

3.2.42.

$a \in \mathbb{R}$

$a.i. \quad V = \{v_1, v_2, v_3\}^1$

$v_1 = (a, 1, 1)$

$v_2 = (1, a, 1)$

$v_3 = (1, 1, a)$

, V bază V bază $\Rightarrow V$ liniar independentă

fie $\alpha_1, \alpha_2, \alpha_3$ a.i. $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$

$\Rightarrow \alpha_1(a, 1, 1) + \alpha_2(1, a, 1) + \alpha_3(1, 1, a) =$

$= (\alpha_1 a + \alpha_2 + \alpha_3, \alpha_1 + \alpha_2 a + \alpha_3, \alpha_1 + \alpha_2 + \alpha_3 a) = (0, 0, 0)$

$$\Rightarrow \begin{cases} \alpha_1 a + \alpha_2 + \alpha_3 = 0 \\ \alpha_1 + \alpha_2 a + \alpha_3 = 0 \\ \alpha_1 + \alpha_2 + \alpha_3 a = 0 \end{cases} \oplus$$

$\alpha_1(a+2) + \alpha_2(a+2) + \alpha_3(a+2) = 0$

$\Rightarrow (a+2)(\alpha_1 + \alpha_2 + \alpha_3) = 0$

I $a+2=0 \Rightarrow a=-2 \Rightarrow \exists \alpha_1, \alpha_2, \alpha_3$ a.i. $\alpha_1, \alpha_2, \alpha_3 \neq 0$

și $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0 \Rightarrow V$ nu e bază

$\Rightarrow a \neq -2$

II $\alpha_1 + \alpha_2 + \alpha_3 = 0 \Rightarrow \alpha_1 = -\alpha_2 - \alpha_3$

$$\Rightarrow \begin{cases} a(-\alpha_2 - \alpha_3) + \alpha_2 + \alpha_3 = 0 \\ -\alpha_2 + \alpha_2 a = 0 \Rightarrow \alpha_2(a-1) = 0 \\ -\alpha_3 + \alpha_3 a = 0 \Rightarrow \alpha_3(a-1) = 0 \end{cases}$$

ii) $a-1=0 \Rightarrow V$ liniar dependentă $\Rightarrow V$ nu e bază

$\Rightarrow a \neq -1$

$$\text{ii) } a-1 \neq 0 \Rightarrow \alpha_2 = \alpha_3 = 0$$

$$\text{dar } \alpha_1 = -\alpha_2 - \alpha_3 = -0 - 0 = 0$$

\Rightarrow v liniar independentă

$$\langle v \rangle = \{ \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 \mid \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R} \} =$$

$$= \{ (\alpha_1 a + \alpha_2 + \alpha_3, \alpha_1 + \alpha_2 a + \alpha_3, \alpha_1 + \alpha_2 + \alpha_3 a) \mid \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R} \} = \mathbb{R}^3$$

$\Rightarrow v$ e bază pentru $\forall a \in \mathbb{R} \setminus \{-2, 1\}$