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Gr. 212

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Lucrare

Pr. 1 Det. tang. la hip. $7x^2-2y^2=14$ care sunt perp. pe $d: x+2y-3=0$ și det. dist. dintre ele. $d: y=-\frac{1}{2}x+\frac{3}{2}$

Fie tg_1, tg_2 cele 2 tangente

$$\left.\begin{aligned} tg_1, tg_2 \perp d \\ m_d = -\frac{1}{2} \end{aligned}\right\} \Rightarrow m_{tg_1} = m_{tg_2} = \frac{-1}{m_d} = 2$$

$$h: 7x^2-2y^2=14 \iff \frac{x^2}{2}-\frac{y^2}{7}=1$$

$$\Rightarrow a^2=2 \quad b^2=7$$

ec. tg. la o hiperbolă:

$$y = Kx \pm \sqrt{a^2K^2-b^2}$$

Știm: $K = m_{tg_1} = m_{tg_2} = 2$, $K^2=4$

$a^2=2$

$b^2=7$

$\Rightarrow tg_1: \ y = 2x+\sqrt{8-7}$

$y=2x+1 \Rightarrow tg_1: 2x-y+1=0$

$tg_2: \ y=2x-\sqrt{8-7}$

$y=2x-1 \Rightarrow tg_2: 2x-y-1=0$

Fie $M(0, 1) \in tg_1$, $tg_2: 2x - y - 1 = 0$

$dist(tg_1, tg_2) \equiv dist(M, tg_2) =$

$$= \frac{|2 \cdot 0 - 1 \cdot 1 - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

P.1.③ Să se det. ec. supr. cilindrice ale cărei gen. sunt paral. cu dr.

$$(\Delta) \begin{cases} x + y + z = 0 \\ x + 2y + 3z = 0 \end{cases}$$ și care intersectează

$$(C) \begin{cases} x^2 + y^2 + z^2 - 2z = 0 \\ y = 0 \end{cases}$$

Obținem ecuațiile generatoarelor:

$$(G) \begin{cases} x + y + z = \lambda \\ x + 2y + 3z = \mu \end{cases}$$

Încercăm să obținem x, y, z în funcție de λ și μ

$$\begin{cases} x = \lambda - z - y \\ x = -2y - 3z + \mu \end{cases} \Rightarrow \lambda - z - y = -2y - 3z + \mu$$

$$2z = -y + \mu - \lambda$$

$$x = \lambda - \frac{\mu - \lambda - y}{2} - y \Leftarrow z = \frac{\mu - \lambda - y}{2}$$

Știm că $y = 0$, deci (din ec. (G) + (C))

$$x = \lambda - \frac{\mu - \lambda}{2}$$

$$z = \frac{\mu - \lambda}{2}$$

Înlocuind în ec. sferei:

$$\left(\lambda - \frac{\mu - \lambda}{2}\right)^2 + 0 + \left(\frac{\mu - \lambda}{2}\right)^2 - 2\,\frac{\mu - \lambda}{2} = 0$$

$$\lambda^2 - 2\lambda\,\frac{\mu - \lambda}{2} + \left(\frac{\mu - \lambda}{2}\right)^2 \cdot 2 - \mu + \lambda = 0$$

$$\lambda^2 - \lambda\mu + \lambda^2 + 2\,\frac{\mu^2 - 2\lambda\mu + \lambda^2}{4} - \mu - \lambda = 0$$

$$\cdot 2 /\ 2\lambda^2 + \frac{\mu^2 - 2\lambda\mu + \lambda^2}{2} - \mu - \lambda - \lambda\mu = 0$$

$$4\lambda^2 + \mu^2 - 2\lambda\mu + \lambda^2 - 2\mu - 2\lambda - 2\lambda\mu = 0$$

$$5\lambda^2 + \mu^2 - 4\lambda\mu - 2\mu - 2\lambda = 0$$

Înlocuind λ și μ, obținem

$$5(x+y+z)^2 + (x+2y+3z)^2 - 4(x+y+z)$$
$$(x+2y+3z) - 2(x+2y+3z) - 2(x+y+z) = 0$$

- ec. supr. cilindrice

Pr. 4 Se consideră ABCD cu

$A(0,0)$, $B(3,0)$, $C(3,3)$, $D(0,3)$.

Det. imaginea $\square$ printr-o refl. relativ la dr. care trece prin $P(1,2)$ și are vectorul director $u(3,4)$, apoi forfecare rel. la O în dir. vect. $v(1,2)$ de 60°. Desen

I Reflexia

$\vec{w} = (3,4) \Rightarrow \vec{w} = \left(\frac{3}{5}, \frac{4}{5}\right)$

vector dir. versor dir. w_1 w_2

$q_1 = 1$

$q_2 = 2$

Obținem, conform formulei:

$$Mirror(P, w) = \begin{pmatrix} 1-2w_2^2 & 2w_1w_2 & 2(q_1w_2^2 - q_2w_1w_2) \\ 2w_1w_2 & 1-2w_1^2 & 2(-q_1w_1w_2 + q_2w_1^2) \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{32}{25} & \frac{24}{25} & 2\left(\frac{16}{25} - \frac{24}{25}\right) \\ \frac{24}{25} & 1-\frac{18}{25} & 2\left(-\frac{12}{25} + \frac{18}{25}\right) \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -7 & \frac{24}{25} & -\frac{16}{25} \\ \frac{24}{25} & \frac{7}{25} & \frac{12}{25} \\ 0 & 0 & 1 \end{pmatrix}$$

ii Forfecare

$\vec{u}(1,2) \Rightarrow \vec{w}\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ $\quad tg\, 60^\circ = \sqrt{3}$

