

Lecture 13: Attention

Midterm

Grades will be out in ~1 week

Please do not discuss midterm questions on Piazza

Someone left a waterbottle in exam room – Post on Piazza if it is yours

Assignment 4

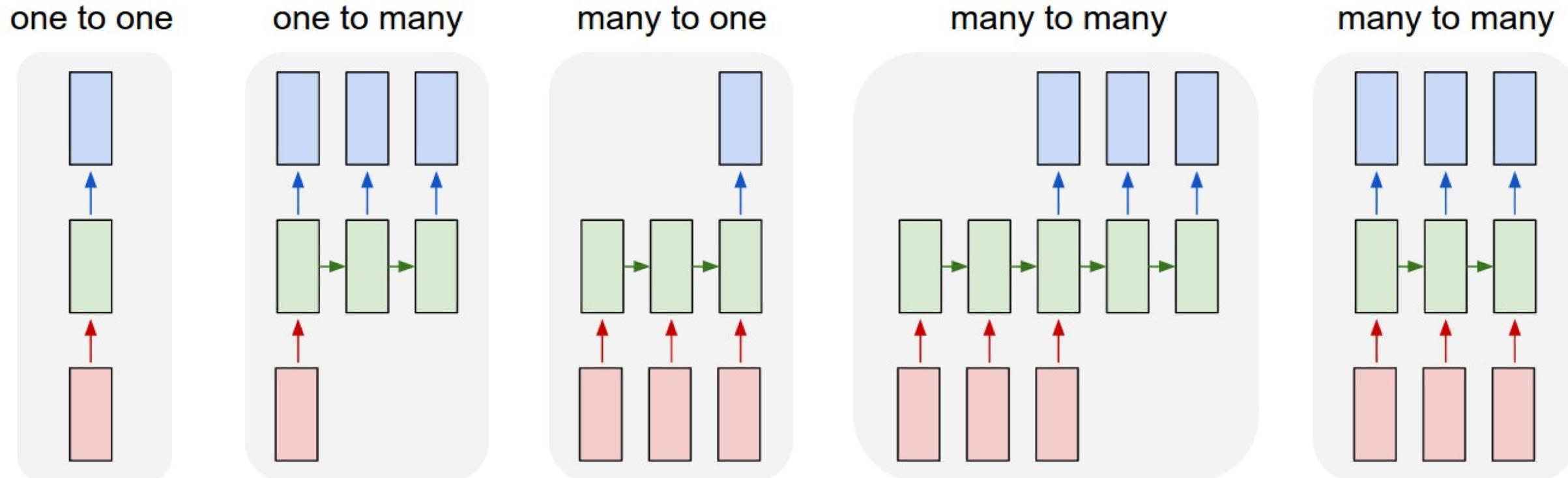
A4 will be released today or tomorrow

Due 2 weeks from the time it is released

Will cover:

- PyTorch autograd
- Residual networks
- Recurrent neural networks
- Attention
- Feature visualization
- Style transfer
- Adversarial examples

Last Time: Recurrent Neural Networks

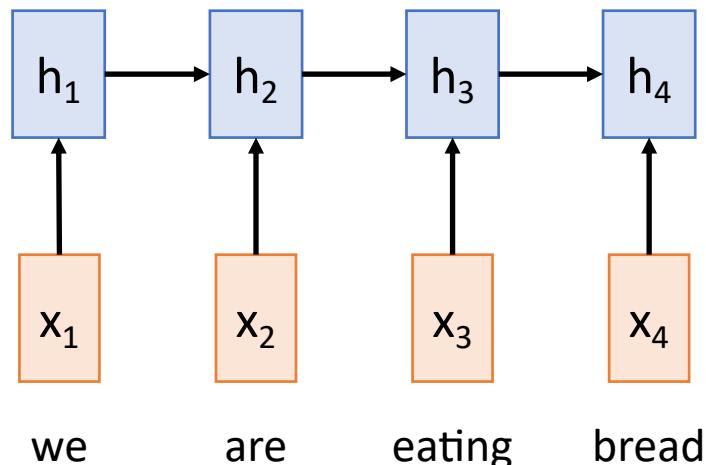


Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Encoder: $h_t = f_w(x_t, h_{t-1})$



Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

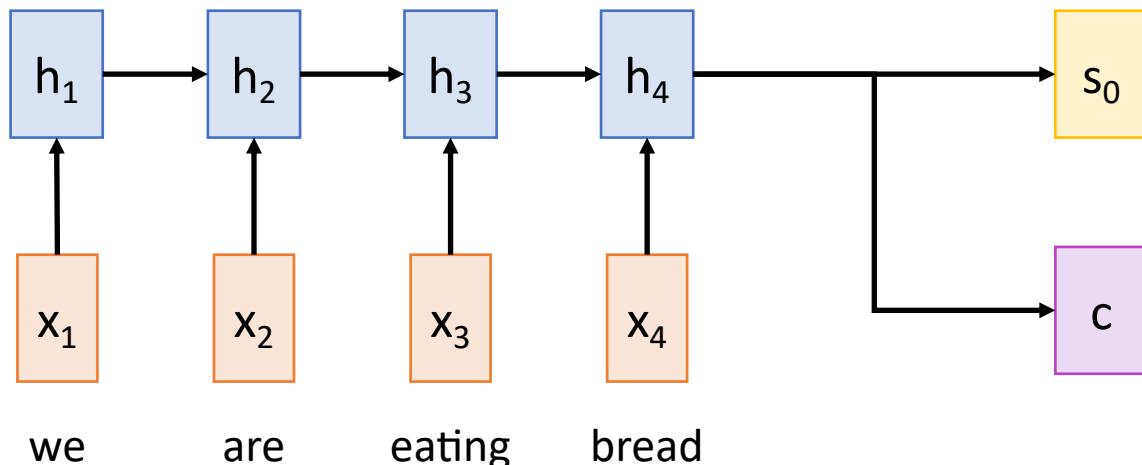
Output: Sequence $y_1, \dots, y_{T'}$

Encoder: $h_t = f_w(x_t, h_{t-1})$

From final hidden state predict:

Initial decoder state s_0

Context vector c (often $c=h_T$)



Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

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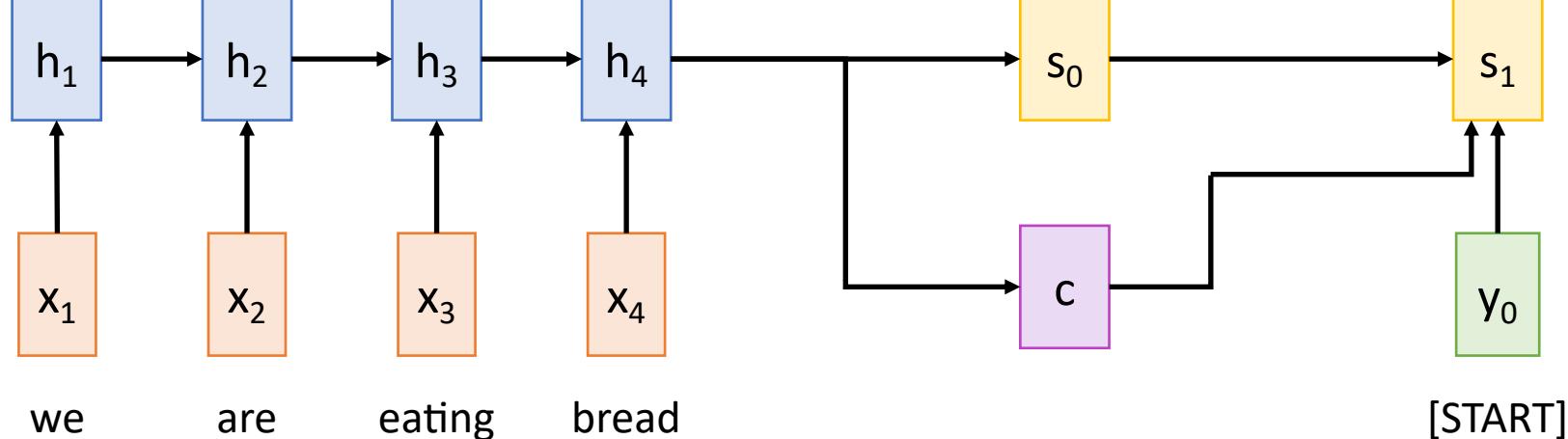
Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

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Sequence-to-Sequence with RNNs

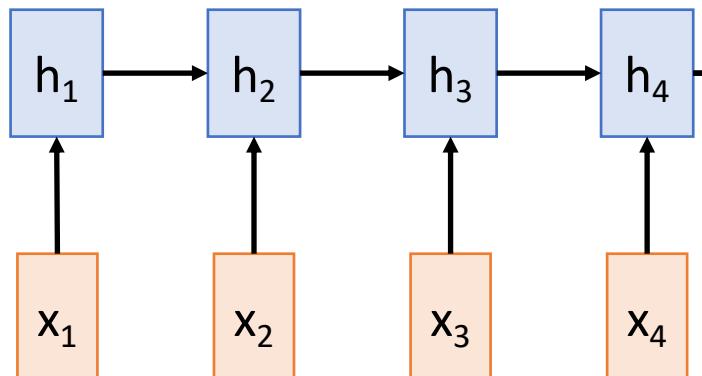
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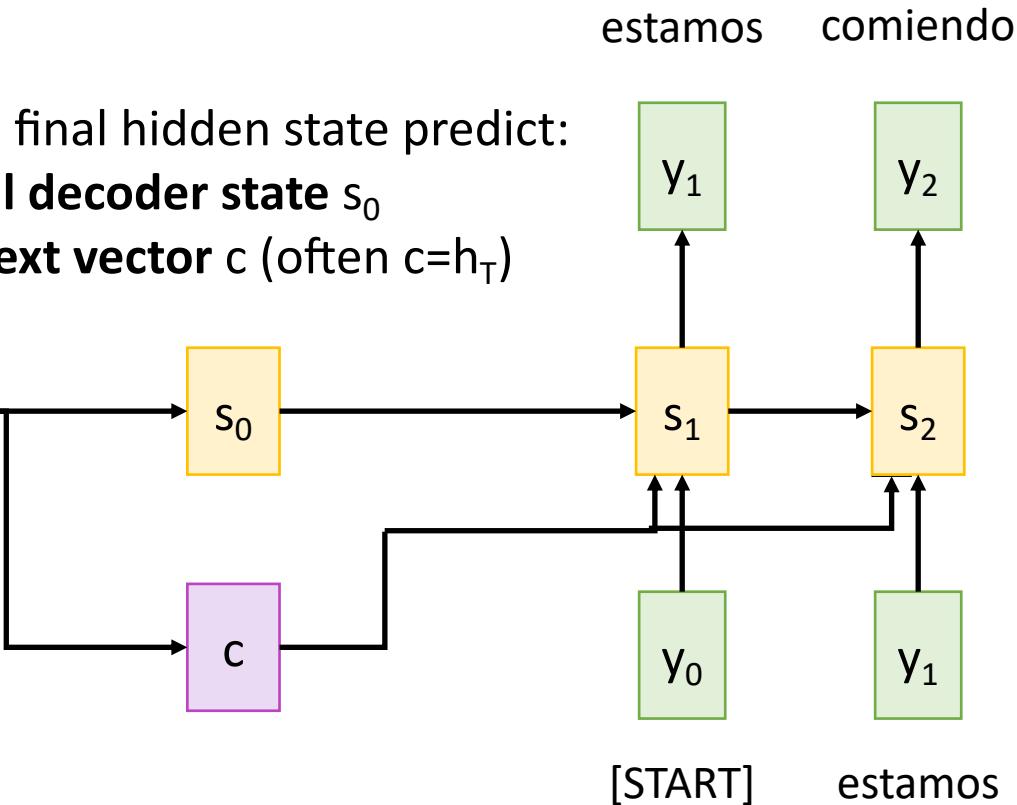
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we are eating bread



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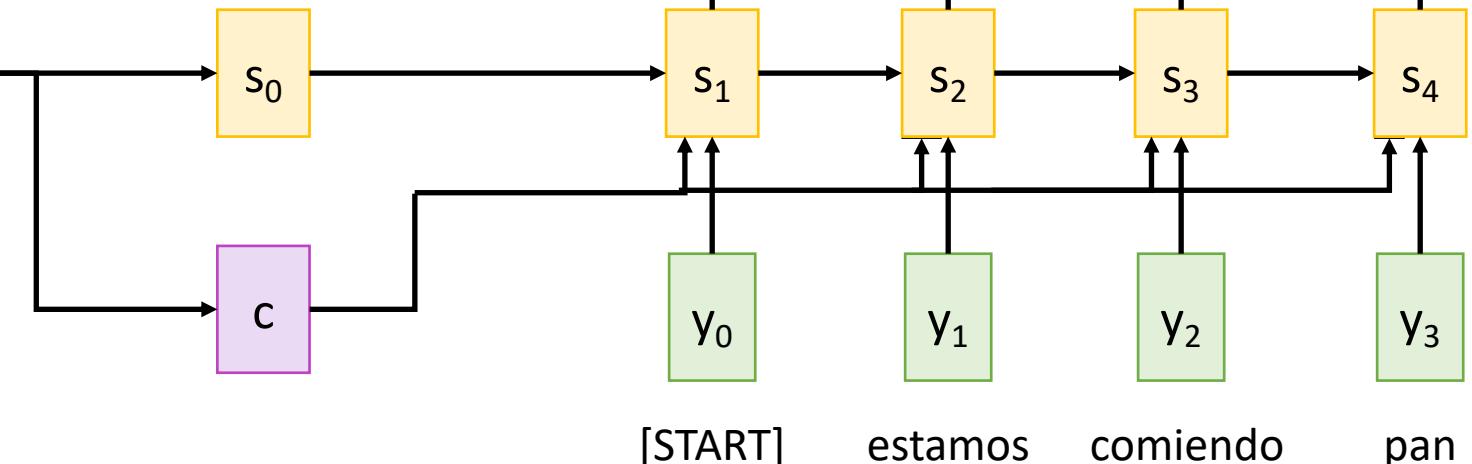
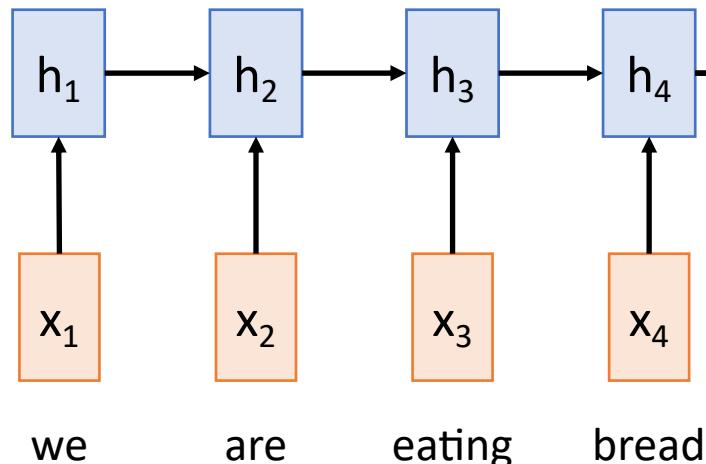
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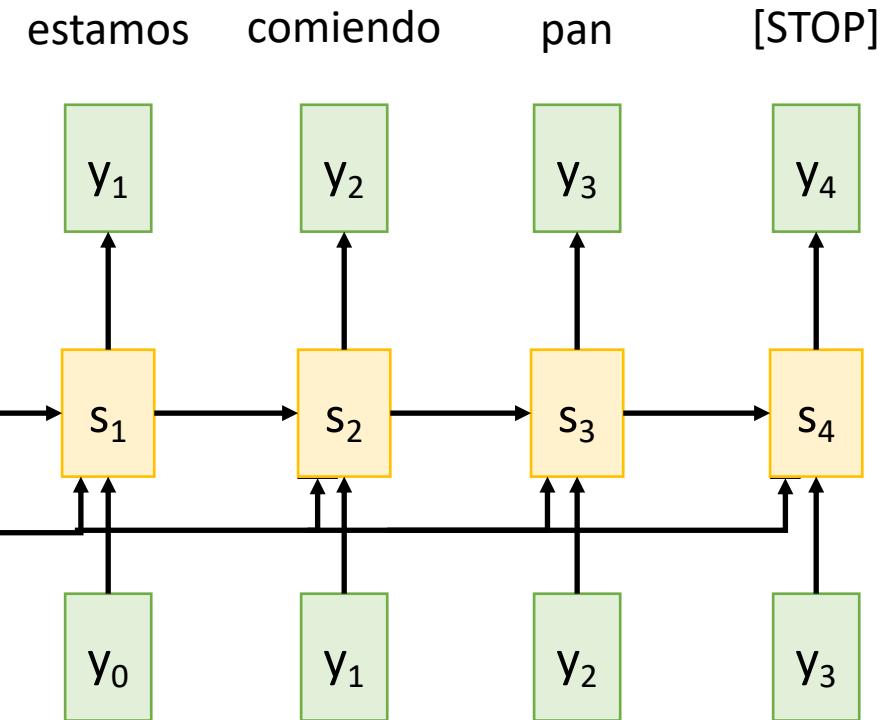
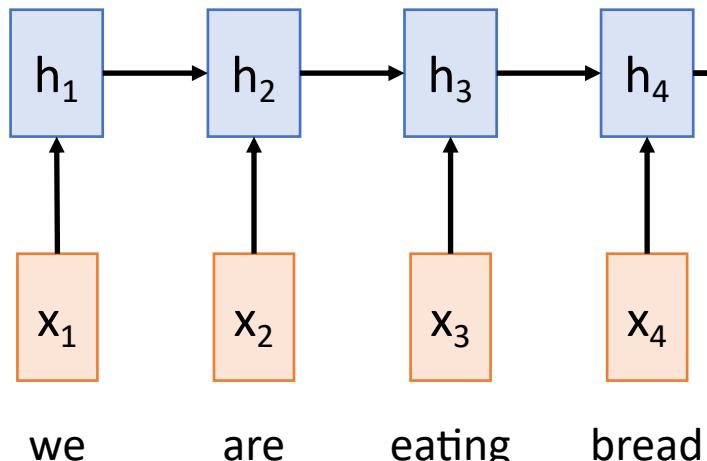
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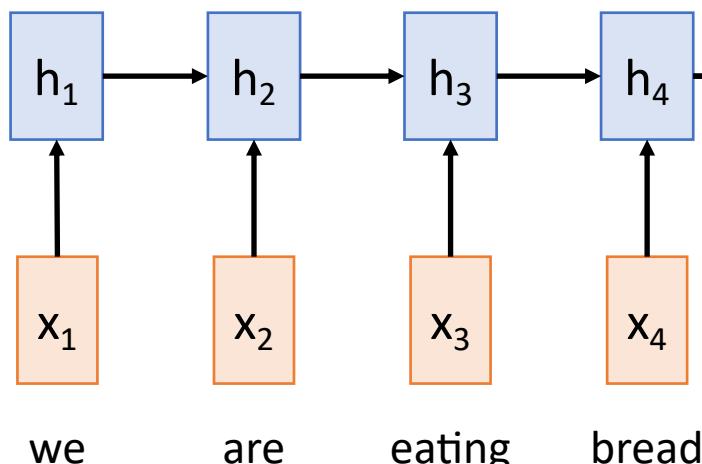
Problem: Input sequence
bottlenecked through fixed-
sized vector. What if $T=1000$?

