TEMA 2

1. b - bile albastre
   r - bile rosii

a) $P(2B) = P(2B \mid 1r) P(r) + P(2B \mid 1B) P(B) =$

$= \frac{b}{b+r} \cdot \frac{b+d}{b+r+d} + \frac{r}{b+r} \cdot \frac{b}{r+b+d} =$

$= \frac{b(b+d+r)}{(b+r)(b+r+d)} = \frac{b}{b+r}$

b) $P(1B \mid 2B) = \frac{b+d}{r+b+d} \cdot \frac{b}{r+b} = \frac{b+d}{r+b+d}$

$\frac{b}{b+r}$

c) $P(B_n) = P(B_1), \forall n \geq 1$

Pas 1: $P(B_1) = P(B_2)$ (A) (am arătat la punctul a))

⋮

Pas k-1: $P(B_{k-1}) = P(B_k)$ (A)

Pas k: $P(B_k) = P(B_1) = \frac{b}{b+r}$

o(1) $P(B_1 | B_2 \cap B_3 \cap \dots \cap B_{n+1}) =$

$= \dfrac{P(B_1 \cap B_2 \cap \dots \cap B_{n+1})}{P(B_2 \cap B_3 \cap \dots \cap B_{n+1})} =$

$= \dfrac{\frac{b}{r+b} \cdot \frac{b+d}{r+b+d} \cdot \frac{b+2d}{r+b+2d} \cdot \dots \cdot \frac{b+nd}{r+b+nd}}{\frac{b}{r+b} \cdot \frac{b+d}{r+b+d} \cdot \dots \cdot \frac{b+nd}{r+b+nd} + \frac{r}{r+b} \cdot \frac{b}{r+b+d} \cdot \frac{b+d}{r+b+2d}} =$

$= \dfrac{b \cdot (b+d)(b+2d) \cdots (b+nd)}{b \cdot (b+d) \cdots (b+(n-1)d) \cdot (b+nd+r)} =$

$= \dfrac{b+nd}{b+nd+r} = 1 - \dfrac{r}{b+nd+r} \xrightarrow{n \to \infty} 1$

2. a) $P(A_1) = P(A_2) = \dfrac{n\alpha + n\beta}{2n} = \dfrac{\alpha+\beta}{2}$

b) $P(A_1 A_2) = \dfrac{n\alpha^2 + n\beta^2}{2n} = \dfrac{\alpha^2+\beta^2}{2}$

c) $P(A_2 | A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{P(A_1 A_2)}{P(A_1)} =$

$= \dfrac{\frac{\alpha^2+\beta^2}{2}}{\frac{\alpha+\beta}{2}} = \dfrac{\alpha^2+\beta^2}{\alpha+\beta}$; $P(A_2|A_1) \geq P(A_1) \Leftrightarrow (\alpha-\beta)^2 \geq 0$

d) $P = \dfrac{nB}{n\cdot\alpha + n\cdot\beta} = \dfrac{nB}{n(\alpha+\beta)} = \dfrac{B}{\alpha+\beta}$