Solutions to hw5 homework on Convex Optimization https://web.stanford.edu/class/ee364a/homework.html

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5.17

Robust linear programming with polyhedral uncertainty. Consider the robust LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \sup_{a \in P_i} a^T x \le b_i, \ i = 1, ..., m \end{array}$$

with variable $x \in R^n$, where $P_i = \{a : \ C_i a \preceq d_i\}$. The problem data are $a \in R^n$, $C_i \in R^{m_i \times n}$, $d_i \in R^{m_i}$, and $b \in R^m$. We asume the polyhedra P_i are nonempty. Show that this problem is equivalent to the LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & d_i^T z_i \le b_i, \ i = 1, ..., m \\ & C_i z_i = x, \ i = 1, ..., m \\ & z_i \succeq 0, \ i = 1, ..., m \end{array}$$

with variables $x \in R^n$, $z_i \in R^{m_i}, \ i = 1, ..., m$. Hint: find the dual of the problem of maximizing $a_i^T x$ over $a_i \in P_i$ (with variable a_i).

Solution:

The problem of maximizing $a_i^T x$ over $a_i \in P_i$ (with variable a_i) is:

$$\begin{array}{ll} \text{maximize} & a_i^T x \\ \text{subject to} & a_i \in P_i, \text{where } P_i = \{a : \ C_i a \preceq d_i\} \end{array}$$

or

$$\begin{array}{ll} \text{minimize} & -a_i^T x \\ \text{subject to} & C_i a_i \preceq d_i \end{array}$$

The Lagrange dual of this problem is:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{m} \lambda_i d_i \\ \text{subject to} & C_i \lambda_i = x \\ & \lambda_i \succeq 0 \end{array}$$

The optimal value of this problem is less or equal to b_i, so we have the equivalent problem to our LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & d_i^T \lambda_i \leq b_i, \ i = 1, ..., m \\ & C_i \lambda_i = x, \ i = 1, ..., m \\ & \lambda_i \succeq 0, \ i = 1, ..., m \end{array}$$

5.40

E - optimal experiment design. A variation on two optimal experiment design problems of exercise 5.10 is the E - optimal design problem:

$$\begin{array}{ll} \text{minimize} & \lambda_{max}(\sum_{i=1}^{p} x_i v_i v_i^T)^{-1} \\ \text{subject to} & x \succeq 0, \ \mathbf{1}^T x = 1 \end{array}$$

(See also §7.5.) Derive a dual fro this problem first by reformulating it as:

$$
\begin{array}{ll}
\text{minimize} & 1/t \\
\text{subject to} & \displaystyle\sum_{i=1}^{p} x_i v_i v_i^T \succeq t\boldsymbol{I} \\
& x \succeq 0,\ \mathbf{1}^T x = 1
\end{array}
$$

with variables $t \in R$, $x \in R^p$ and domain $R_{++} \times R^p$, and applying Lagrange duality. Simplify the dual problem as much as you can.