

## Lesson 5

Тест, 8 вопроса

1

Баллы

1.

Which of the following descriptions matches the process of Gibbs sampling for multiple random variables?

- ☐ Draw candidates for all  $J$  variables simultaneously using a multivariate proposal distribution. For each variable, calculate the acceptance ratio  $\alpha_j$  using the joint (unnormalized) density. Accept each candidate with probability  $\min\{1, \alpha_j\}$  for  $j = 1, \dots, J$ . Repeat this cycle for many iterations.
  - ☒ Cycle through the variables, drawing a sample from the full conditional distribution of each variable while substituting in the current values of all other variables. Repeat this cycle for many iterations.
  - ☐ Draw candidates for all variables simultaneously using a multivariate proposal distribution. Calculate the acceptance ratio  $\alpha$  using the joint (unnormalized) density. Accept the candidates with probability  $\min\{1, \alpha\}$ . Repeat this step for many iterations.
  - ☐ Cycle through the variables, drawing from a proposal distribution for each variable and accepting the candidate with probability equal to the ratio of the candidate draw to the old value of the variable. Repeat this cycle for many iterations.
- 

1

Баллы

2.

Suppose we have a joint probability distribution for four variables,  $p(w, x, y, z)$ . Which of the following expresses the full conditional distribution for variable  $x$ ?

- ☐  $p(x)$
  - ☐  $p(w, y, z \mid x)$
  - ☒  $p(x \mid w, y, z)$
  - ☐  $p(x \mid y)$
- 

1

# Lesson 5

Тест, 8 вопроса

Suppose we have the following joint distribution for  $x, y$ , and  $z$ :

$$p(x, y, z) = 5e^{-5z} I_{z \geq 0} \frac{\Gamma(z+3)}{\Gamma(z)\Gamma(3)} y^{z-1} (1-y)^2 I_{0 < y < 1} \binom{10}{x} y^x (1-y)^{10-x} I_{x \in \{1, \dots, 10\}}.$$

The density for the full conditional distribution of  $z$  is proportional to which of the following?

**Hint:** The full conditional for  $z$  is proportional to the full joint distribution  $p(x, y, z)$  where  $x$  and  $y$  are just constants.

- ☒  $p(z | x, y) \propto e^{-5z} \frac{\Gamma(z+3)}{\Gamma(z)} y^{z-1} I_{z \geq 0}$
- ☐  $p(z | x, y) \propto 5e^{-5z} I_{z \geq 0}$
- ☐  $p(z | x, y) \propto \binom{10}{x} y^x (1-y)^{10-x} I_{x \in \{1, \dots, 10\}}$
- ☐  $p(z | x, y) \propto y^{z-1} (1-y)^2 y^x (1-y)^{10-x} I_{0 < y < 1}$

1

Баллы

4.

The full conditional distribution in Question 3 is not a standard distribution that we can easily sample. Fortunately, it turns out that we can do a Metropolis-Hastings step *inside* our Gibbs sampler step for  $z$ .

If we employ that strategy in a Gibbs sampler for  $y$  and  $z$  (always conditioning on  $x$ ), then the algorithm would look like this:

```

1  For iteration i in 1 to m, repeat:
2
3    1.
4      a) Draw  $z^*$  from a proposal distribution  $q$ .
5      b) Calculate the acceptance ratio ( $\alpha$ ) using the full
6         conditional distribution for  $z|x, y$  and the candidate distribution  $q$ ,
           plugging in the previous iteration's value  $y_{i-1}$  for  $y$ .
7      c) Accept the candidate with probability  $\min\{1, \alpha\}$  and
           set the value for  $z_i$  accordingly.
8
9
10   2. _____.
11
12  end.
```

What would go in step 2 to complete the Gibbs sampler?

- ☐ Draw  $y_i$  from the full conditional  $p(y | x, z)$ , plugging in the previous iteration's value  $z_{i-1}$  for  $z$ .
- ☐ Draw  $y_i$  from the marginal distribution  $p(y)$ .

# Lesson 5

Тест, 8 вопроса

Draw  $y_i$  from the full conditional  $p(y | x, z)$ , plugging in the candidate  $z^*$  for  $z$ .

Draw  $y_i$  from the full conditional  $p(y | x, z)$ , plugging in the value  $z_i$  just drawn in step 1 for  $z$ .

1

Баллы

5.

