ALG 2 GRAFURI
CURS 4

Grafuri bipartite

1. Fie $G=(V,E)$ graf simplu

$c : V \longrightarrow \{0,1,2,\dots,p-1\}$

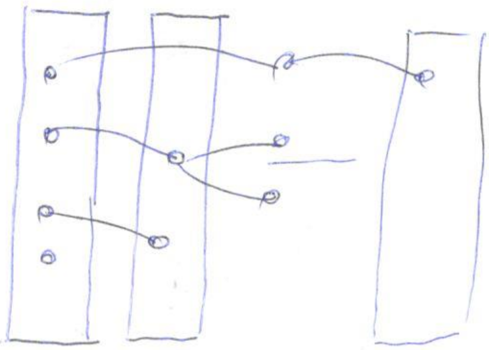
c - p colorare proprie a vârfurilor $\iff \forall e = xy \in E : c(x) \neq c(y)$

$c^{-1}(i)$ este mulț. independentă de vf. $\forall i \in \{0,1,2,\dots,p-1\}$

$A \subseteq V$ A s.n. mulț. independentă $\iff \forall x,y \in A,\ x \neq y : xy \notin E$

$V = c^{-1}(0) \cup c^{-1}(1) \cup \dots \cup c^{-2}(p-1)$ orice parte monocromă este independ. (stabilă)

Fie c o p-colorare proprie.



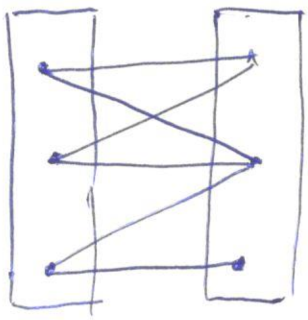
$c^{-1}(0)$ $c^{-1}(1)$ $c^{-2}(p-1)$

$\boxed{\chi_v(G) = \min\{p \mid \exists\, c\ p\text{-col prop a lui } G\}}$

$\chi_v(G) = 2 \iff G$ bipartit

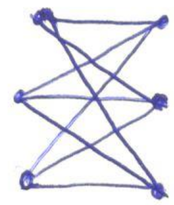
G bipartit $\iff \exists\, c : V \to \{0,1\}$

c - $\underbrace{\text{bicolorare}}_{2}$ proprie



$V_0 = c^{-1}(0)$ $V_1 = c^{-1}(1)$

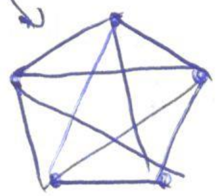
ex:



— graf bipartit $K_{3,3}$.

$\chi_v(K_{3,3}) = 2$

K_5



5-partit complet $\chi_v(K_5) = 5$.

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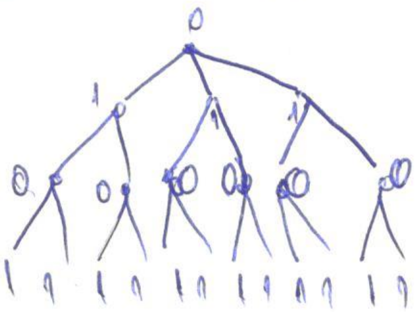
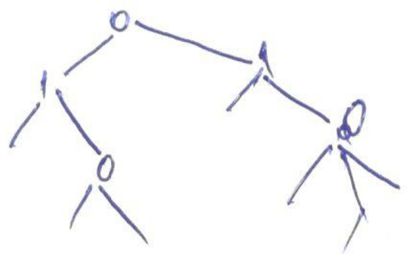
ex: $\forall\, T = (V, E)$ - arbore este bipartit : $|V| \geq 2$

$c : V \longrightarrow \{0, 1\}$



$\forall\, x \neq y \in V \ \exists^{*} P$ x, y - lanț.



2. <u>Teorema lui KÖNIG de caracterizare a grafurilor bipartite)</u>

Fie $G=(V,E)$ graf bipartit, $|V| \geq 2$, (conex)

G graf bipartit $\Leftrightarrow$ Orice ciclu elem din G este par.

Un ciclu este par dacă are un nr par de vf/muchii

$|vf| = |muchii|$

Dem: $\Rightarrow$ G bipartit.

