Temă seminar 12

12.6

Determinați imaginea triunghiului ABC prin reflexia relativă la dreapta BC, urmată de o forfecare, de unghi de 60°, relativă la punctul A, în direcția vectorului $\vec{v}(1,1)$.

$A(1,1)$, $B(4,1)$, $C(2,3)$

$\vec{BC}(x_C - x_B, y_C - y_B) \to \vec{BC}(-2, 2)$

$|\vec{BC}| = \sqrt{a^2 + b^2} = \sqrt{4+4} = 2\sqrt{2}$

versorul lui $\vec{BC}$, not. cu $\vec{u} = \dfrac{\vec{BC}}{|\vec{BC}|}$

$\Leftrightarrow |\vec{BC}| \cdot \vec{u} = \vec{BC} \Leftrightarrow \vec{BC} = 2\sqrt{2} \cdot \vec{u}$

$\Leftrightarrow \begin{cases} u_1 \cdot 2\sqrt{2} = -2 \\ u_2 \cdot 2\sqrt{2} = 2 \end{cases} \to$

$\to \vec{u}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Matr reflexiei:

$$\text{Mrefl} = (B, \vec{u}) = \begin{pmatrix} 1 - 2 \cdot \frac{1}{2} & 2\left(-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} & 2\left(4 \cdot \frac{1}{2} - 1 \cdot \left(-\frac{1}{2}\right)\right) \\ 2 \cdot \left(-\frac{1}{2}\right) & 1 - 2 \cdot \frac{1}{2} & 2\left(-4\left(-\frac{1}{2}\right) + 1 \cdot \frac{1}{2}\right) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\to \text{Mrefl}(B, \vec{u}) = \begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

Calculăm versorul vectorului $\vec{v}(1,1)$, pe care îl vom nota cu $\vec{a}(a_1, a_2)$.

$$\Leftrightarrow \vec{a} = \frac{\vec{v}}{|\vec{v}|} \quad (1)$$

$$\vec{v} = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2} \quad (2)$$

$$\overset{(1)}{\underset{(2)}{\Rightarrow}} \sqrt{2}\cdot\vec{a} = \vec{v} \Leftrightarrow \begin{cases} a_1\cdot\sqrt{2} = 1 \\ a_2\cdot\sqrt{2} = 1 \end{cases} \Rightarrow \begin{cases} a_1 = \frac{1}{\sqrt{2}} \\ a_2 = \frac{1}{\sqrt{2}} \end{cases}$$

$$\Leftrightarrow \vec{a}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \Rightarrow$$

$\Rightarrow$ Calc. matr. forfecării:

$$Shear(A, \vec{a}, 60^\circ) = \begin{pmatrix} 1-\frac{1}{2}\cdot\sqrt{3} & \frac{1}{2}\cdot\sqrt{3} & 0 \\ -\frac{1}{2}\cdot\sqrt{3} & 1+\frac{1}{2}\cdot\sqrt{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow Shear(A, \vec{a}, 60^\circ) = \begin{pmatrix} 1-\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 1+\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow [A'B'C'] = Shear \cdot Shear \cdot [ABC]$$

$$\Leftrightarrow [A'B'C'] = \begin{pmatrix} 4 & \frac{8-3\sqrt{3}}{2} & \frac{4+\sqrt{3}}{2} \\ 4 & \frac{2-3\sqrt{3}}{2} & \frac{6+\sqrt{3}}{2} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow A'(4,4);\ B'\left(\frac{8-3\sqrt{3}}{2}, \frac{2-3\sqrt{3}}{2}\right);\ C'\left(\frac{4+\sqrt{3}}{2}, \frac{6+\sqrt{3}}{2}\right)$$