1. Let $A_0,\dots,A_n$ be the vertices of a polygon. Determine $\overrightarrow{A_0A_1}+\overrightarrow{A_1A_2}+\cdots+\overrightarrow{A_{n-1}A_n}+\overrightarrow{A_nA_0}$.

2. In each of the following cases, decide if the indicated vectors $\mathbf{u},\mathbf{v},\mathbf{w}$ can be represented with the vertices of a triangle:

a) $\mathbf{u}(7,3)$, $\mathbf{v}(-2,-8)$, $\mathbf{w}(-5,5)$.

b) $\mathbf{u}(7,3)$, $\mathbf{v}(2,8)$, $\mathbf{w}(-5,5)$.

c) $\|\mathbf{u}\|=7$, $\|\mathbf{v}\|=3$, $\|\mathbf{w}\|=11$.

d) $\mathbf{u}(1,0,1)$, $\mathbf{v}(0,1,0)$, $\mathbf{w}(2,2,2)$.

3. Let $ABCDEF$ be a regular hexagon centered at O.

a) Express the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$ in terms of $\overrightarrow{OE}$ and $\overrightarrow{OF}$.

b) Show that $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=3\overrightarrow{AD}$.

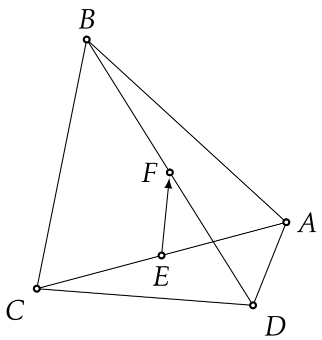
4. Let $ABCD$ be a quadrilateral. Let M, N, P, Q be the midpoints of $[AB]$, $[BC]$, $[CD]$ and $[DA]$ respectively. Show that

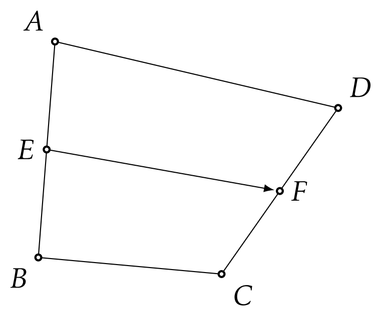
$$\overrightarrow{MN}+\overrightarrow{PQ}=0.$$

Deduce that the midpoints of the sides of an arbitrary quadrilateral form a parallelogram.

5. Let $ABCD$ be a quadrilateral. Let E be the midpoint of $[AC]$ and let F be the midpoint of $[BD]$. Show that

$$\overrightarrow{EF}=\frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{CD}\right)=\frac{1}{2}\left(\overrightarrow{AD}+\overrightarrow{CB}\right).$$





6. Let $ABCD$ be a quadrilateral. Let E be the midpoint of $[AB]$ and let F be the midpoint of $[CD]$. Show that

$$\overrightarrow{EF}=\frac{1}{2}\left(\overrightarrow{AD}+\overrightarrow{BC}\right).$$

Deduce that the length of the midsegment in a trapezoid is the arithmetic mean of the lengths of the bases.

7. Let $k = \frac{|CA|}{|CB|}$ be the ratio in which the point $C \in [AB]$ divides the segment $[AB]$. Show that for any point O we have

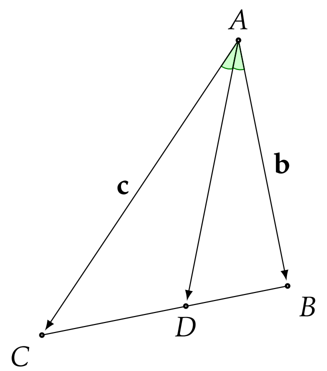
$$\overrightarrow{OC} = \frac{1}{1+k}\left(\overrightarrow{OA} + k\,\overrightarrow{OB}\right).$$

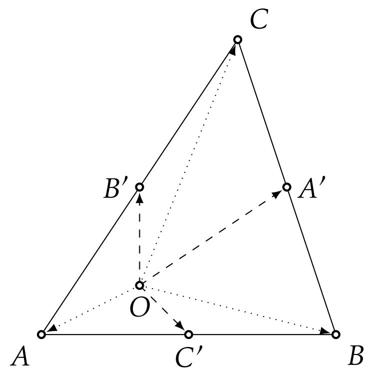
Deduce that C has coordinates

$$\left(\frac{a_1 + kb_1}{1+k}, \frac{a_2 + kb_2}{1+k}, \ldots, \frac{a_n + kb_n}{1+k}\right)$$

where $A = A(a_1, \ldots, a_n)$ and $B = B(b_1, \ldots, b_n)$.

8. Let ABC be a triangle and let $D \in [BC]$ be such that AD is an angle bisector. Express $\overrightarrow{AD}$ in terms of $\mathbf{b} = \overrightarrow{AB}$ und $\mathbf{c} = \overrightarrow{AC}$.

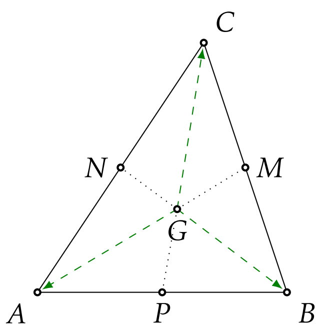




9. Let A', B' and C' be midpoints of the sides of a triangle ABC. Show that for any point O we have

$$\overrightarrow{OA'} + \overrightarrow{OB'} + \overrightarrow{OC'} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}.$$

10. Show that the medians in a triangle intersect in one point and deduce the ratio in which the common intersection point divides the medians.



11. In each of the following cases, decide if the given points are collinear:

a) $P(3,-5)$, $Q(-1,2)$, $R(-5,9)$.

b) $A(11,2)$, $B(1,-3)$, $C(31,13)$.

c) $P(1,0,-1)$, $Q(0,-1,2)$, $R(-1,-2,5)$.

d) $A(-1,-1,-4)$, $B(1,1,0)$, $C(2,2,2)$.

12. Let $ABCD$ be a tetrahedron. Determine the sums

a) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$,

b) $\overrightarrow{AD}+\overrightarrow{BC}+\overrightarrow{DB}$,

c) $\overrightarrow{AB}+\overrightarrow{CD}+\overrightarrow{BC}+\overrightarrow{DA}$.

13. Let $ABCD$ be a tetrahedron. Show that

$$\overrightarrow{AD}+\overrightarrow{BC}=\overrightarrow{BD}+\overrightarrow{AC}.$$

14. Let $SABCD$ be a pyramid with apex S and base the parallelogram $ABCD$. Show that

$$\overrightarrow{SA}+\overrightarrow{SB}+\overrightarrow{SC}+\overrightarrow{SD}=4\overrightarrow{SO}$$

where O is the center of the parallelogram.

15. Give the coordinates of the vertices of the parallelepiped whose faces lie in the coordinate planes and in the planes $x=1$, $y=3$ and $z=-2$.

16. In $\mathbb{E}^3$ consider the parallelograms $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$. Show that the midpoints of the segments $[A_1B_1]$, $[A_2B_2]$, $[A_3B_3]$ and $[A_4B_4]$ are the vertices of a parallelogram.