Sequence-to-Sequence with RNNs

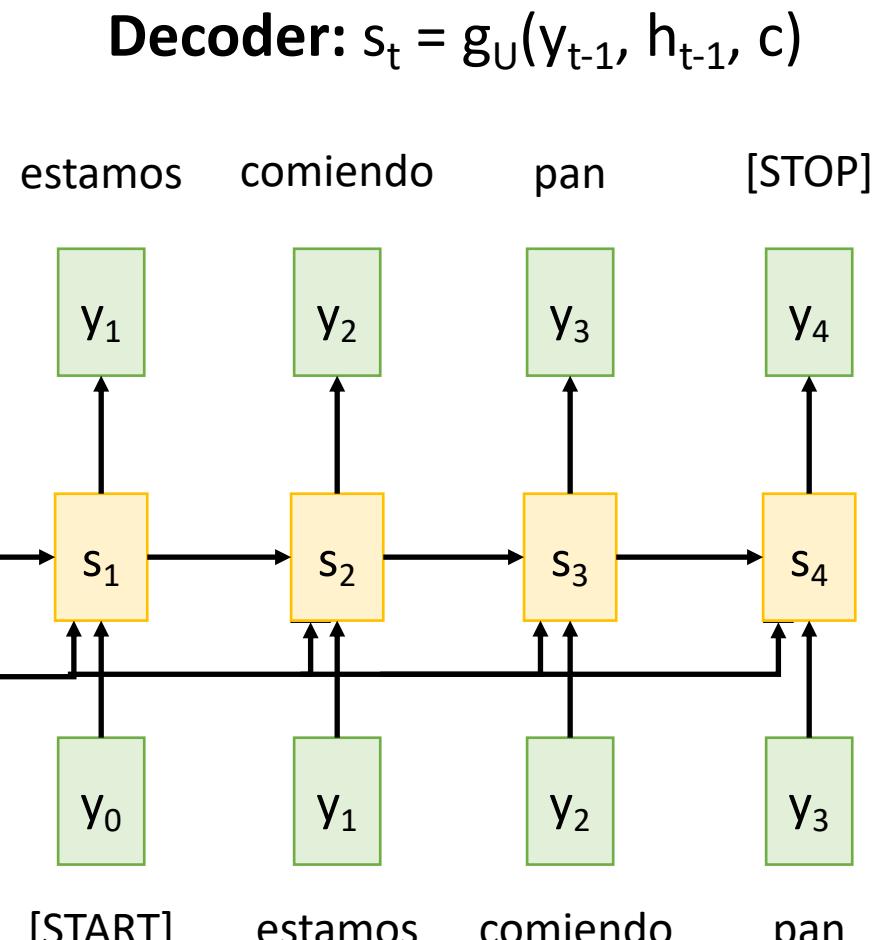
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Encoder: $h_t = f_w(x_t, h_{t-1})$



From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)



**Problem: Input sequence
bottlenecked through fixed-
sized vector. What if $T=1000$?**

**Idea: use new context vector
at each step of decoder!**

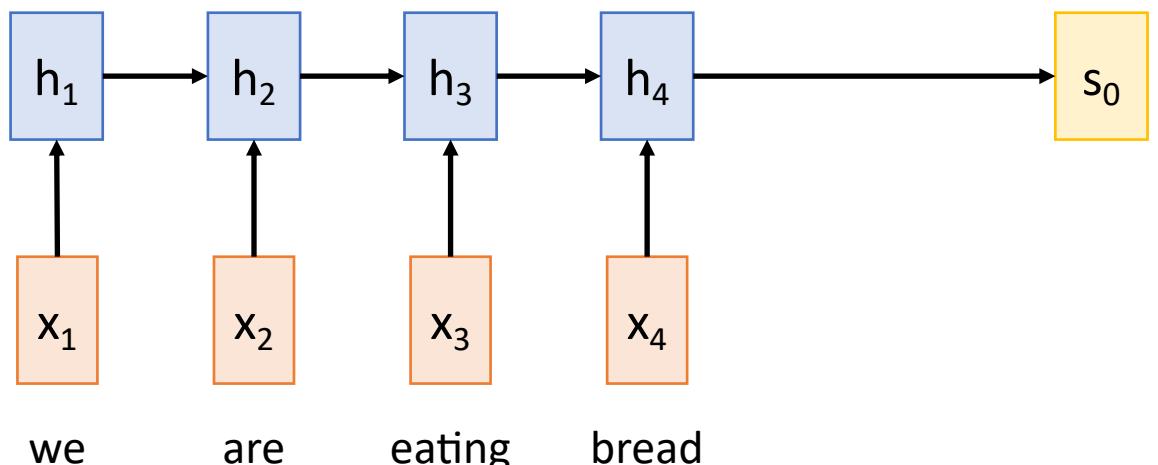
Sequence-to-Sequence with RNNs and Attention

Input: Sequence x_1, \dots, x_T

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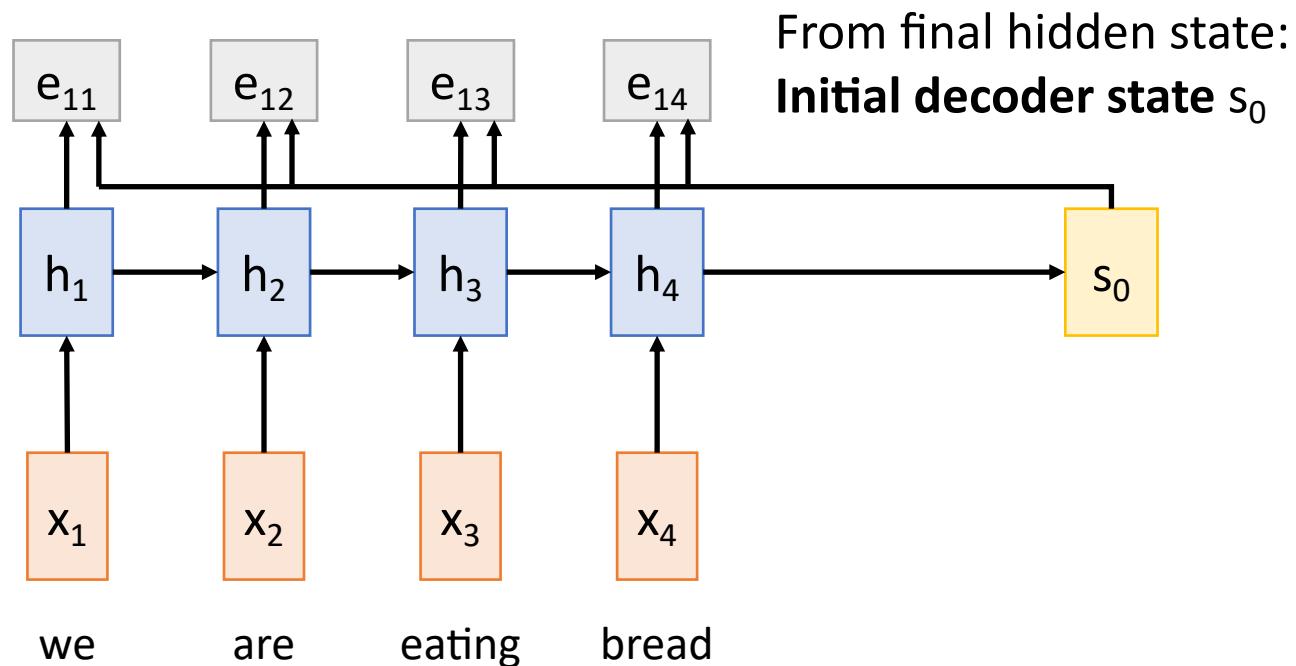
From final hidden state:
Initial decoder state s_0



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

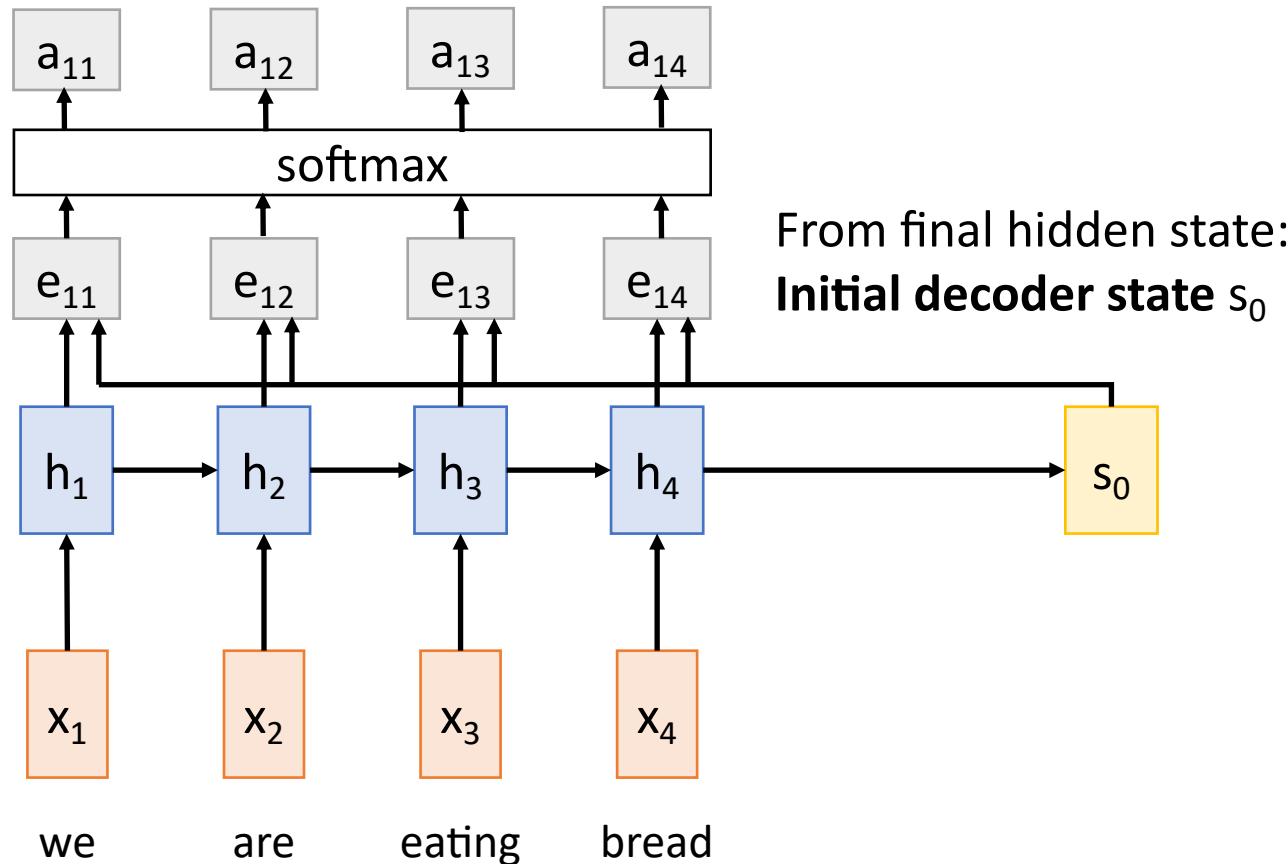
Sequence-to-Sequence with RNNs and Attention

Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

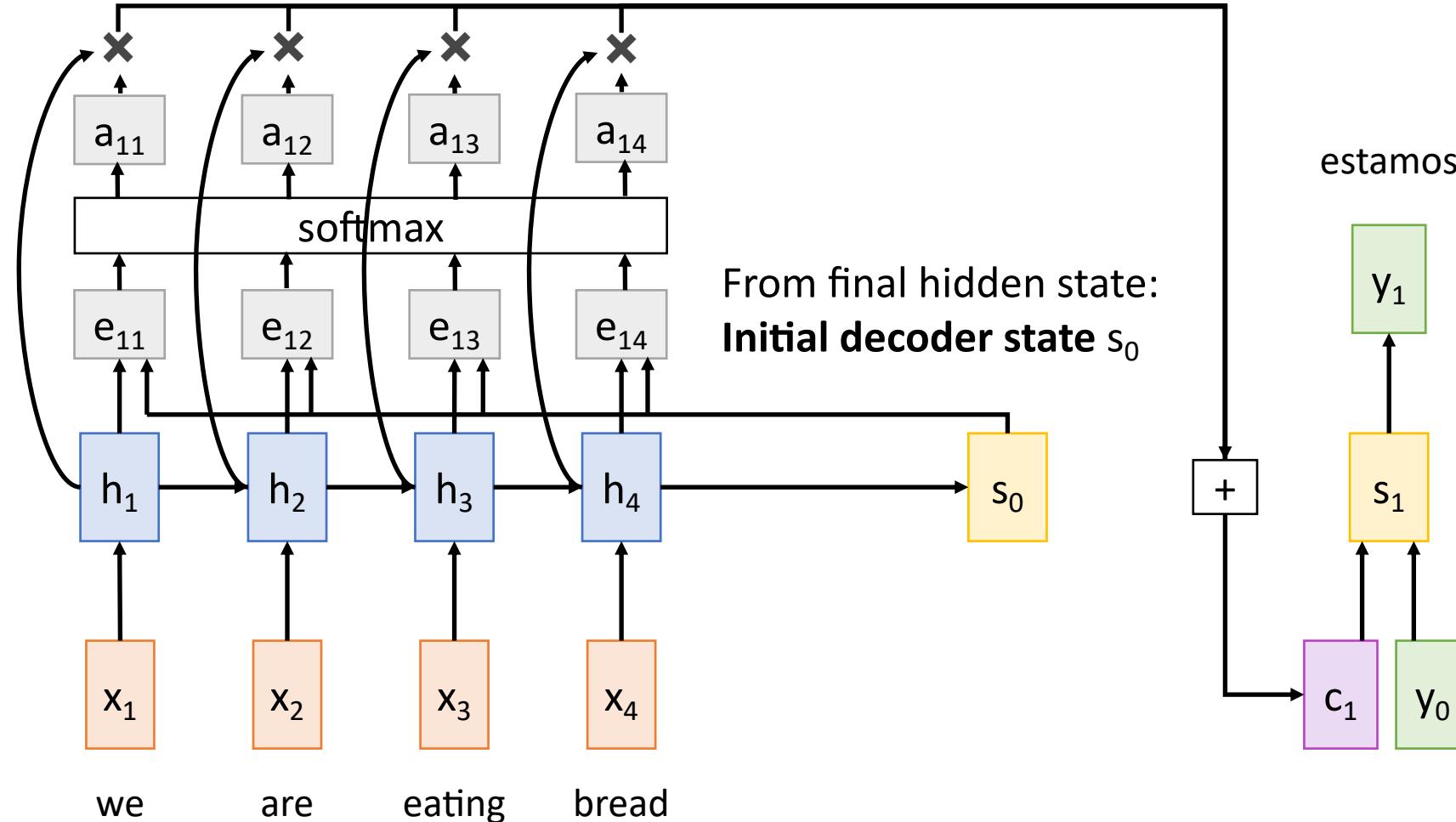
Sequence-to-Sequence with RNNs and Attention



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Normalize alignment scores
to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 0$

Sequence-to-Sequence with RNNs and Attention



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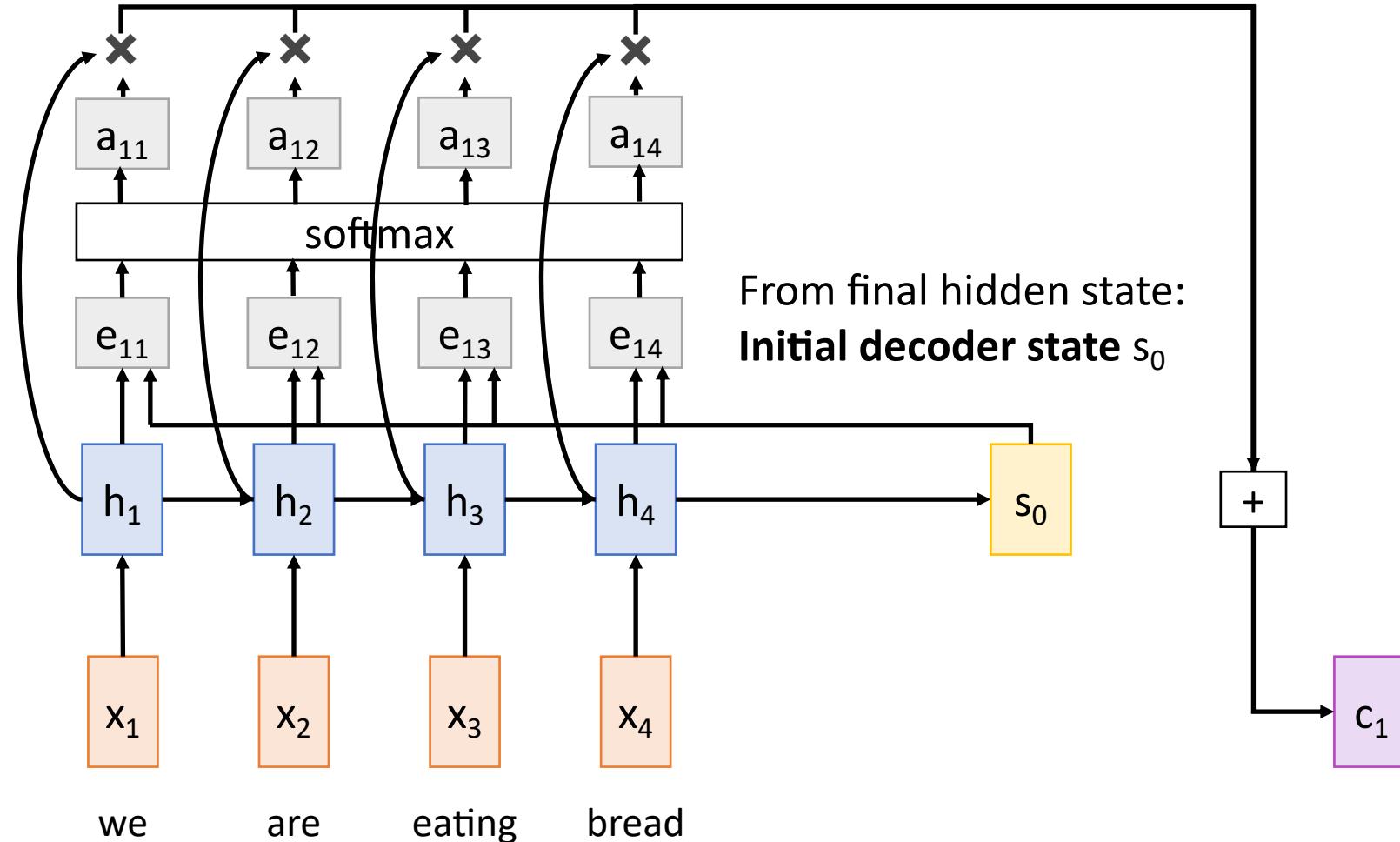
Normalize alignment scores
to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 0$

Compute context vector as linear
combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Use context vector in
decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

This is all differentiable! Do not
supervise attention weights –
backprop through everything

Sequence-to-Sequence with RNNs and Attention

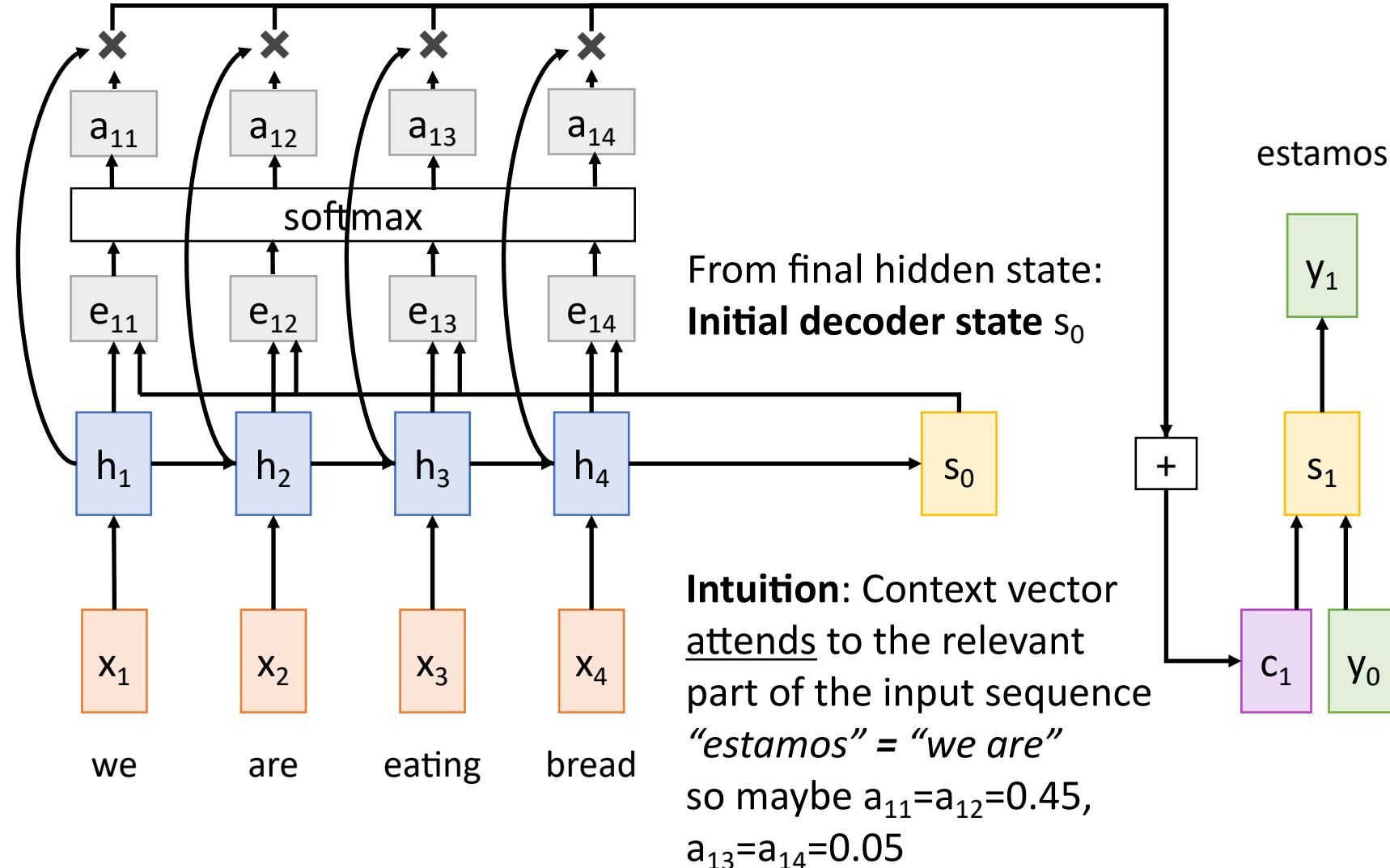


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Sequence-to-Sequence with RNNs and Attention



