Temă - Seminar 2

2.2

Să se găsească unghiul dintre vectorii $\vec{a} = 2\vec{m} + 4\vec{n}$ și $\vec{b} = \vec{m} - \vec{n}$, unde $\vec{m}$ și $\vec{n}$ vectori unitari și $\angle(\vec{m}, \vec{n}) = 120^\circ$.

Rezolvare:

Cum $m(\angle(\vec{m}, \vec{n})) = 120^\circ$, aplicăm:

$$\left.\begin{array}{l} \cos(\widehat{\vec{m}, \vec{n}}) = \dfrac{\vec{m} \cdot \vec{n}}{\|\vec{m}\| \cdot \|\vec{n}\|} \\ \vec{m} \text{ și } \vec{n} \text{ vectori unitari} \Rightarrow \|\vec{m}\| = \|\vec{n}\| = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \vec{m} \cdot \vec{n} = \cos(\widehat{\vec{m}, \vec{n}}) = \cos(120^\circ) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -1/2.$$

Calculăm $\|\vec{a}\|$ și $\|\vec{b}\|$:

$$\|\vec{a}\| = \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{(2\vec{m} + 4\vec{n})^2} = \sqrt{4\underbrace{\vec{m}^2}_{=1} + 16\underbrace{\vec{n}^2}_{=1} + 16\underbrace{\vec{m} \cdot \vec{n}}_{=-1/2}} =$$

(vectori unitari)

$$= \sqrt{4 + 16 - 16 \cdot \frac{1}{2}} = \sqrt{20 - 8} = \sqrt{12} = 2\sqrt{3}. \Rightarrow \|\vec{a}\| = 2\sqrt{3}$$

$$\|\vec{b}\| = \sqrt{\vec{b} \cdot \vec{b}} = \sqrt{(\vec{m} - \vec{n})^2} = \sqrt{\underbrace{\vec{m}^2}_{=1} + \underbrace{\vec{n}^2}_{=1} - 2\vec{m}\vec{n}} = \sqrt{2 + \cancel{2} \cdot \frac{1}{\cancel{2}}} = \sqrt{3}.$$

(vectori unitari) $\Rightarrow \|\vec{b}\| = \sqrt{3}$.

$$\Rightarrow \|\vec{a}\| \cdot \|\vec{b}\| = 2\sqrt{3} \cdot \sqrt{3} = 2 \cdot 3 = 6$$

$$\vec{a} \cdot \vec{b} = (2\vec{m} + 4\vec{n})(\vec{m} - \vec{n}) = 2\underbrace{\vec{m}^2}_{=1} - 4\underbrace{\vec{n}^2}_{=1} - 2\vec{m}\vec{n} + 4\vec{n}\vec{m} =$$

(vectori unitari)

$$= 2 - 4 + 2\vec{m}\vec{n} = -2 - 2 \cdot 1/2 = -3$$

Folosind $\cos(\widehat{\vec{a}, \vec{b}}) = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} \Rightarrow \cos(\widehat{\vec{a}, \vec{b}}) = \dfrac{(-3)}{6} = -\dfrac{3}{6} = -1/2$

$$\Rightarrow \cos(\widehat{\vec{a}, \vec{b}}) = -1/2 \Rightarrow m(\widehat{\vec{a}, \vec{b}}) = \frac{2\pi}{3} = 120^\circ.$$

$$\left(\pi - \frac{\pi}{3}\right)$$