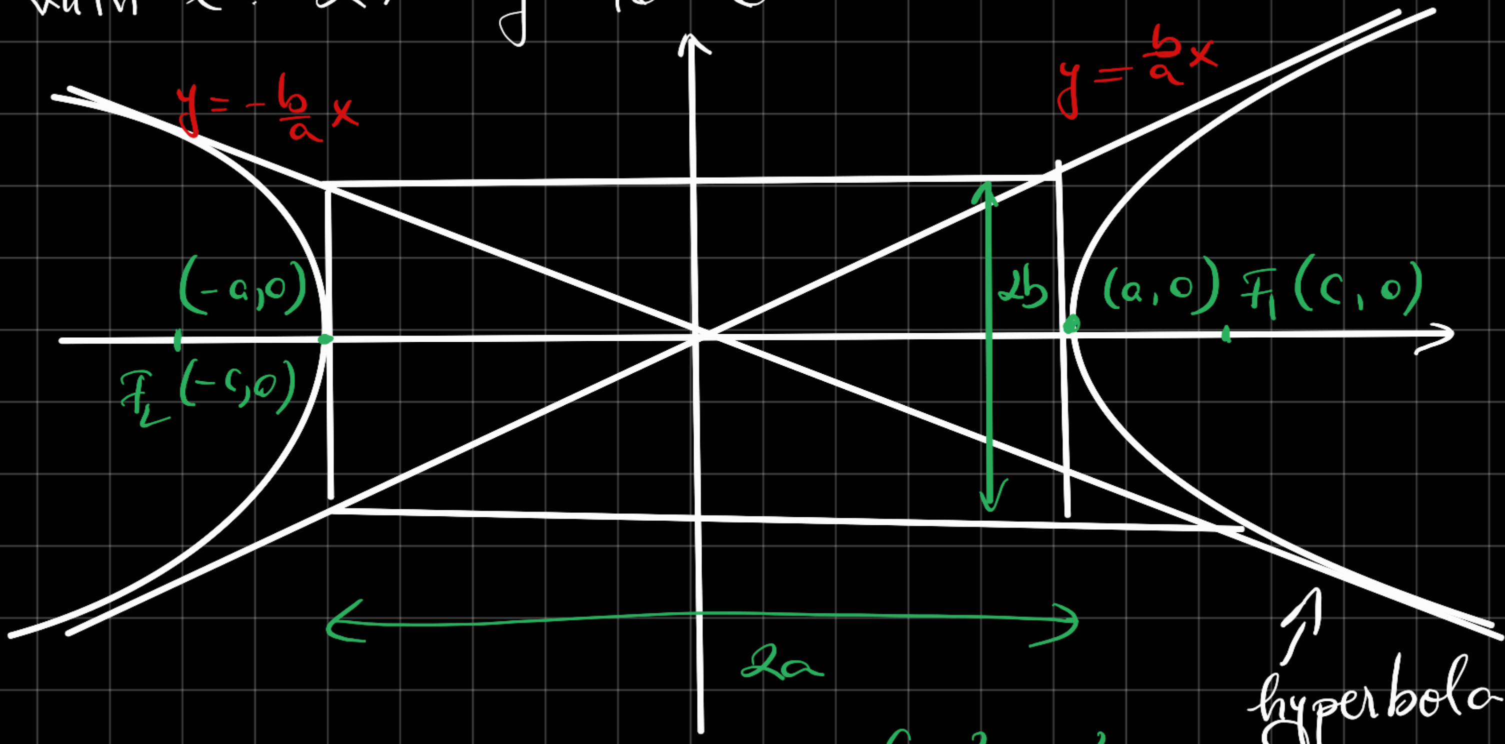
① Get the intersection of $H: \frac{x^2}{20} - \frac{y^2}{5} = 1$ with $l: 2x - y - 10 = 0$



Just solve the system $\begin{cases} \frac{x^2}{20} - \frac{y^2}{5} = 1 \\ 2x - y - 10 = 0 \end{cases}$

② Get tangent lines to $H: \frac{x^2}{16} - \frac{y^2}{8} - 1 = 0$ parallel to $l: 4x + 2y - 5 = 0$

! $T_m H: y = mx \pm \sqrt{a^2m^2 - b^2}$ equation of a tangent line $\boxed{m \in \left(-\infty, -\frac{b}{a}\right) \cup \left(\frac{b}{a}, \infty\right)}$

$\Rightarrow m \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

③ $H: x^2 - y^2 = 16$ $M \in (-1, 7)$. Get tangents to H containing M

$(x_0, y_0) \in \mathcal{H}$

$$T_{(x_0,y_0)}\mathcal{H}: \frac{x x_0}{a^2} - \frac{y y_0}{b^2} = 1$$

Replace M in there

$$\frac{x x_0}{16} - \frac{y y_0}{16} = 1$$

$$-\frac{x_0}{16} - \frac{7 y_0}{16} = 1$$

$$x_0 + 7y_0 = -16$$

$$\boxed{x_0 = -16 - 7y_0}$$

$$\begin{cases} x_0 = -16 - 7y_0 & \text{- tangent} \\ x_0^2 - y_0^2 = 16 & \text{- hyperbola} \end{cases}$$