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Normalize alignment scores to get **attention weights**
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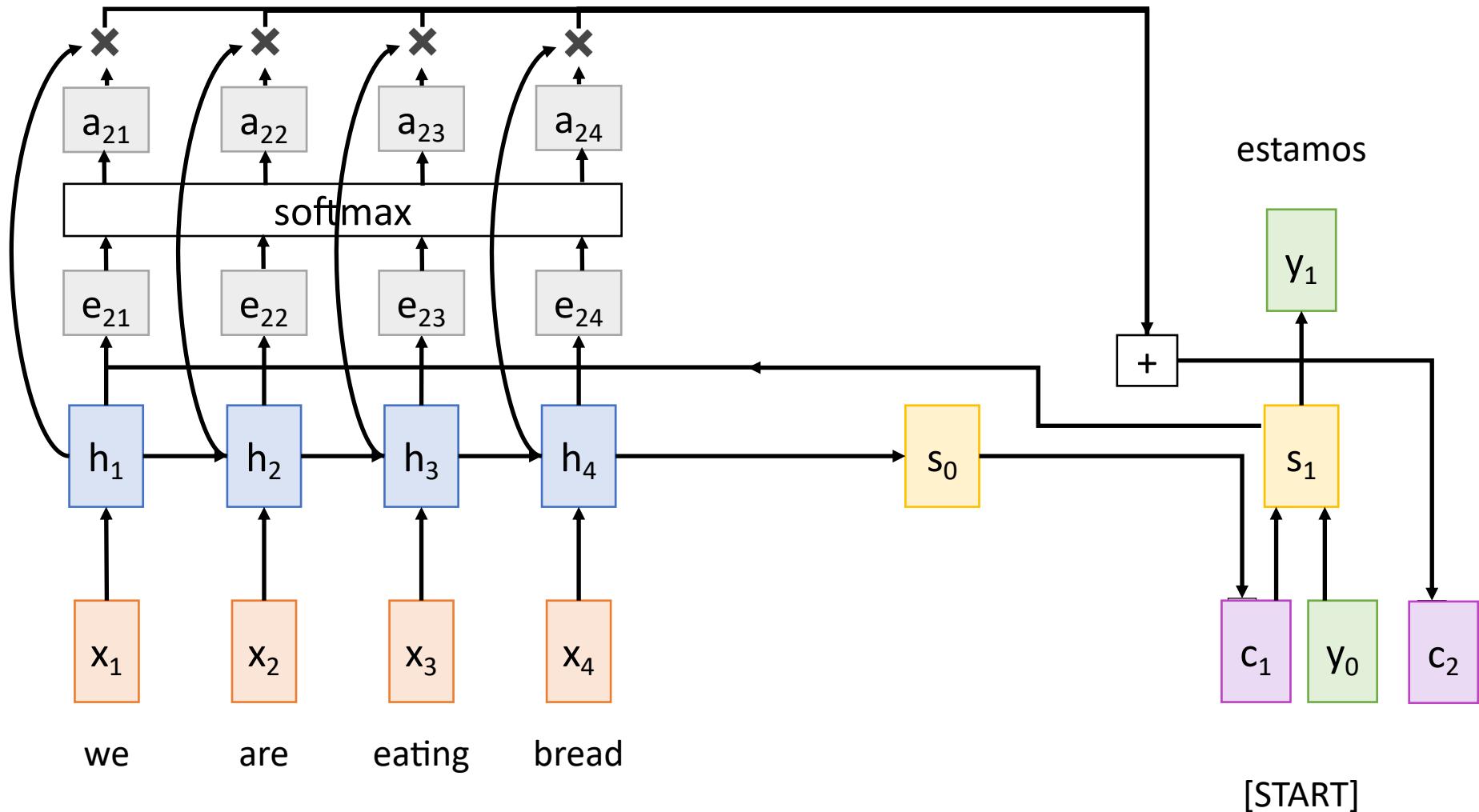
Compute context vector as linear combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Use context vector in decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

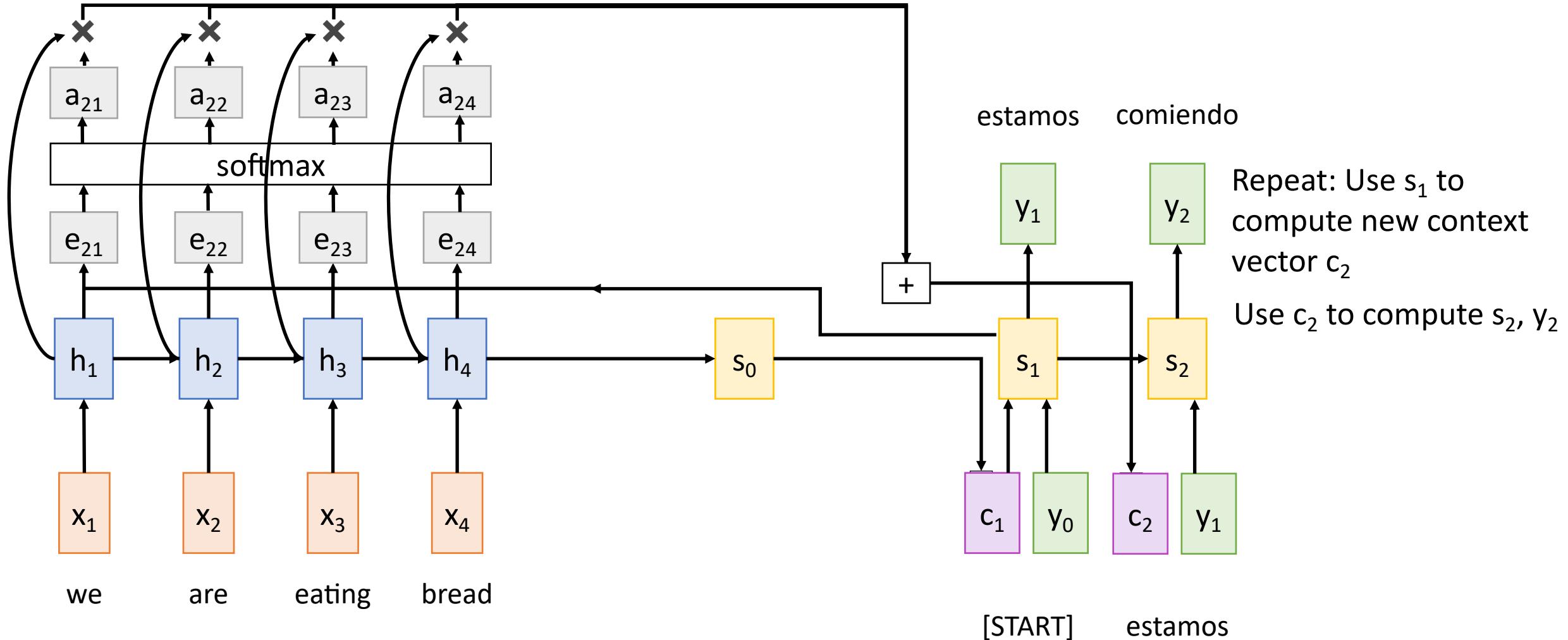
This is all differentiable! Do not supervise attention weights – backprop through everything

Sequence-to-Sequence with RNNs

Repeat: Use s_1 to compute new context vector c_2

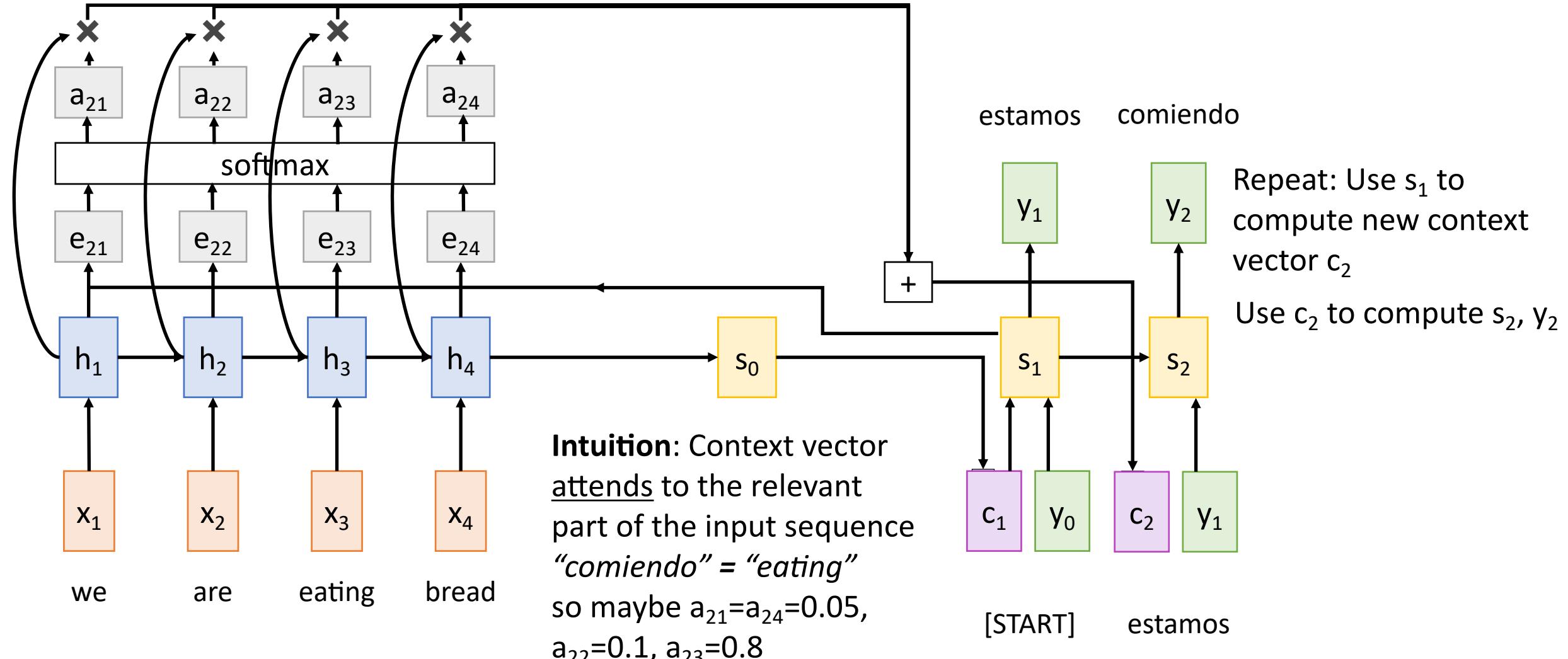


Sequence-to-Sequence with RNNs and Attention



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

Sequence-to-Sequence with RNNs and Attention

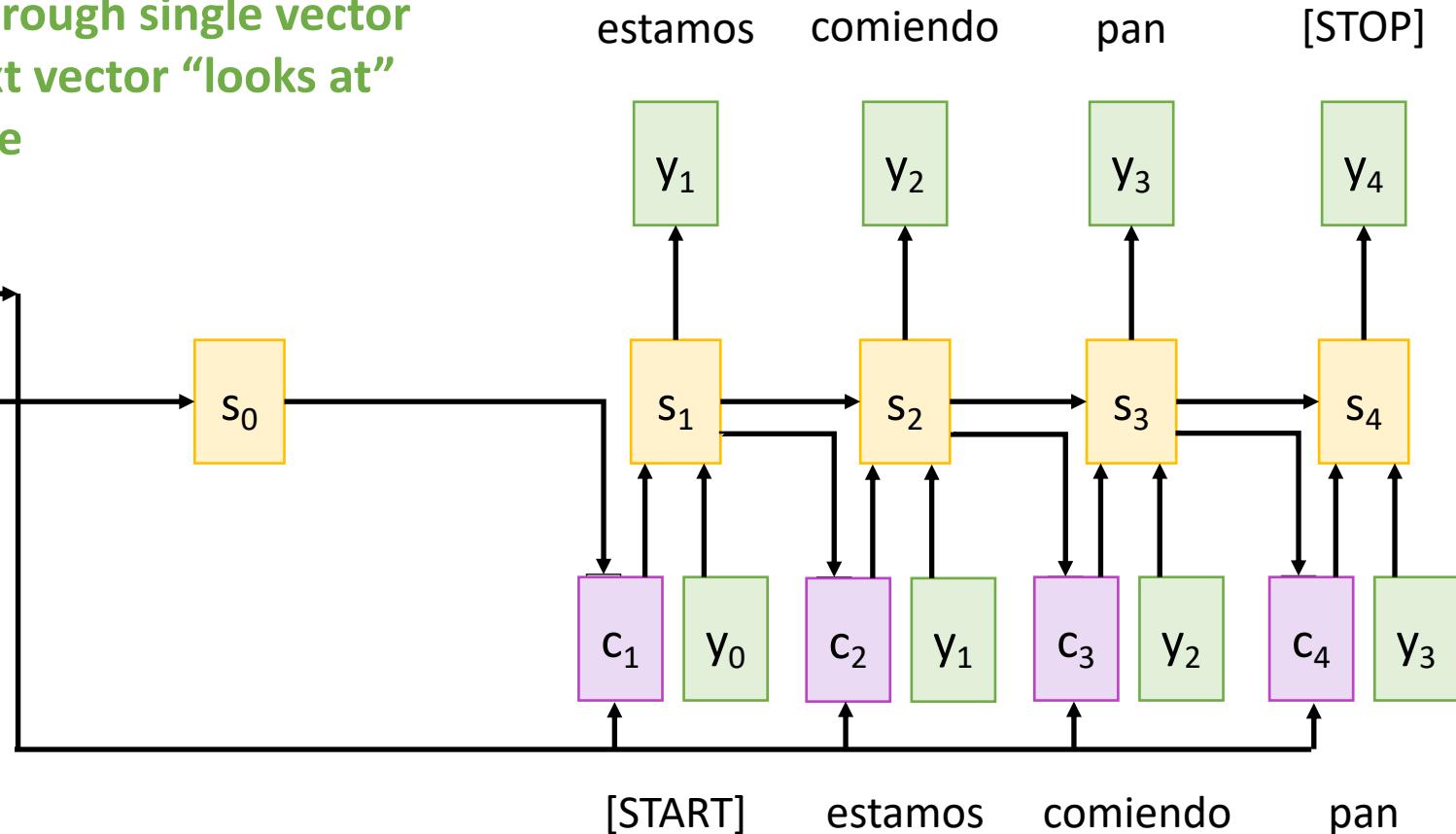
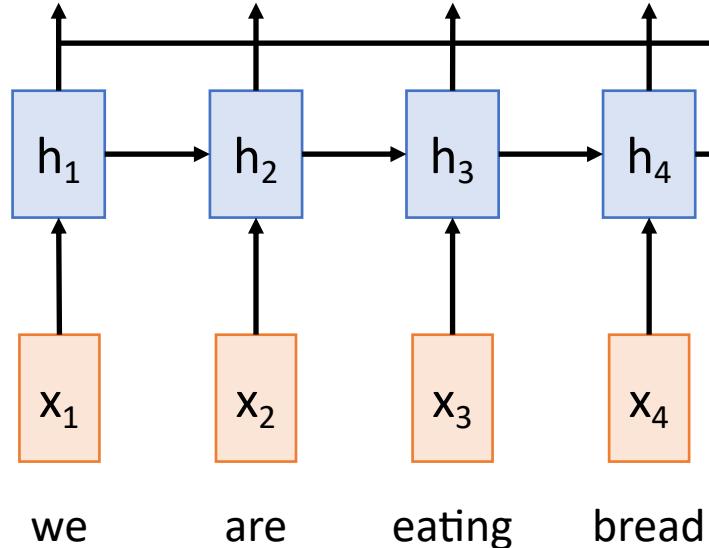


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Sequence-to-Sequence with RNNs and Attention

Use a different context vector in each timestep of decoder

- Input sequence not bottlenecked through single vector
- At each timestep of decoder, context vector “looks at” different parts of the input sequence



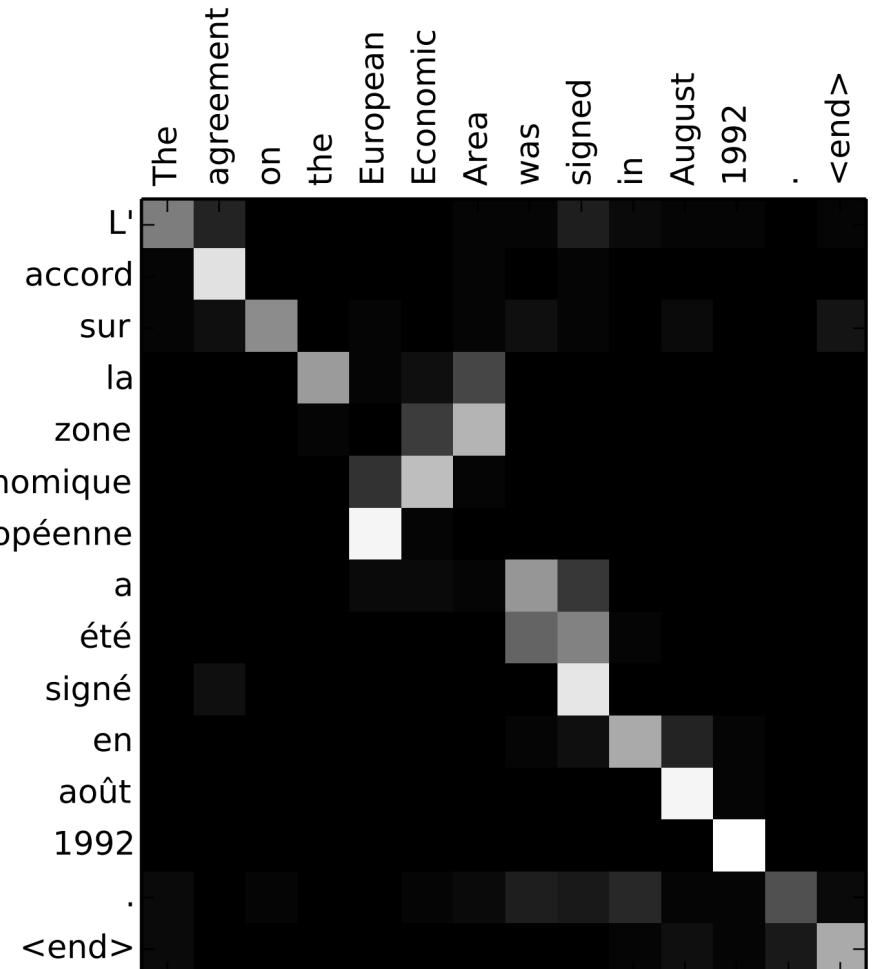
Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$



Bahdanau et al, “Neural machine translation by jointly learning to align and translate”, ICLR 2015

Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

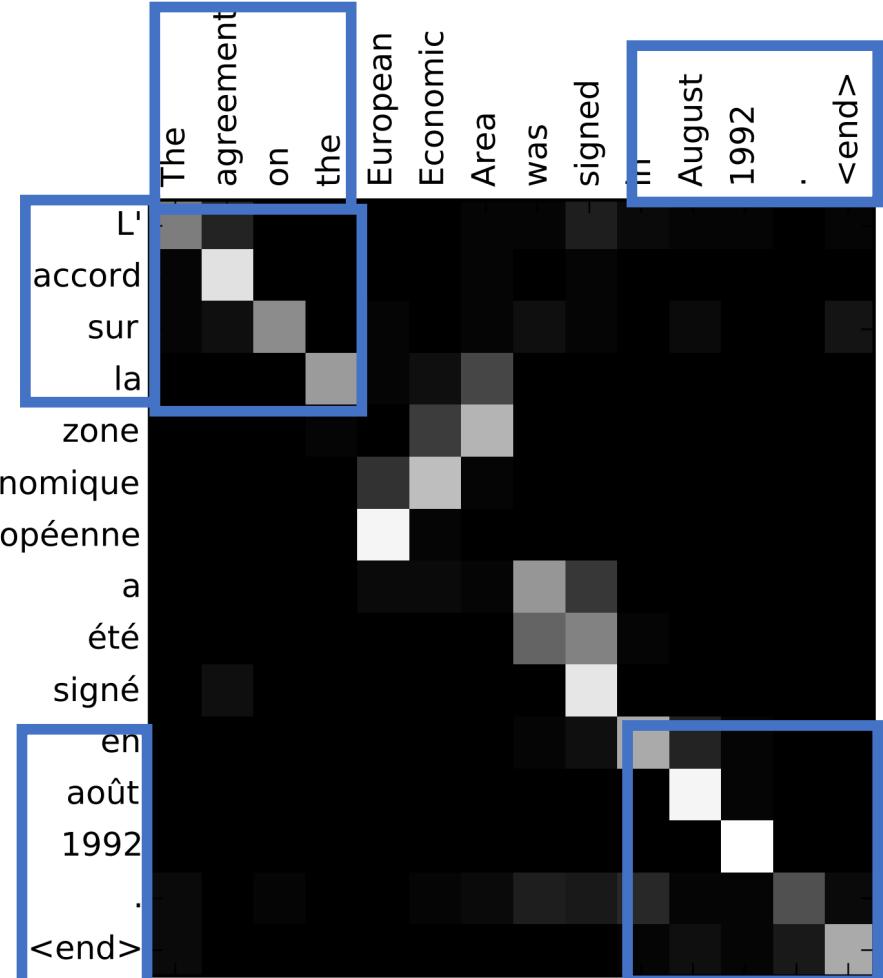
Input: “**The agreement on the European Economic Area was signed in August 1992.**”

Output: “**L'accord sur la zone économique européenne a été signé en août 1992.**”

