10.12.2020

Temă seminar

Problema 9.2.19

Utilizând rezoluția liniară, demonstrați:

4. distributivitatea cuantificatorului „$\exists$" față de „$\vee$":

$$\vdash (\exists x)(P(x) \vee Q(x)) \leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$$

1) „$\vdash (\exists x)(P(x) \vee Q(x)) \rightarrow (\exists x)P(x) \vee (\exists x)Q(x)$"

$\overset{TD}{\Rightarrow}$ $(\exists x)(P(x) \vee Q(x)) \overset{?}{\vdash} (\exists x)P(x) \vee (\exists x)Q(x)$

$U_1 = (\exists x)(P(x) \vee Q(x)) \quad [x \leftarrow a]$

$U_1^C = P(a) \vee Q(a)$

$U_2 = \neg((\exists x)P(x) \vee (\exists x)Q(x)) =$

$= (\forall x)\neg P(x) \wedge (\forall x)\neg Q(x) =$

$= (\forall x)(\neg P(x) \wedge \neg Q(x)) = U_2^P$

$U_2^{Sk} = (\forall x)(\neg P(x) \wedge \neg Q(x))$

$U_2^C = \neg P(x) \wedge \neg Q(x)$

$S_1 = \{ \overset{C_1}{P(a) \vee Q(a)}, \overset{C_2}{\neg P(x)}, \overset{C_3}{\neg Q(x)} \}$

$\boxed{C_1 = P(a) \vee Q(a)}$ $\boxed{C_2 = \neg P(x)}$

[x <- a]

$\boxed{C_4 = Q(a)}$ $\boxed{C_3 = \neg Q(x)}$

[x <- a]

$\boxed{C_5 = \square}$ $\overset{TCC}{\Longrightarrow}$ S_1 inconsistentă

$\overset{TD}{\Rightarrow} \vdash (\exists x)(P(x) \vee Q(x)) \rightarrow (\exists x)P(x) \vee (\exists x)Q(x)$

(pe baza teoremei din curs) că $U \vdash V$ și aplicând TD

2) „$\vdash (\exists x)P(x) \vee (\exists x)Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$"

$\overset{iTD}{\Rightarrow} (\exists x)P(x) \vee (\exists x)Q(x) \overset{?}{\vdash} (\exists x)(P(x) \vee Q(x))$

$U_1 = (\exists x)P(x) \vee (\exists x)Q(x) =$ $[x \leftarrow a]; [x \leftarrow b]$

$\cancel{= (\exists x)(p(x) \vee q(x)) = U_1^P}$

$U_1^{Sk} = P(a) \vee Q(b) = U_1^C$

$U_2 = \neg(\exists x)(P(x) \vee Q(x)) =$
$= (\forall x)\neg(P(x) \vee Q(x)) =$
$= (\forall x)(\neg P(x) \wedge \neg Q(x)) = U_1^P$

$U_2^{Sk} = \cancel{(\forall x)} \neg P(x) \wedge \neg Q(x))$

$U_2^C = \neg P(x) \wedge \neg Q(x)$

$S_2 = \{ \overset{C_1}{P(a) \vee Q(b)}, \overset{C_2}{\neg P(x)}, \overset{C_3}{\neg Q(x)} \} = S_1$

~~$\Rightarrow$ analog, $\vdash (\exists x)p(x) \vee (\exists x)q(x) \rightarrow (\exists x)(p(x) \vee q(x))$~~

~~Deci, din 1) și 2) $\Rightarrow$~~
~~$\Rightarrow \vdash (\exists x)(p(x) \vee q(x)) \leftrightarrow (\exists x)p(x) \vee (\exists x)q(x)$~~

$\boxed{C_1 : P(a) \vee Q(b)}$ $\boxed{C_2 : \neg P(x)}$

[x <- a] $\boxed{C_4 : Q(b)}$ $\boxed{C_3 : \neg Q(x)}$

[x <- b] $\boxed{C_5 : \square}$ $\overset{TCC}{\Rightarrow} S_2$ inconsistentă

$\overset{TD}{\Rightarrow} \vdash (\exists x)P(x) \vee (\exists x)Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$

(pe baza teoremei din curs) că $U \vdash V$ și aplicând TD

1) + 2) $\Rightarrow$

$$\vdash (\exists x)(P(x) \vee Q(x)) \leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$$