

# Temă seminar

## Problema 9.2.19

Utilizând rezoluția liniară, demonstrați:

4. distributivitatea cuantificatorului "∃" față de "∨" :

$$\vdash (\exists x) (P(x) \vee Q(x)) \leftrightarrow (\exists x) P(x) \vee (\exists x) Q(x)$$

$$1) \vdash (\exists x) (P(x) \vee Q(x)) \rightarrow (\exists x) P(x) \vee (\exists x) Q(x)$$

$$\stackrel{ITD}{\Rightarrow} (\exists x) (P(x) \vee Q(x)) \stackrel{?}{\vdash} (\exists x) P(x) \vee (\exists x) Q(x)$$

$$U_1 = (\exists x) (P(x) \vee Q(x)) \quad [x \leftarrow a]$$

$$U_1^c = P(a) \vee Q(a)$$

$$U_2 = \neg ((\exists x) P(x) \vee (\exists x) Q(x)) =$$

$$= (\forall x) \neg P(x) \wedge (\forall x) \neg Q(x) =$$

$$= (\forall x) (\neg P(x) \wedge \neg Q(x)) = U_2^p$$

$$U_2^{yh} = (\forall x) (\neg P(x) \wedge \neg Q(x))$$

$$U_2^c = \neg P(x) \wedge \neg Q(x)$$

$$S_1 = \{ P(a)^{c_1} \vee Q(a), \neg P(x)^{c_2}, \neg Q(x)^{c_3} \}$$

$$c_1 = P(a) \vee Q(a)$$

$$c_2 = \neg P(x)$$

[x ← a]

$$c_4 = Q(a)$$

$$c_3 = \neg Q(x)$$

[x ← a]

$$c_5 = \square$$

$\stackrel{TCC}{\Rightarrow}$

$S_1$  inconsistentă



TD  $\Rightarrow$

$\vdash (\exists x)(P(x) \vee Q(x)) \rightarrow (\exists x)P(x) \vee (\exists x)Q(x)$   
(pe baza teoremei din curs) cã  $V \vdash V$  ni aplicând TD

2) " $\vdash (\exists x)P(x) \vee (\exists x)Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$ "

iTD  $\Rightarrow (\exists x)P(x) \vee (\exists x)Q(x) \vdash^? (\exists x)(P(x) \vee Q(x))$

$$U_1 = (\exists x)P(x) \vee (\exists x)Q(x) =$$

$$= \cancel{(\exists x)(P(x) \vee Q(x))} = U_1^P \quad [x \leftarrow a]; [x \leftarrow b]$$

$$U_1^{yh} = P(a) \vee Q(b) = U_1^c$$

$$U_2 = \neg (\exists x)(P(x) \vee Q(x)) =$$

$$= (\forall x) \neg (P(x) \vee Q(x)) =$$

$$= (\forall x) (\neg P(x) \wedge \neg Q(x)) = U_1^P$$

$$U_2^{yh} = \cancel{(\forall x)P(x) \wedge \neg Q(x)}$$

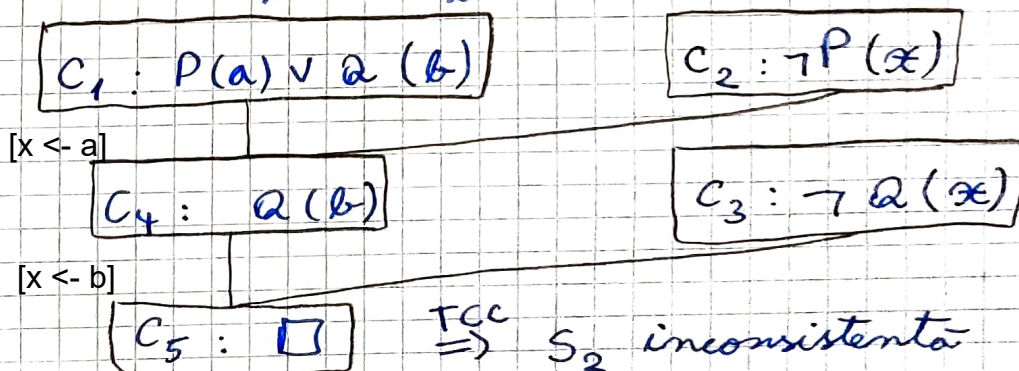
$$U_2^c = \neg P(x) \wedge \neg Q(x)$$

$$S_2 = \{ P(a) \vee Q(b), \neg P(x), \neg Q(x) \} = S_1$$

~~$\Rightarrow$  analog,  $\vdash (\exists x)P(x) \vee (\exists x)Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$~~

~~Deci, din 1) si 2)  $\Rightarrow$~~

~~$\Rightarrow \vdash (\exists x)(P(x) \vee Q(x)) \leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$~~



TD  $\Rightarrow \vdash (\exists x)P(x) \vee (\exists x)Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$   
(pe baza teoremei din curs) cã  $V \vdash V$  ni aplicând TD

1) + 2)  $\Rightarrow$

$$\vdash (\exists x)(P(x) \vee Q(x)) \leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$$