ANALIZĂ MATEMATICĂ clasa a XI-a
1.Limite de şiruri

Să se calculeze limitele:

1.	$\lim\limits_{n\to\infty} n^3 = \infty$	$\lim\limits_{n\to\infty}(-n^2+n-1)$
2.	$\lim\limits_{n\to\infty}\dfrac{1}{\sqrt{n+1}+\sqrt{n}} = \lim\limits_{n\to\infty}\dfrac{1}{2\sqrt{n}} = 0$	$\lim\limits_{n\to\infty}\dfrac{n}{\sqrt{4n^2+1}}$
3.	$\lim\limits_{n\to\infty}\dfrac{(n-1)^3}{2n^3} = \frac{\infty}{\infty} = \lim\limits_{n\to\infty}\dfrac{n^3}{2n^3} = \dfrac{1}{2}$	$\lim\limits_{n\to\infty}\dfrac{n^2}{(1-2n)^2}$
4.	$\lim\limits_{n\to\infty}\dfrac{1+2+\cdots+n}{n^2} = \frac{\infty}{\infty} = \lim\limits_{n\to\infty}\dfrac{n(n+1)}{2n^2} = \dfrac{1}{2}$	$\lim\limits_{n\to\infty}\dfrac{1^2+2^2+\cdots+n^2}{n^3}$
5.	$\lim\limits_{n\to\infty}\left(\sqrt{n^2+5n+1}-n\right) = \infty-\infty$ $a-b = \dfrac{a^2-b^2}{a+b}$ $= \lim\limits_{n\to\infty}\dfrac{n^2+5n+1-n^2}{\sqrt{n^2+5n+1}+n} = \lim\limits_{n\to\infty}\dfrac{5n}{2n} = \dfrac{5}{2}$	$\lim\limits_{n\to\infty}\left(n-\sqrt{n^2-4n+1}\right)$
6.	$\lim\limits_{n\to\infty}\left(\sqrt{n^2+5n+1}-\sqrt{n^2-7n+3}\right) = \infty-\infty$ $= \lim\limits_{n\to\infty}\dfrac{n^2+5n+1-n^2+7n-3}{n+n} = \lim\limits_{n\to\infty}\dfrac{12n}{2n} = 6$	$\lim\limits_{n\to\infty}\left(\sqrt{n^2-1}-\sqrt{n^2+9n}\right)$
7.	$\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+5n^2+1}-n\right) = \infty-\infty$ $a-b = \dfrac{a^3-b^3}{a^2+ab+b^2}$ $= \lim\limits_{n\to\infty}\dfrac{n^3+5n^2+1-n^3}{\sqrt[3]{(n^3+5n^2+1)^2}+n\sqrt[3]{n^3+5n^2+1}+n^2}$ $= \lim\limits_{n\to\infty}\dfrac{5n^2+1}{n^2+n^2+n^2} = \lim\limits_{n\to\infty}\dfrac{5n^2}{3n^2} = \dfrac{5}{3}$	$\lim\limits_{n\to\infty}\left(n-\sqrt[3]{n^3-6n^2+1}\right)$
8.	$\lim\limits_{n\to\infty}\left(\dfrac{n}{n+1}\right)^n = 1^\infty =$	$\lim\limits_{n\to\infty}\left(\dfrac{n-1}{n}\right)^n$

