Lab 1 - notițe

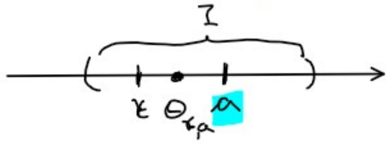
Wednesday, March 1, 2023 12:22 PM

Formula lui Taylor

I = interval deschis; $C^k(I) = \{ f : I \to \mathbb{R} : \exists f^{(i)}, i=\overline{0,k};\ f^{(k)} \text{ cont.} \}$

$\boxed{T1}$ (formula lui Taylor cu restul Lagrange)

$f \in C^{n+1}(I)$, $a \in I \Rightarrow f(x) = \underbrace{T_n f(x)}_{\text{pol. Taylor de ord. } n \text{ în jurul lui } a} + \underbrace{R_n f(x)}_{\text{rest}}$, $x \in I$, unde



$$T_n f(x) = f(a) + \frac{f'(a)}{1!}\cdot(x-a) + \dots + \frac{f^{(n)}(a)}{n!}\cdot(x-a)^n$$

$$R_n f(x) = \frac{f^{(n+1)}(\theta_{x,a})}{(n+1)!}\cdot(x-a)^{n+1}, \quad \theta_{x,a} = \text{pct. intermediar este între } x \text{ și } a$$

$T_n f(x)$ = taylor(f, x, a, 'order', n+1) → apel în Octave

>> help @sym/taylor

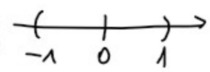
Serii de puteri MacLaurin ($a=0$)

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots, \quad x \in \mathbb{R}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n}{(2n+1)!}\cdot x^{2n+1} + \dots, \quad x \in \mathbb{R}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n}{(2n)!}\cdot x^{2n} + \dots, \quad x \in \mathbb{R}$$

$$\underbrace{\ln}_{\log}(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{n-1}}{n}\cdot x^n + \frac{(-1)^n}{n+1}\cdot x^{n+1} + \dots, \quad |x|<1$$



$$\frac{1}{1+x} = 1 - x + x^2 - \dots + (-1)^n \cdot x^n + \dots, \quad |x|<1$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots, \quad |x|<1$$

$$(1+x)^k = 1 + \sum_{n=0}^{\infty} \underbrace{\binom{k}{n}}_{\frac{k(k-1)\cdot\ldots\cdot(k-(n-1))}{n!}} \cdot x^n, \quad |x|<1$$

$(k \in \mathbb{R})$

ex: $(1+x)^{-\frac{1}{2}} = \dfrac{1}{\sqrt{1+x}}$

$\boxed{T_2}$ (crit. lui Leibniz pt. serii alternante)

$$S = \underbrace{\sum_{k=0}^{\infty} (-1)^k a_k}_{\text{conv.}}, \quad \text{unde } (a_k)_{k\in\mathbb{N}} \text{ desc. cu } \lim = 0 \quad (a_k \searrow 0,\ k \to \infty)$$

$$S_n = \sum_{k=0}^{n} (-1)^k a_k \Rightarrow |S - S_n| \le a_{n+1}.$$

$\boxed{T_3}$ (teor. lui Abel pt. serii)

$$f(x) = \sum_{k=0}^{\infty} a_k \cdot x^k, \quad |x| < r$$

$$\sum_{k=0}^{\infty} a_k \cdot r^k \text{ conv.} \Rightarrow \lim_{x \nearrow r} f(x) = \sum_{k=0}^{\infty} a_k \cdot r^k.$$

Pb. 5: $\ln 2 \simeq ?$ cu precizia de 5 zecimale

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots, \quad |x| < 1$$

Seria in $x = 1$: $1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n-1}}{n} + \dots$ este alternantă.

$\boxed{T_2} \Rightarrow$ seria conv.

$\boxed{T_3} \Rightarrow \underbrace{\lim_{x \nearrow 1} \ln(1+x)}_{\ln 2} = 1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n-1}}{n} + \dots$

$$\boxed{T_2} \Rightarrow \left| \ln 2 - \left(1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n-1}}{n}\right) \right| \le \frac{1}{n+1}$$

$$? \quad \frac{1}{n+1} < \frac{1}{10^5} \Rightarrow n \geq 10^5.$$

ex:

$1{,}000001 \simeq 1 \qquad |1{,}000001 - 1| < \frac{1}{10^5}$

$0{,}999999 \simeq 1 \qquad |0{,}999999 - 1| < \frac{1}{10^5}$

Temă: rez. pb. 8 completând codurile

taylor_coef.m

pade_sym.m