Problema 12.4

Fie triunghiul ABC cu varfurile $A(1,\ 1)\quad B(4,\ 1)\quad C(2,\ 3)$.
Determinati imaginea triunghiului ABC prin reflexia relativ la dreapta $2x\ +\ 3y\ -\ 5\ =\ 0$.

Formula pentru matricea omogena a transformarii in forma explicita care trece prin punctul $Q(q_1,\ q_2)$ si are versorul director w:

$$Mirror(Q,\ w)\ =\ \begin{bmatrix} 1-2w_2^2 & 2w_1w_2 & 2\left(q_1w_2^2-q_2w_1w_2\right) \\ 2w_1w_2 & 1-2w_1^2 & 2\left(-q_1w_1w_2+q_2w_1^2\right) \\ 0 & 0 & 1 \end{bmatrix}.$$

Gasim pentru dreapta noastra vectorul director $v(-3,\ 2)$. Deci, versorul director w va fi:

$$w\ =\left(\frac{-3}{|\vec{v}|},\ \frac{2}{|\vec{v}|}\right)$$

$$w\ =\left(\frac{-3}{\sqrt{13}},\ \frac{2}{\sqrt{13}}\right)$$

Si punctul $Q(1,\ 1)$ aflat pe dreapta.

Calculam pe rand elementele matricii:

$$1-2w_2^2\ =\ 1\ -\ 2\,\frac{4}{13}\ =\ \frac{5}{13}$$

$$2w_1w_2\ =\ 2\ \frac{-3}{\sqrt{13}}\ \frac{2}{\sqrt{13}}\ =\ 2\ \frac{-6}{13}\ =\ \frac{-12}{13}$$

$$2\left(q_1w_2^2-q_2w_1w_2\right)\ =\ 2\left(\frac{4}{13}\ +\ \frac{6}{13}\right)\ =\ 2\ \frac{10}{13}\ =\ \frac{20}{13}$$

$$1-2w_1^2\ =\ 1\ -\ 2\,\frac{9}{13}\ =\ \frac{-5}{13}$$

$$2\left(-q_1 w_1 w_2 + q_2 w_1^2\right) = 2\left(\frac{6}{13} + \frac{9}{13}\right) = 2\,\frac{15}{13} = \frac{30}{13}$$

Avem deci:

$$Mirror(Q,\ w) = \begin{bmatrix} \frac{5}{13} & \frac{-12}{13} & \frac{20}{13} \\ \frac{-12}{13} & \frac{-5}{13} & \frac{30}{13} \\ 0 & 0 & 1 \end{bmatrix}$$

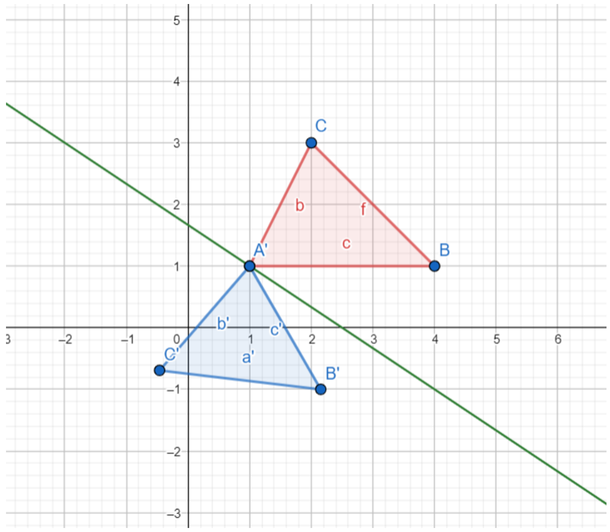
Asadar, imaginea triunghiului $A'B'C'$ va fi data de:

$$[A'\ B'\ C'] = Mirror(Q,\ w) \cdot [\ A\ B\ C\ ]$$

$$[A'\ B'\ C'] = \begin{bmatrix} \frac{5}{13} & \frac{-12}{13} & \frac{20}{13} \\ \frac{-12}{13} & \frac{-5}{13} & \frac{30}{13} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[A'\ B'\ C'] = \begin{bmatrix} 1 & \frac{28}{13} & -\frac{6}{13} \\ 1 & -\frac{23}{13} & -\frac{9}{13} \\ 1 & 1 & 1 \end{bmatrix}$$

Rezultand coordonatele: $A'(1,\ 1),\ B\left(\frac{28}{13},\ -\frac{23}{13}\right),\ C'\left(-\frac{6}{13},\ -\frac{9}{13}\right)$.



C
b
f
c
A'
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