

Temă

6.7 Determinați simetricul punctului $Q(4, -5, 4)$ relativ la planul care trece prin dreptele:

$$d_1 \begin{cases} x + y + z - 3 = 0 \\ x - y + z - 1 = 0 \end{cases} \text{ și } d_2 \begin{cases} x + z = 0 \\ y = 0 \end{cases}$$

$$\vec{d}_{11} = (1, 1, 1)$$

$$\vec{d}_{21} = (1, 0, 1)$$

$$\vec{d}_{12} = (1, -1, 1)$$

$$\vec{d}_{22} = (0, 1, 0)$$

$$\vec{d}_{11} \times \vec{d}_{12} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i} + \vec{j} - \vec{k} - \vec{k} + \vec{i} - \vec{j} = 2\vec{i} - 2\vec{k}$$

$$\Rightarrow \vec{d}_1 = (2, 0, -2)$$

$$\vec{d}_{21} \times \vec{d}_{22} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 + 0 + \vec{k} - 0 - \vec{i} - 0 = -\vec{i} + \vec{k}$$

$$\vec{d}_2 = (-1, 0, 1)$$

$$\begin{cases} x + y + z = 3 \\ x - y + z = 1 \end{cases} \Rightarrow 2y = 2 \Rightarrow y = 1$$

$$\Rightarrow x + z = 2$$

$$\text{Fie } x = z = 1 \Rightarrow A(1, 1, 1) \in d_1$$

$$\text{Fie } B(1, 0, -1) \in d_2$$

$$\vec{AB} = (0, -1, -2)$$

$$\Pi: \begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 0 & -2 \\ 0 & -1 & -2 \end{vmatrix} = -2z + 2 - 2x + 2 + y - 1 = 0$$

$$\Pi: -2x + y - 2z = 0$$

$$\Pi: x - 2y + z = 0$$

d Fie P simetriul lui Q fata de Π .

$$O \in \Pi \text{ a } PO = OQ$$

$\vec{n}(1, -2, 1)$ - vectorul director al planului

$$\begin{cases} x = 1 + t \\ y = -5 - 2t \\ z = 1 + t \end{cases} \Rightarrow 1 + t - 2(-5 - 2t) + 1 + t = 0$$

$$\Rightarrow 8 + 2x + 10 + 4x = 0$$

$$\Rightarrow 18 + 6x = 0 \Rightarrow x = -3$$

$$\Rightarrow \begin{cases} x = 1 \\ y = 1 \\ z = 1 \end{cases} \Rightarrow O(1, 1, 1)$$

$$\Rightarrow x_0 = \frac{x_p + x_q}{2}$$

$$1 = \frac{4 + x_p}{2} \Rightarrow x_p = -2$$

$$1 = \frac{y_p + y_q}{2}$$

$$1 = \frac{-5 + y_p}{2} \Rightarrow y_p = +7$$

$$1 = \frac{z_p + z_q}{2}$$

$$1 = \frac{4 + z_p}{2} \Rightarrow z_p = -2$$

$$\Rightarrow P(-2, 7, -2)$$