Confidence intervals: Discrete-event simulations are stochastic 随机的, so all outputs are random variables and each an **observation** of some measure, θ, of interest

If $X_i, 1 \leq i \leq n, n \geq 1$ are steady-state observations from a simulation then a **point estimate** for θ is the sample mean: $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

If we draw a large number of samples, $100(1-\alpha)\%$ of them will have the mean value that lies under this interval.

For any desired coverage probability level $1-\alpha$ we can define the $\mathbf{100(1-\alpha)}\%$ **confidence interval** for μ by $[\overline{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}]$ where z_α is the α-quantile of the standard normal.

For 95% confidence level: $\overline{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$.

If n>1, a $\mathbf{100\alpha}\%$ **confidence interval** estimate for θ is $\overline{X} \pm t_{n-1,(1-\alpha)/2}\frac{S}{\sqrt{n}}$ where $t_{v,\alpha}$ is the α-quantile of the Student's t distribution with v degrees of freedom.

Confidence intervals in simulation: How do we apply confidence intervals to simulation outputs?

- Running n independent replications, possibly in parallel, guarantees independence of the X_i.