Diagonal attention means words correspond in order

Diagonal attention means words correspond in order

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

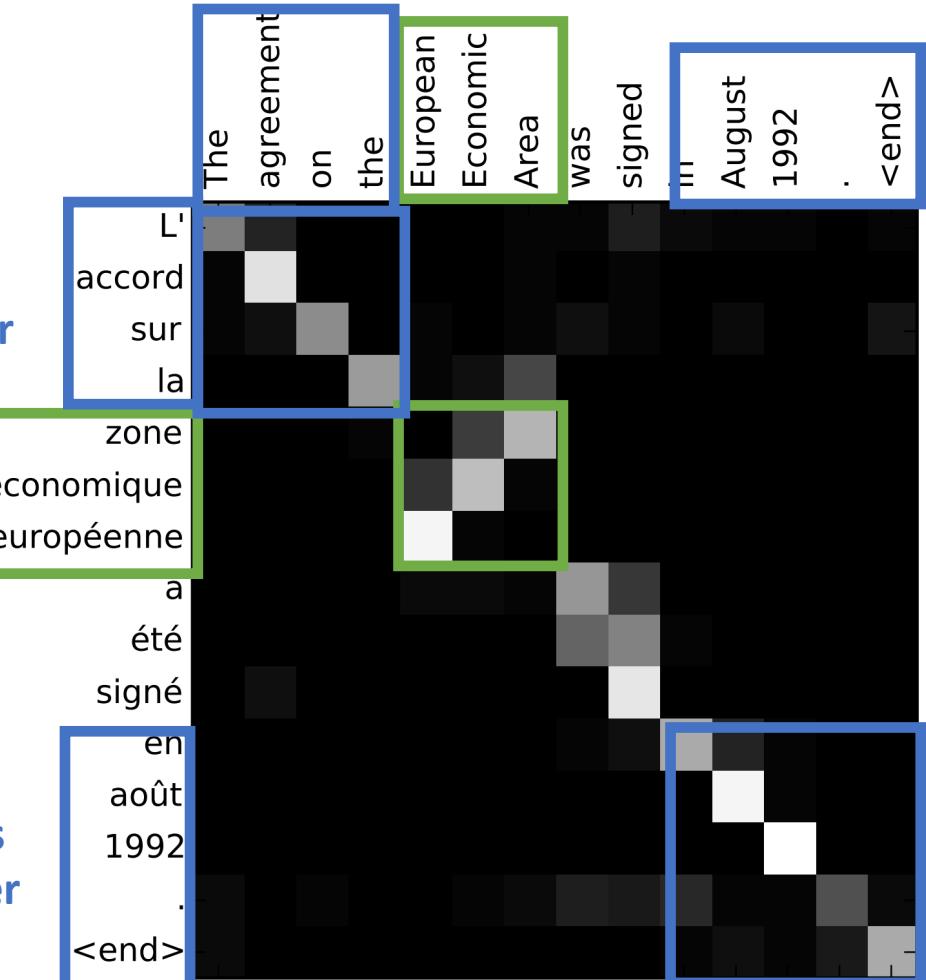
Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

Diagonal attention means words correspond in order

Attention figures out different word orders

Diagonal attention means words correspond in order

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

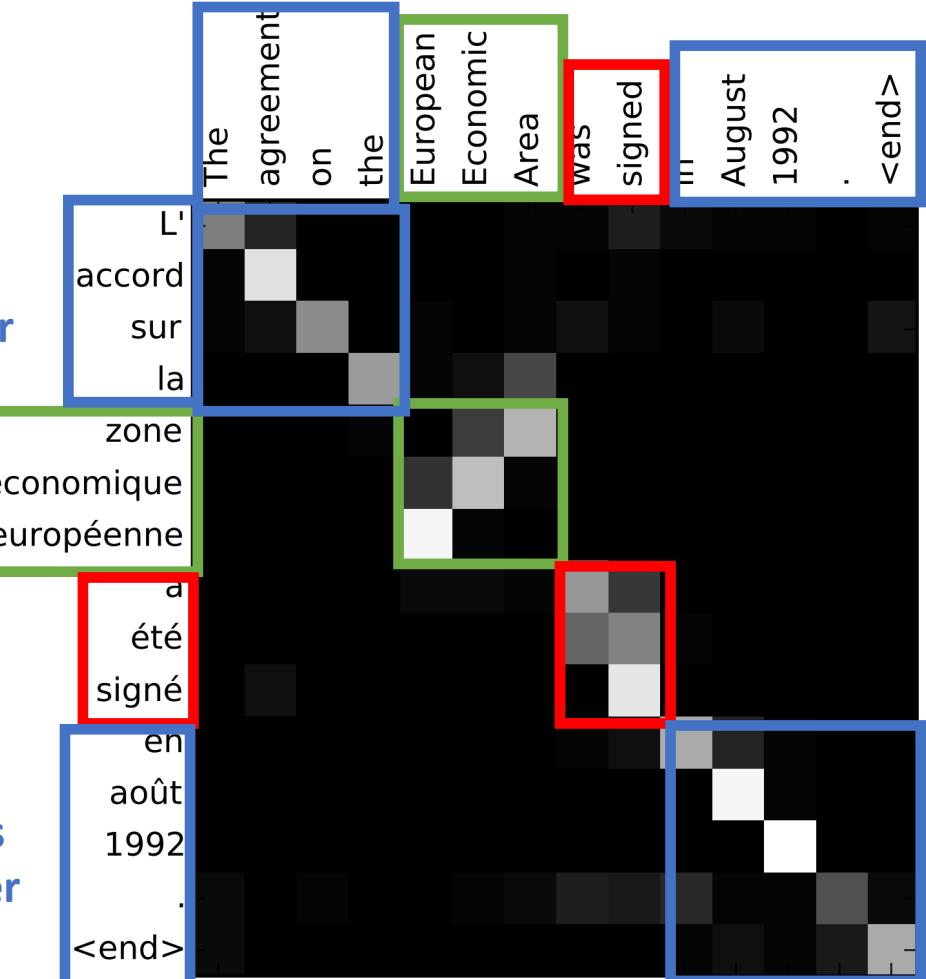
Diagonal attention means words correspond in order

Attention figures out different word orders

Verb conjugation

Diagonal attention means words correspond in order

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

The decoder doesn't use the fact that h_i form an ordered sequence – it just treats them as an unordered set $\{h_i\}$

Can use similar architecture given any set of input hidden vectors $\{h_i\}$!

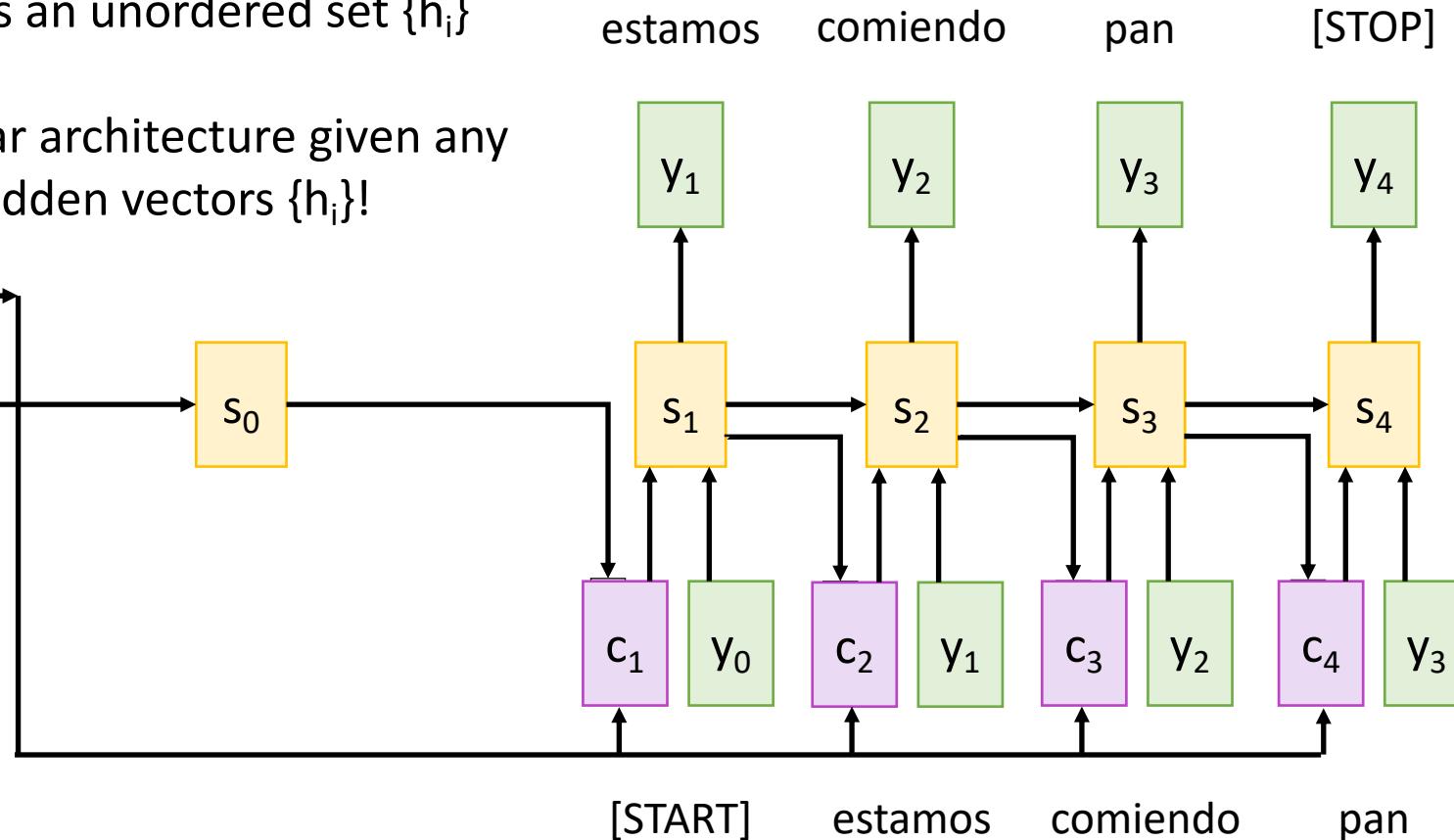
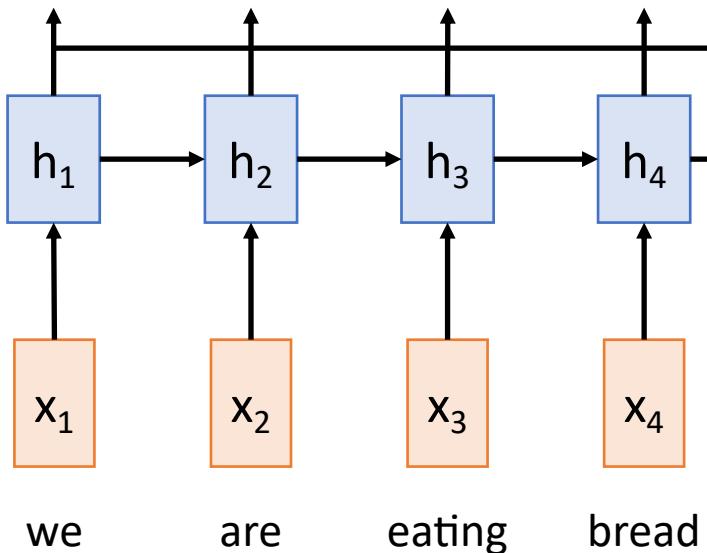
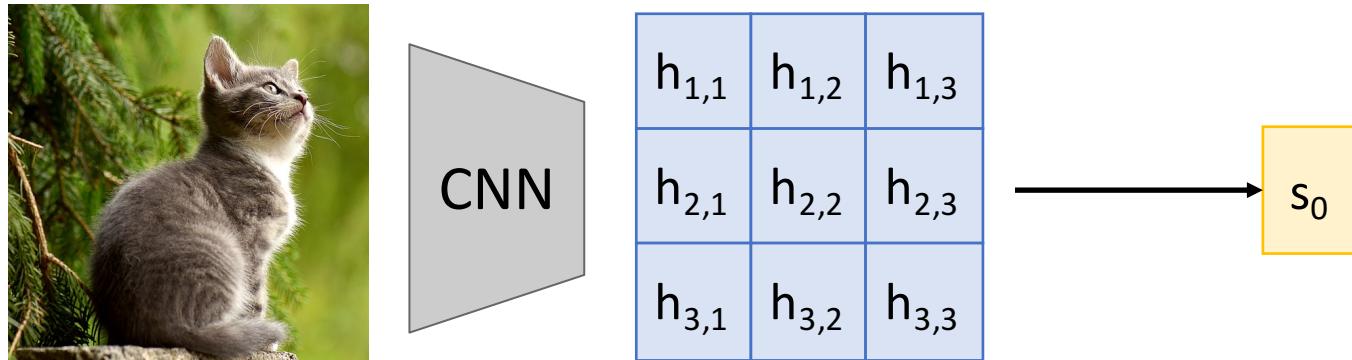


Image Captioning with RNNs and Attention



Use a CNN to compute a
grid of features for an image

[Cat image](#) is free to use under the [Pixabay License](#)

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$

Alignment scores

$e_{1,1,1}$	$e_{1,1,2}$	$e_{1,1,3}$
$e_{1,2,1}$	$e_{1,2,2}$	$e_{1,2,3}$
$e_{1,3,1}$	$e_{1,3,2}$	$e_{1,3,3}$



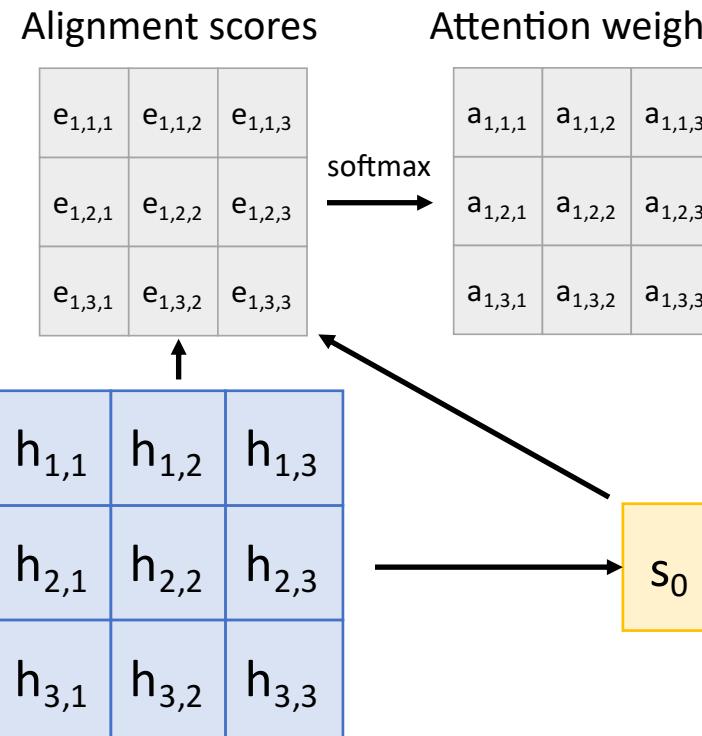
$h_{1,1}$	$h_{1,2}$	$h_{1,3}$
$h_{2,1}$	$h_{2,2}$	$h_{2,3}$
$h_{3,1}$	$h_{3,2}$	$h_{3,3}$



Use a CNN to compute a grid of features for an image

Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{att}(s_{t-1}, h_{i,j})$$
$$a_{t,:,:} = \text{softmax}(e_{t,:,:})$$



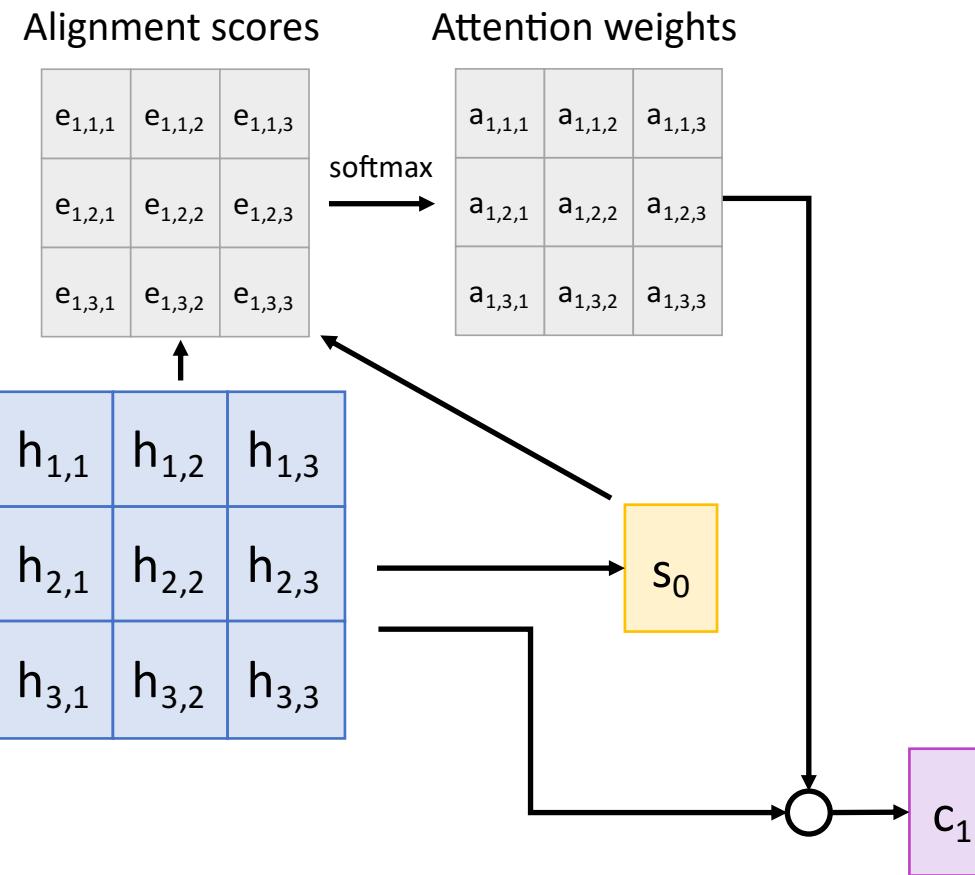
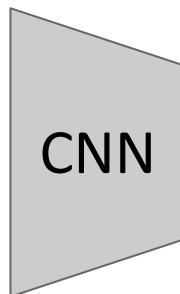
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$$e_{t,i,j} = f_{att}(s_{t-1}, h_{i,j})$$

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$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$



Use a CNN to compute a grid of features for an image

Image Captioning with RNNs and Attention

$$\begin{aligned} e_{t,i,j} &= f_{att}(s_{t-1}, h_{i,j}) \\ a_{t,:,:} &= \text{softmax}(e_{t,:,:}) \\ c_t &= \sum_{i,j} a_{t,i,j} h_{i,j} \end{aligned}$$



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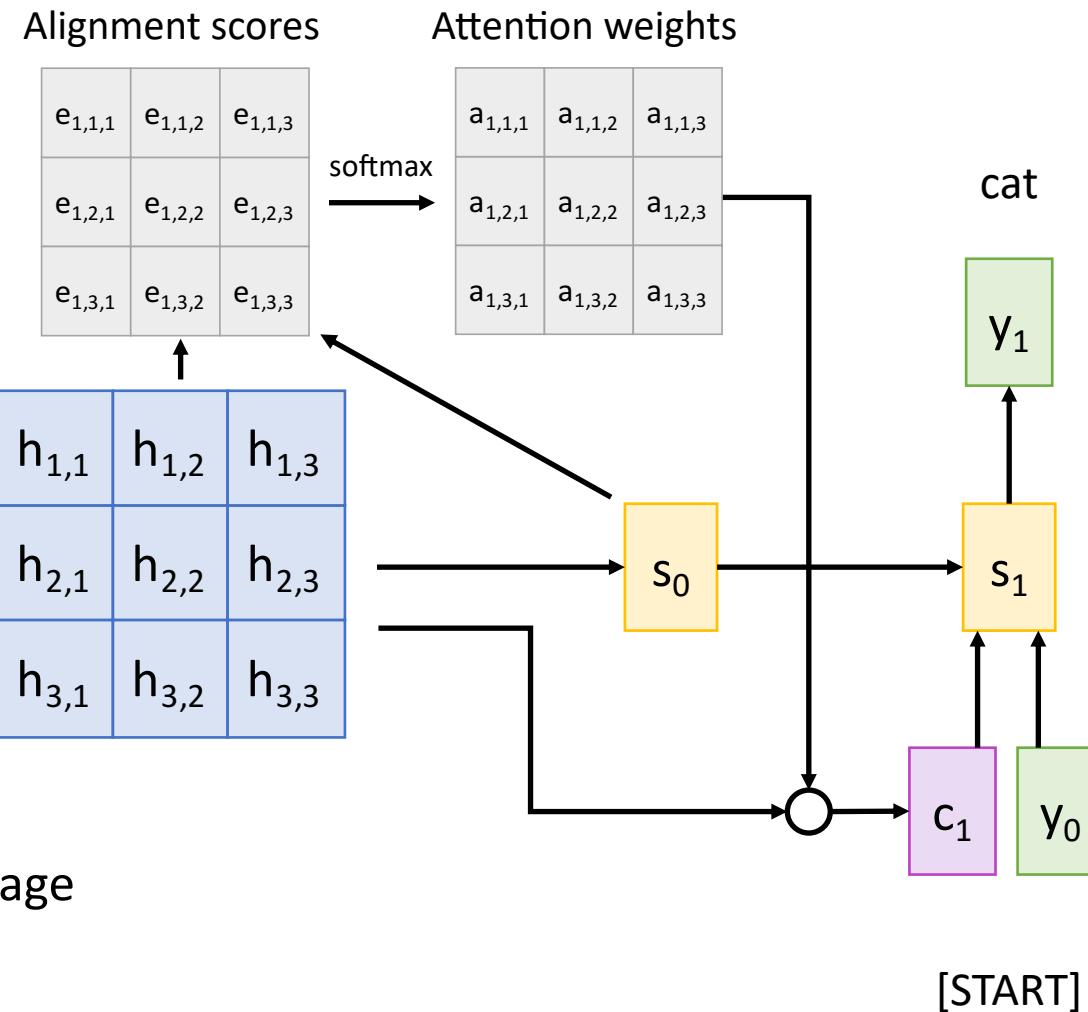
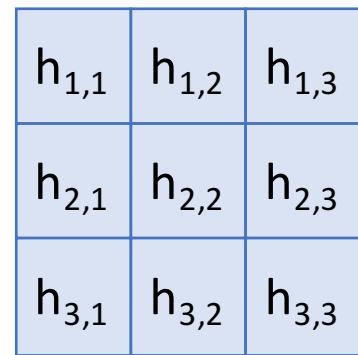
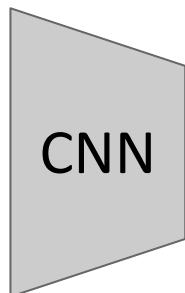


