\mathbb{R}$ has pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$, for some $\mu, \sigma \in \mathbb{R}, \sigma > 0$. X hash mean μ and variance σ^2, and $X \sim N(\mu, \sigma^2)$.

$\phi_X(t) = e^{itu - \frac{1}{2}\sigma^2 t^2}, M_X(t) = e^{t\mu + \frac{1}{2}\sigma^2 t^2}$

The cdf of X does not have an analytically tractable form for any (μ, σ), so we can only write: $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\{-\frac{(t-\mu)^2}{2\sigma^2}\} dt.$

Setting $\mu = 0, \sigma = 1, Z \sim N(0,1)$ gives the special case of the **standard Normal** random variable, with simplified pdf $f(z) \equiv \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$. Again for cdf we can only write: $F(z) \equiv \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt.$

Linear Transformations of Normal Random Variables: To **standardise** any Normal random variable: $X \sim N(\mu, \sigma^2) \Longrightarrow \frac{X-\mu}{\sigma} \sim N(0,1)$. Cdf: $F_X(x) = P\left(Z \leq \frac{x-\mu}{\sigma}\right) = \Phi(\frac{x-\mu}{\sigma})$.