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or

$$y = mx \pm \sqrt{16m^2 - 16}$$

$$7 = -m \pm \sqrt{16m^2 - 16}$$

$$7 + m = \sqrt{\quad} \quad \Big| (\ )^2$$

$$49 + 14m + m^2 = 16m^2 - 16$$

Check if correct and $m_{1,2} \in \left(-\infty, -\frac{b}{a}\right) \cup \left[\frac{b}{a}, \infty\right.$

4\. Find the area of Δ, det. asymptote of

$$\mathcal{H}: \frac{x^2}{4} - \frac{y^2}{9} = 1 \quad \text{and} \quad \ell: 9x + 2y - 24 = 0$$

$\Rightarrow$ Eq. of asymptote: $(a_1)\ y = -\frac{b}{a}x = -\frac{3}{2}x$

$(a_2)\ y = \frac{3}{2}x$

$$\begin{cases} 9x + 2y - 24 = 0 \\ 2y + 3x = 0 \end{cases} \Big| (-)$$

$6x - 24 = 0$

$x = \frac{24}{6} = \boxed{4.}$

$2y + 12 = 0$

$\boxed{y = -6}$

$\boxed{A_1(4, -6)}$

$$\begin{cases} 9x + 2y - 24 = 0 \\ -3x + 2y = 0 \end{cases} (-)$$

$12x - 24 = 0$

$\boxed{x = 2}$

$-6 + 2y = 0$

$y = 3$

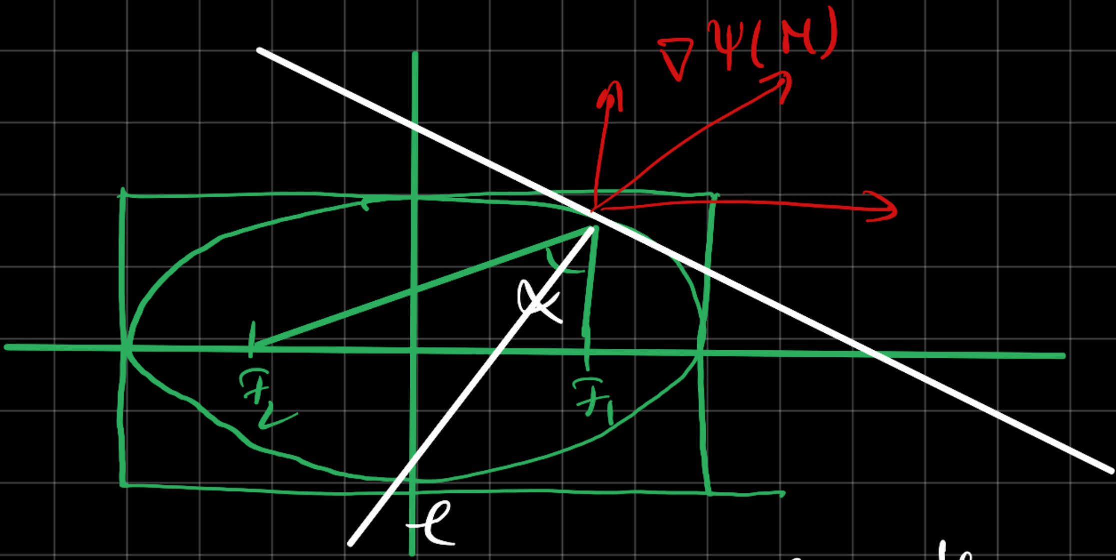
$\boxed{A_2(2, 3)}$

$O(0, 0)$

$\Delta OA_1A_2 \Rightarrow$ Area: $\frac{1}{2} \left\| \begin{vmatrix} 0 & 0 & 1 \\ 4 & -6 & 1 \\ 2 & 3 & 1 \end{vmatrix} \right\| = \boxed{12.}$

Any normal vector n to T & given the

direction of the angle bisector of $\sphericalangle F_1MF_2$



ℓ is the angle bisector of α, where $\ell \perp$ tangent

$$\mathcal{E} = \psi^{-1}(0)$$

$$\psi(M) = d(F_1, M) + d(F_2, M) - 2a$$

$$n = \nabla \psi(x, y)$$

$$\psi(M) = \sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} - 2a$$

$$\nabla\psi(x,y) = \frac{x-c}{\underbrace{\sqrt{(x-c)^2+y^2}}_{\|\overrightarrow{F_1M}\|}} + \frac{x+c}{\underbrace{\sqrt{(x+c)^2+y^2}}_{\|\overrightarrow{F_2M}\|}} + \frac{y}{\sqrt{(x-c)^2+y^2}}$$

$$+ \frac{y}{\sqrt{(x+c)^2+y^2}}$$

$$= \frac{\overrightarrow{F_1M}}{\|\overrightarrow{F_1M}\|} + \frac{\overrightarrow{F_2M}}{\|\overrightarrow{F_2M}\|}$$

$\|F_2 M\|$

$\Rightarrow$ $\nabla \Psi(M)$ gives direction of angle bisector of $\measuredangle F_1 M F_2$

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4. $\mathcal{P}: y^2 - 8x = 0$

$\ell: \ 2x + 2y - 3 = 0$

$$\boxed{y = mx + \frac{p}{2m}}$$

$\rightarrow y^2 = 2px$

$\rightarrow$ slope

$$\Rightarrow y = -x + \frac{4}{-2} \Leftrightarrow \boxed{y = -x - 2}$$