Image Captioning with RNNs and Attention

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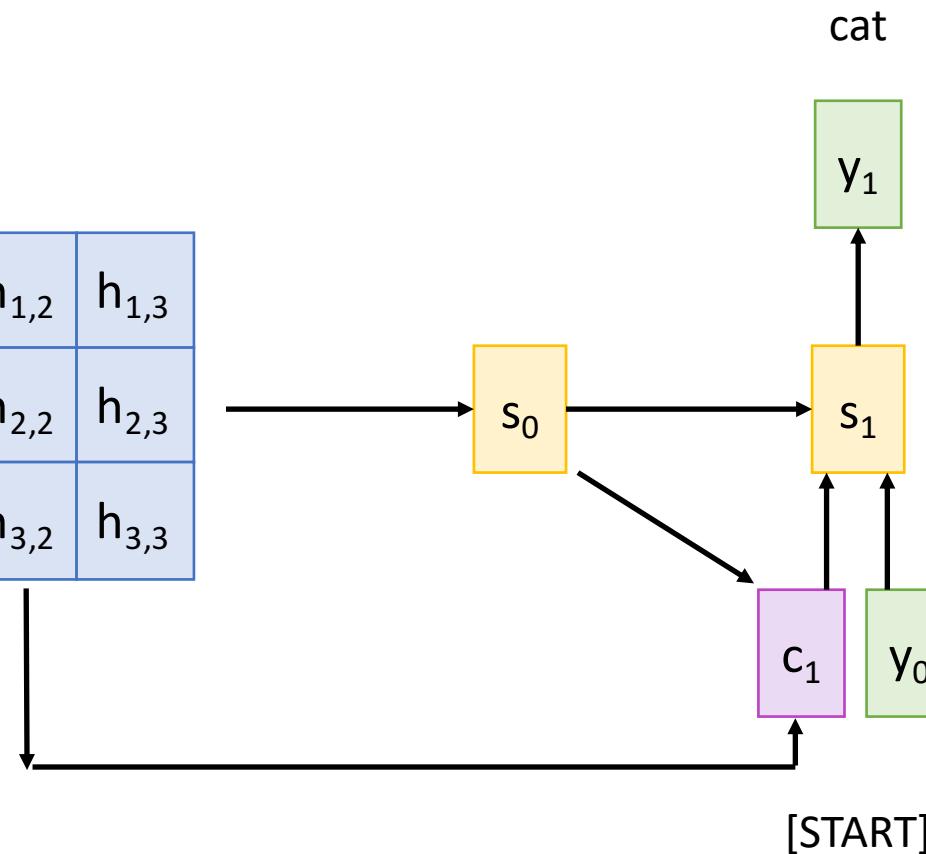


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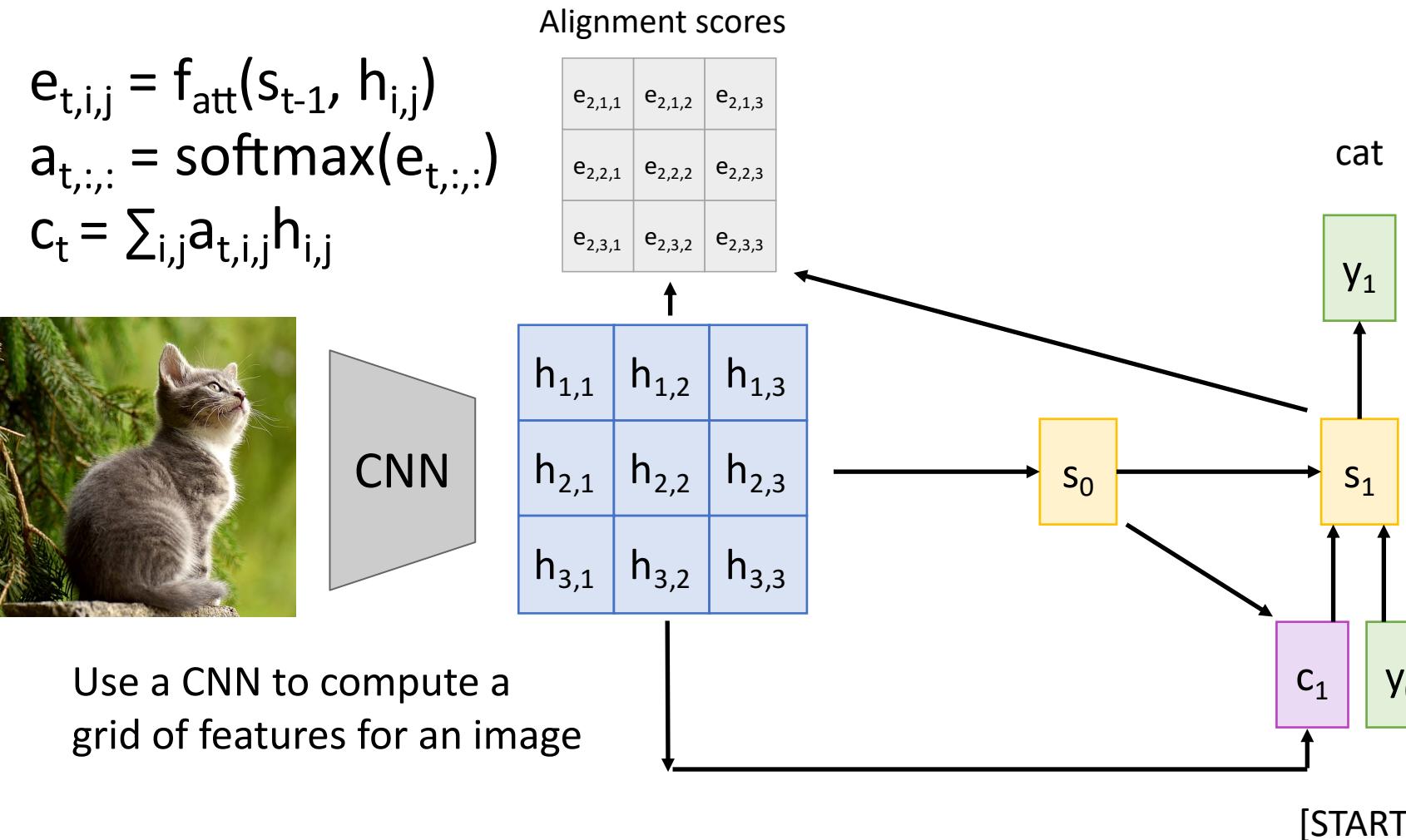


Image Captioning with RNNs and Attention

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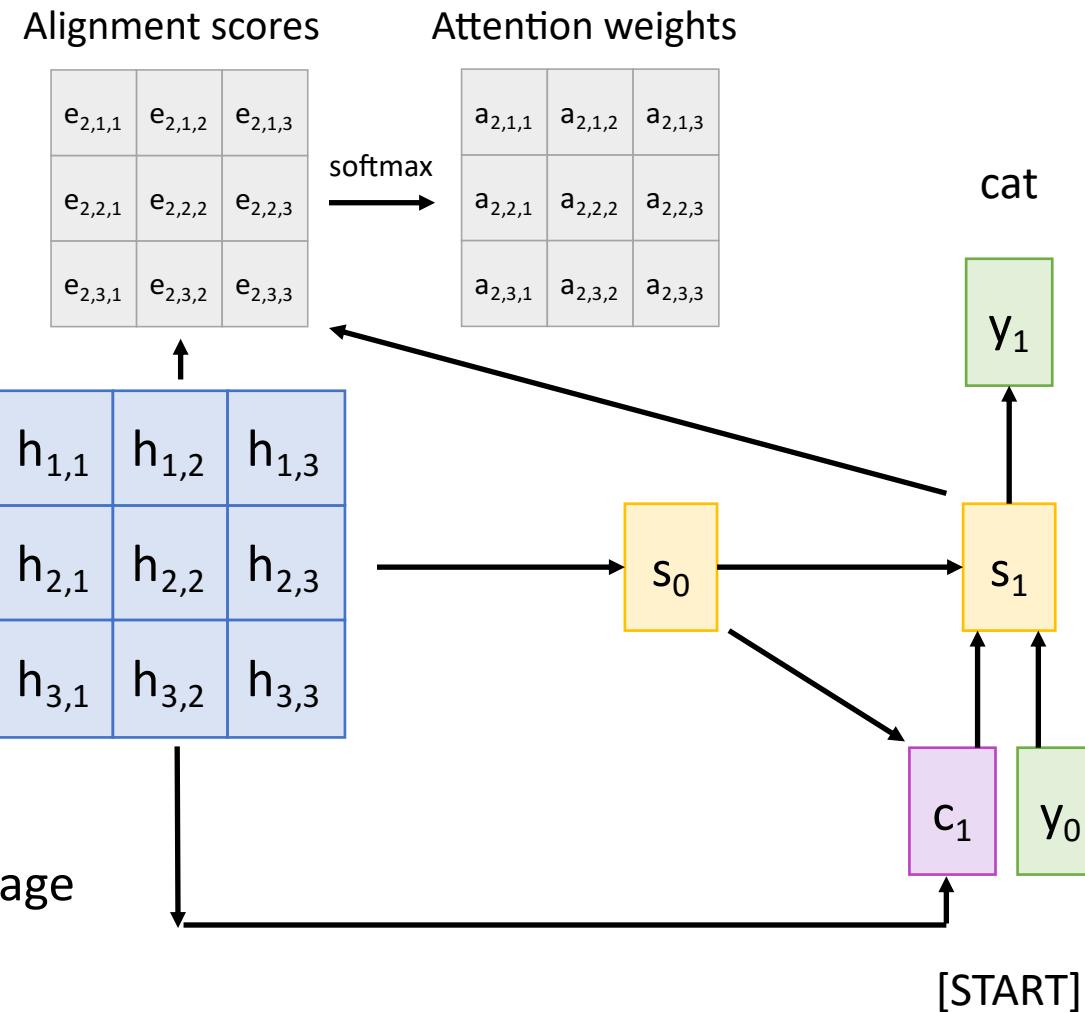
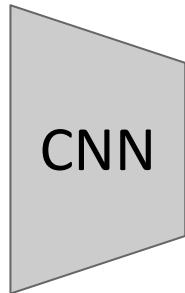
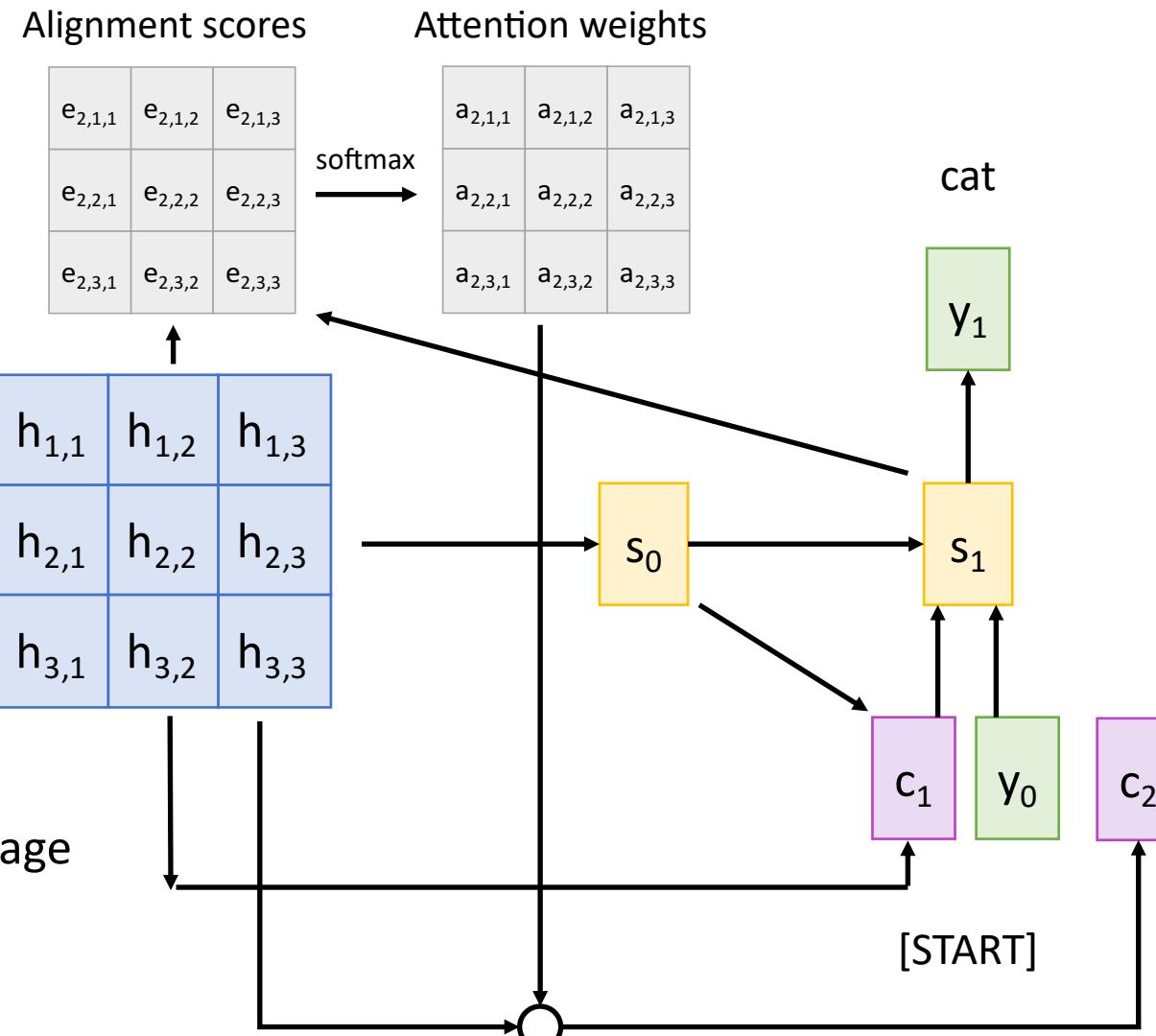


Image Captioning with RNNs and Attention

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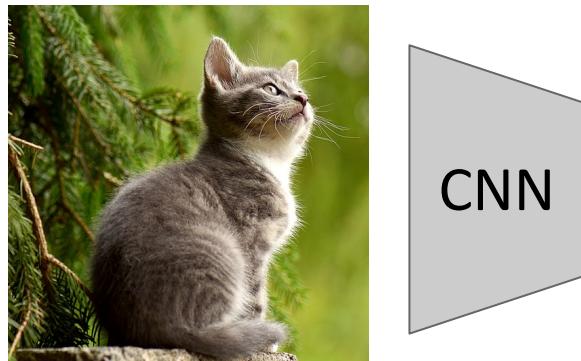
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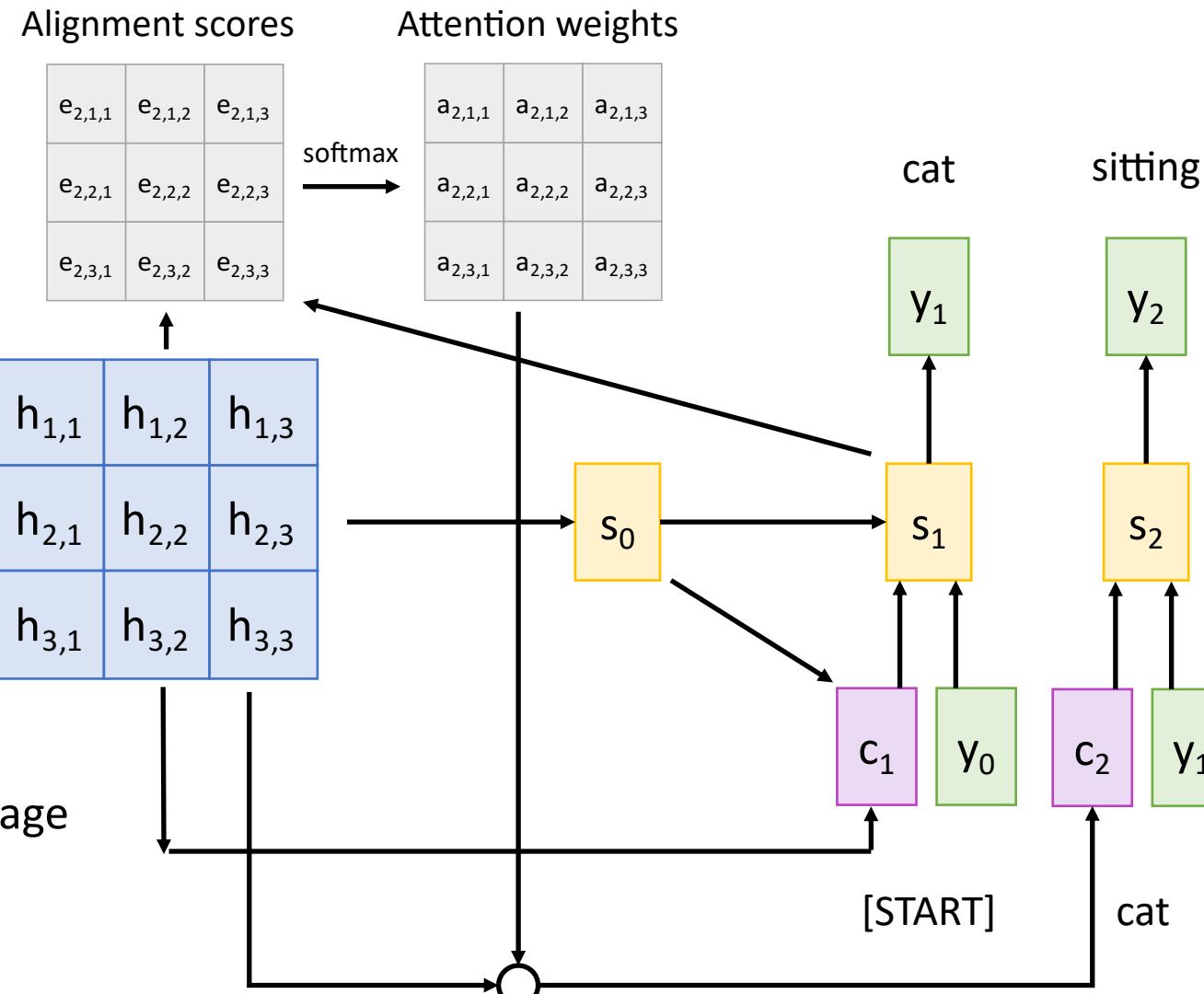
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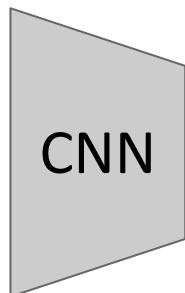
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Image Captioning with RNNs and Attention

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Use a CNN to compute a grid of features for an image

$h_{1,1}$	$h_{1,2}$	$h_{1,3}$
$h_{2,1}$	$h_{2,2}$	$h_{2,3}$
$h_{3,1}$	$h_{3,2}$	$h_{3,3}$

Each timestep of decoder uses a different context vector that looks at different parts of the input image

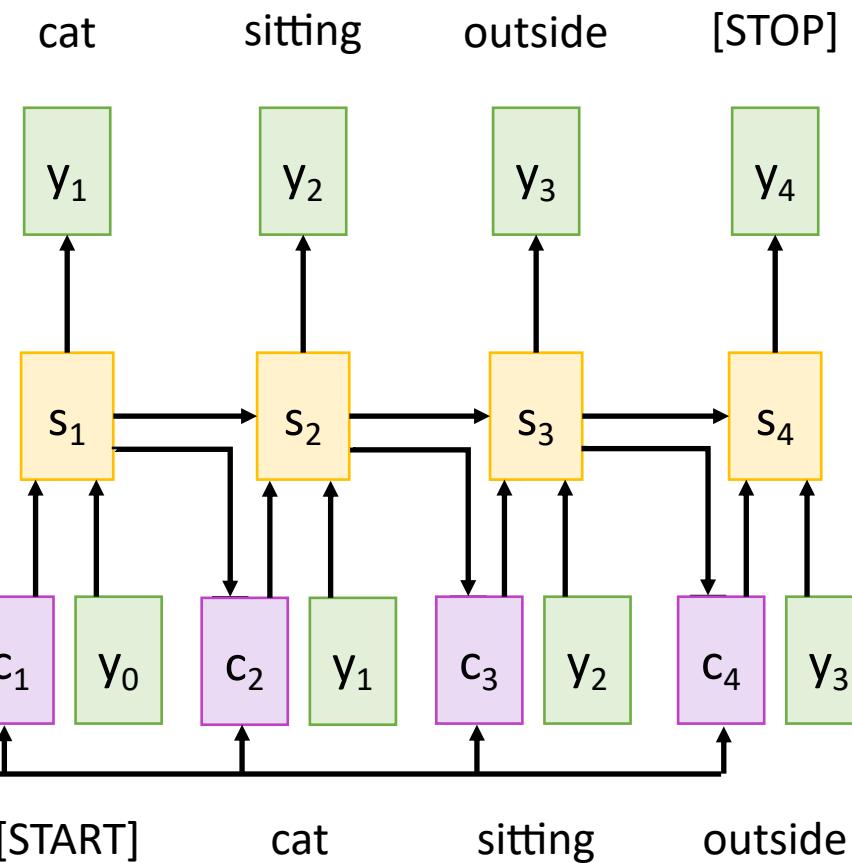


Image Captioning with RNNs and Attention

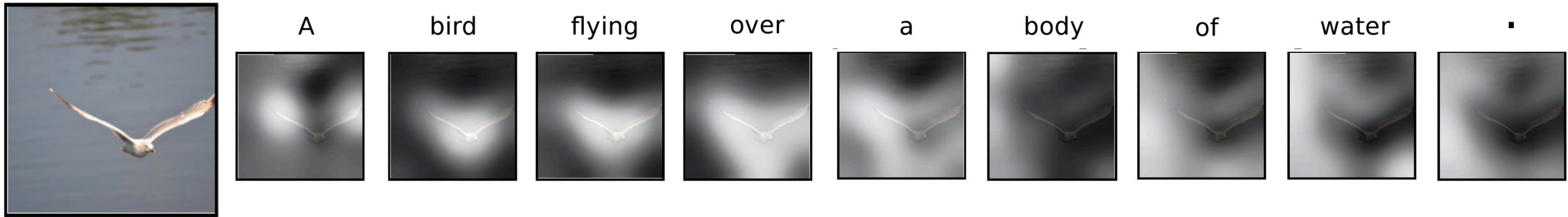


Image Captioning with RNNs and Attention



A dog is standing on a hardwood floor.



