

Solutions to hw3 homework on Convex
Optimization
<https://web.stanford.edu/class/ee364b/homework.html>

Andrei Keino

March 1, 2021

3.1 (4 points)

Consider the optimization problem

$$\begin{aligned} \underset{\{x_j\}_{j=1}^J}{\text{minimize}} \quad & f(x_1, \dots, x_J) := \frac{1}{2} \|b - \sum_{j=1}^J A_j x_j\|_2^2 + \lambda \cdot \sum_{j=1}^J \|x_j\|_2, \\ \text{s.t.} \quad & A_j x_j \geq 0, \quad \forall j \in \{1, 2, \dots, J\} \end{aligned}$$

with variable $x_1, \dots, x_J \in R^n$, and problem data $A_1, \dots, A_J \in R^{m \times n}$, $b \in R^m$, and $\lambda > 0$. for constrained optimization given on page 11 (really p. 12) of the lecture slides for subgradient methods for constrained problems.

Let $J = 3$, $n = 100$, $m = 10$ and $\lambda = 0.5$. Generate random matrices $A_1, \dots, A_J \in R^{m \times n}$ with independent uniformly distributed entries in the interval $[0, \frac{1}{\sqrt{m}})$, and, random vectors $x_1, \dots, x_J \in R^n$ uniformly distributed entries in the interval $[0, \frac{1}{\sqrt{n}})$, then set $b = \sum_{j=1}^J A_j x_j$. Plot convergence in terms of the objective $f(x_1^{(k)}, \dots, x_J^{(k)})$. Try different step length schedules. Also, plot the maximal violation for the linear constraints at each step.

Solution:

The code is in the file `solution_3_1_b.m`. The code is nearly the same as one for the task 2.4, the only difference that on every step of the gradient descent we are calculating the constraint violation vector and if there are at least one violation, we replace the gradient of the minimized function with the gradient of any constraint violation found.

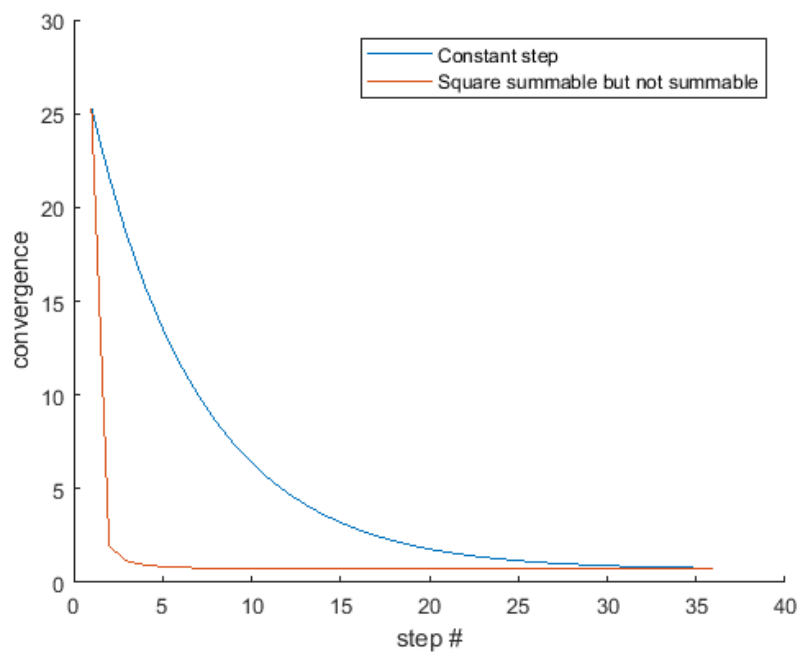


Figure 1: Convergence with different step length.

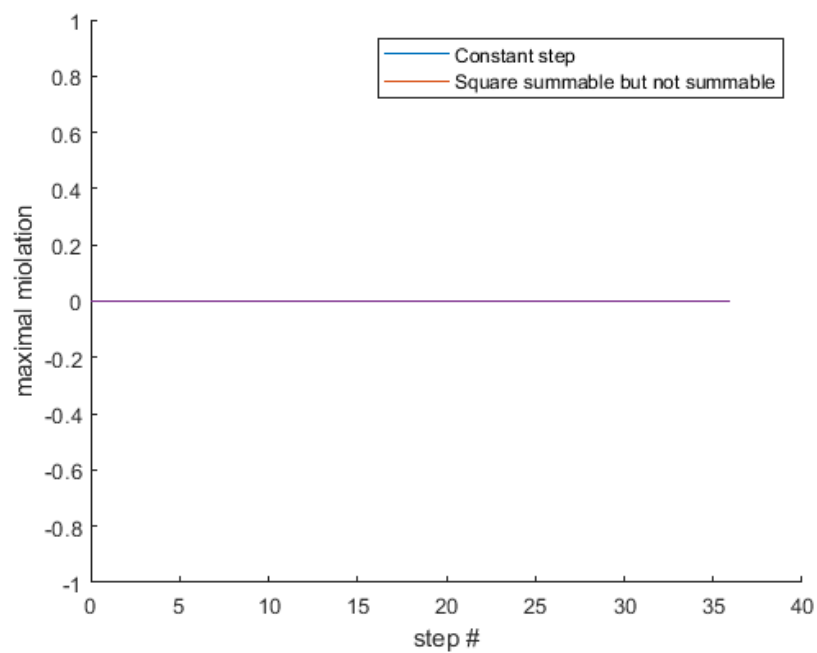


Figure 2: Maximal violation for the linear constraints.