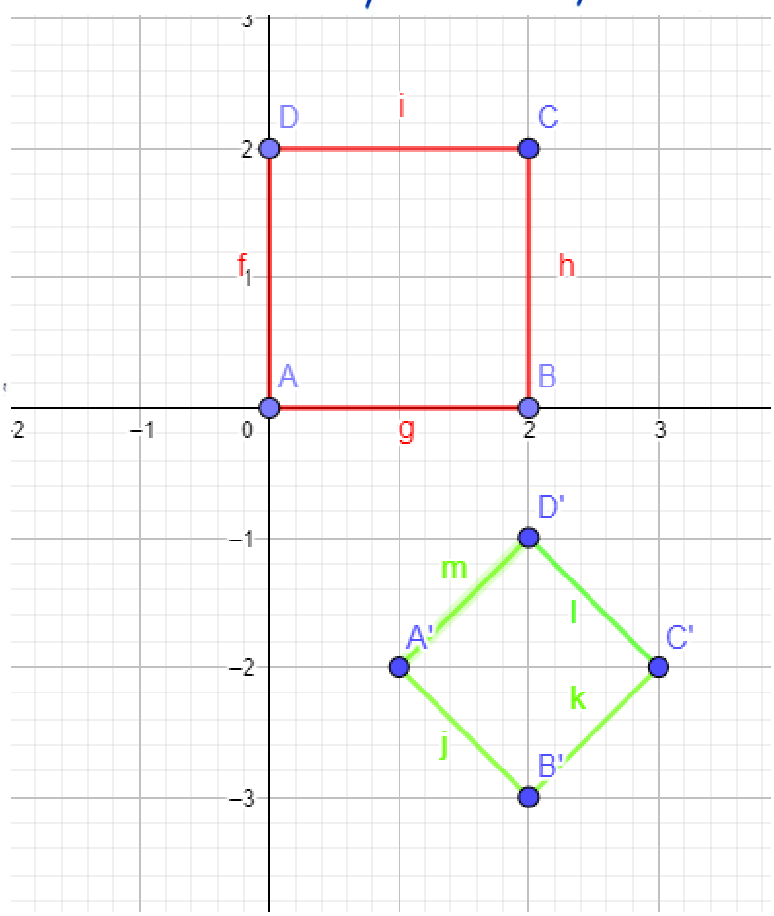
Problema 11.8

Se consideră pătratul $AB C\Delta$, de vârfuri $A(0,0)$, $B(2,0)$, $C(2,2)$, $\Delta(0,2)$. Demonstrați că patrulaterul $A'B'C'\Delta'$ cu $A'(1,-2)$, $B'(2,-3)$, $C'(3,-2)$, $\Delta'(2,-1)$ este, de asemenea, un pătrat și indicați o secvență de transformări geometrice care transformă primul pătrat în cel de-al doilea.



Demonstrație că $A'B'C'\Delta'$ este pătrat.

$$\left.\begin{aligned} m_{A'\Delta'} &= \frac{y_{\Delta'}-y_{A'}}{x_{\Delta'}-x_{A'}} = \frac{-1+2}{2-1} = 1 \\ m_{B'C'} &= \frac{y_{C'}-y_{B'}}{x_{C'}-x_{B'}} = \frac{-2+3}{3-2} = 1 \end{aligned}\right\} \Rightarrow A'\Delta' \parallel B'C'$$

$$\left.\begin{aligned} m_{A'B'} &= \frac{y_{B'}-y_{A'}}{x_{B'}-x_{A'}} = \frac{-3+2}{2-1} = -1 \\ m_{C'\Delta'} &= \frac{y_{C'}-y_{\Delta'}}{x_{C'}-x_{\Delta'}} = \frac{2-1}{-3+2} = -1 \end{aligned}\right\} \Rightarrow A'B' \parallel \Delta'C'$$

$\Rightarrow$ laturile opuse sunt paralele (1)

$$m_{A'D'} = \frac{-1}{m_{A'B'}} \Rightarrow A'B' \perp A'D'$$

$$m_{B'C'} = \frac{-1}{m_{D'C'}} \Rightarrow B'C' \perp D'C'$$

$\Rightarrow$ laturile alăturate sunt perpendiculare (2)

$$\left.\begin{aligned} A'B' &= \sqrt{1+1} = \sqrt{2} \\ A'D' &= \sqrt{1+1} = \sqrt{2} \\ C'D' &= \sqrt{1+1} = \sqrt{2} \\ B'C' &= \sqrt{1+1} = \sqrt{2} \end{aligned}\right\} \text{ laturile sunt egale } (3)$$

(1), (2), (3) $\Rightarrow A'B'C'D'$ este pătrat.

Transformări: Translație, Scalare, Rotație.

