1. Let $(\mathbf{i},\mathbf{j},\mathbf{k})$ be a right oriented orthonormal basis of $\mathbb{V}^3$. Consider the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 7\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$. Determine $\mathbf{a}\times\mathbf{b}$ in terms of the given basis vectors.

2. With respect to a right oriented orthonormal basis of $\mathbb{V}^3$ consider the vectors $\mathbf{a}(3,-1,-2)$ and $\mathbf{b}(1,2,-1)$. Calculate

$$\mathbf{a}\times\mathbf{b},\quad (2\mathbf{a}+\mathbf{b})\times\mathbf{b},\quad (2\mathbf{a}+\mathbf{b})\times(2\mathbf{a}-\mathbf{b}).$$

3. Determine the distances between opposite sides of a parallelogram spanned by the vectors $\overrightarrow{AB}(6,0,1)$ and $\overrightarrow{AC}(1.5,2,1)$ if the coordinates of the vectors are given with respect to a right oriented orthonormal basis.

4. Consider the vectors $\mathbf{a}(2,3,-1)$ and $\mathbf{b}(1,-1,3)$ with respect to an orthonormal basis.

a) Determine the vector subspace $\langle\mathbf{a},\mathbf{b}\rangle^{\perp}$.

b) Determine the vector $\mathbf{p}$ which is orthogonal to $\mathbf{a}$ and $\mathbf{b}$ and for which $\mathbf{p}\cdot(2\mathbf{i}-3\mathbf{j}+4\mathbf{k}) = 51$.

5. Consider the points $A(1,2,0)$, $B(3,0,-3)$ and $C(5,2,6)$ with respect to an orthonormal coordinate system.

a) Determine the area of the triangle ABC.

b) Determine the distance from C to AB.

6. Let $ABCD$ be a quadrilateral in $\mathbb{E}^3$ and let E, F be the midpoints of $[AB]$ and $[CD]$ respectively. Denote by K, L, M and N the midpoints of the segments $[AF]$, $[CE]$, $[BF]$ and $[DE]$ respectively. Prove that $KLMN$ is a parallelogram.

7. Let ABC be a triangle and let $\mathbf{u} = \overrightarrow{AB}$, $\mathbf{v} = \overrightarrow{BC}$, $\mathbf{w} = \overrightarrow{CA}$. Show that

$$\mathbf{u}\times\mathbf{v} = \mathbf{v}\times\mathbf{w} = \mathbf{w}\times\mathbf{u}.$$

and deduce the law of sines in a triangle.

8. With respect to a right oriented orthonormal coordinate system consider the vectors $\mathbf{a}(2,-3,1)$, $\mathbf{b}(-3,1,2)$ and $\mathbf{c}(1,2,3)$. Calculate $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ and $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$.

9. Fix $\mathbf{v}\in\mathbb{V}^3$ and let $\psi:\mathbb{V}^3\to\mathbb{V}^3$ be the map $\phi(\mathbf{w}) = \mathbf{v}\times\mathbf{w}$. Is the map linear? Explain why. Give the matrix of ϕ relative to a right oriented orthonormal basis. What changes if we define ϕ by $\phi(\mathbf{w}) = \mathbf{w}\times\mathbf{v}$?

10. Let $(\mathbf{i},\mathbf{j},\mathbf{k})$ be a right oriented orthonormal basis. Determine the matrices of the linear maps $\phi,\psi:\mathbb{V}^3\to\mathbb{V}^3$ defined by $\phi(\mathbf{v}) = \mathbf{w}\times\mathbf{v}$ and $\psi(\mathbf{v}) = \mathbf{v}\times\mathbf{u}$ where

a) $\mathbf{w} = -\mathbf{i}+3\mathbf{j}+\mathbf{k}$,

b) $\mathbf{w} = \mathbf{i}+\mathbf{k}$,

c) $\mathbf{u} = 2\mathbf{i}-\mathbf{j}$,

d) $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

11. Prove the following identities:

a) the Jacobi identity,

b) the Lagrange identity,

c) the formula for the cross product of two cross products.

12. Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a right oriented orthonormal basis. Consider the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} - \mathbf{k}$ and $\mathbf{c} = \mathbf{k}$. Determine if

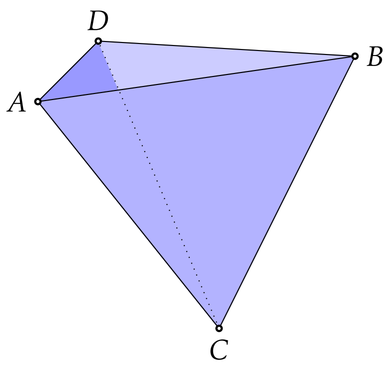
a) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ is a basis of $\mathbb{V}^3$,

b) if it is a basis, decide if it is left or right oriented.

13. The points $A(1,2,-1)$, $B(0,1,5)$, $C(-1,2,1)$ and $D(2,1,3)$ are given with respect to an orthonormal coordinate system. Are the four points coplanar?

14. Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be an orthonormal basis and consider the vectors $\mathbf{u} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{k}$. Determine the matrix of the linear map $\phi : \mathbb{V}^3 \to \mathbb{R}$ defined by $\phi(\mathbf{v}) = [\mathbf{v}, \mathbf{u}, \mathbf{w}]$.

15. Determine the volume of the tetrahedron with vertices $A(2,-1,1)$, $B(5,5,4)$, $C(3,2,-1)$ and $D(4,1,3)$ given with respect to an orthonormal system.



16. The volume of a tetrahedron $ABCD$ is 5. With respect to an orthonormal system $Oxyz$ the vertices are $A(2,1,-1)$, $B(3,0,1)$, $C(2,-1,3)$ and $D \in Oy$. Determine the coordinates of D.

17. With respect to an orthonormal system consider the vectors $\mathbf{a}(8,4,1)$, $\mathbf{b}(2,2,1)$ and $\mathbf{c}(1,1,1)$. Determine a vector $\mathbf{d}$ satisfying the following properties

a) the angles of $\mathbf{d}$ with $\mathbf{a}$ and with $\mathbf{b}$ are congruent,

b) $\mathbf{d}$ is orthogonal to $\mathbf{c}$,

c) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ and $(\mathbf{a}, \mathbf{b}, \mathbf{d})$ have the same orientation.