Lab 3

Wednesday, March 15, 2023 11:49 AM

# Nr. de condiționare

$$x, x^* \longrightarrow \boxed{f} \longrightarrow y, y^*$$

Er. abs.: $|x - x^*| = |\Delta x|$

$|y - y^*| = |\Delta y|$

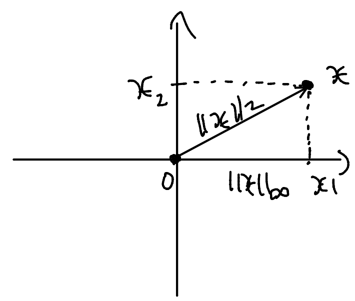
Er. rel.: $|\delta x| = \dfrac{|\Delta x|}{|x|}$

$|\delta y| = \dfrac{|\Delta y|}{|y|}$

$$\frac{|\delta y|}{|\delta x|} \underset{\simeq}{\leq} \operatorname{cond} f(x)$$

$f : \mathbb{R}^m \to \mathbb{R}^n$

$x = [x_1 \ \dots \ x_m]^T$




Ex. norme (p-norme)

- Euclidiană: $\|x\|_2 = \sqrt{x_1^2 + \dots + x_m^2}$
- Cebîșev/supremum:
  $\|x\|_\infty = \max\{|x_1|, \dots, |x_m|\}$
- Taxi/Manhattan:
  $\|x\|_1 = |x_1| + \dots + |x_m|$

$A \in \mathcal{M}_{m,n}(\mathbb{R})$, $\quad \|A\|_p = \max\limits_{\substack{x \in \mathbb{R}^m \\ x \neq 0_m}} \dfrac{\|A \cdot x\|_p}{\|x\|_p} \to$ normă matriceală

2 def. ale nr. cond. pt. $f = (f_1, \dots, f_n)$

1) $\Gamma(x) = \left( \dfrac{x_j \cdot \frac{\partial f_i}{\partial x_j}(x)}{f_i(x)} \right)_{\substack{i = \overline{1,n} \\ j = \overline{1,m}}}$ $\qquad \operatorname{cond}_p f(x) = \|\Gamma(x)\|_p$

2) $\dfrac{\partial f}{\partial x}(x) = \left( \dfrac{\partial f_i}{\partial x_j}(x) \right)_{\substack{i = \overline{1,n} \\ j = \overline{1,m}}}$ $\qquad \operatorname{cond}_p f(x) = \dfrac{\|x\|_p \cdot \left\| \frac{\partial f}{\partial x}(x) \right\|_p}{\|f(x)\|_p}$.

Ex.:
- $f : \mathbb{R} \to \mathbb{R}$, $x \in \mathbb{R}^*$, $f(x) \neq 0 \Rightarrow \operatorname{cond} f(x) = \dfrac{|x| \cdot |f'(x)|}{|f(x)|}$
- $\left.\begin{array}{l} A \cdot y = b \ (\text{sist.}) \\ A \in \mathcal{M}_{n,n}(\mathbb{R}), \det A \neq 0 \end{array}\right\} \Rightarrow y = A^{-1} \cdot b$

$y = f(b) = A^{-1} \cdot b$

$A \in \mathcal{M}_{n,n}(\mathbb{R}), \det A \neq 0$

$$b \longrightarrow \boxed{f} \longrightarrow y \qquad y = f(b) = A^{-1} \cdot b$$

$$\mathrm{cond}_p f(b) \underset{\text{Def. 2)}}{=} \frac{\|b\|_p \cdot \|A^{-1}\|_p}{\|A^{-1} b\|_p}$$

$$\mathrm{cond}_p A := \max_{\substack{b \in \mathbb{R}^n \\ b \neq 0_n}} \mathrm{cond}_p f(b) = \|A\|_p \cdot \|A^{-1}\|_p .$$

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>> cond(A,p)
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A Vandermonde matrix has the form:

   c(1)^(n-1)  ... c(1)^2  c(1)   1
   c(2)^(n-1)  ... c(2)^2  c(2)   1
       .       .     .       .    .
       .         .   .       .    .
       .           . .       .    .
   c(n)^(n-1)  ... c(n)^2  c(n)   1
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**Temă:** ec. alg.: $p(x) = 0$, unde $p(x) = x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$ are o răd. nenulă și simplă $\xi(a)$

$$(\xi(a) \neq 0,\ p(\xi(a)) = 0,\ p'(\xi(a)) \neq 0)$$

ex.: $p(x) = x^3 - x^2 = x^2(x-1) \Rightarrow \xi = 1$ răd. simplă

$$(p(1) = 0;\ p'(x) = 3x^2 - 2x;\ p'(1) = 1)$$

Completați condpol.m a.î. să returneze

$$\mathrm{cond}_1 \xi(a) \underset{\text{Def. 2)}}{=} \frac{\sum_{j=1}^{n} |a_j \cdot \xi(a)^{n-j}|}{|\xi(a) \cdot p'(\xi(a))|}$$

$a = [a_1 \dots a_n]$

$$a \longrightarrow \boxed{\xi} \longrightarrow \xi(a)$$