<u>Translația</u>

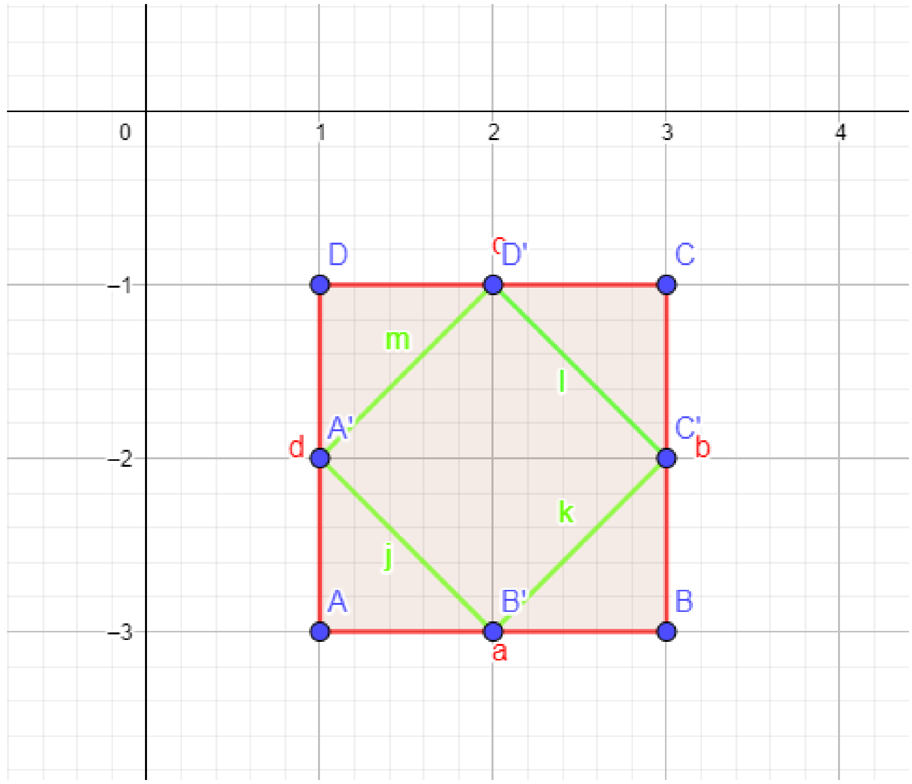
M centrul pătratului $ABCD \Rightarrow M(1,1)$

N centrul pătratului $A'B'C'D' \Rightarrow N(2,-2)$

$$T(x,y) = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} \quad \text{cu } \vec{v}(x,y)$$

$$\overrightarrow{MN}(x_N - x_M, y_N - y_M) \Rightarrow \overrightarrow{MN}(1,-3)$$

$$\Rightarrow T(1,-3) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$



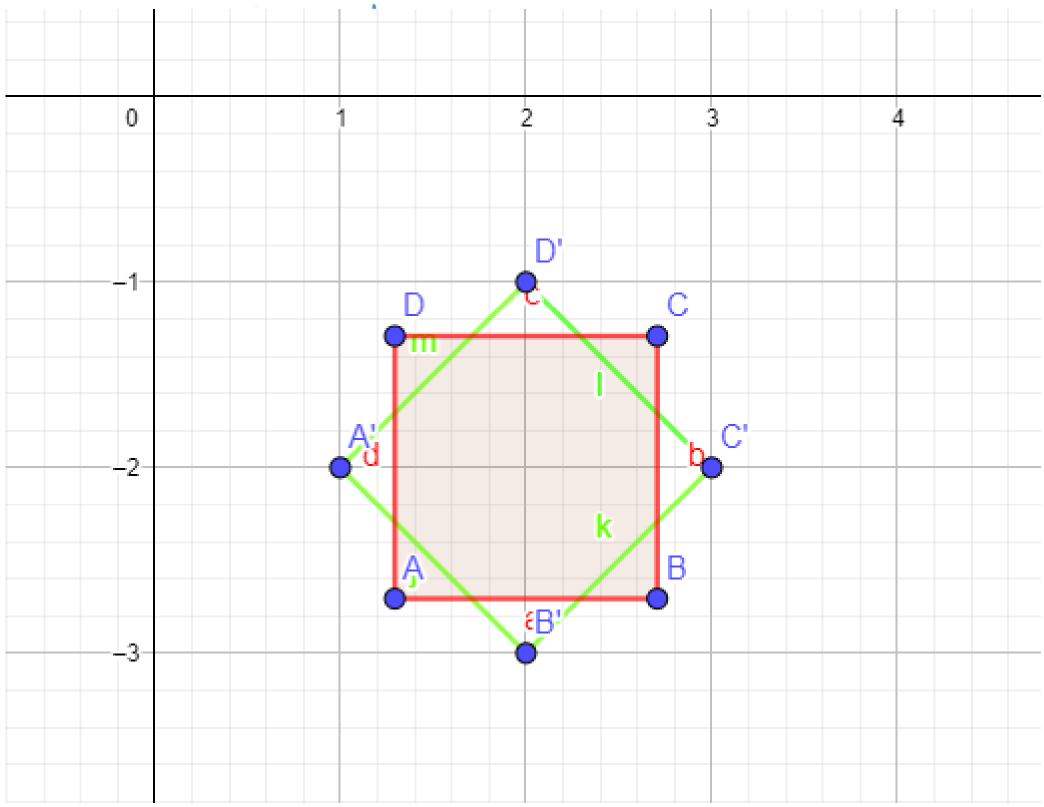
Scalarea

scalare uniformă $\Rightarrow s_x = s_y = s$

$$\Rightarrow S(Q, s) = \begin{pmatrix} s & 0 & (1-s)q_1 \\ 0 & 1 & (1-s)q_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$A'B' = \sqrt{2}$ $\quad AB = 2 \Rightarrow s = \frac{A'B'}{AB} = \frac{\sqrt{2}}{2}$

$$S\left(1, 1, \frac{\sqrt{2}}{2}\right) = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 1 - \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 1 - \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix}$$