$\exists c : V \to \{0,1\}$ bicolorare proprie.

$V = c^{-1}(0) \dot{\cup} c^{-1}(1)$

Fie $C = [x_0, x_1, x_2 \dots x_{p-1}, x_0]$ un ciclu elementar cu p vf.

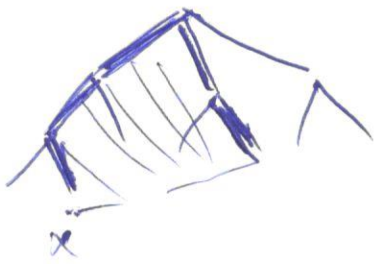
"$\Rightarrow$" $\begin{array}{ccccc} c(x_0) & c(x_1) & c(x_i) \dots & c(x_p) & c(x_0) \\ \| & \| & \| & \| & \| \\ 0 & \dots 1 & \dots 0 \dots & 1 \dots & 0 \end{array}$ $\to p$ este par.

"$\Leftarrow$" $\forall$ C - ciclu element din G este par.

G conex $\to \exists T = (V, E')$ arb parțial în G $E' \subseteq E$

Fie $c : V \to \{0,1\}$ bicolorare proprie a lui T

Vom demonstra că c este bicolorare proprie în G



Fie $e = xy \in E(G)$

$c(x) \neq c(y)$

a) $e \in E(T) (= E') \Rightarrow c(x) \neq c(y)$ (Știu!)

b) $e \in E(G) - E(T) \overset{?}{\Rightarrow} c(x) \neq c(y)$

25

Fie P x,y-lanțul din T

$C = [x \xrightarrow{P} y, x]$ este ciclu elementar în G $\Rightarrow$ C are un nr. par de vf.

$c : [x \ldots y\ x]$

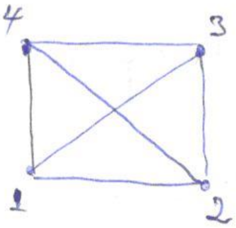
$c(v) : 0 \ldots \quad 1 \Leftarrow 0$

$c(x) = c(y)$ c bicol. propr.

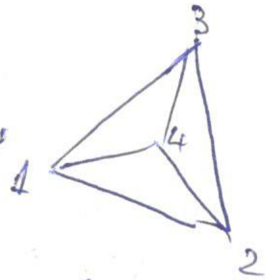
Grafuri planare

1. Def. Notații, Ex

K_4

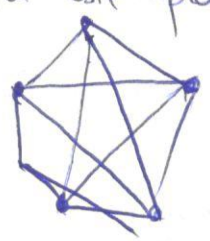


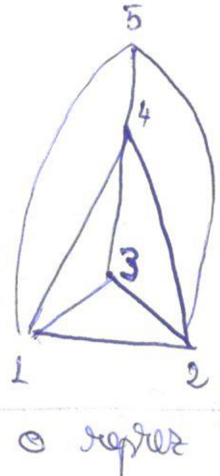
$\simeq$



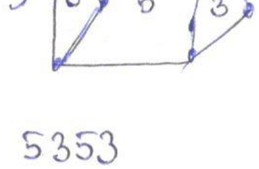
hartă

K_5 - nu este planar





O hartă este o repr. în ... muchiile nu se intersect.



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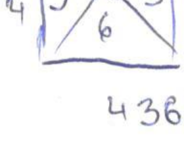
$\neq$



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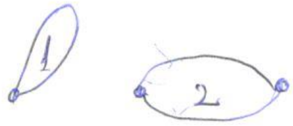
graf planar

hartă

fețe

grad al unei fețe.



$$G=(V,E) \to M=(V,E,F)$$

$M=(V,E,F)$ - hartă în plan.

<u>Notații</u>:

$$V_i = \{x \mid x\in V,\ d_M(x)=i\}.$$

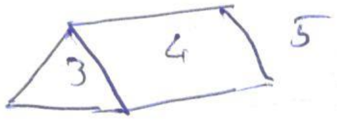
$$F_i = \{f \mid f\in F,\ d_M(f)=i\}.$$ - fețe.