$Q = O \Rightarrow q_1 = q_2 = 0 \quad \theta = 60^\circ$

$\cancel{1 - w_1 w_2\, tg\,\theta \quad w_1^2\, tg\,\theta}$

$$Shear(O, w, \theta) = \begin{pmatrix} 1 - w_1 w_2\, tg\,\theta & w_1^2\, tg\,\theta & (q_1 w_1 w_2 - q_2 w_1^2)\, tg\,\theta \\ -w_2^2\, tg\,\theta & 1 + w_1 w_2\, tg\,\theta & (q_1 w_2^2 - q_2 w_1 w_2)\, tg\,\theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{2\sqrt{3}}{5} & \frac{\sqrt{3}}{5} & 0 \\ -\frac{4\sqrt{3}}{5} & 1 + \frac{2\sqrt{3}}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{5 - 2\sqrt{3}}{5} & \frac{\sqrt{3}}{5} & 0 \\ -\frac{4\sqrt{3}}{5} & \frac{5 + 2\sqrt{3}}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A\,B\,C\,D] = \begin{pmatrix} 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

~~[A' B' C' D'] =~~

$$Tr = Shear \cdot Mirror =$$

$$= \begin{pmatrix} \frac{5-2\sqrt{3}}{5} & \frac{\sqrt{3}}{5} & 0 \\ -\frac{4\sqrt{3}}{5} & \frac{5+2\sqrt{3}}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -7 & \frac{24}{25} & -\frac{16}{25} \\ \frac{24}{25} & \frac{7}{25} & \frac{12}{25} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{-7(5-2\sqrt{3})}{5} \Big/ \cdot(25) + \frac{24\sqrt{3}}{125} & \frac{24(5-2\sqrt{3})+7\sqrt{3}}{125} & \frac{-16(5-2\sqrt{3})+12\sqrt{3}}{125} \\ \frac{28\sqrt{3}}{5} \Big/ \cdot(25) + \frac{24(5+2\sqrt{3})}{125} & \frac{-96\sqrt{3}+7(5+2\sqrt{3})}{125} & \frac{64\sqrt{3}+12(5+2\sqrt{3})}{125} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{-175(5-2\sqrt{3})+24\sqrt{3}}{125} & \dfrac{24(5-2\sqrt{3})+7\sqrt{3}}{125} & \dfrac{-16(5-2\sqrt{3})+12\sqrt{3}}{125} \\ \dfrac{700\sqrt{3}+24(5+2\sqrt{3})}{125} & \dfrac{-96\sqrt{3}+7(5+2\sqrt{3})}{125} & \dfrac{64\sqrt{3}+12(5+2\sqrt{3})}{125} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{-875+374\sqrt{3}}{125} & \dfrac{120-41\sqrt{3}}{125} & \dfrac{-80+44\sqrt{3}}{125} \\ \dfrac{120+748\sqrt{3}}{125} & \dfrac{35-82\sqrt{3}}{125} & \dfrac{60+88\sqrt{3}}{125} \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A'B'C'D'] = Tr \cdot [ABCD] =$$

$$\begin{pmatrix} \dfrac{-875+374\sqrt{3}}{125} & \dfrac{120-41\sqrt{3}}{125} & \dfrac{-80+44\sqrt{3}}{125} \\ \dfrac{120+748\sqrt{3}}{125} & \dfrac{35-82\sqrt{3}}{125} & \dfrac{60+88\sqrt{3}}{125} \\ 0 & 0 & 1 \end{pmatrix} \cdot ABCD =$$

$$\cancel{= \frac{-80+44\sqrt{5}}{125}} \qquad \cancel{\frac{3\cdot(-875+374\sqrt{5})}{125} + \frac{-80+44\sqrt{5}}{125}}$$

De unde rezultă $A'B'C'D'$

<u>Pr. 2</u> Det. gen. rect. ale parab. hip. $x^2-y^2=2z$ care sunt paral. cu pl. $x+y+z=0$

$$P: \frac{x^2}{p} - \frac{y^2}{q} = 2z \Rightarrow p=q=1$$

Ec. poate fi rescrisă sub forma

$$\left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right)\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = 2z\cdot 1$$

Obținem familia de drepte

$$\begin{cases} \lambda\left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right) = 2\mu z \\ \mu\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = \lambda \end{cases}$$

$$\begin{cases} \lambda(\cancel{m}\ x - y) = 2\mu z \\ \mu(x+y) = \lambda \end{cases}$$

$$\frac{3(-875+74\sqrt{5})+3(1126-41\sqrt{5})+44\sqrt{5}-80}{125}$$

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precum și

$$\begin{cases} \alpha(x+y) = 2\beta z \\ \beta(x-y) = \alpha \end{cases}$$