Rotatie

$$R(Q, \theta) = \begin{pmatrix} \cos\theta & -\sin\theta & q_1(1-\cos\theta) + q_2 \sin\theta \\ \sin\theta & \cos\theta & -q_1 \sin\theta + q_2(1-\cos\theta) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta = 2\pi - \frac{\pi}{4} \Rightarrow \sin\left(2\pi - \frac{\pi}{4}\right) = \sin 2\pi \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos 2\pi =$$

$$= -\frac{\sqrt{2}}{2}$$

$$\cos\left(2\pi - \frac{\pi}{4}\right) = \cos 2\pi \cos\frac{\pi}{4} + \sin 2\pi \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$R(1, 1, 315^\circ) = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1-\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_r = T(1,-3) \cdot S\left(1,1,\frac{\sqrt{2}}{2}\right) \cdot R(1,1,315^\circ)$$

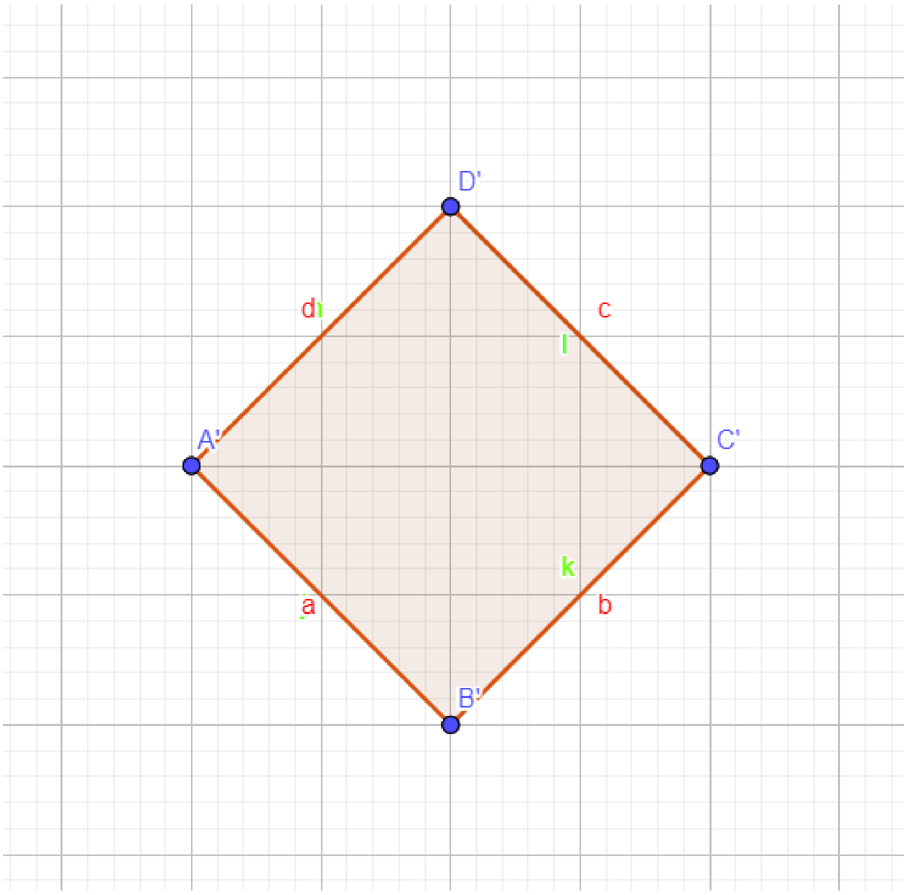
$$T(1,-3) \cdot S\left(1,1,\frac{\sqrt{2}}{2}\right) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 1-\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 1-\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 2-\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -2-\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$T \cdot S \cdot R = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 2-\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -2-\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1-\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & (1-\sqrt{2})\left(2-\frac{\sqrt{2}}{2}\right) + \left(-2-\frac{\sqrt{2}}{2}\right) + 1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} - 2 - \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ -\frac{1}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix} = T_r$$



Verificare

$$(A'B'C'D') = Tr \cdot (ABCD)$$

$$(A'B'C'D') = \begin{pmatrix} 1 & 2 & 3 & 2 \\ -2 & -3 & -2 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$(ABCD) = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$Tr = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{-1}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Tr \cdot (ABCD) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{-1}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 2 & 3 & 2 \\ -2 & -3 & -2 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$