Changing reference frames.

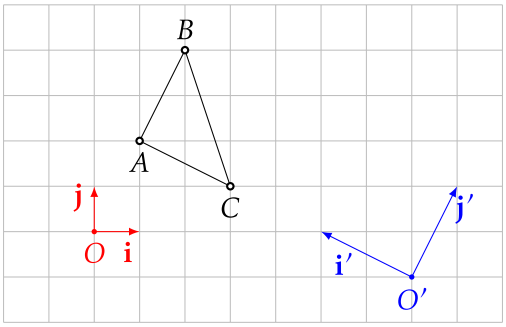
1. We consider two coordinate systems $\mathcal{K} = (O, \mathbf{i}, \mathbf{j})$ and $\mathcal{K}' = (O', \mathbf{i}', \mathbf{j}')$ where

$$[O']_{\mathcal{K}} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}, \quad [\mathbf{i}']_{\mathcal{K}} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad [\mathbf{j}']_{\mathcal{K}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

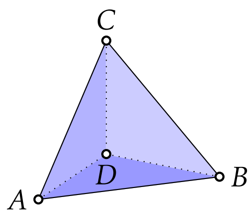
Determine the base change matrix from $\mathcal{K}$ to $\mathcal{K}'$ and the coordinates of the points

$$[A]_{\mathcal{K}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad [B]_{\mathcal{K}} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad [C]_{\mathcal{K}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

in the system $\mathcal{K}'$. Further, determine the base change matrix from $\mathcal{K}'$ to $\mathcal{K}$ and use it with the previously obtained coordinates to calculate $[A]_{\mathcal{K}}$, $[B]_{\mathcal{K}}$ and $[C]_{\mathcal{K}}$.



2. With the assumptions in the previous exercise, give parametric equations and Cartesian equations for the lines AB, AC, BC both in the coordinate system $\mathcal{K}$ and in the coordinate system $\mathcal{K}'$.



3. Consider the tetrahedron $ABCD$ and the coordinate systems

$$\mathcal{K}_A = (A, \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}), \quad \mathcal{K}'_A = (A, \overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AC}), \quad \mathcal{K}_B = (B, \overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{BD}).$$

Determine

a) the coordinates of the vertices of the tetrahedron in the three coordinate systems,

b) the base change matrix from $\mathcal{K}_A$ to $\mathcal{K}'_A$,

c) the base change matrix from $\mathcal{K}_B$ to $\mathcal{K}_A$.

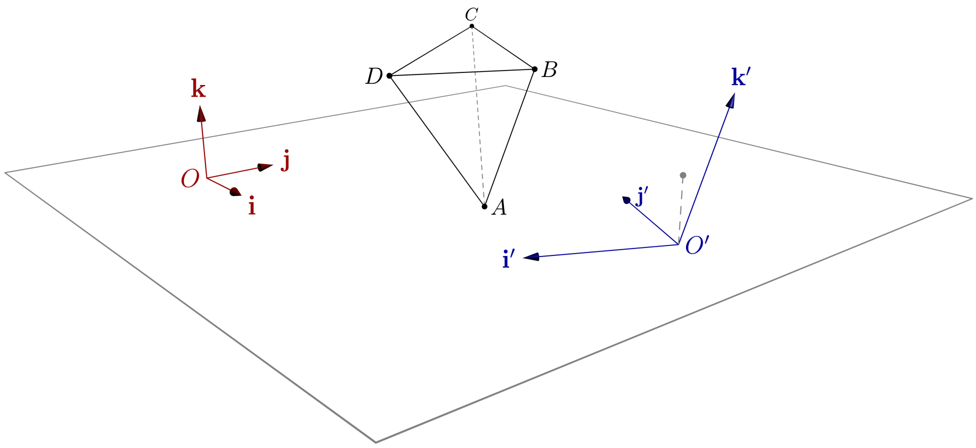
4. We consider the coordinate systems $\mathcal{K} = (O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and $\mathcal{K}' = (O', \mathbf{i}', \mathbf{j}', \mathbf{k}')$ where

$$[O']_{\mathcal{K}} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}, \quad [\mathbf{i}']_{\mathcal{K}} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}, \quad [\mathbf{j}']_{\mathcal{K}} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \quad [\mathbf{k}']_{\mathcal{K}} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

