

Geometric(p): [Memoryless] Consider a potentially infinite sequence of independent Bernoulli(p) random variables X_1, X_2, \dots . Suppose we define a quantity X by $X = \min \{i | i \geq 1, X_i = 1\}$ to be the **index of the first Bernoulli trial to result in a 1**. Then X is a random variable taking values in $\text{supp}(X) = \{1, 2, \dots\}$; $X \sim \text{Geometric}(p)$.

Pmf: $p(x) = p(1-p)^{x-1}, x = 1, 2, \dots$. **Mean:** $\mu = \frac{1}{p}$. **Variance:**

$\sigma^2 = \frac{1-p}{p^2}$. **Skewness:** $\gamma_1 = \frac{2-p}{\sqrt{1-p}}$ and is always positive.

E.g. Tossing a coin: X is the number of tosses until the first head is obtained.

Poi(λ) (Poisson): Let X be a random variable on $\mathbb{N} = \{0, 1, 2, \dots\}$.

Define **$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots, (i.e. x \in \mathbb{N}),$** for some **$\lambda > 0$,**

otherwise $p(x) = 0$. X is said to follow a Poisson distribution with parameter λ : $X \sim \text{Poi}(\lambda)$.

Mean: $\mu = \lambda$. **Variance:** $\sigma^2 = \lambda$. **Skewness:** $\gamma_1 = \frac{1}{\sqrt{\lambda}}$ is always positive but decreasing as λ grows.