3.6.

$A(1, -1, 2)$

$B(5, -6, 2)$

$C(1, 3, -1)$

Determinați lungimea înălțimii triunghiului ABC, coborâte din vârful B pe latura AC a triunghiului.

$\vec{a} = \overrightarrow{AB} = (x, y, z)$

$\vec{b} = \overrightarrow{AC} = (x', y', z')$

$$A_{\Delta ABC} = \frac{1}{2}\|\vec{a} \times \vec{b}\| = \sqrt{\begin{vmatrix} y & z \\ y' & z' \end{vmatrix}^2 + \begin{vmatrix} x & z \\ x' & z' \end{vmatrix}^2 + \begin{vmatrix} x & y \\ x' & y' \end{vmatrix}^2}$$

$A(1, -1, 2)$, $B(5, -6, 2)$ $\Rightarrow \overrightarrow{AB} = (5-1, -6-(-1), 2-2)$

$\overrightarrow{AB} = (4, -5, 0) = (x, y, z)$

$A(1, -1, 2)$, $C(1, 3, -1)$ $\Rightarrow \overrightarrow{AC} = (0, 4, -3) = (x', y', z')$

$$\begin{vmatrix} y & z \\ y' & z' \end{vmatrix} = \begin{vmatrix} -5 & 0 \\ 4 & -3 \end{vmatrix} = 15$$

$$\begin{vmatrix} x & z \\ x' & z' \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 0 & -3 \end{vmatrix} = -12$$

$$\begin{vmatrix} x & y \\ x' & y' \end{vmatrix} = \begin{vmatrix} 4 & -5 \\ 0 & 4 \end{vmatrix} = 16$$

$$A_{\Delta ABC} = \sqrt{15^2 + (-12)^2 + 16^2} \cdot \frac{1}{2}$$

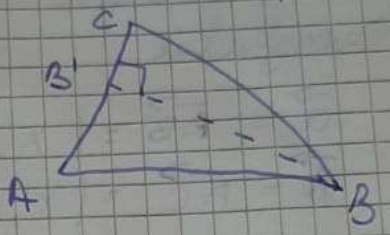
$$= \sqrt{225 + 144 + 256} \cdot \frac{1}{2}$$

$$= \sqrt{625} \cdot \frac{1}{2}$$

$$= 25 \cdot \frac{1}{2} \quad (1)$$

$$A_{\Delta ABC} = \frac{g \cdot h}{2} \quad (2)$$

$$= \frac{AC \cdot BB'}{2}$$



$$AC = \| \vec{AC} \|$$

$$= \sqrt{0^2 + 4^2 + (-3)^2}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5 \quad (3)$$

$$\xrightarrow{(1)(2)(3)} \frac{25}{2} = \frac{5 \cdot BB'}{2}$$

$$\Rightarrow 5 = BB'$$

$$\Rightarrow BB' = 5$$