Circles

1. Find the equation of the circle:

a) of diameter $[A,B]$, with $A(1,2)$ and $B(-3,-1)$,

b) with center $I(2,-3)$ and radius $R=7$,

c) with center $I(-1,2)$ and passing through $A(2,6)$,

d) centered at the origin and tangent to $\ell : 3x-4y+20=0$,

e) passing through $A(3,1)$ and $B(-1,3)$ and having the center on the line $\ell : 3x-y-2=0$,

f) passing through $A(1,1)$, $B(1,-1)$ and $C(2,0)$,

g) tangent to both $\ell_1 : 2x+y-5=0$ and $\ell_2 : 2x+y+15=0$ if one tangency point is $M(3,-1)$.

2. For a circle $\mathcal{C}$ of radius R:

a) Use the parametrization $x \mapsto (x, \pm\sqrt{R^2-x^2})$ to deduce a parametrization of tangent lines to $\mathcal{C}$.

b) Use the parametrization $\theta \mapsto (R\cos(\theta), R\sin(\theta))$ to deduce a parametrization of tangent lines to $\mathcal{C}$.

c) Compare these to the equation of the tangent line $xx_0+yy_0=R^2$ where $(x_0,y_0)\in\mathcal{C}$.

Ellipses

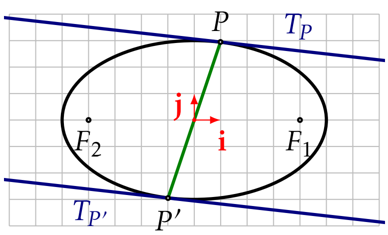
3. Determine the foci (focal points) of the Ellipse $9x^2+25y^2-225=0$

4. Determine the intersection of the line $\ell : x+2y-7=0$ and the ellipse $\mathcal{E} : x^2+3y^2-25=0$.

5. Determine the position of the line $\ell : 2x+y-10=0$ relative to the ellipse $\mathcal{E} : \frac{x^2}{9}+\frac{y^2}{4}-1=0$.

6. Determine an equation of a line which is orthogonal to $\ell : 2x-2y-13=0$ and tangent to the ellipse $\mathcal{E} : x^2+4y^2-20=0$.

7. A *diameter* of an ellipse is the line segment determined by the intersection points of the ellipse with a line passing through the center of the ellipse. Show that the tangent lines to an ellipse at the endpoints of a diameter are parallel.

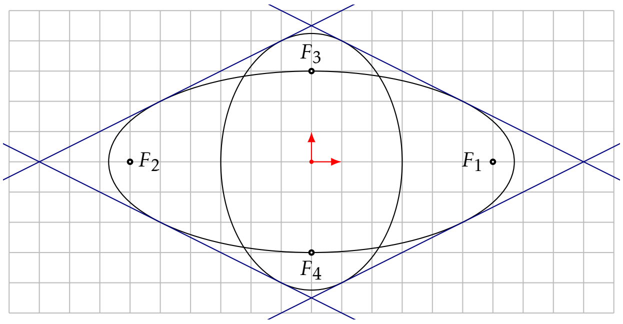


8. Consider the family of ellipses $\mathcal{E}_a : \frac{x^2}{a^2} + \frac{y^2}{16} = 1$. For what value $a \in \mathbb{R}$ is $\mathcal{E}_a$ tangent to the line $\ell : x - y + 5 = 0$?

9. Consider the family of lines $\ell_c : \sqrt{5}x - y + c = 0$. For what values $c \in \mathbb{R}$ is ℓ_c tangent to the ellipse $\mathcal{E} : x^2 + \frac{y^2}{4} = 1$?

10. Determine the common tangents to the ellipses

$$\frac{x^2}{45} + \frac{y^2}{9} = 1 \quad \text{and} \quad \frac{x^2}{9} + \frac{y^2}{18} = 1.$$



11. Consider the ellipse $\mathcal{E} : \frac{x^2}{4} + y^2 - 1 = 0$ with focal points F_1 and F_2. Determine the points M, situated on the ellipse, for which

a) the angle $\measuredangle F_1MF_2$ is right;

b) the angle $\measuredangle F_1MF_2$ is θ;

c) the angle $\measuredangle F_1MF_2$ is maximal.

12. Using a rotation of the coordinate system, find the equation of an ellipse centered at the origin, with focal points on the line $x = y$ and having the large diameter equal to 4 and the distance between the focal points equal to $2\sqrt{3}$.

13. Consider the ellipse $\mathcal{E} : x^2 + 4y^2 = 25$. Find the chords on the ellipse which have the point $A(7/2, 7/4)$ as midpoint.

14. Consider the ellipse $\mathcal{E} : \frac{x^2}{25} + \frac{y^2}{9} = 1$. Determine the geometric locus of the midpoints of the chords on the ellipse which are parallel to the line $\ell : x + 2y = 1$.

15. Using the gradient, prove the reflective properties of an ellipse.