Metoda celor mai mici pătrate

$\langle f, g \rangle$ = prod. scalar pt. f şi g (funcţii/vectori)

$\|f-g\|$ = distanţa dintre f şi g (normă)

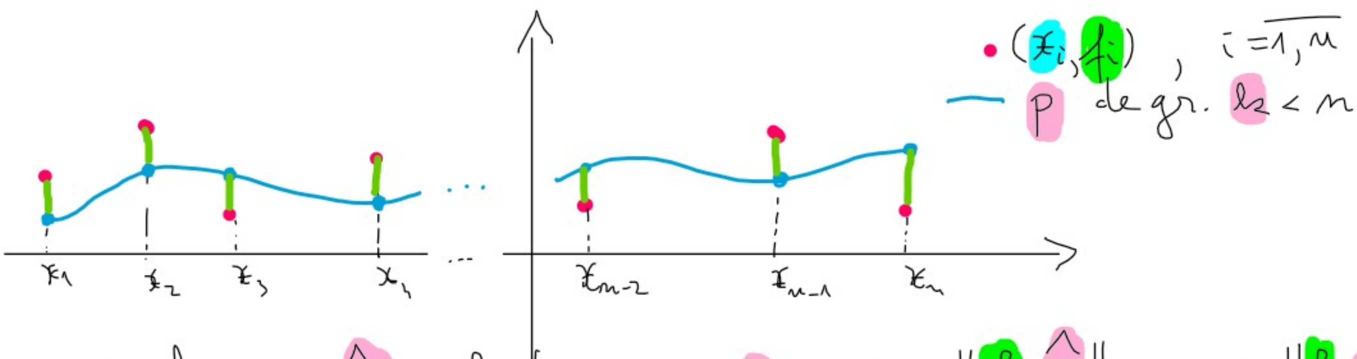
$= \sqrt{\langle f-g, f-g \rangle}$

Ex.: $\langle f, g \rangle = f_1 \cdot g_1 + \ldots + f_n \cdot g_n$ = prod. scal. Euclidian

$\|f-g\| = \sqrt{(f_1-g_1)^2 + \ldots + (f_n-g_n)^2}$ = norma Euclid.

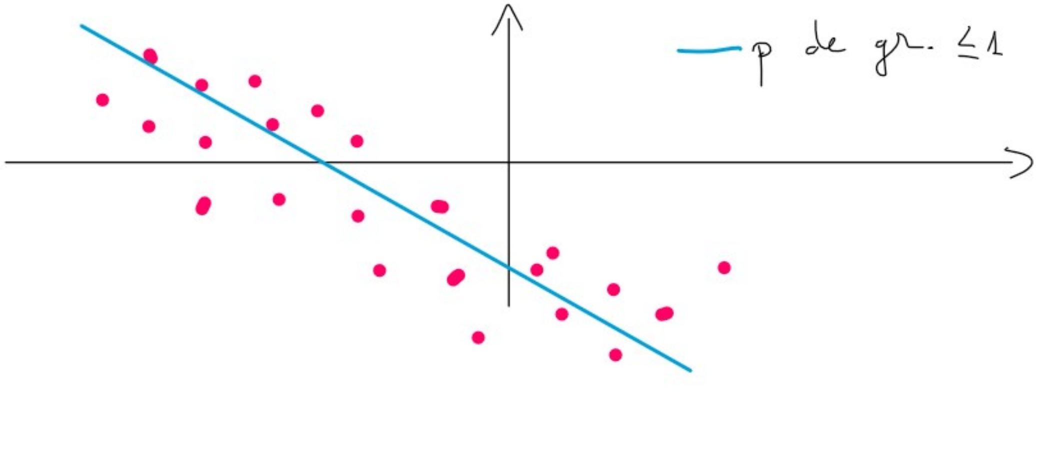
Ex.: $\langle f, g \rangle = \int_a^b f(x) \cdot g(x)\, dx$

M.c.m.m.p. discretă (norma Euclidiană)



? Căutăm $\hat{p}$ pol. de gr. $\leq k$ a.î. $\|f - \hat{p}\| = \min\limits_{p \in \mathbb{P}_k} \|f - p\|$.

