Notable Continuous Distributions

Uniform Distribution U(a, b): A continuous random variable X with range (a, b) has a uniform distribution on the interval (a, b) if its pdf is $\boldsymbol{f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & otherwise \end{cases}}$ or equivalently its cdf is $F(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$. We write $X \sim U(a, b)$.

To map $X \sim U(0, 1)$ to $Y \sim U(a, b)$: $Y = a + (b - a)X$.

Mean: $E(X) = \frac{a+b}{2}$. **Variance:** $Var(X) = \frac{(b-a)^2}{12}$.

Exponential Distribution Exp(λ): Consider a random variable X with $supp(X) = [0, \infty)$ and pdf $f(x) = \lambda e^{-\lambda x}, x \ge 0$ for some $\lambda > 0$. Then X is a exponential (or negative exponential) random variable with rate parameter λ, and $X \sim Exp(\lambda)$.

Integration between 0 and x leads to the **cdf:**

$F(x) = 1 - e^{-\lambda x}, x \ge 0.$

Mean: $E(x) = \frac{1}{\lambda}$. **Variance:** $Var(X) = \frac{1}{\lambda^2}$.

Memoryless (Lack of memory) Property of the Exponential:

The **complementary cumulative distribution function** (or survival function, or tail distribution) is $P(X > x) = e^{-\lambda x}$.