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Gr. 212

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Ex. 1) $\sqrt{x^2+1}\cdot y' - y = 0$

$$y' = y \cdot \frac{1}{\sqrt{x^2+1}}$$

$$y' = g(y)\cdot f(x), \quad g(y) = y$$

$$f(x) = \frac{1}{\sqrt{x^2+1}}$$

$$y' = \frac{dy}{dx}$$

I $\quad g(y) \neq 0$

$$\frac{dy}{dx} = y \cdot \frac{1}{\sqrt{x^2+1}} \;\Big|\; \cdot dx \;\; : y \neq 0$$

$$\frac{dy}{y} = \frac{dx}{\sqrt{x^2+1}} \;\Big|\; \int$$

$$\int \frac{dy}{y} = \int \frac{dx}{\sqrt{x^2+1}}$$

$$\ln|y| = \ln\left|x + \sqrt{x^2+1}\right| + c$$

$c \in \mathbb{R}$

$$|y| = e^{\ln|x+\sqrt{x^2+1}|} \cdot e^{c}$$

$$y = c_1\left(x + \sqrt{x^2+1}\right), \quad c_1 \in \mathbb{R}^*$$

II $g(y) = 0 \Leftrightarrow y = 0$ - sol. sing.

$$y = c\left(x + \sqrt{x^2+1}\right),\ c \in \mathbb{R}$$

- sol. gen.

Ex. 2) $\begin{cases} y' + 2xy = x \\ y(0) = -\frac{1}{2} \end{cases}$

$$y' = x - 2xy = x(1-2y)$$

$$y' = \underbrace{f(x)}_{x} \cdot \underbrace{g(y)}_{1-2y}$$

I $g(y) \neq 0$

$$\frac{dy}{dx} = x(1-2y) \quad \Big/ \cdot dx \ : (1-2y) \neq 0$$

$$\frac{dy}{1-2y} = x\,dx \quad / \int$$

$$\int \frac{1}{1-2y}\,dy = \int x\,dx$$

$$-\frac{1}{2} \cdot \ln|2y-1| = \frac{x^2}{2} + c,\ c \in \mathbb{R}$$

$$\ln|2y-1| = -x^2 + c_1,\ c_1 \in \mathbb{R}$$

$$|2y-1| = e^{-x^2} \cdot c_2 \,, \quad c_2 \in \mathbb{R}$$

$$2y-1 = \pm c_2 \cdot e^{-x^2}$$

$$2y-1 = c_3 \cdot e^{-x^2} \,, \quad c_3 \in \mathbb{R}$$

$$y = \frac{c_3 \cdot e^{-x^2}}{2} \,, \quad c_3 \in \mathbb{R}$$

$$y(0) = -\frac{1}{2} \Rightarrow \frac{c_3 \cdot e^0}{2} = -\frac{1}{2}$$

$$c_3 = -1 \Rightarrow$$

$$\Rightarrow y(x) = -\frac{1}{2} \cdot e^{-x^2}$$

Ex. 4) $\begin{cases} y'' - 2y' + 5y = 5x^2 - 4x + 2 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

$$r^2 - 2r + 5 = 0$$

$$r_{1,2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm i\sqrt{16}}{2} = \frac{2 \pm 4i}{2} =$$

$$= \begin{cases} r_1 = 1 + 2i \Rightarrow \alpha = 1 \quad \beta = 2 \\ r_2 = 1 - 2i \end{cases}$$

$$\Rightarrow y_1 = e^x \cos 2x$$

$$y_2 = e^x \sin 2x$$

$$y(x) = c_1 e^{x} \cos 2x + c_2 e^{x} \sin 2x,$$

$$c_1, c_2 \in \mathbb{R}$$

$$f(x) = 5x^2 - 4x + 2 \Rightarrow P_2(x)$$

$$y_p = ax^2 + bx + c$$

$$y_p' = 2ax + b$$

$$y_p'' = 2a$$

$$2a - 2(2ax + b) + 5(ax^2 + bx + c) =$$
$$= 5x^2 - 4x + 2$$

$$2a - 4ax - 2b + 5ax^2 + 5bx + 5c =$$
$$= 5x^2 - 4x + 2$$

$$5ax^2 + (5b - 4a)x + 2a - 2b + 5c =$$
$$= 5x^2 - 4x + 2$$

$$\begin{cases} 5a = 5 & \Rightarrow a = 1 \\ 5b - 4a = -4 & \Rightarrow b = 0 \\ 2a - 2b + 5c = 2 & \Rightarrow c = 0 \end{cases}$$

$$\Rightarrow y_p = x^2$$

$$y = y_p + y_0 = c_1 e^x \cos 2x + c_2 e^x \sin 2x +$$

$$+ x^2 , \quad c_1 , c_2 \in \mathbb{R}$$

$$y(0) = 1 \Rightarrow \cancel{c_1 + c_2 = 1} \quad c_1 = 1$$

$$y' = -2 c_1 e^x \sin 2x + c_1 e^x \cos 2x +$$

$$+ 2 c_2 e^x \cos 2x + c_2 e^x \sin 2x + 2x$$

$$y'(0) = 1 \Rightarrow 0 + c_1 + 2 c_2 + 0 + 0 = 1$$

$$1 + 2 c_2 = 1 \quad c_2 = 0$$

$$\Rightarrow y = e^x \cos 2x + x^2 ,$$

Ex. 5) $\begin{cases} y_1' = 2 y_1 + y_2 \\ y_2' = y_1 + 2 y_2 \end{cases}$

$$y_2 = y_1' - 2 y_1$$

$$y_2' = y_1'' - 2 y_1'$$

$$y_1'' - 2 y_1' = y_1 + 2 y_1' - 4 y_1$$

$$y_1'' - 4 y_1' + 3 y_1 = 0$$

$$r^2 - 4 r + 3 = 0$$

$$(r - 1)(r - 3) = 0 \Rightarrow r_1 = 1$$

$$r_2 = 3$$

$y_1 = c_1 e^{x} + c_2 e^{3x}, \quad c_1, c_2 \in \mathbb{R}$

$y_1' = c_1 e^{x} + 3 c_2 e^{3x}$

$y_2 = y_1' - 2 y_1 = c_1 e^{x} + 3 c_2 e^{3x} -$

$- 2 c_1 e^{x} - 2 c_2 e^{3x} =$

$= - c_1 e^{x} + c_2 e^{3x}$

Ex. 3) $y'' = 1 - (y')^2$