

Poisson random variables are concerned with the number of random events occurring per **unit** of time or space, if there is a constant rate of random events occurring across this unit.

E.g. the number of patients arriving at an emergency room in a hour.

If we have a **non-unit interval (or space) of length t**:  $\lambda t$  can be

used in the pmf instead of  $\lambda$ , so that  $p(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$ ,  $x =$

$0, 1, 2, \dots$ , and  $X \sim Poi(\lambda t)$ . Now we see  $\lambda$  as the rate at which random events occur and  $\lambda t$  as the mean number of events in  $t$ .

**Uniform  $U(\{1, 2, \dots, n\})$ :** Let  $X$  be a random variable on  $\{1, 2, \dots, n\}$  with pmf  $p(x) = \frac{1}{n}$ ,  $x = 1, 2, \dots, n$ . Then  $X$  is said to follow a discrete uniform distribution and  $X \sim U(\{1, 2, \dots, n\})$ .

**Mean:**  $\mu = \frac{n+1}{2}$ , **Variance:**  $\sigma^2 = \frac{n^2-1}{12}$ .

Distribution	rv	pmf	$\mu$	$\sigma^2$
Bernoulli( $p$ )	$X \in \{0, 1\}$	$p^x(1-p)^{1-x}$	$p$	$p(1-p)$
Binomial( $n, p$ )	$X \in \{0, \dots, n\}$	$\binom{n}{x} p^x(1-p)^{n-x}$	$np$	$np(1-p)$
Geometric( $p$ )	$X \in \{1, 2, \dots\}$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$
Alternative Geometric( $p$ )	$Y = X - 1$ $X \sim \text{Geometric}(p)$	$p(1-p)^y$	$\frac{(1-p)}{p}$	$\frac{(1-p)}{p^2}$
Poisson( $\lambda$ )	$X \in \{0, 1, \dots\}$	$\frac{e^{-\lambda} \lambda^x}{x!}$	$\lambda$	$\lambda$
Uniform( $1, n$ )	$X \in \{1, \dots, n\}$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$