12.3 $A(0,0)$, $B(3,0)$, $C(3,3)$, $D(0,3)$

Determinați imaginea pătratului $ABCD$ prin forfecarea de vector $v(1,1)$, de unghi $60°$, relativ la origine

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$$Shear(Q, \vec{w}, \theta) = \begin{pmatrix} 1 - w_1 w_2 \operatorname{tg}\theta & w_1^2 \operatorname{tg}\theta & (q_1 w_1 w_2 - q_2 w_1^2)\operatorname{tg}\theta \\ -w_2^2 \operatorname{tg}\theta & 1 + w_1 w_2 \operatorname{tg}\theta & (q_1 w_2^2 - q_2 w_1 w_2)\operatorname{tg}\theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q = O(0,0), \quad \vec{w} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \quad \operatorname{tg}\theta = \operatorname{tg} 60° = \sqrt{3}$$

$$Shear(O, \vec{w}, 60°) = \begin{pmatrix} 1 - \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 1 + \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A'B'C'D'] = Shear(O, \vec{w}, 60°) \cdot [ABCD]$$

$$[A'B'C'D'] = \begin{pmatrix} 1 - \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 1 + \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 3 - \frac{3\sqrt{3}}{2} & 3 & \frac{3\sqrt{3}}{2} \\ 0 & -\frac{3\sqrt{3}}{2} & 3 & 3 + \frac{3\sqrt{3}}{2} \\ 1 & 1 & 1 & 1 \end{pmatrix} \Rightarrow \begin{array}{l} A'(0,0) \\ B'\left(3 - \frac{3\sqrt{3}}{2}, -\frac{3\sqrt{3}}{2}\right) \\ C'(3,3) \\ D'\left(\frac{3\sqrt{3}}{2}, 3 + \frac{3\sqrt{3}}{2}\right) \end{array}$$