Suppose we have a joint probability distribution for three variables:  $p(x, y, z)$ . Identify the algorithm to perform Gibbs sampling for all three variables.

```

1  For iteration i in 1 to m, repeat:
2
3      1. Draw candidates  $x^*, y^*, z^*$  from a joint proposal
4         distribution  $q$ .
5
6      2. Calculate the acceptance ratio  $\alpha$  using
7          $g(x, y, z) = p(x|y, z)p(y|x, z)p(z|x, y)$  and  $q$ .
8
9      3. Accept the candidates with probability  $\min\{1, \alpha\}$ 
10         and set  $x_i, y_i, z_i$  accordingly.
11
12  end.
```

```

1  For iteration i in 1 to m, repeat:
2
3      1. Draw candidates  $x^*, y^*, z^*$  from a joint proposal
4         distribution  $q$ .
5
6      2. a) i) Calculate the acceptance ratio  $\alpha_x$  using
7         the full conditional  $p(x|y, z)$  and  $q$ , plugging in the
8         candidates  $y^*, z^*$  for  $y$  and  $z$ .
9         ii) Accept  $x^*$  with probability  $\min\{1, \alpha_x\}$ 
10            and set  $x_i$  accordingly.
11
12     b) i) Calculate the acceptance ratio  $\alpha_y$  using
13         the full conditional  $p(y|x, z)$  and  $q$ , plugging in  $x_i, z^*$ 
14         for  $x$  and  $z$ .
15         ii) Accept  $y^*$  with probability  $\min\{1, \alpha_y\}$ 
16            and set  $y_i$  accordingly.
17
18     c) i) Calculate the acceptance ratio  $\alpha_z$  using
19         the full conditional  $p(z|x, y)$  and  $q$ , plugging in  $x_i, y_i$ 
20         for  $x$  and  $y$ .
21         ii) Accept  $z^*$  with probability  $\min\{1, \alpha_z\}$ 
22            and set  $z_i$  accordingly.
23  end.
```

## Lesson 5

Тест, 8 вопроса

```

1  For iteration i in 1 to m, repeat:
2
3      1. Draw  $x_i$  from the full conditional distribution for
4          $x|y,z$ , plugging in the previous iteration's values
5          $y_{i-1}$ ,  $z_{i-1}$  for  $y$  and  $z$ .
6
7      2. Draw  $y_i$  from the full conditional distribution for
8          $y|x,z$ , plugging in the previous iteration's values
9          $x_{i-1}$ ,  $z_{i-1}$  for  $x$  and  $z$ .
10
11     3. Draw  $z_i$  from the full conditional distribution for
12         $z|x,y$ , plugging in the previous iteration's values
13         $x_{i-1}$ ,  $y_{i-1}$  for  $x$  and  $y$ .
14
15  end.
```



```

1  For iteration i in 1 to m, repeat:
2
3      1. Draw  $x_i$  from the full conditional distribution for
4          $x|y,z$ , plugging in the previous iteration's values
5          $y_{i-1}$ ,  $z_{i-1}$  for  $y$  and  $z$ .
6
7      2. Draw  $y_i$  from the full conditional distribution for
8          $y|x,z$ , plugging in the previous iteration's value
9          $z_{i-1}$  for  $z$  and this iteration's value  $x_i$  for  $x$ .
10
11     3. Draw  $z_i$  from the full conditional distribution for
12         $z|x,y$ , plugging in this iteration's values
13         $x_i$ ,  $y_i$  for  $x$  and  $y$ .
14
15  end.
```

1

Баллы

6.

For Questions 6 to 8, consider the example from the lesson where the data are percent change in total personnel since last year for  $n = 10$  companies.

In our model with normal likelihood and unknown mean  $\mu$  and unknown variance  $\sigma^2$ , we chose a normal prior for the mean and an inverse-gamma prior for the variance. What was the major advantage of selecting these particular priors?



Each prior was conjugate in the case where the other parameter was known, causing the full conditional distributions to come from the same distribution families as the priors (and therefore easy to sample).



Because these priors are conjugate for their respective parameters, they guarantee the smallest possible Monte Carlo standard error for posterior mean estimates.



These priors allowed us to bypass MCMC, providing a joint conjugate posterior for  $\mu$  and  $\sigma^2$ .

## Lesson 5

Тест, 8 вопроса

Because these priors are conjugate for their respective parameters, they guarantee the most accurate posterior distribution possible for the given likelihood.

1

Баллы

7.

Suppose we repeat the analysis for  $n = 6$  companies in another industry and the data are:

```
1 y = c(-0.2, -1.5, -5.3, 0.3, -0.8, -2.2)
```

Re-run the Gibbs sampler in R for these new data (5000 iterations using the same priors and initial values as in the Lesson) and report the posterior mean for  $\mu$ . Round your answer to two decimal places.

-0.99

1

Баллы

8.

An industry expert is surprised by your results from Question 7 and insists that growth in this sector should be positive on average. To accommodate this expert's prior beliefs, you adjust the prior for  $\mu$  to be normal with a mean 1.0 and variance 1.0. This is a fairly informative and optimistic prior (the prior probability that  $\mu > 0$  is about 0.84).

What happens to the posterior mean of  $\mu$ ? Re-run the analysis on the new data with this new prior. Again, use 5000 iterations and the same prior for  $\sigma^2$  and initial values as before).

- ☒ The posterior mean for  $\mu$  is less than  $-0.25$ , suggesting that despite the optimistic prior, the data strongly favor estimating growth to be negative in this industry.
- ☐ The posterior mean for  $\mu$  is between  $-0.25$  and  $0.25$ , suggesting that the data are not as optimistic about growth as the prior, but we are inconclusive about whether growth is positive or negative.
- ☐ The posterior mean for  $\mu$  is between  $0.25$  and  $1.0$ , suggesting that the data are not informative enough to contradict this expert's opinion.
- ☐ The posterior mean for  $\mu$  is above  $1.0$ , suggesting that the optimistic prior was actually not optimistic enough.

# Lesson 5

Оплатить курс

Тест, 8 вопроса

