## Lab Nr. 3, Probability and Statistics

## Continuous random variables; CDF and inverse CDF; Quantiles; Approximations of the Binomial distribution

- **1.** Let X have one of the following distributions:  $X \in N(\mu, \sigma)$  (Normal),  $X \in T(n)$  (Student),  $X \in \chi^2(n)$ , or  $X \in F(m, n)$  (Fisher) (all continuous). Compute the following:
- a)  $P(X \le 0)$  and  $P(X \ge 0)$ ;
- b)  $P(-1 \le X \le 1)$  and  $P(X \le -1 \text{ or } X \ge 1)$ ;
- c) the value  $x_{\alpha}$  such that  $P(X < x_{\alpha}) = P(X \le x_{\alpha}) = \alpha$ , for  $\alpha \in (0,1)$  ( $x_{\alpha}$  is called the *quantile* of order  $\alpha$ );
- d) the value  $x_{\beta}$  such that  $P(X > x_{\beta}) = P(X \ge x_{\beta}) = \beta$ , for  $\beta \in (0,1)$   $(x_{\beta})$  is the quantile of order  $1 \beta$ ).
- 2. Approximations of the Binomial distribution
  - **Normal** approximation of the Binomial distribution: For moderate values of p (0.05  $\leq p \leq$  0.95) and large values of n ( $n \to \infty$ ),

$$\operatorname{Bino}(n,p) \approx \operatorname{Norm}\left(\mu = np, \sigma = \sqrt{np(1-p)}\right).$$

Write a Matlab code to visualize how the Binomial distribution gradually takes the shape of the Normal distribution as  $n \to \infty$ .

• **Poisson** approximation of the Binomial distribution: If  $n \ge 30$  and  $p \le 0.05$ , then

$$Bino(n, p) \approx Poisson(\lambda = np).$$

Compare graphically the two pdf's.