05.04.2016.

CURS 7
AG

Cuplaje

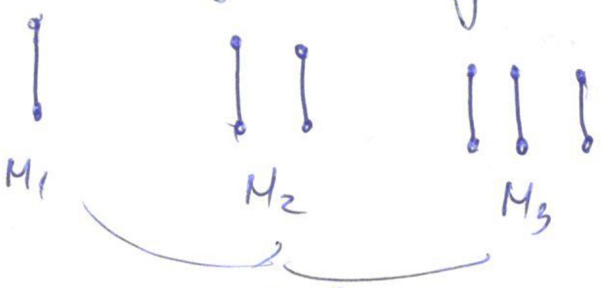
1) Def. Notații. Scop. Exemple

$G = (V, E)$ graf simplu

$M \subseteq E$

M cuplaj $\Leftrightarrow \forall e, f \in M,\ e \neq f : e \cap f = \emptyset$

ex:



M^* cuplaj de cardinal maxim în G

$$V(M) := \bigcup_{e \in M} e$$

$v \in V(G)$, M cuplaj

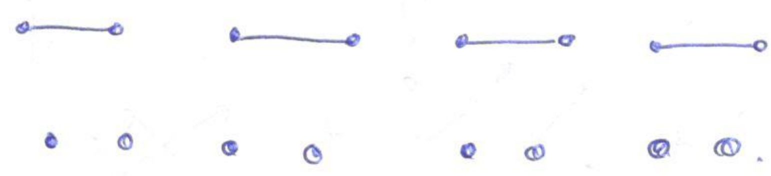
v M-saturat $\Leftrightarrow v \in V(M)$

$\Leftrightarrow \exists\, e \in M$ cu $v \in e$ $:\Leftrightarrow \forall v \in V(G)$ v este saturat

$V(G) = V(M)$

vf. M-saturate $V(M)$

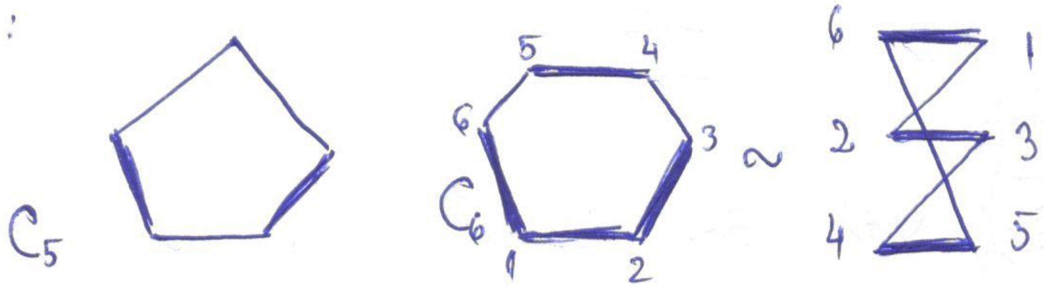
vf. M-nesaturate $\overline{V(M)}$



$\overline{V(M)} = V(G) - V(M)$

M cuplaj perfect

ex:



$|M^*| = \lfloor \frac{n}{2} \rfloor$ pt. C_n

ex: Problema perechilor (a seratei)

$B = \{b_1, b_2, b_3 — b_n\}$ - băieți

$F = \{f_1, f_2, f_3 --- f_n\}$ - fete.

$\forall\, b_i$ - k fete

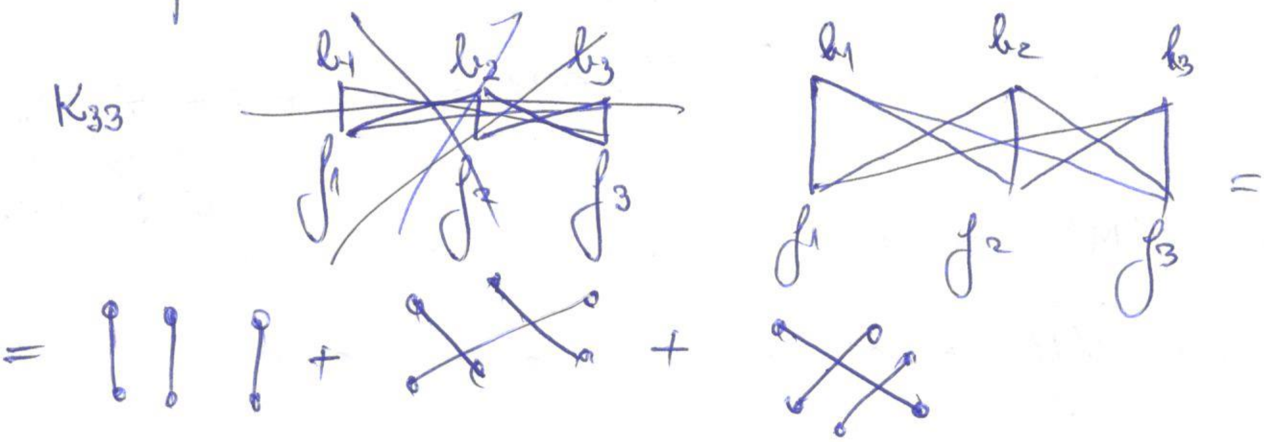
$\forall\, f_i$ - k băieți.

