$$\begin{cases} s - x = 0 \\ y - t = 0 \end{cases} \Rightarrow s - x = 0 = y - t \Rightarrow s = x \text{ și } t = y$$

$\Downarrow$

scrierea lui v e unică

ii) $\Rightarrow$ i) $S \cap T \leq_K S \Rightarrow 0 \in S \cap T$.

Fie $v \in S \cap T \Rightarrow v = \underset{\substack{\uparrow \\ S}}{v} + \underset{\substack{\uparrow \\ T}}{0} = 0 + v \xrightarrow[\text{ipot}]{\text{unicitate}}$

$v = 0$ Deci $S \cap T = \{0\}$

<u>3.1.3.1.</u> $\mathbb{R}_+^* = (0, +\infty)$ este un $\mathbb{R}$-spațiu vectorial

$\boxplus : \mathbb{R}_+^* \times \mathbb{R}_+^* \to \mathbb{R}_+^*$, $x \boxplus y = xy$, $\forall x, y \in \mathbb{R}_+^*$

$\boxdot : \mathbb{R} \times \mathbb{R}_+^* \to \mathbb{R}_+^*$, $\alpha \boxdot x = x^\alpha$, $\forall \alpha \in \mathbb{R}$
$\forall x \in \mathbb{R}_+^*$

I $(\mathbb{R}_+^*, \boxplus)$ grup abelian.

1) Operație internă

$x, y \in \mathbb{R}_+^* \Rightarrow \begin{matrix} x > 0 \\ y > 0 \end{matrix}_{(\cdot)} \Big| \; x \cdot y > 0 \Rightarrow x \boxplus y > 0$

$\Downarrow$

$x \boxplus y \in \mathbb{R}_+^*$

2) Comutativitate: $x, y \in \mathbb{R}_+^* \Rightarrow x \boxplus y = y \boxplus x$

$x \boxplus y = xy = y \cdot x = y \boxplus x$

3) Asociativitate

$x, y, z \in \mathbb{R}_+^* \Rightarrow (x \boxplus y) \boxplus z = x \boxplus (y \boxplus z)$

– evident deoarece „$\cdot$" uzuală este asoc pe $\mathbb{R}$

4) Element neutru $\theta \in \mathbb{R}_+^*$

$\forall x \in \mathbb{R}_+^* \Rightarrow x \boxplus \theta = \theta \boxplus x = x$

$x \boxplus \theta = x \Leftrightarrow x\theta = x \Rightarrow x\theta - x = 0 \Rightarrow$
$x(\theta - 1) = 0 \Rightarrow \theta = 1.$

5) Elemente simetrizabile (inversabile)

Fie $x \in \mathbb{R}_+^* \Rightarrow \exists x' \in \mathbb{R}_+^*$ a.î. $x \boxplus x' =$
$= x' \boxplus x = \theta$

$x \boxplus x' = \theta \Leftrightarrow x \cdot x' = 1 \Rightarrow x' = \frac{1}{x} \in \mathbb{R}_+^*$
$\Rightarrow (\mathbb{R}_+^*)^x = \mathbb{R}_+^*$

Din 1) 2) 3) 4) 5) $\Rightarrow (\mathbb{R}_+^*, \boxplus)$ grup abelian

$\underline{\text{II}}$ Satisfacerea axiomelor.

$\boxdot : \mathbb{R} \times \mathbb{R}_+^* \to \mathbb{R}_+^*, \ \alpha \boxdot x = x^\alpha$

1) $\underline{\alpha \boxdot (x \boxplus y)} = (\alpha \boxdot x) \boxplus (\alpha \boxdot y)$

$\alpha \boxdot (x \boxplus y) = \alpha \boxdot xy = (xy)^\alpha = x^\alpha \cdot y^\alpha =$

$= (\alpha \boxdot x) \cdot (\alpha \boxdot y) = (\alpha \boxdot x) \boxplus (\alpha \boxdot y)$

2) $(\alpha + \beta) \boxdot x = \alpha \boxdot x \boxplus \beta \boxdot x$

$(\alpha + \beta) \boxdot x = x^{\alpha+\beta} = x^\alpha \cdot x^\beta = (\alpha \boxdot x) \cdot (\beta \boxdot x)$

$= (\alpha \boxdot x) \boxplus (\beta \boxdot x)$

3) $\alpha \boxed{\cdot} (\beta \boxed{\cdot} x) = (\alpha \cdot \beta) \boxed{\cdot} x$

$\alpha \boxed{\cdot} (\beta \boxed{\cdot} x) = \alpha \boxed{\cdot} x^{\beta} = (x^{\beta})^{\alpha} = x^{\beta \cdot \alpha} =$

$= x^{\alpha \cdot \beta} = (\alpha \beta) \boxed{\cdot} x$

4) $1 \boxed{\cdot} x = x$

$1 \boxed{\cdot} x = x^{1} = x$

Din I și II $\Rightarrow \mathbb{R}_+^*$ este un $\mathbb{R}$ spațiu vectorial