~~v - liniar independentă~~

3.2.41

$b = [b_1, b_2, b_3, b_4]^t$

$b_1 = [1, 2, -1, 2]$ $\quad b_2 = [1, 2, 1, 4]$ $\quad b_3 = [2, 3, 0, -1]$

$b_4 = [1, 3, -1, 0]$

(1) b este bază a lui $\mathbb{R}^4$, $\dim \mathbb{R}^4 = 4 \overset{\text{Teorema alternativei}}{\Longrightarrow}$
verif. $\langle b \rangle = \mathbb{R}^4$ este suficientă

Fie $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 3 & 3 \\ -1 & 1 & 0 & -1 \\ 2 & 4 & -1 & 0 \end{pmatrix}$

$\det A = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 3 & 3 \\ -1 & 1 & 0 & -1 \\ 2 & 4 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 1 \\ -1 & -1 & -3 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 4 & -1 & 0 \end{vmatrix} =$

$-3L_1 + L_2$

$L_1 + L_3$

$= \begin{vmatrix} -1 & -1 & -3 \\ 0 & 2 & 2 \\ 2 & 4 & -1 \end{vmatrix} = 18 \neq 0 \Rightarrow \operatorname{rang} A = 4 = \max$

$\Rightarrow \langle b \rangle = \mathbb{R}^4 \overset{\text{T. alt.}}{\Longrightarrow} b$ - bază

(2) $x = [2, 3, 2, 10]$

Coord. lui x sunt $\overset{\alpha_1, \alpha_2, \alpha_3, \alpha_4}{\cancel{\alpha, \beta, \gamma, \delta}} \Leftrightarrow$

$$X = \alpha_1[1,2,-1,2] + \alpha_2[1,2,1,4] + \alpha_3[2,3,0,-1] + \alpha_4[1,3,-1,0]$$

$$[2,3,2,10] = [\alpha_1 + \alpha_2 + 2\alpha_3 + \alpha_4,\ 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4,\ -\alpha_1 + \alpha_2 - \alpha_4,\ 2\alpha_1 + 4\alpha_2 - \alpha_3]$$

$$\Rightarrow \begin{cases} \alpha_1 + \alpha_2 + 2\alpha_3 + \alpha_4 = 2 \\ 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4 = 3 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \\ 2\alpha_1 + 4\alpha_2 - \alpha_3 = 10 \end{cases}$$

$$\begin{cases} \alpha_1 + \alpha_2 + 2\alpha_3 + \alpha_4 = 2 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \end{cases}$$

$$/ \quad 2\alpha_2 + 2\alpha_3 = 4 \Rightarrow \alpha_2 + \alpha_3 = 2 \Rightarrow \alpha_3 = 2 - \alpha_2$$

$$\begin{cases} \alpha_1 + \alpha_2 + 4 - 2\alpha_2 + \alpha_4 = 2 \\ 2\alpha_1 + 2\alpha_2 + 6 - 3\alpha_2 + 3\alpha_4 = 3 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \\ 2\alpha_1 + 4\alpha_2 - 2 + \alpha_2 = 10 \end{cases}$$

$$\begin{cases} \alpha_1 - \alpha_2 + \alpha_4 = -2 \\ 2\alpha_1 - \alpha_2 + 3\alpha_4 = -3 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \\ 2\alpha_1 + 5\alpha_2 = 12 \end{cases}$$

$$\begin{cases} 2\alpha_1 - \alpha_2 + 3\alpha_4 = -3 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \\ 2\alpha_1 + 5\alpha_2 = 12 \end{cases}$$

$$\begin{cases} 2\alpha_1 - \alpha_2 + 3\alpha_4 = -3 \\ -\alpha_1 + \alpha_2 - \alpha_4 = 2 \end{cases}$$

$$\alpha_1 \quad / \quad + 2\alpha_4 = -1 \Rightarrow \alpha_1 = -1 - 2\alpha_4$$

$$\begin{cases} -2 - 4\alpha_4 - \alpha_2 + 3\alpha_4 = -3 \\ 1 + 2\alpha_4 + \alpha_2 - \alpha_4 = 2 \\ -2 - 4\alpha_4 + 5\alpha_2 = 12 \end{cases}$$

$$\begin{cases} -\alpha_2 - \alpha_4 = -1 \\ \alpha_2 + \alpha_4 = 1 \\ 5\alpha_2 - 4\alpha_4 = 14 \end{cases} \Leftrightarrow \begin{cases} \alpha_2 + \alpha_4 = 1 \quad | \cdot 4 \\ 5\alpha_2 - 4\alpha_4 = 14 \end{cases}$$

$$\begin{cases} 4\alpha_2 + 4\alpha_4 = 4 \\ 5\alpha_2 - 4\alpha_4 = 14 \end{cases}$$

$$9\alpha_2 \quad / \quad = 18 \Rightarrow \alpha_2 = 2 \Rightarrow \alpha_4 = -1$$

$\Rightarrow \alpha_1 = 1 \Rightarrow \alpha_3 = 0$

$\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 0, \alpha_4 = -1$

$\Rightarrow$ coordonatele lui x în baza b sunt

$x = [b_1, 2b_2, 0, -b_4]$

<u>3.2.42</u>

$v = [v_1, v_2, v_3]^t$

$v_1 = (a, 1, 1), v_2 = (1, a, 1) \quad v_3 = (1, 1, a)$

v este bază a lui $\mathbb{R}^3$, $\dim \mathbb{R}^3 = 3$

T. alt. $\Rightarrow$ verif. $< v > = \mathbb{R}^3$ este suficientă

Fie $A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$

$\det A = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3 - 3a + 2 \neq 0$

$\Leftrightarrow \quad a^3 - 3a + 2 = 0 \qquad a_1 = a_2 = 1$

$a_3 = -2$

$\Rightarrow \det A \neq 0 \Leftrightarrow a \neq 1$ și $a \neq -2$

$\Rightarrow v$ - bază $\Leftrightarrow a \in \mathbb{R} \setminus \{-2, 1\}$