1. For each of the equations in Table 8.1 of Chapter 8 of the lecture notes, discuss the geometric locus of points satisfying them.

2. For each of the following matrices A, write down a quadratic equation with associated matrix A and find the matrix $M \in SO(2)$ which diagonalizes A.

a) $\begin{bmatrix} 6 & 2 \\ 2 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 5 & -13 \\ -15 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 7 & -2 \\ -2 & 5/3 \end{bmatrix}$

3. Check the calculations in examples 8.3.2, 8.3.3 and 8.3.4 of Chapter 8 of the lecture notes.

4. For each of the following equations write down the associated matrix and bring the equation in canonical form.

a) $-x^2 + xy - y^2 = 0$,

b) $6xy + x - y = 0$.

5. In each of the following cases, decide the type of the quadratic curve based on the parameter $a \in \mathbb{R}$.

a) $x^2 - 4xy + y^2 = a$,

b) $x^2 + 4xy + y^2 = a$.

6. Consider the rotation R_{90° of $\mathbb{E}^2$ around the origin and the translation $T_{\mathbf{v}}$ of $\mathbb{E}^2$ with vector $\mathbf{v}(1,0)$.

a) Give the algebraic form of the isometries R_{90°, $T_{\mathbf{v}}$ and $T_{\mathbf{v}} \circ R_{90^\circ}$.

b) Determine the equations of the hyperbola $\mathcal{H} : \frac{x^2}{4} - \frac{y^2}{9} - 1 = 0$ and the parabola $\mathcal{P} : y^2 - 8x = 0$ after transforming them with R_{90° and with $T_{\mathbf{v}} \circ R_{90^\circ}$ respectively.

7. Find the canonical equation for each of the following cases

a) $5x^2 + 4xy + 8y^2 - 32x - 56y + 80 = 0$,

b) $8y^2 + 6xy - 12x - 26y + 11 = 0$,

c) $x^2 - 4xy + y^2 - 6x + 2y + 1 = 0$.

8. For each of the conics in the previous exercise, indicate the affine change of coordinates which brings the equation in canonical form.

9. Discuss the type of the curve

$$x^2 + \lambda xy + y^2 - 6x - 16 = 0$$

in terms of $\lambda \in \mathbb{R}$.

10. Using the classification of quadrics, decide what surfaces are described by the following equations.

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$,

b) $xy + yz + zx = 1$,

c) $x^2 + xy + yz + zx = 1$,

d) $xy + yz + zx = 0$.