MCMC

Тест, 5 вопроса

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1.
For Questions 1 through 3, consider the following model for data that take on values between 0 and 1:

$$x_i \mid \alpha, \beta \overset{\text{iid}}{\sim} \text{Beta}(\alpha, \beta), \quad i = 1, \ldots, n,$$
$$\alpha \sim \text{Gamma}(a, b),$$
$$\beta \sim \text{Gamma}(r, s),$$

where α and β are independent a priori. Which of the following gives the full conditional density for α up to proportionality?

- ( ) $p(\alpha \mid \beta, x) \propto \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(\alpha>0)}$
- ( ) $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n \Gamma(\beta)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \left[\prod_{i=1}^n (1-x_i)\right]^{\beta-1} \alpha^{a-1} e^{-b\alpha} \beta^{r-1} e^{-s\beta} I_{(0<\alpha<1)} I_{(0<\beta<1)}$
- ( ) $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(0<\alpha<1)}$
- (•) $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(\alpha>0)}$

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2.
Suppose we want posterior samples for α from the model in Question 1. What is our best option?

- ( ) The joint posterior for α and β is a common probability distribution which we can sample directly. Thus we can draw Monte Carlo samples for both parameters and keep the samples for α.
- ( ) The full conditional for α is proportional to a common distribution which we can sample directly, so we can draw from that.
- (•) The full conditional for α is not proportional to any common probability distribution, and the marginal posterior for β is not any easier, so we will have to resort to a Metropolis-Hastings sampler.
- ( ) The full conditional for α is not a proper distribution (it doesn't integrate to 1), so we cannot sample from it.

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3.
If we elect to use a Metropolis-Hastings algorithm to draw posterior samples for α, the Metropolis-Hastings candidate acceptance ratio is computed using the full conditional for α as

$$\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} \alpha^{*\,a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} \alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$$

where α^* is a candidate value drawn from proposal distribution $q(\alpha^*|\alpha)$. Suppose that instead of the full conditional for α, we use the full joint posterior distribution of α and β and simply plug in the current (or known) value of β. What is the Metropolis-Hastings ratio in this case?

- ( ) $\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$
- (•)

$$\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} {\alpha^*}^{a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} \alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$$

- $$\frac{\Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*-1} \left[\prod_{i=1}^n (1-x_i)\right]^{\beta-1} {\alpha^*}^{a-1} e^{-b\alpha^*} \beta^{r-1} e^{-s\beta} q(\alpha|\alpha^*) I_{(0<\alpha^*)} I_{(0<\beta)}}{\Gamma(\alpha^*)^n \Gamma(\beta)^n q(\alpha^*|\alpha)}$$

- $$\frac{{\alpha^*}^{a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$$

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4.
For Questions 4 and 5, re-run the Metropolis-Hastings algorithm from Lesson 4 to draw posterior samples from the model for mean company personnel growth for six new companies: (-0.2, -1.5, -5.3, 0.3, -0.8, -2.2). Use the same prior as in the lesson.

Below are four possible values for the standard deviation of the normal proposal distribution in the algorithm. Which one yields the best sampling results?

- 0.5
- 1.5
- 3.0
- 4.0

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5.
Report the posterior mean point estimate for μ, the mean growth, using these six data points. Round your answer to two decimal places.

-1.48

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