$G = (B \cup F, E)$ - graf bipartit de $n \times n$, k-regulat

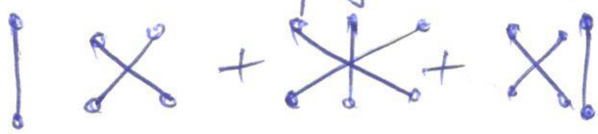
$E = \{\{b_i, f_j\} \mid b_i \in B, f_j \in F,$ "b_i prieten f_j" $\}$

Caz particular

K_{33}



sau alte cuplaje



2. <u>Pregătiri</u> $G=(V,E)$

M^*

$M \subseteq E$ cuplaj

$P=$ x,y-lanț M-alternant $\Rightarrow$

P lanț elementar x,y,

$\underline{M}$

$\underline{\overline{M}}$

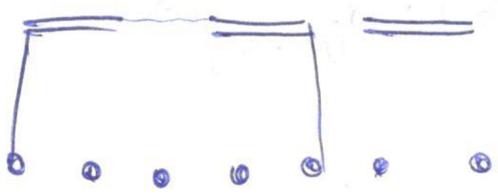
$\boxed{\overline{M} := E - M.}$

P x,y-lanț M-alternant deschis/crescător.

$\Leftrightarrow$ x,y M-nesaturate

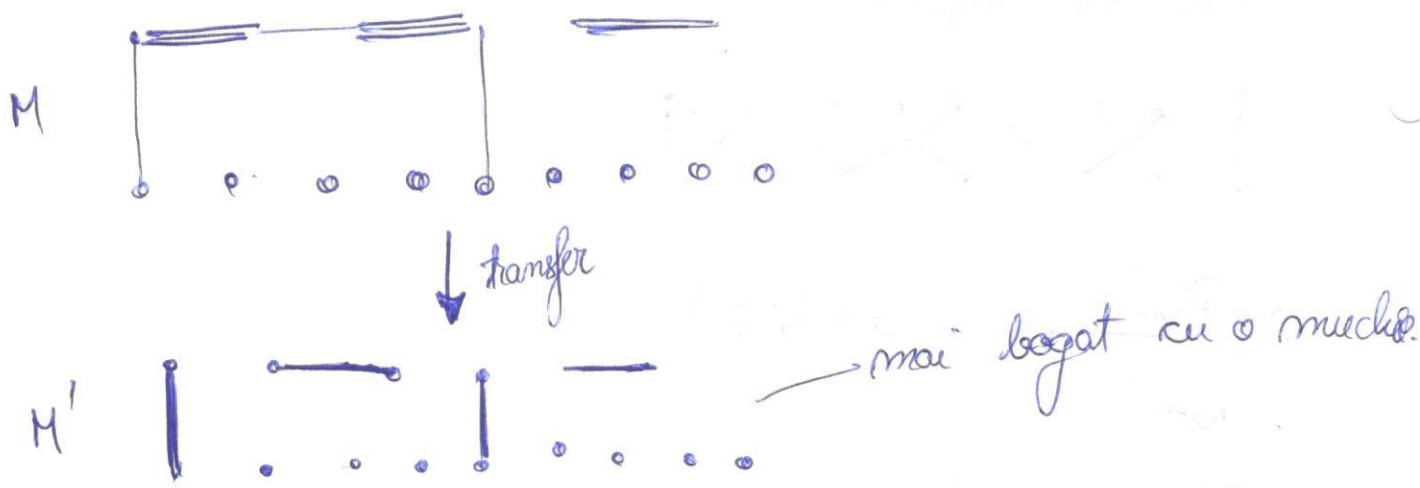
$V(M)$

$\overline{V(M)}$



<u>Propr</u>:

transfer de-a lungul unui lanț M-alt deschis



$|M'| = |M| + 1$

$$\boxed{M' = M \Delta E(P)}$$

<u>Diferența simetrică a două mulțimi</u>

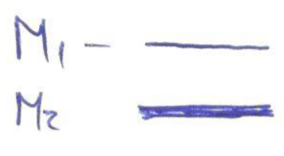
$X, Y \qquad X \Delta Y := (X - Y) \cup (Y - X)$