$\mathbb{P}_k$ – sp. pol. gr. $\leq k$



$$p(X) = c_k \cdot X^k + \dots + c_1 X + c_0 \cdot 1$$

$$\begin{cases} p(x_1) = c_k \cdot x_1^k + \dots + c_1 \cdot x_1 + c_0 \cdot 1 \simeq f_1 \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ p(x_n) = c_k \cdot x_n^k + \dots + c_1 \cdot x_n + c_0 \cdot 1 \simeq f_n \end{cases}$$

$k+1$ necunoscute
$\wedge\!\backslash$
n ec.

$$\underbrace{\begin{bmatrix} x_1^k & \dots & x_1 & 1 \\ \vdots & & & \vdots \\ x_n^k & \dots & x_n & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} c_k \\ \vdots \\ c_1 \\ c_0 \end{bmatrix}}_{c} \simeq \underbrace{\begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}}_{f}$$

Sist. supradeterminat: $A \cdot c \simeq f$, $A \in \mathcal{M}_{n,m}(\mathbb{R})$

$n \geq m \; (= k+1)$

Căutăm $\hat{c} \in \mathbb{R}^m$ a.î. $\|A \cdot \hat{c} - f\| = \min\limits_{c \in \mathbb{R}^m} \|A \cdot c - f\|$.

Descompunerea QR: $\forall A \in \mathcal{M}_{n,m}(\mathbb{R})$, $n \geq m$,

$\exists Q \in \mathcal{M}_{n,n}(\mathbb{R})$, $Q^{-1} = Q^T$, $\exists R \in \mathcal{M}_{n,m}(\mathbb{R})$ triung. superior,

$$R = \begin{array}{c} m \\ \begin{bmatrix} \begin{matrix} \diagdown \\ 0 \end{matrix} \\ \hline 0 \end{bmatrix} \end{array} \begin{matrix} \}m \\ \}n-m \end{matrix}, \quad \text{a.î. } A = Q \cdot R.$$

- $Q^{-1} = Q^T \Rightarrow \|Q \cdot v\| = \|v\|, \forall v \in \mathbb{R}^n$
- $\|A c - f\| = \|Q(Rc) - f\| = \|Q\overbrace{(Rc - Q^T \cdot f)}^{v}\|$

$$\|A\cdot c - f\| = \|Q(R c) - f\| = \|Q(Rc - Q^T f)\|$$

$$= \|\underbrace{R\cdot c}_{y} - \underbrace{Q^T\cdot f}_{b}\| = \left\| \begin{bmatrix} \hat{y} - \hat{b} \\ -\tilde{b} \end{bmatrix} \right\|$$

$$\overbrace{\begin{bmatrix} R \\ 0 \end{bmatrix}}^{R} \cdot \begin{bmatrix} c \end{bmatrix} = \overbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}}^{y} \begin{matrix} \left.\vphantom{\begin{matrix} y_1 \\ \vdots \\ y_m \end{matrix}}\right\} \hat{y} \\ \left.\vphantom{\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}}\right\} 0 \end{matrix} \qquad \overbrace{\begin{bmatrix} b_1 \\ \vdots \\ b_m \\ b_{m+1} \\ \vdots \\ b_n \end{bmatrix}}^{b} \begin{matrix} \left.\vphantom{\begin{matrix} b_1 \\ \vdots \\ b_m \end{matrix}}\right\} \hat{b} \\ \left.\vphantom{\begin{matrix} b_{m+1} \\ \vdots \\ b_n \end{matrix}}\right\} \tilde{b} \end{matrix}$$

Sol. sist. supradet. $A\cdot c \approx f$ este

$$\hat{y} = \hat{b}$$

$$\Updownarrow$$

$$R(1:m,:)\cdot \hat{c} = b(1:m)$$

$$\|A\cdot \hat{c} - f\| = \underbrace{\min_{c\in\mathbb{R}^m} \|Ac - f\|}_{} \leftarrow \boxed{\hat{c} = R(1:m,:)\backslash b(1:m)}$$

$$\|\tilde{b}\| = \|b(m+1:n)\|$$