

Solutions to hw5 homework on Convex
Optimization
<https://web.stanford.edu/class/ee364a/homework.html>

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July 10, 2020

5.17

Robust linear programming with polyhedral uncertainty. Consider the robust LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \sup_{a \in P_i} a^T x \leq b_i, \quad i = 1, \dots, m \end{array}$$

with variable $x \in R^n$, where $P_i = \{a : C_i a \preceq d_i\}$. The problem data are $a \in R^n$, $C_i \in R^{m_i \times n}$, $d_i \in R^{m_i}$, and $b \in R^m$. We assume the polyhedra P_i are nonempty. Show that this problem is equivalent to the LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & d_i^T z_i \leq b_i, \quad i = 1, \dots, m \\ & C_i z_i = x, \quad i = 1, \dots, m \\ & z_i \succeq 0, \quad i = 1, \dots, m \end{array}$$

with variables $x \in R^n$, $z_i \in R^{m_i}$, $i = 1, \dots, m$. Hint: find the dual of the problem of maximizing $a_i^T x$ over $a_i \in P_i$ (with variable a_i).

Solution:

The problem of maximizing $a_i^T x$ over $a_i \in P_i$ (with variable a_i) is:

$$\begin{array}{ll}
\text{maximize} & a_i^T x \\
\text{subject to} & a_i \in P_i, \text{ where } P_i = \{a : C_i a \preceq d_i\}
\end{array}$$

or

$$\begin{array}{ll}
\text{minimize} & -a_i^T x \\
\text{subject to} & C_i a_i \preceq d_i
\end{array}$$

The Lagrange dual of this problem is:

$$\begin{array}{ll}
\text{minimize} & \sum_{i=1}^m \lambda_i d_i \\
\text{subject to} & C_i \lambda_i = x \\
& \lambda_i \succeq 0
\end{array}$$

The optimal value of this problem is less or equal to b_i , so we have the equivalent problem to our LP:

$$\begin{array}{ll}
\text{minimize} & c^T x \\
\text{subject to} & d_i^T \lambda_i \leq b_i, \ i = 1, \dots, m \\
& C_i \lambda_i = x, \ i = 1, \dots, m \\
& \lambda_i \succeq 0, \ i = 1, \dots, m
\end{array}$$