1) $\sum_{i\geq 0} |V_i| = |V|$

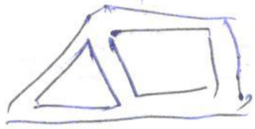
2) $\sum_{i\geq 1} |F_i| = F$

3) $\sum_{i\geq 1} i|V_i| = 2|E|$ - suma gradelor

4) $\sum_{i\geq 1} i|F_i| = 2|E|$

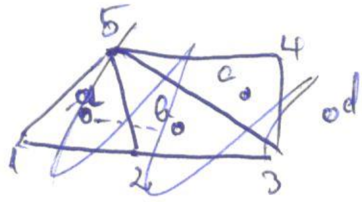


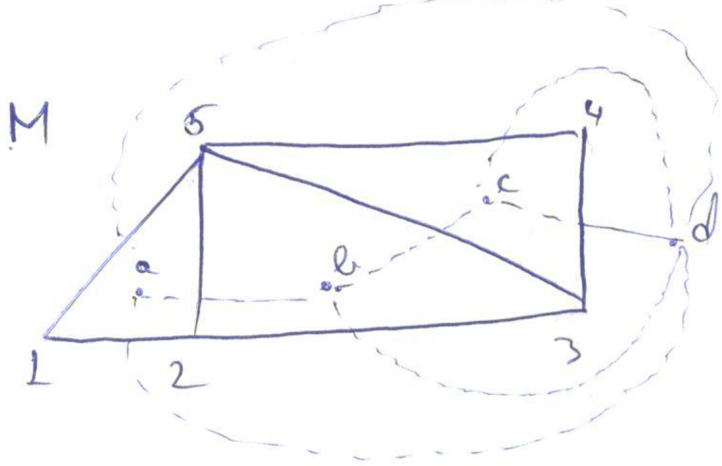
$|E| = 6$ $\quad 3+4+5 = 2\cdot 6.$



$M^* = (F,E,V)$ - harta duală a lui M

fețe vf.





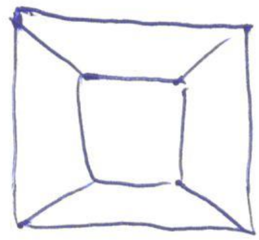
$(M^*)^* \sim M$

2. Teorema poliedrală a lui Euler

$M = (V, E, F)$ o hartă conexă

Avem: $|V| - |E| + |F| = 2$

ex:

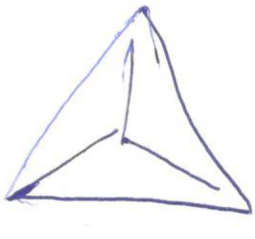


$|V| = 8$

$|E| = 12$

$|F| = 6$

$8 - 12 + 6 = 2$ ✓



$|V| = 4$

$|E| = 6$

$|F| = 4$

$4 - 6 + 4 = 2$ ✓

Dem:

M conexă $\longrightarrow \exists\ T = (V, E')$ arbore parțial în M $(T < M)$

$E' \subseteq E \qquad E - E' = \{e_1, e_2, \dots, e_p\}$

Notăm: $M_0 = T$

$M_1 = M_0 + e_1$

$M_2 = M_1 + e_2 = M_0 + e_1 + e_2$

$$M_p = M_{p-1} + e_p = M_0 + e_1 + e_2 + \dots + e_p.$$

Aveu $T = M_0 < M_1 < M_2 < \dots < M_p = M$.

Avem $M_i = (V, E_i)$

$$E_0 \subseteq E_1 \subseteq \dots \subseteq E_p$$

$T = M_0$: $|V| - |E_0| + |F_0| = |V| - (|V| - 1) + 1 = 2.$

M_i $\quad |V| - |E_i| + |F_i| = |V| - (|E_{i-1}| + 1) + (|F_{i-1}| + 1)$

$$= |V| - |E_{i-1}| + |F_{i-1}|$$

$$|V| - |E| + |F| = |V| - |E_p| - |F_p| = |V| - |E_{p-1}| + |F_{p-1}| = \dots$$
$$\dots = |V| - |E_0| + |F_0| = 2 \square.$$

$M_0 = T$.