A stop sign is on a road with a mountain in the background.

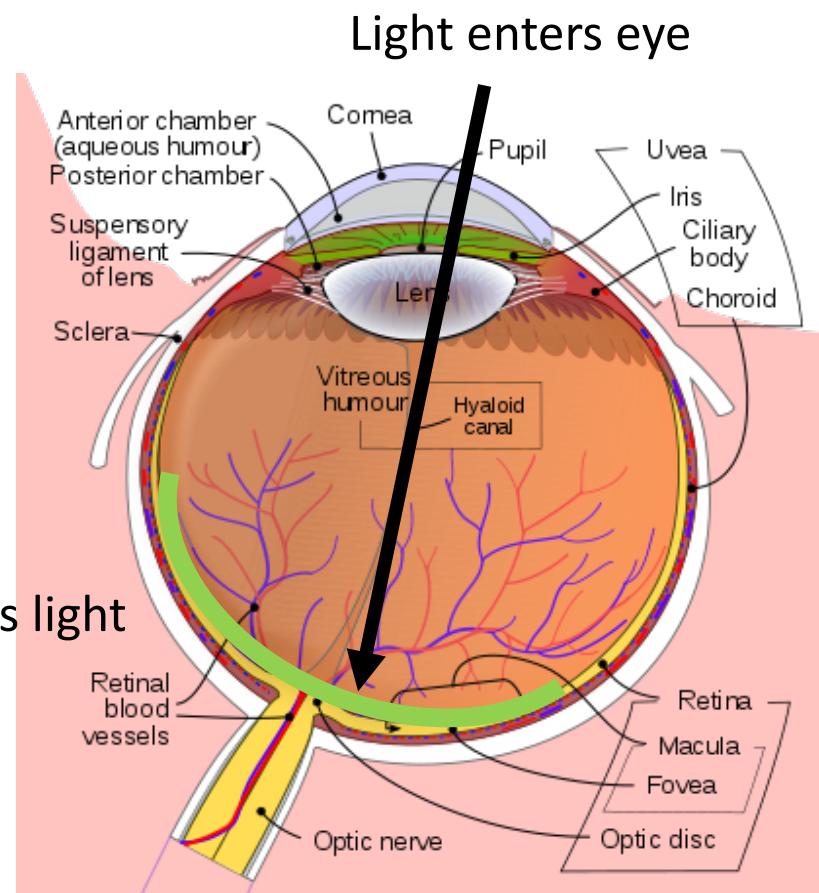


A group of people sitting on a boat in the water.



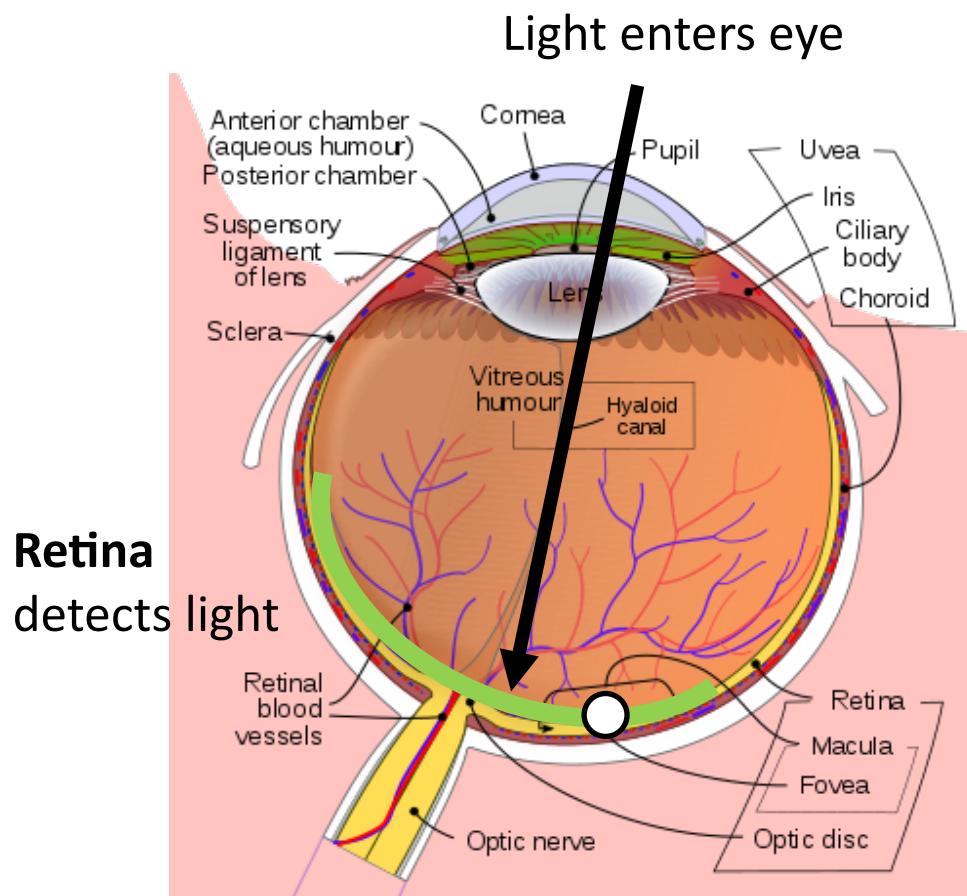
A giraffe standing in a forest with trees in the background.

Human Vision: Fovea



[Acuity graph](#) is licensed under [CC A-SA 3.0 Unported](#)

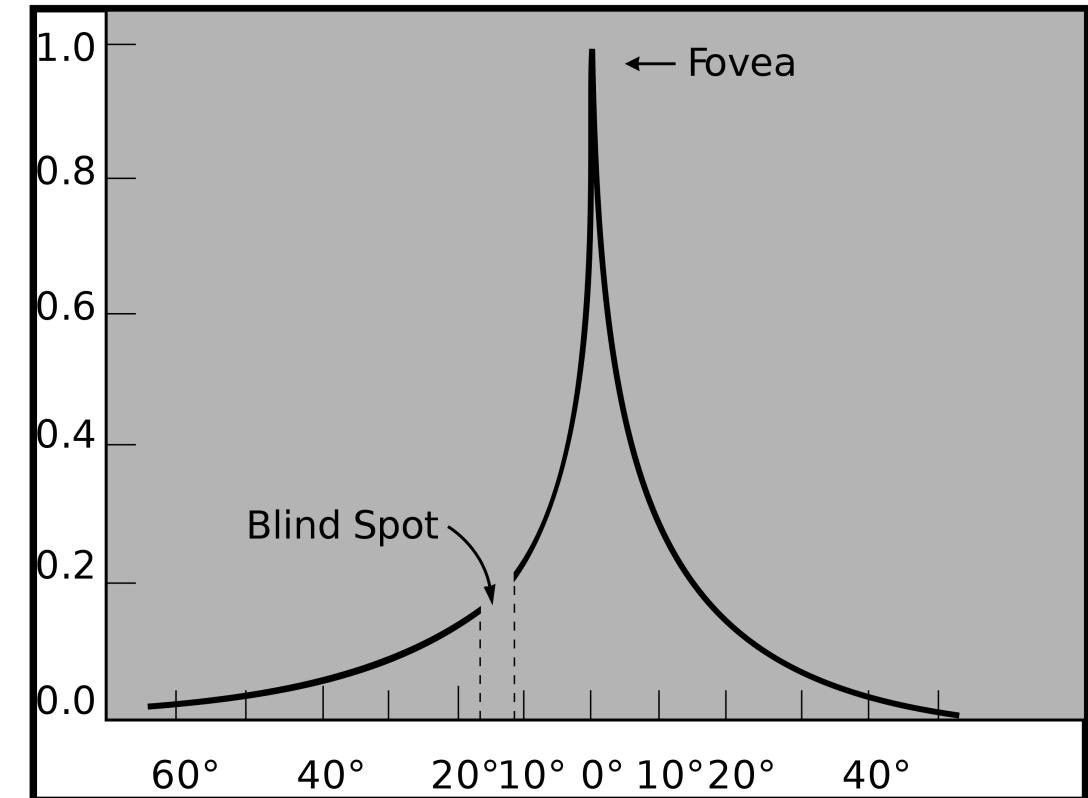
Human Vision: Fovea



Light enters eye

Retina
detects light

The **fovea** is a tiny region of the retina that can see with high acuity



Human Vision: Saccades

Human eyes are constantly moving so we don't notice



The **fovea** is a tiny region of the retina that can see with high acuity

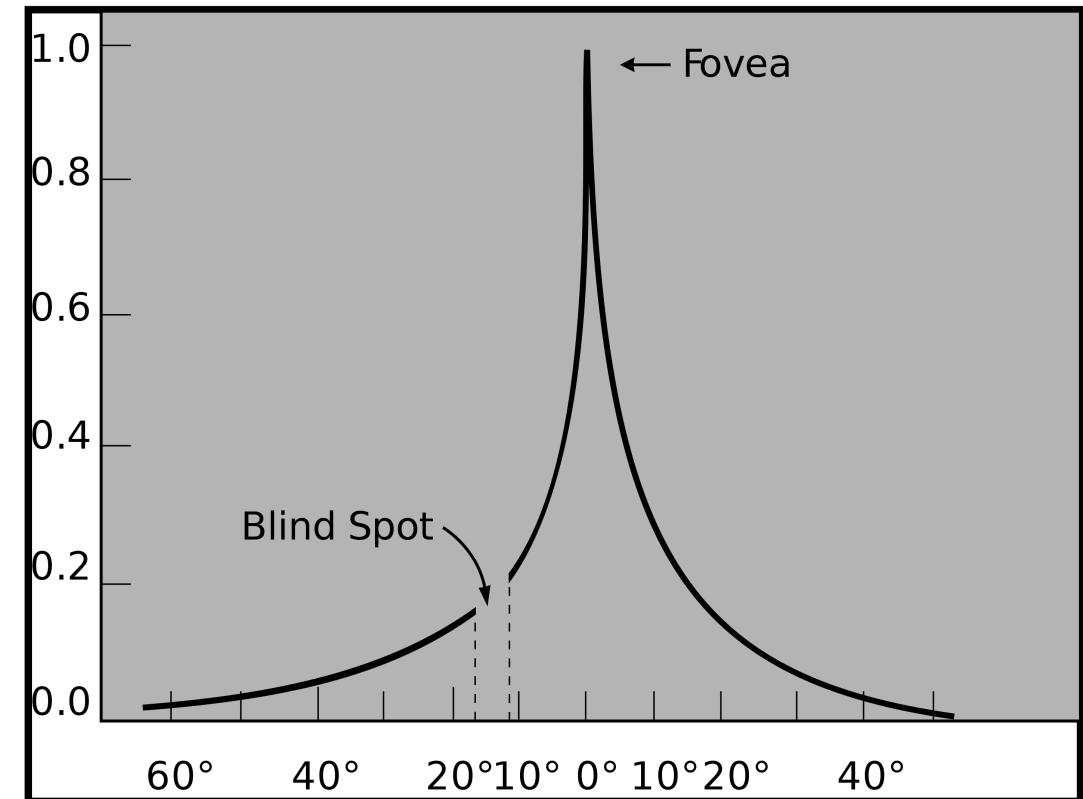
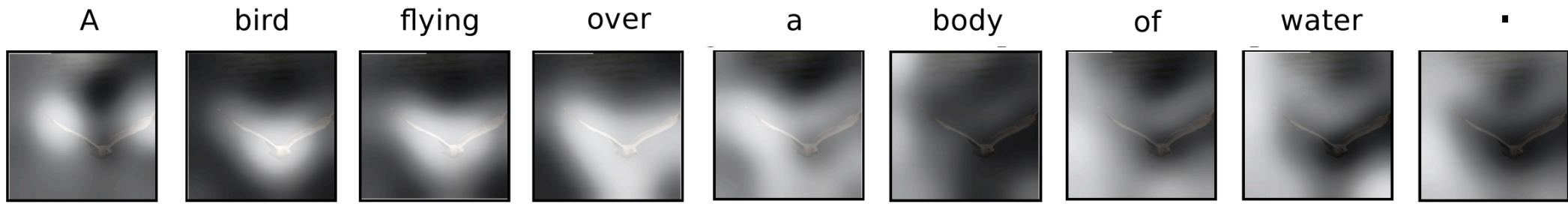


Image Captioning with RNNs and Attention



Attention weights at each timestep kind of like saccades of human eye



X, Attend, and Y

“Show, attend, and tell” (*Xu et al, ICML 2015*)

Look at image, attend to image regions, produce question

“Ask, attend, and answer” (*Xu and Saenko, ECCV 2016*)

“Show, ask, attend, and answer” (*Kazemi and Elqursh, 2017*)

Read text of question, attend to image regions, produce answer

“Listen, attend, and spell” (*Chan et al, ICASSP 2016*)

Process raw audio, attend to audio regions while producing text

“Listen, attend, and walk” (*Mei et al, AAAI 2016*)

Process text, attend to text regions, output navigation commands

“Show, attend, and interact” (*Qureshi et al, ICRA 2017*)

Process image, attend to image regions, output robot control commands

“Show, attend, and read” (*Li et al, AAAI 2019*)

Process image, attend to image regions, output text

Attention Layer

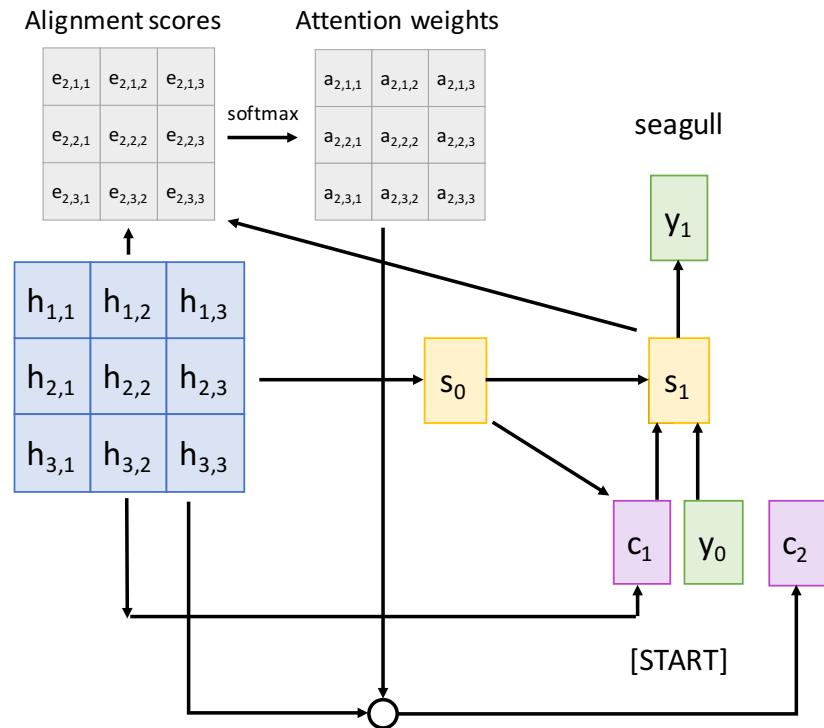
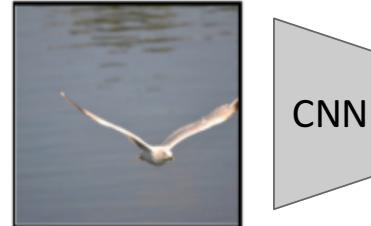
Inputs:

Query vector: q (Shape: D_Q)

Input vectors: X (Shape: $N_x \times D_x$)

Similarity function: f_{att}

$$e_{t,i,j} = f_{att}(s_{t-1}, h_{i,j})$$
$$a_{t,:,:} = \text{softmax}(e_{t,:,:})$$
$$c_t = \sum_i a_{t,i,j} h_{i,j}$$



Computation:

Similarities: e (Shape: N_x) $e_i = f_{att}(q, X_i)$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_x)

Output vector: $y = \sum_i a_i X_i$ (Shape: D_x)

Attention Layer

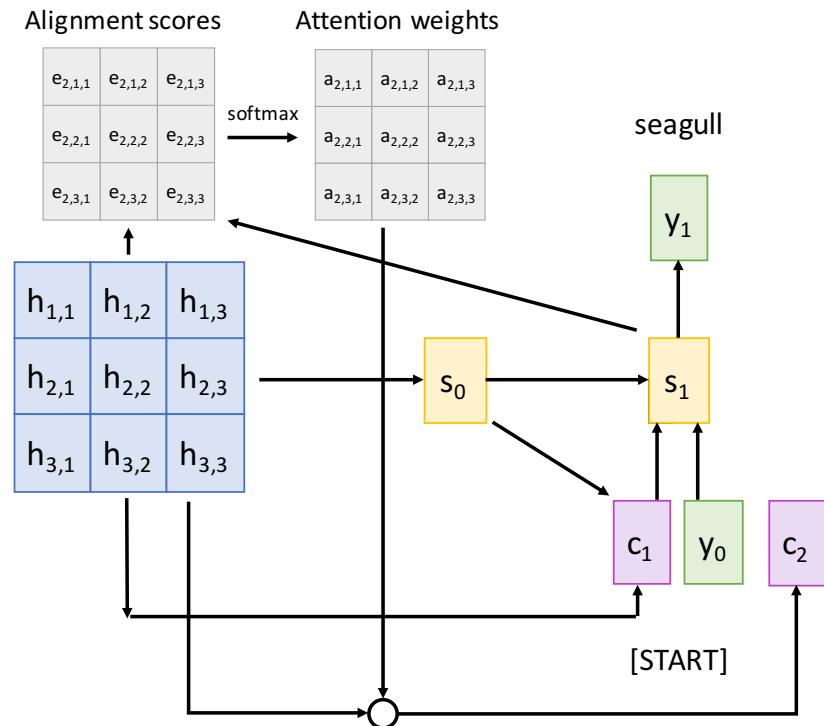
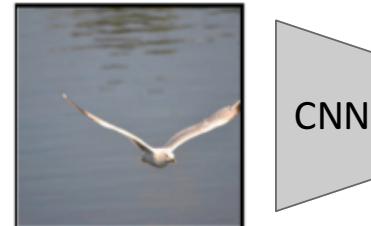
Inputs:

Query vector: q (Shape: D_Q)

Input vectors: X (Shape: $N_x \times D_Q$)

Similarity function: dot product

$$e_{t,i,j} = f_{att}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:,:})$$
$$c_t = \sum_i a_{t,i,j} h_{i,j}$$



Computation:

Similarities: e (Shape: N_x) $e_i = q \cdot X_i$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_x)

Output vector: $y = \sum_i a_i X_i$ (Shape: D_X)

Changes:

- Use dot product for similarity

Attention Layer

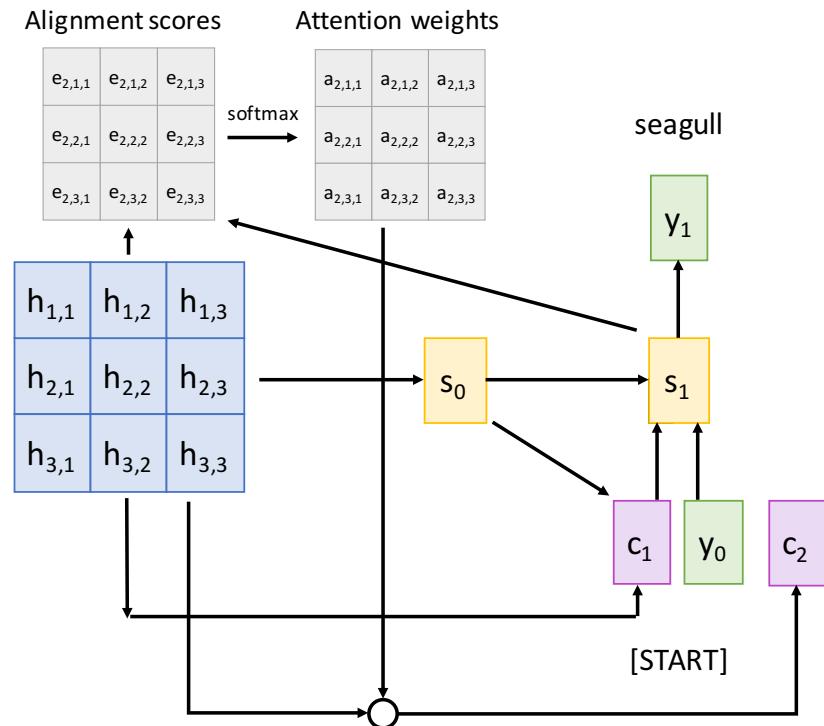
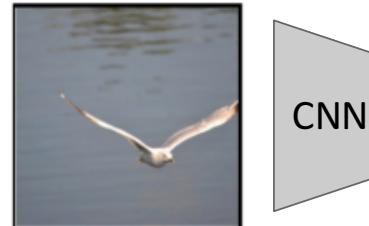
Inputs:

Query vector: q (Shape: D_Q)

Input vectors: X (Shape: $N_x \times D_Q$)

Similarity function: scaled dot product

$$e_{t,i,j} = f_{att}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:,:})$$
$$c_t = \sum_i a_{t,i,j} h_{i,j}$$



Computation:

Similarities: e (Shape: N_x) $e_i = q \cdot X_i / \sqrt{D_Q}$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_x)

Output vector: $y = \sum_i a_i X_i$ (Shape: D_X)

Changes:

- Use **scaled** dot product for similarity

Attention Layer

Inputs:

Query vector: q (Shape: D_Q)

Input vectors: X (Shape: $N_x \times D_Q$)

Similarity function: scaled dot product

Large similarities will cause softmax to saturate and give vanishing gradients

Recall $a \cdot b = |a| |b| \cos(\text{angle})$

Suppose that a and b are constant vectors of dimension D

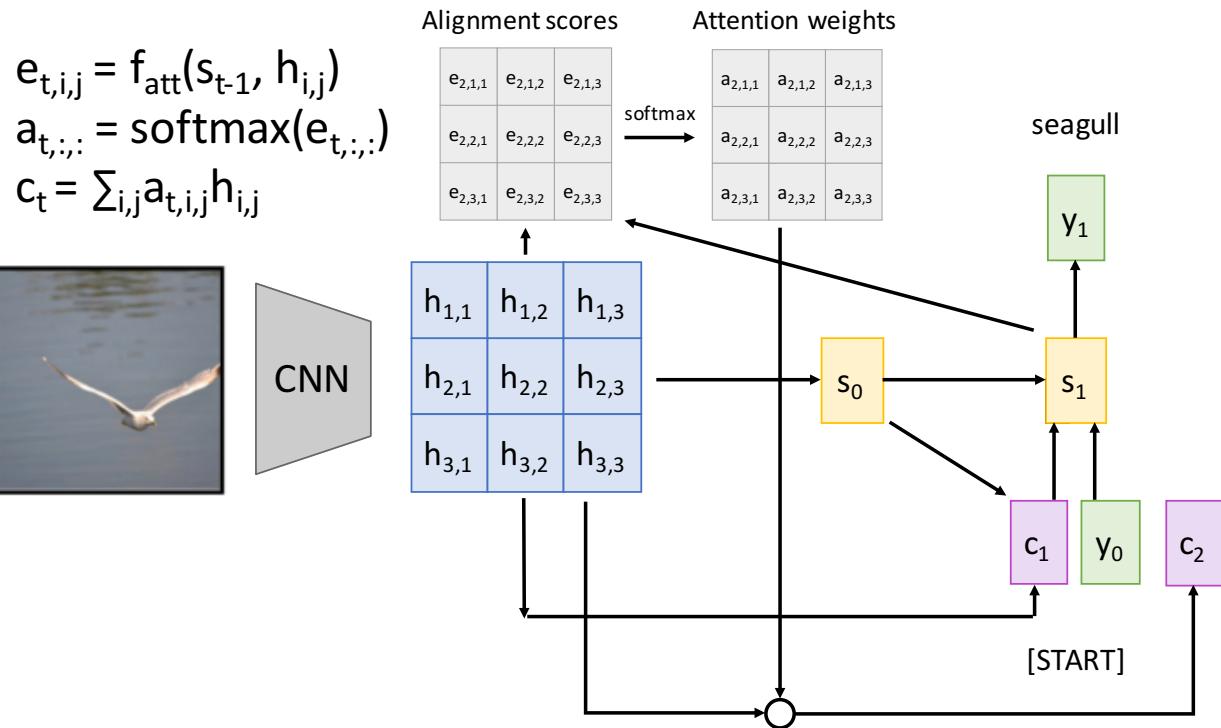
Then $|a| = (\sum_i a_i^2)^{1/2} = \sqrt{D}$

Computation:

Similarities: e (Shape: N_x) $e_i = q \cdot X_i / \sqrt{D_Q}$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_x)

Output vector: $y = \sum_i a_i X_i$ (Shape: D_X)



Changes:

- Use scaled dot product for similarity

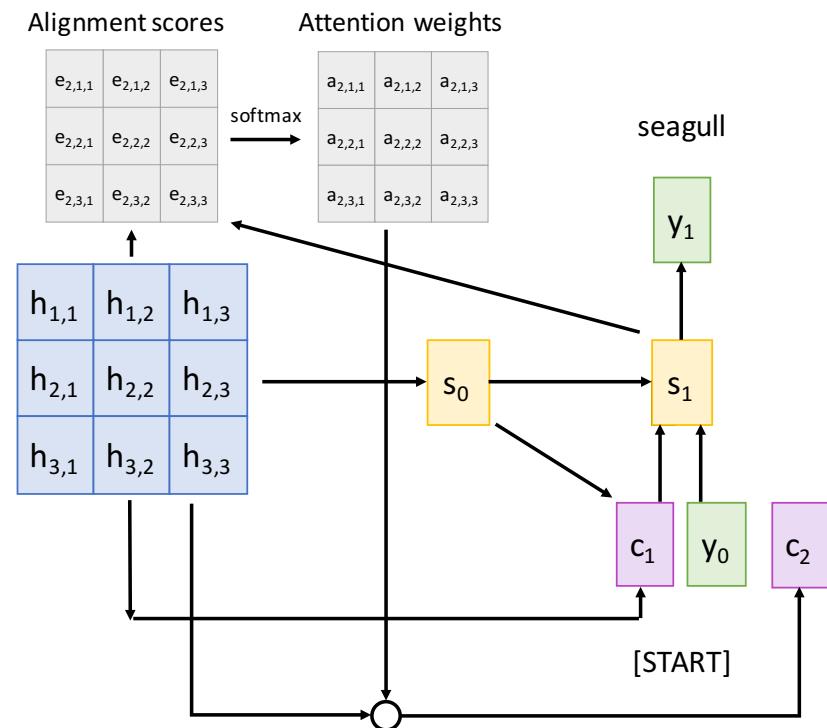
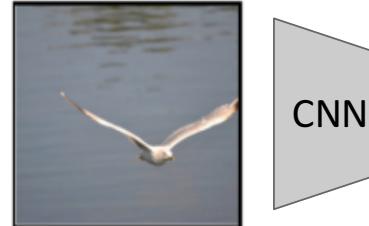
Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

$$\begin{aligned} e_{t,i,j} &= f_{att}(s_{t-1}, h_{i,j}) \\ a_{t,:,:} &= \text{softmax}(e_{t,:,:}) \\ c_t &= \sum_i a_{t,i,j} h_{i,j} \end{aligned}$$



Computation:

Similarities: $E = \mathbf{Q}\mathbf{X}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{X}_j / \sqrt{D_Q}$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = A\mathbf{X}$ (Shape: $N_Q \times D_X$) $Y_i = \sum_j A_{i,j} X_j$

Changes:

- Use dot product for similarity
- Multiple query vectors

Attention Layer

Inputs:

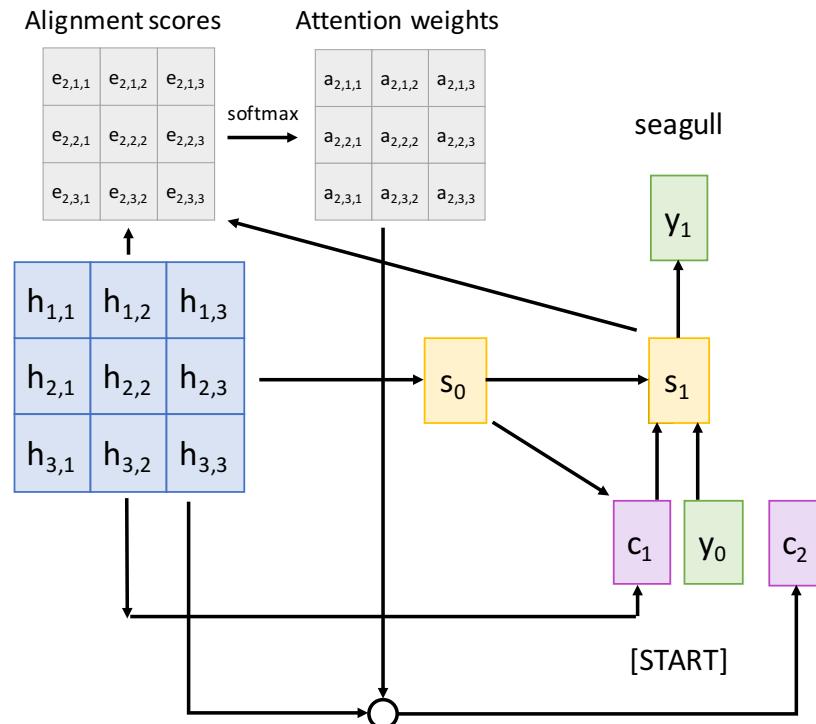
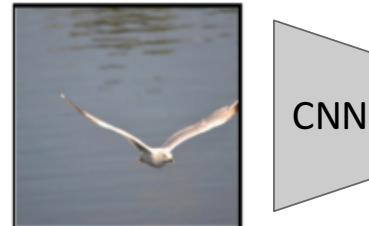
Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

$$\begin{aligned} e_{t,i,j} &= f_{att}(s_{t-1}, h_{i,j}) \\ a_{t,:,:} &= \text{softmax}(e_{t,:,:}) \\ c_t &= \sum_i a_{t,i,j} h_{i,j} \end{aligned}$$



Computation:

Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \sqrt{D_Q}$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $Y = A\mathbf{V}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Changes:

- Use dot product for similarity
- Multiple **query** vectors
- Separate **key** and **value**

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_Q \times D_V$) $\mathbf{Y}_i = \sum_j A_{i,j} \mathbf{V}_j$

\mathbf{X}_1

\mathbf{X}_2

\mathbf{X}_3

\mathbf{Q}_1

\mathbf{Q}_2

\mathbf{Q}_3

\mathbf{Q}_4

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

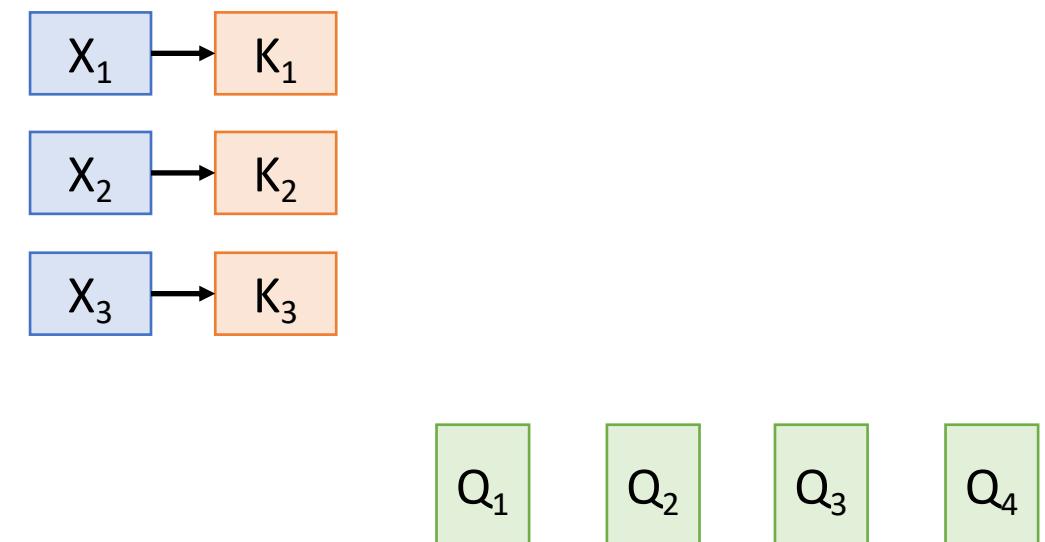
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

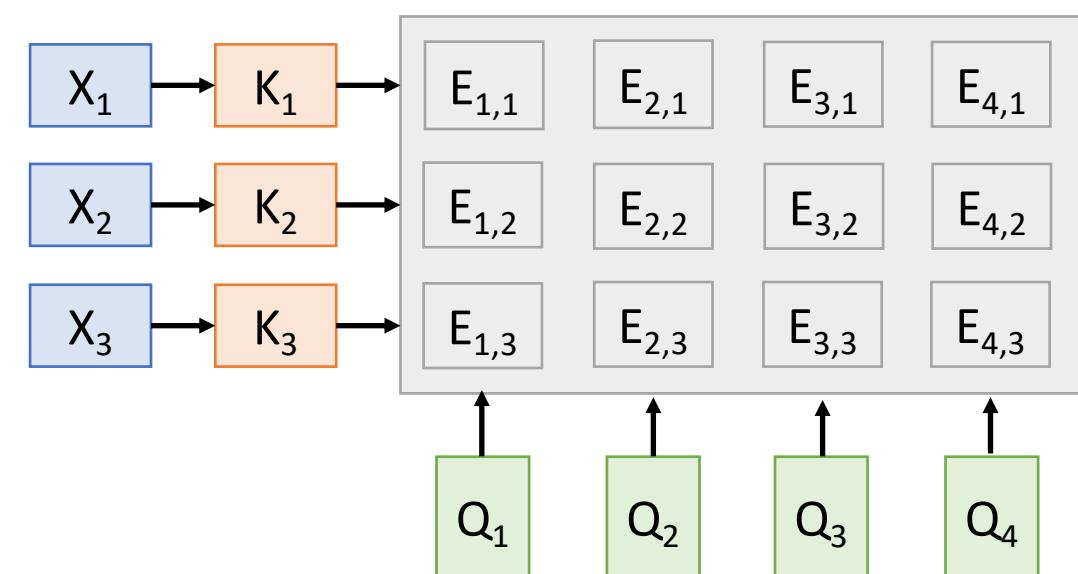
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_Q \times D_V$) $\mathbf{Y}_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

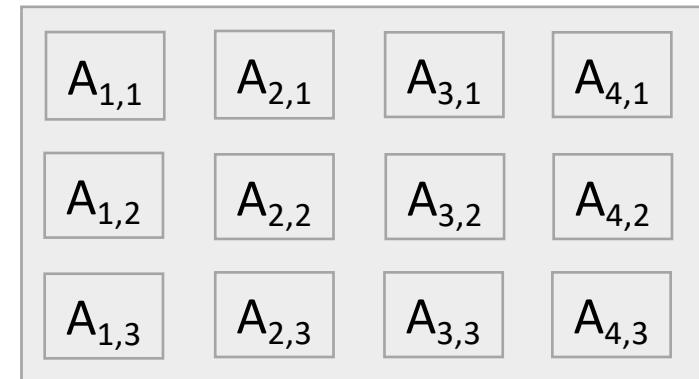
Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)



Computation:

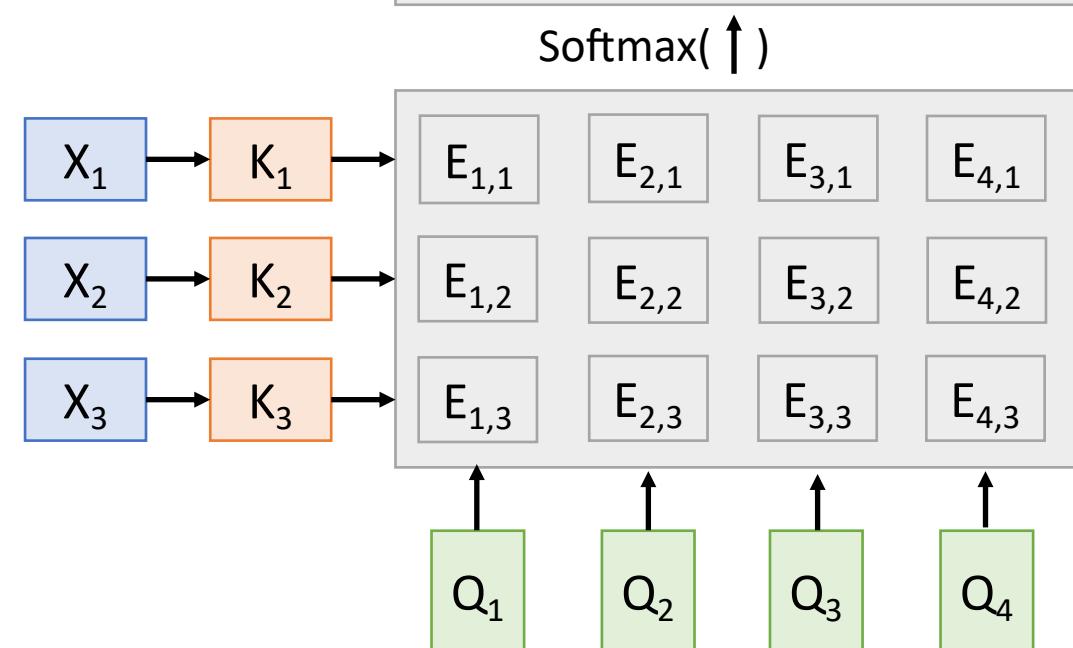
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_Q \times D_V$) $\mathbf{Y}_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

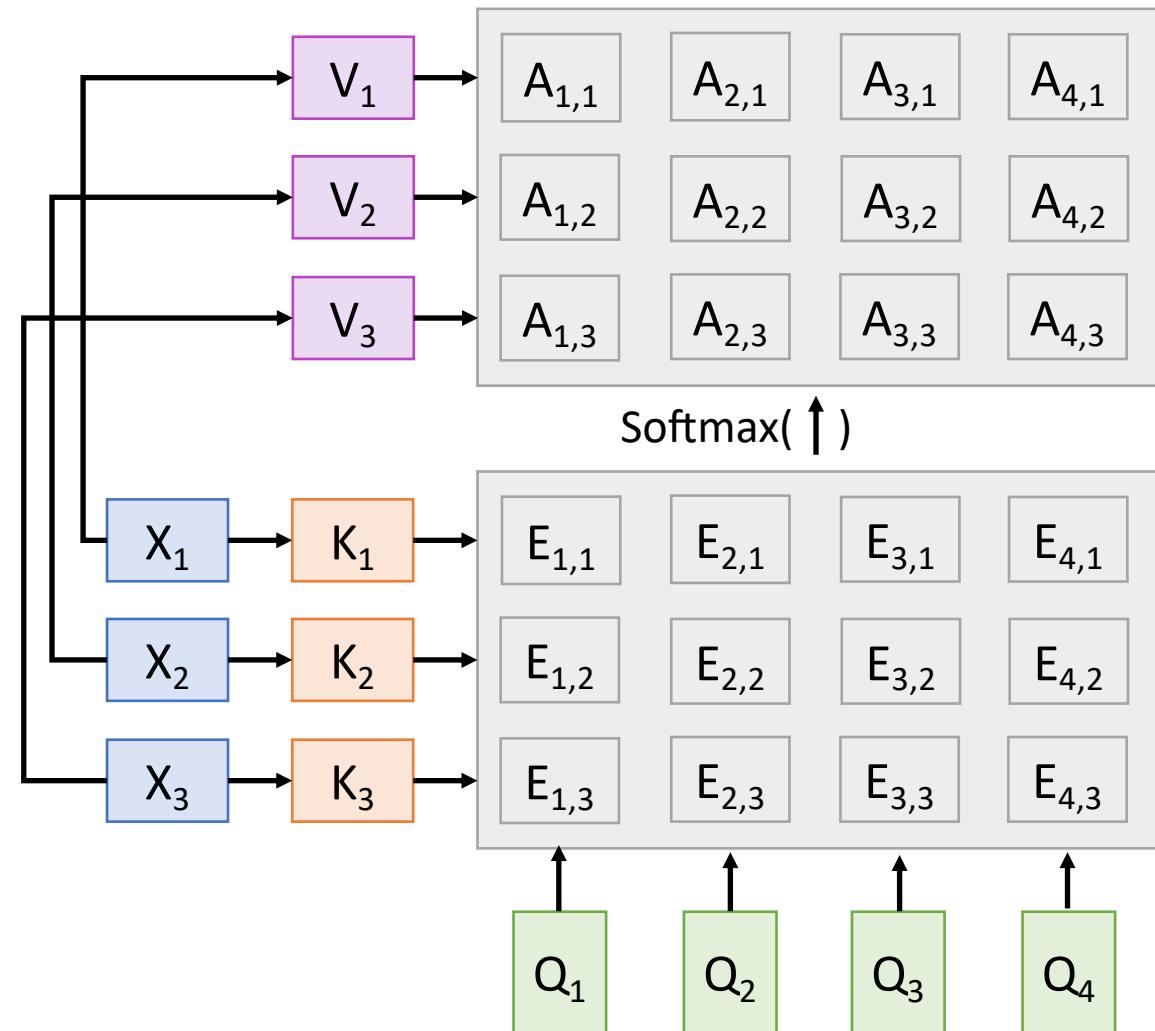
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{A}\mathbf{V}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