M_1, M_2 cuplaje diferite

$[M_1 \Delta M_2]$

graful indus de $M_1 \Delta M_2$

<u>Obs</u>: Orice vârf. $V[M_1 \Delta M_2]$: $d_{[M_1 \Delta M_2]}(v)$



o comp. conexă din $[M_1 \Delta M_2]$ este $\begin{cases} \text{lanț elem } M_1, M_2\text{-alt.} \\ \text{ciclu elem } M_1, M_2\text{-alt} \end{cases}$

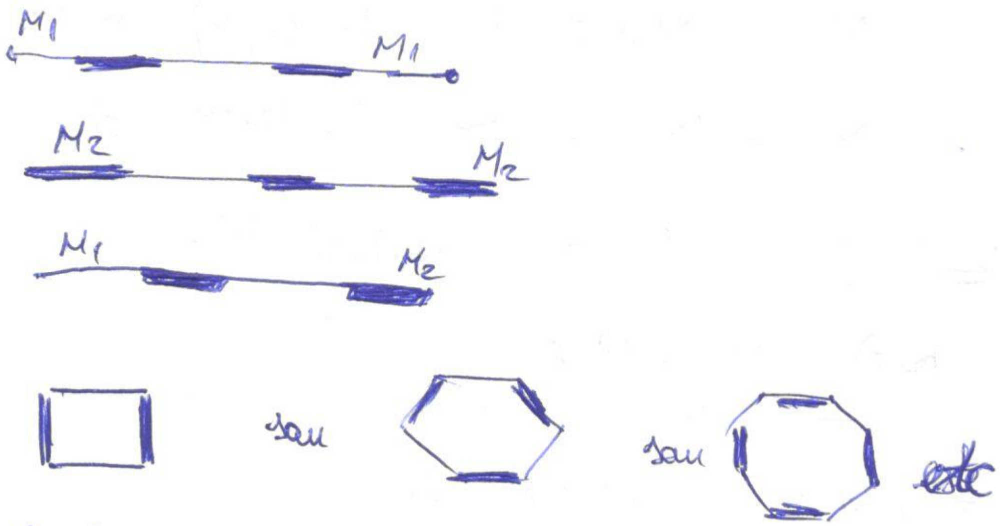
tipuri:

t_1 :

t_2 :

t_3 :

t_4 :



la tipul t_i

	M_1	M_2
t_1	>	
t_2	<	
t_3	=	
t_4	=	

(3.) Caract. cuplajelor de cardinal maxim, M^*. Th. lui Berge

(T) (Berge) $G=(V,E)$ $M \subseteq E$ cuplaj

M cuplaj de cardinal maxim $\Leftrightarrow$ G nu conț. lanțuri M-alternante deschise

Dem:

"$\Rightarrow$" Fie $|M| = |M^*|$

R. A $\exists$. P lanț M-alt. deschis.

$M' := M \Delta E(P)$ cu M' cuplaj în G

$|M'| = |M| + 1 > |M| = |M^*|$ ↯

"$\Leftarrow$" G nu conține lanțuri M-alternate deschise

R.A M nu est cuplaj de card. maxim.

Fie M^* un cuplaj de card maxim.

$\Rightarrow M \neq M^* \Rightarrow M \Delta M^* \neq \emptyset$

$[M \Delta M^*]$ $\qquad M = (M \cap M^*) \dot\cup (M - M^*)$

$M^* = (M \cap M^*) \dot\cup (M^* - M)$

$|M^*| > |M| \Rightarrow \exists$ o comp conexă cu $[M \Delta M^*]$ care cont. mai multe muchii din M^* decât M.

	M	M^*
t_1		>
t_2		<
t_3		=
t_4		=

$\Rightarrow \exists$ un lanț M, M^*-alternant de tip t_2 (care are în capete muchii din M^*) : P

P este lanț M alt. deschis. ↯

Deci M e cupl. de card maxim.

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