12.5. Determinați imaginea triunghiului ABC prin reflexia relativ la dreapta AB.

$A(1,1)$

$B(4,1)$

$C(2,3)$

Matricea omogenă a reflexiei față de dreapta care trece prin Q și are versorul director $\vec{w}$ este:

$$\text{Mirror}(Q,\vec{w}) = \begin{pmatrix} I_2 - 2(\vec{w}^{\perp} \otimes \vec{w}^{\perp}) & 2(\vec{w}^{\perp} \otimes \vec{w}^{\perp}) \cdot Q \\ 0 & 1 \end{pmatrix} \quad (1)$$

Calculăm ec. dreptei AB:

$$AB: \frac{y-1}{1-1} = \frac{x-1}{4-1} \Rightarrow AB: \frac{y-1}{0} = \frac{x-1}{3} \Leftrightarrow AB: y=1$$

$\Rightarrow$ vectorul director al dreptei $\vec{v}(1,0) = \vec{i}$

Înlocuim în (1) punctul Q cu A și versorul director $\vec{w}$ cu $\vec{i}$:

$$\text{Mirror}(A,\vec{i}) = \begin{pmatrix} I_2 - 2(\vec{i}^{\perp} \otimes \vec{i}^{\perp}) & 2(\vec{i}^{\perp} \otimes \vec{i}^{\perp}) \cdot A \\ 0 & 1 \end{pmatrix}$$

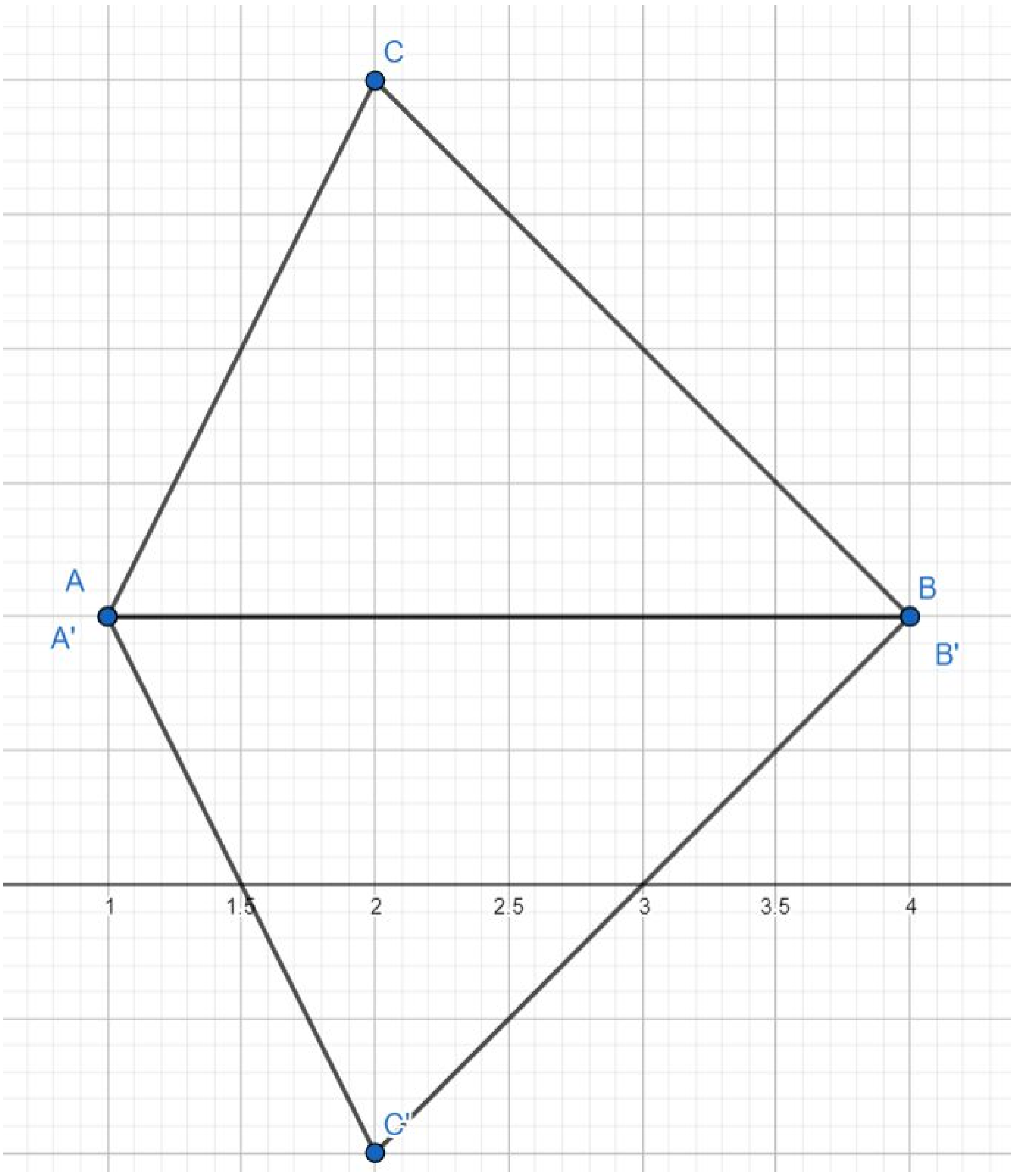
$$\vec{i}^{\perp} = (0,1) \Rightarrow \vec{i}^{\perp} \otimes \vec{i}^{\perp} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0\ 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_2 - 2(\vec{i}^{\perp} \otimes \vec{i}^{\perp}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$2(\vec{i}^{\perp} \otimes \vec{i}^{\perp}) \cdot A = 2 \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\Rightarrow \text{Mirror}(A,\vec{i}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow (A'\ B'\ C') = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow (A'\ B'\ C') = \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow A'(1,1);\ B'(4,1);\ C'(2,-1)$$


C
A
A'
B
B'
C'
1
1.5
2
2.5
3
3.5
4