	$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$ $= \lim\limits_{n\to\infty}\left(1+\frac{n}{n+1}-1\right)^{n} = \lim\limits_{n\to\infty}\left(1+\frac{-1}{n+1}\right)^{n} =$ $= \lim\limits_{n\to\infty}\left[\left(1+\frac{-1}{n+1}\right)^{\frac{n+1}{-1}}\right]^{\frac{-1}{n+1}n} = e^{\lim_{n\to\infty}\frac{-n}{n+1}} = e^{-1}$	
9.	$\lim\limits_{n\to\infty}\left(\frac{n^2-1}{n^2+n+1}\right)^{\frac{(3n-1)^2}{n}} = 1^{\infty} =$ $= \lim\limits_{n\to\infty}\left(1+\frac{n^2-1}{n^2+n+1}-1\right)^{\frac{(3n-1)^2}{n}}$ $= \lim\limits_{n\to\infty}\left(1+\frac{-n-2}{n^2+n+1}\right)^{\frac{(3n-1)^2}{n}}$ $= \lim\limits_{n\to\infty}\left[\left(1+\frac{-n-2}{n^2+n+1}\right)^{\frac{n^2+n+1}{-n-2}}\right]^{\frac{-n-2}{n^2+n+1}\cdot\frac{(3n-1)^2}{n}}$ $= e^{\lim_{n\to\infty}\frac{-n}{n^2}\cdot\frac{9n^2}{n}} = e^{-9}$	$\lim\limits_{n\to\infty}\left(\frac{3n^2+1}{n^2+n+1}\right)^{n}$
10.	$\lim\limits_{n\to\infty}(1+2^n+3^n) = \lim\limits_{n\to\infty}3^n = \infty, \qquad 3>1$ $\lim\limits_{n\to\infty}a^n = \begin{cases}\infty, a>1\\ 1, a=1\\ 0, \lvert a\rvert<1\\ \nexists, a\le -1\end{cases}$	$\lim\limits_{n\to\infty}\left(1+\left(\frac{1}{2}\right)^{n}+\left(\frac{1}{3}\right)^{n}\right)$
11.	$\lim\limits_{n\to\infty}\frac{3^n+2^n}{\left(\frac{2}{3}\right)^{n}+2^n+3^{n-1}} = \lim\limits_{n\to\infty}\frac{3^n}{3^{n-1}} = 3$	$\lim\limits_{n\to\infty}\frac{3^n-5^n}{4^n+3^n+2^n}$
12.	$\lim\limits_{n\to\infty}\frac{1+2+2^2+\cdots+2^{n-1}}{1+3+3^2+\cdots+3^{n-1}}$ $1+q+q^2+\cdots+q^{n-1} = \frac{q^n-1}{q-1}$	$\lim\limits_{n\to\infty}\frac{1+\frac{1}{2}+\left(\frac{1}{2}\right)^{2}+\cdots+\left(\frac{1}{2}\right)^{n-1}}{1+\frac{1}{3}+\left(\frac{1}{3}\right)^{2}+\cdots+\left(\frac{1}{3}\right)^{n-1}}$

	$= \lim\limits_{n\to\infty} \dfrac{\dfrac{2^n-1}{2-1}}{\dfrac{3^n-1}{3-1}} = \lim\limits_{n\to\infty} \dfrac{2\cdot 2^n}{3^n} = \lim\limits_{n\to\infty} 2\cdot\left(\dfrac{2}{3}\right)^n = 0,$ $\dfrac{2}{3} \in (-1,1)$ atunci $\left(\dfrac{2}{3}\right)^n \to 0$	
13.	$\lim\limits_{n\to\infty}\big(ln(8n-1)-ln(n+8)\big) = \infty-\infty =$ $= \lim\limits_{n\to\infty} ln\dfrac{8n-1}{n+8} = \lim\limits_{n\to\infty} ln\dfrac{8n}{n} = ln8$	$\lim\limits_{n\to\infty}(ln(e^{2n}+4)-n)$
14.	$\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}\right) =$ *utilizăm șirul remarcabil* $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\ln n\right) = c,$ $c = 0{,}57\ldots$ *constanta lui Euler* $= \lim\limits_{n\to\infty}\left(1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\ln n+\ln n\right) =$ $= \lim\limits_{n\to\infty}(c+\ln n) = \infty$	$\lim\limits_{n\to\infty}\left(\dfrac{1}{n+1}+\dfrac{1}{n+2}+\cdots+\dfrac{1}{2n}\right)$
	Lema Stolz-Cesaro *Dacă șirurile* $(a_n),(b_n)$ *au proprietățile:* 1) (b_n)*are termeni nenuli și este crescător și nemărginit,* 2) $\lim\limits_{n\to\infty}\dfrac{a_{n+1}-a_n}{b_{n+1}-b_n} = a,$ *atunci* $\lim\limits_{n\to\infty}\dfrac{a_n}{b_n} = a.$	
15.	$\lim\limits_{n\to\infty}\dfrac{1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}}{n} = \dfrac{\infty}{\infty} = Stolz-Cesaro$ $= \lim\limits_{n\to\infty}\dfrac{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+\frac{1}{n+1}-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)}{n+1-n}$ $= \lim\limits_{n\to\infty}\dfrac{1}{n+1} = 0$	$\lim\limits_{n\to\infty}\dfrac{1+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+\cdots+\dfrac{1}{\sqrt{n}}}{\sqrt{n}}$
	Criteriul raportului *Fie șirul* (a_n) *de numere strict pozitive* $a.$î. $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = a.$ *Atunci*: 1)*dacă* $a<1 \Rightarrow \lim\limits_{n\to\infty} a_n = 0,$ 2)*dacă* $a>1 \Rightarrow \lim\limits_{n\to\infty} a_n = \infty$	

