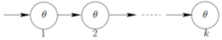
3. The Convolution Method: Some random variables are defined as the sum of two or more independent random variables.
We can **sample the individual distributions and sum the results.**

- Example: An Erlang(k, θ) random variable, X say, is defined as the sum of k independent exponentially-distributed random variables X_i, each with rate parameter θ



- Notice that

$$E[X] = E[X_1 + \ldots + X_k] = \frac{1}{\theta} + \frac{1}{\theta} + \cdots + \frac{1}{\theta} = \frac{k}{\theta}$$

- We can generate Erlang(k, θ) samples using the sampler for the exponential distribution: if $X_i \sim \exp(\theta)$ then

$$X = \sum_{i=1}^{k} X_i \sim \text{Erlang}(k, \theta)$$

- If $U_i \sim U(0,1)$ then X_i is sampled using $-\log U_i/\theta$
- We can save the more expensive log calculations in the summation by turning the sum into a product:

$$X = \sum_{i=1}^{k} -\frac{\log U_i}{\theta} = -\frac{1}{\theta} \log \prod_{i=1}^{k} U_i$$