TEMA 3

1) a) $i, j > 0$

$$P(x > i + j \mid x > i) = \frac{P(x > i + j \cap x > i)}{P(x > i)} =$$

$$\frac{P(x > i + j)}{P(x > i)} \; ; \quad P(x)^{<k} = P(1-p)^{(k-1)}$$

$$P(x \leq i) = P(x=1) + P(x=2) + \dots + P(x=i) =$$

$$= \sum_{k=1}^{i} P(1-p)^{k-1}$$

$$P(x \leq i + j) = \sum_{k=1}^{j+i} p(1-p)^{k}$$

$$P(x > i) = 1 - P(x \leq i) = 1 + (1-p)^{i} - 1 = (1-p)^{i}$$

$$P(x > i + j) = (1-p)^{i+j}$$

$$P(x > i + j \mid x > i) = \frac{P(x > i+j)}{P(x > i)} = \frac{(1-p)^{i+j}}{(1-p)^{i}} =$$

$$= (1-p)^{j} = P(x > j)$$

b) $P(x = k) = (1-p)^{k-1} p$

$$E\left[\frac{1}{x}\right] = \sum_{x} x\, f(x) \; ; \; f(x) = P\left(x = \frac{1}{x}\right) = P\left(x = \frac{1}{x}\right) =$$

$$= \frac{p}{1-p}(\ln 1 - \ln p) = (1 - \ln p) \cdot \frac{p}{1-p} = \frac{p}{1-p} \cdot \log p$$

2. a) „$\Rightarrow$" X var Poisson de par λ

$$\Leftrightarrow e^{-\lambda}\cdot\frac{\lambda^{n}}{n!} = P(n)$$

$$\frac{P(n)}{P(n-1)} = \frac{e^{-\lambda}\frac{\lambda^{n}}{n!}}{e^{-\lambda}\frac{\lambda^{n-1}}{(n-1)!}} = \frac{\frac{\lambda^{n}}{\lambda^{n-1}}}{\frac{n!}{(n-1)!}} \Rightarrow$$

$$\Rightarrow \frac{P(n)}{P(n-1)} = \frac{\lambda}{n} \quad (A)$$

„$\Leftarrow$" $\frac{P(n)}{P(n-1)} = \frac{\lambda}{n} \ \forall n$

$$P(2) = \frac{\lambda}{2}\cdot P(1)$$

$$P(3) = \frac{\lambda}{3}\cdot P(2)$$

$$P(n) = \frac{\lambda}{n}\cdot P(n-1) \Rightarrow P(n) = P(1)\cdot\frac{\lambda^{n}}{n!} \Rightarrow$$

$\Rightarrow$ X este o var Poisson de parametrul λ

b) $P(X=k)$ max $\Rightarrow \frac{P(k)}{P(k-1)} > 1$ și $\frac{P(k+1)}{P(k)} < 1$

$\Rightarrow \frac{\lambda}{k} > 1$ și $\frac{\lambda}{k+1} < 1 \Rightarrow \lambda > k$ și $\lambda < k+1 \Rightarrow$

$\Rightarrow k = (\lambda - 1, \lambda) \Rightarrow k = [\lambda]$

2) b) continuare

$$\max\left(e^{-\lambda}\cdot \frac{\lambda^{[\lambda]}}{[\lambda]!}\right) ?$$

funcția de mai sus este continuă în $[k, k+1]$ și crescătoare.

max de intervalele este obținută pe intervalul $(0,1] \Rightarrow \lambda \Rightarrow \lambda = 1 \Rightarrow$

$$\Rightarrow \max\left(e^{-\lambda}\cdot \frac{\lambda^{[\lambda]}}{[\lambda]!}\right) = e^{-1}\cdot \frac{1!}{1!} = e^{-1}$$

3) $$G = (a-b)\cdot \frac{[n(1+2+\dots+n) - 1 - 1^2 - 2 - 2^2 - \dots - (n- \quad -n-1-(n-1)^2]}{(n+1)(n+2)\frac{1}{2}} = \frac{(a-b)\left[\frac{n^2(n+1)}{2} - \frac{(n-1)n}{2} - \frac{(n-1)n(2n-1)}{6}\right]}{(n+1)(n+2)\frac{1}{2}}$$

$$= \frac{(a-b)\left[\frac{n^2(n+1)}{2} - \frac{n(n-1)(2n-1+3)}{6}\right]}{(n+1)(n+2)\cdot\frac{1}{2}} =$$

$$= (a-b)\,\frac{n(n+1)[3n-2n+2]\cdot\frac{1}{6}}{(n+1)(n+2)\cdot\frac{1}{2}} =$$

$$= (a-b)\,\frac{n(n+1)(n+2)}{3(n+1)(n+2)} = (a-b)\,\frac{n}{3}$$

4) $7 - X = \begin{pmatrix} 6 & 5 & 4 & 3 & 2 & 1 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$X(7-X) = \begin{pmatrix} 6 & 10 & 12 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} = Y$

$E(Y) = \frac{1}{3} \cdot 6 + \frac{1}{3} \cdot 10 + \frac{1}{3} \cdot 12 = \frac{28}{3}$

$var(Y) = E(X^2) - E^2(X) = \frac{56}{9}$

cum $Y = \begin{pmatrix} 6 & 10 & 12 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \Rightarrow M_4$, unde

M_4 înseamnă prob că Y_i $i \in \overline{1,4}$ să fie maxim este $\frac{1}{3}$

5) a) $X_1 = \begin{pmatrix} 0 & 1 \\ (1-p) & p \end{pmatrix}$

$X_1 + X_2 = \begin{pmatrix} 0 & 1 & 2 \\ (1-p)^2 & 2p+(1-p) & p^2 \end{pmatrix}$

$S_n = X_1 + X_2 + \dots * X_n = \cancel{\begin{pmatrix} 0 \\ C_n^0 (1-p)^n \end{pmatrix}}$

$= \begin{pmatrix} 0 & 1 & \dots & n \\ C_n^0 (1-p)^n & C_n^1 (1-p)^{n-1} p & & C_n^n p^n \end{pmatrix} \Rightarrow$

$\Rightarrow S_n$ e ~~r~~ ~~ep~~ $B(n, p)$

$E(S_1) = p$

$E(S_2) = 2p - 2p^2 - 2p^2 = 2p$

$\vdots$

$E(S_k) = kp$

$E(S_{k+1}) = (k+1)p \Rightarrow \forall n \in N^* \; E(S_n) = np$

$var(S_n) = E^2(S_n) - E(S_n^2) = (np)^2 - np^2 =$

$= n(np^2 - p^2) = np^2(n-1)$