16.	$\lim\limits_{n\to\infty}\frac{5^n}{n!}=$ *utilizăm Criteriul raportului* $\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}}=\lim\limits_{n\to\infty}\frac{5}{n+1}=0<1\Rightarrow\lim\limits_{n\to\infty}\frac{5^n}{n!}=0$	$\lim\limits_{n\to\infty}\frac{3^n}{n!}$
	Criteriul radicalului.Cauchy-d'Alembert *Fie șirul* (a_n) *de numere strict pozitive.* *Dacă* $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=a,atunci\ \lim\limits_{n\to\infty}\sqrt[n]{a_n}=a.$	
17.	$\lim\limits_{n\to\infty}\sqrt[n]{n^2+n+1}$ *utilizăm Criteriul radicalului* $\lim\limits_{n\to\infty}\frac{(n+1)^2+(n+1)+1}{n^2+n+1}=\lim\limits_{n\to\infty}\frac{n^2}{n^2}=1\Rightarrow$ $\lim\limits_{n\to\infty}\sqrt[n]{n^2+n+1}=1$	$\lim\limits_{n\to\infty}\sqrt[n]{n!}$
	Criteriu de convergență $(a_n)mărginit, b_n\to 0\Rightarrow a_n\cdot b_n\to 0$	
18.	$\lim\limits_{n\to\infty}\frac{\sin n}{n}=0$ *utilizând Criteriul de convergență* $a_n=\sin n\in[-1,1]$ *și* $b_n=\frac{1}{n}\to 0$	$\lim\limits_{n\to\infty}\left(\frac{1}{2}\right)^n\cos n^2$
19.	$\lim\limits_{n\to\infty}\frac{sinx_n}{x_n}=1,x_n\in\mathbb{R}^*,x_n\to 0$ $\lim\limits_{n\to\infty}\frac{sin\frac{1}{n}}{\frac{1}{n}}=1$	$\lim\limits_{n\to\infty}\frac{sin\frac{n+1}{n^2}}{\frac{n+1}{n^2}}$
20.	$\lim\limits_{n\to\infty}\frac{tgx_n}{x_n}=1,x_n\in\mathbb{R}^*,x_n\to 0$ $\lim\limits_{n\to\infty}\frac{tg\frac{n+1}{n^2}}{\frac{n+1}{n^2}}=1$	$\lim\limits_{n\to\infty}\frac{tg\frac{1}{3^n}}{\frac{19}{3^n}}$

21.	$\lim_{n\to\infty}\frac{arcsinx_n}{x_n}=1, x_n\in\mathbb{R}^*, x_n\to 0$ $\lim_{n\to\infty}\frac{arcsin\frac{3}{n}}{\frac{3}{n}}=1$	$\lim_{n\to\infty}\frac{arcsin\frac{\sqrt{2}}{n}}{\frac{1}{n}}$
22.	$\lim_{n\to\infty}\frac{arctgx_n}{x_n}=1, x_n\in\mathbb{R}^*, x_n\to 0$ $\lim_{n\to\infty}\frac{arctg\frac{\sqrt{2}}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{arctg\frac{\sqrt{2}}{n}}{\sqrt{2}\frac{1}{n}}\sqrt{2}=\sqrt{2}$	$\lim_{n\to\infty}\frac{arctg\frac{1}{2\sqrt{n}}}{\frac{1}{\sqrt{n}}}$
23.	$\lim_{n\to\infty}\frac{\ln(1+x_n)}{x_n}=1, x_n\in\mathbb{R}^*, x_n\to 0, 1+x_n>0$ $\lim_{n\to\infty}\frac{\ln(1+\frac{1}{\sqrt{n}})}{\frac{1}{\sqrt{n}}}=1$	$\lim_{n\to\infty}\frac{\ln\left(\frac{n+1}{n}\right)}{\frac{1}{n}}$
24.	$\lim_{n\to\infty}\frac{a^{x_n}-1}{x_n}=\ln a, x_n\in\mathbb{R}^*, x_n\to 0, a>0, a\neq 1$ $\lim_{n\to\infty} n\left(2^{\frac{1}{n}}-1\right)=\ln 2$	$\lim_{n\to\infty}\frac{\sqrt[n]{5}-1}{\frac{1}{n}}$
25.	$\lim_{n\to\infty}\frac{(1+x_n)^r-1}{x_n}=r, x_n\in\mathbb{R}^*, x_n\to 0, r\in\mathbb{R}$ $\lim_{n\to\infty}\frac{\left(\frac{n+3}{n}\right)^7-1}{\frac{3}{n}}=7$	$\lim_{n\to\infty} n^2\left(\sqrt[21]{\frac{n^2+7}{n^2}}-1\right)$