Determine the base change matrix from $\mathcal{K}$ to $\mathcal{K}'$ and the coordinates of the points

$$[A]_{\mathcal{K}} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}, \quad [B]_{\mathcal{K}} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \quad [C]_{\mathcal{K}} = \begin{bmatrix} -3 \\ 7 \\ 1 \end{bmatrix}, \quad [D]_{\mathcal{K}} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}.$$

in the coordinate system $\mathcal{K}'$. Further, determine the base change matrix from $\mathcal{K}'$ to $\mathcal{K}$ and use it with the previously determined coordinates to calculate $[A]_{\mathcal{K}}$, $[B]_{\mathcal{K}}$, $[C]_{\mathcal{K}}$ and $[D]_{\mathcal{K}}$.



5. With the assumptions in the previous exercise, give parametric equations and Cartesian equations for the line AB and the plane ACD both in the coordinate system $\mathcal{K}$ and in the coordinate system $\mathcal{K}'$.

Projections and reflections on/in hyperplanes.

6. Consider $\mathbf{v}(2,1,1) \in \mathbb{V}^3$ and $Q(2,2,2) \in \mathbb{E}^3$.

a) Give the matrix form for the parallel projection on the plane $\pi : z = 0$ along the line $Q + \langle \mathbf{v} \rangle$.

b) Give the matrix form for the parallel reflection in the plane $\pi : z = 0$ along the line $Q + \langle \mathbf{v} \rangle$.

7. Determine the orthogonal projection of the point $A(2,11,-5)$ on the plane $x + 4y - 3z + 7 = 0$ by determining the matrix form of the projection. (Compare your result with the previous seminar.)

8. Determine the orthogonal reflection of the point $P(6,-5,5)$ in the plane $2x-3y+z-4=0$ by determining the matrix form of the reflection. (Compare your result with the previous seminar.)

9. Determine the orthogonal projection of the line $\ell: 2x-y-1=0 \cap x+y-z+1=0$ on the plane $\pi: x+2y-z=0$ by determining the matrix form of the projection. (Compare your result with the previous seminar.)

10. Give Cartesian equations for the line passing through the point $M(1,0,7)$, parallel to the plane $\pi: 3x-y+2z-15=0$ and intersecting the line

$$\ell: \frac{x-1}{4}=\frac{y-3}{2}=\frac{z}{1}.$$

11. In $\mathbb{E}^3$, show that the orthogonal reflection $\mathrm{Ref}_\pi^\perp(x)$ in the plane $\pi: \langle n,x\rangle = p$ is given by

$$\mathrm{Ref}_\pi(x)=Ax+b$$

where $A=\left(I-2\frac{nn^t}{||n||^2}\right)$ and $b=\frac{2p}{||n||^2}n$.

12. Give the matrix form for the orthogonal reflections in the planes

$$\pi_1: 3x-4z=-1 \quad \text{and} \quad \pi_2: 10x-2y+3z=4 \quad \text{respectively.}$$

13. Write down the vector forms and matrix forms for parallel projections and reflections in $\mathbb{E}^3$.

14. In $\mathbb{E}^2$, for the lines/hyperplanes

$$\pi: ax+by+c=0, \quad \ell: \frac{x-x_0}{v_1}=\frac{y-y_0}{v_2}$$

with $\pi \nparallel \ell$, deduce the matrix forms of $\mathrm{Pr}_{\pi,\ell}$ and $\mathrm{Ref}_{\pi,\ell}$.

15. Let H be a hyperplane and let $\mathbf{v}$ be a vector. Use the deduced compact matrix forms to show that

a) $\mathrm{Pr}_{H,\mathbf{v}} \circ \mathrm{Pr}_{H,\mathbf{v}} = \mathrm{Pr}_{H,\mathbf{v}}$ and

b) $\mathrm{Ref}_{H,\mathbf{v}} \circ \mathrm{Ref}_{H,\mathbf{v}} = \mathrm{Id}$.