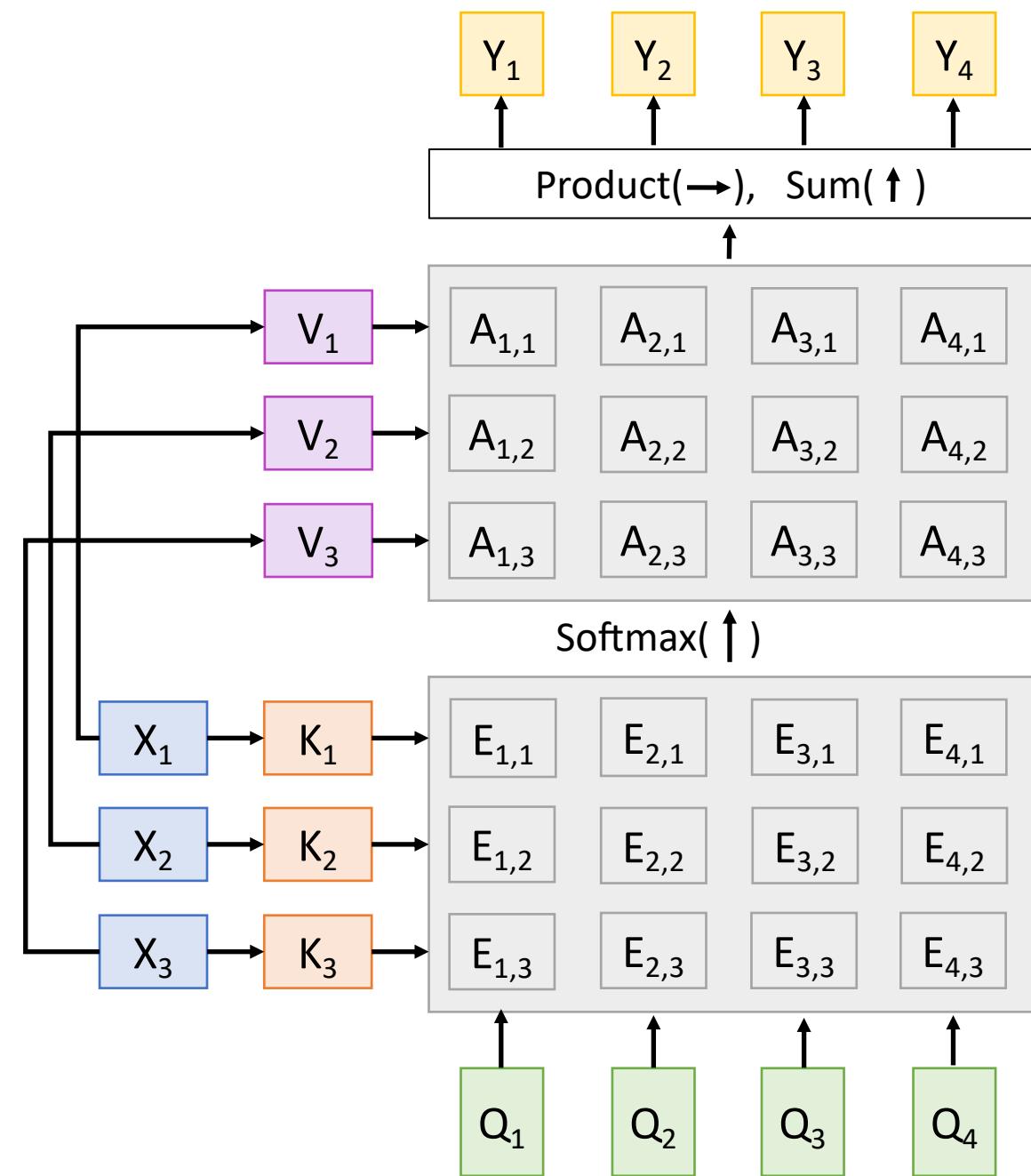
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{A}\mathbf{V}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{X}\mathbf{W}_Q$

Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_x \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_x \times D_V$) $\mathbf{Y}_i = \sum_j A_{i,j} \mathbf{V}_j$

\mathbf{x}_1

\mathbf{x}_2

\mathbf{x}_3

Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{X}\mathbf{W}_Q$

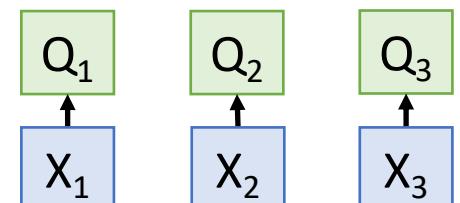
Key vectors: $\mathbf{K} = \mathbf{X}\mathbf{W}_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{X}\mathbf{W}_V$ (Shape: $N_x \times D_V$)

Similarities: $E = \mathbf{Q}\mathbf{K}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = A\mathbf{V}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

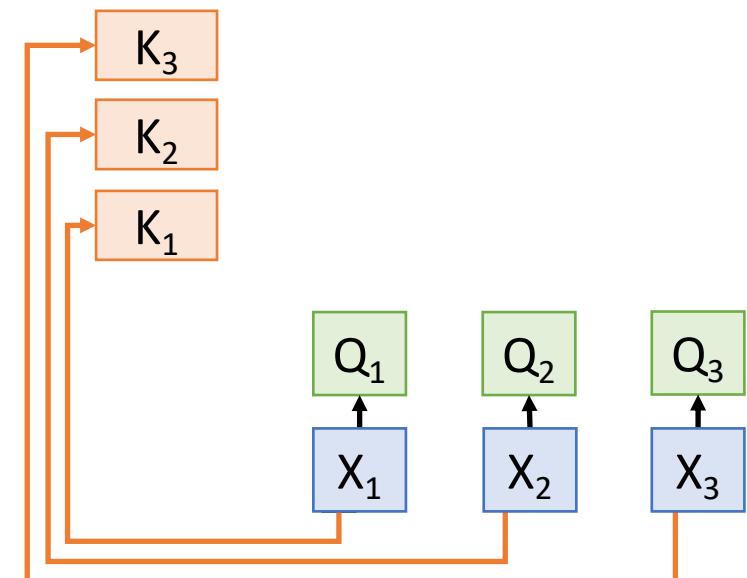
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

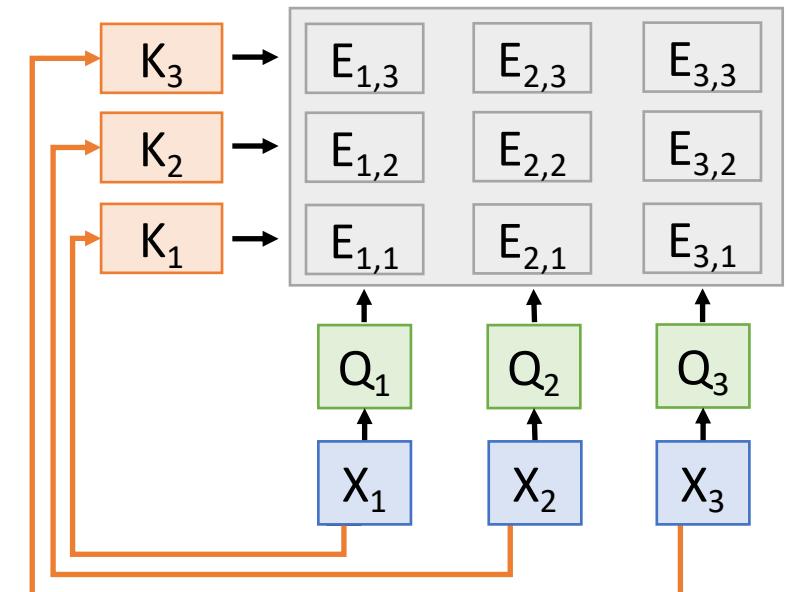
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

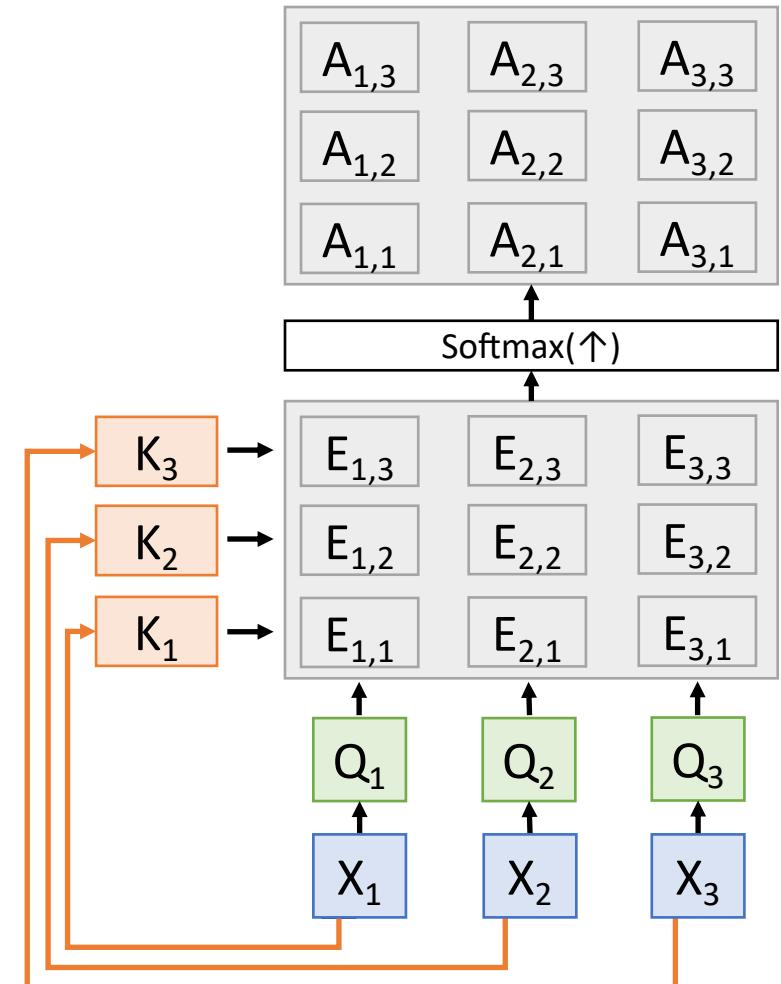
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

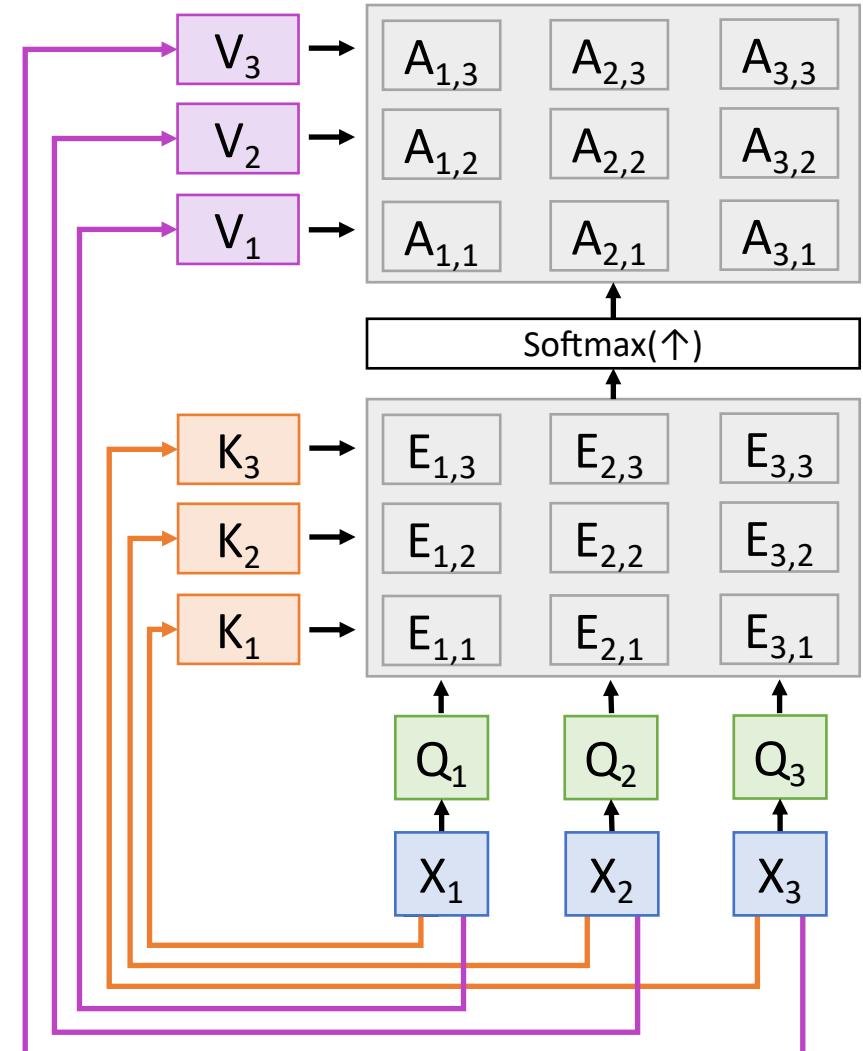
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

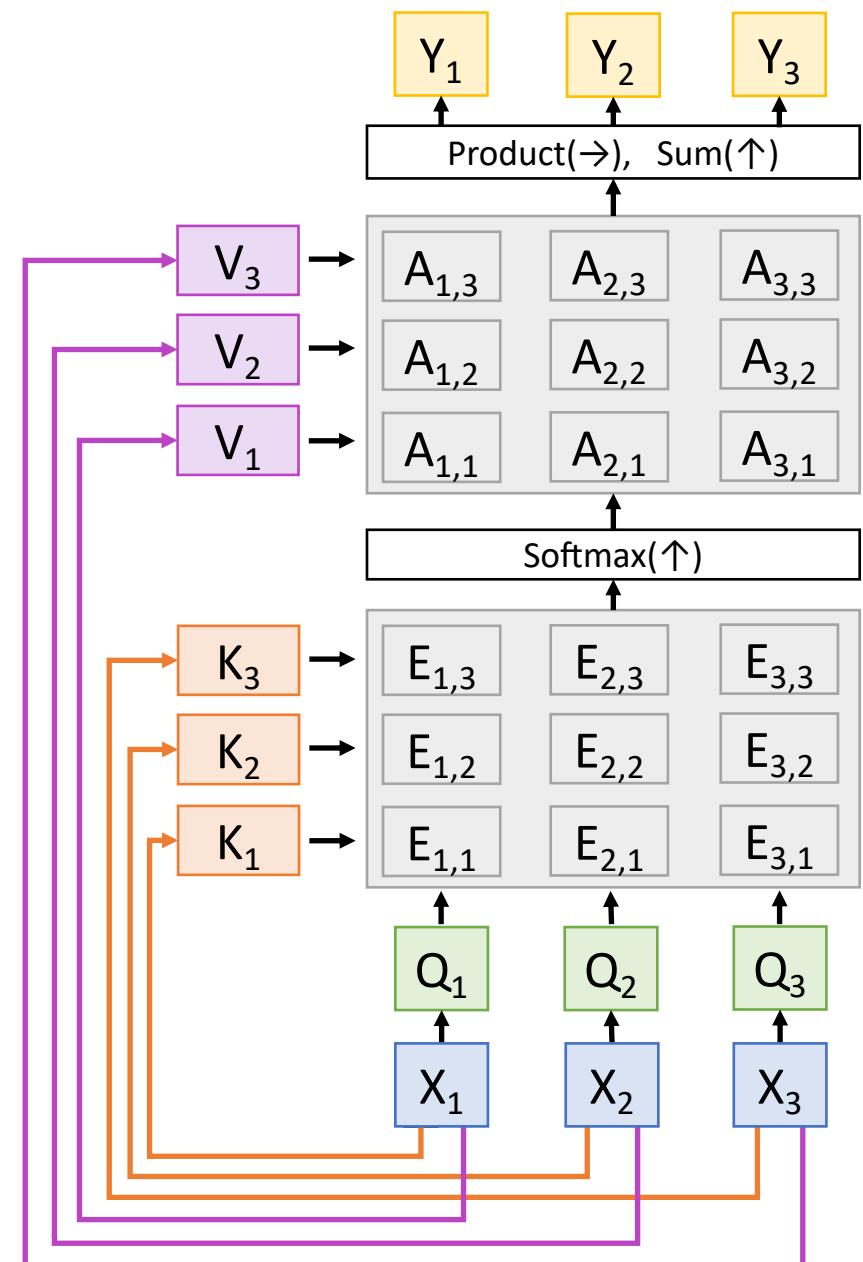
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

Consider **permuting** the input vectors:

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

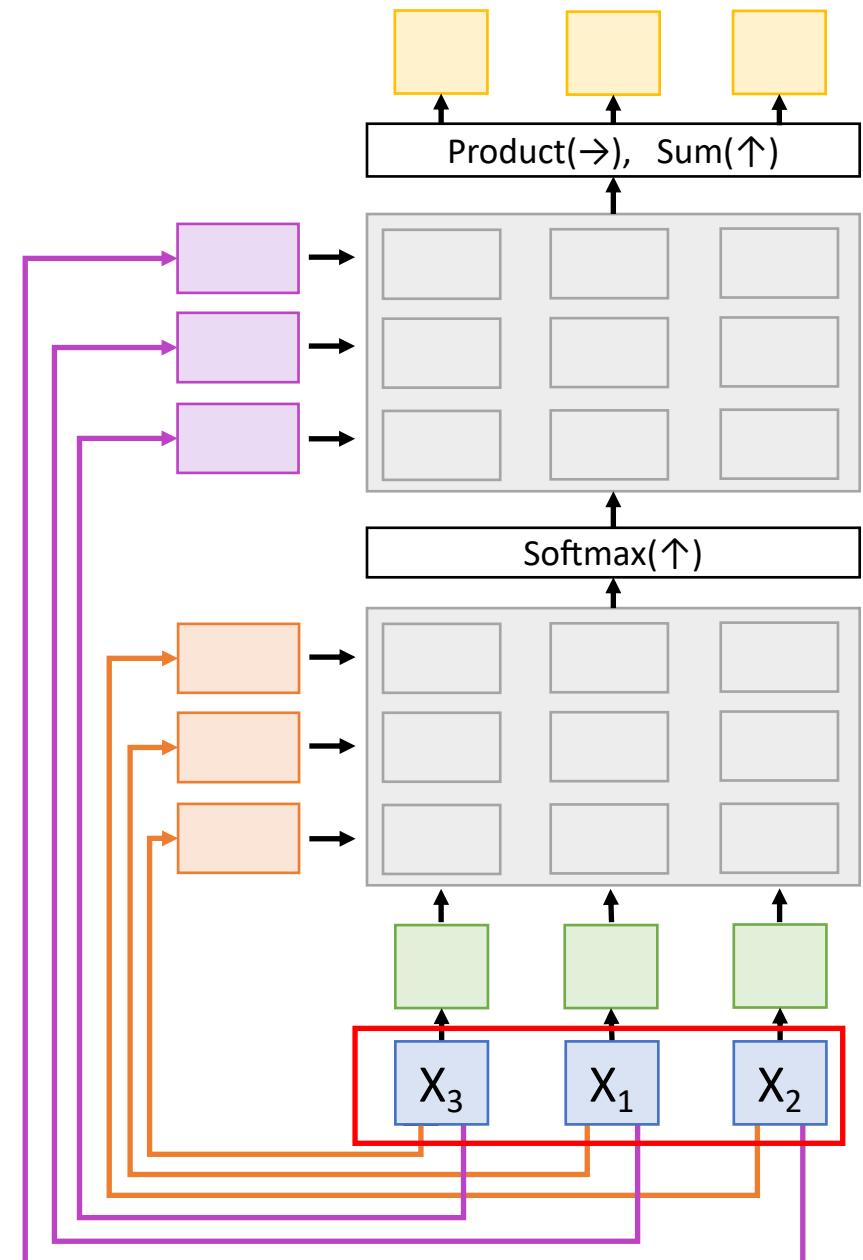
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

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Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

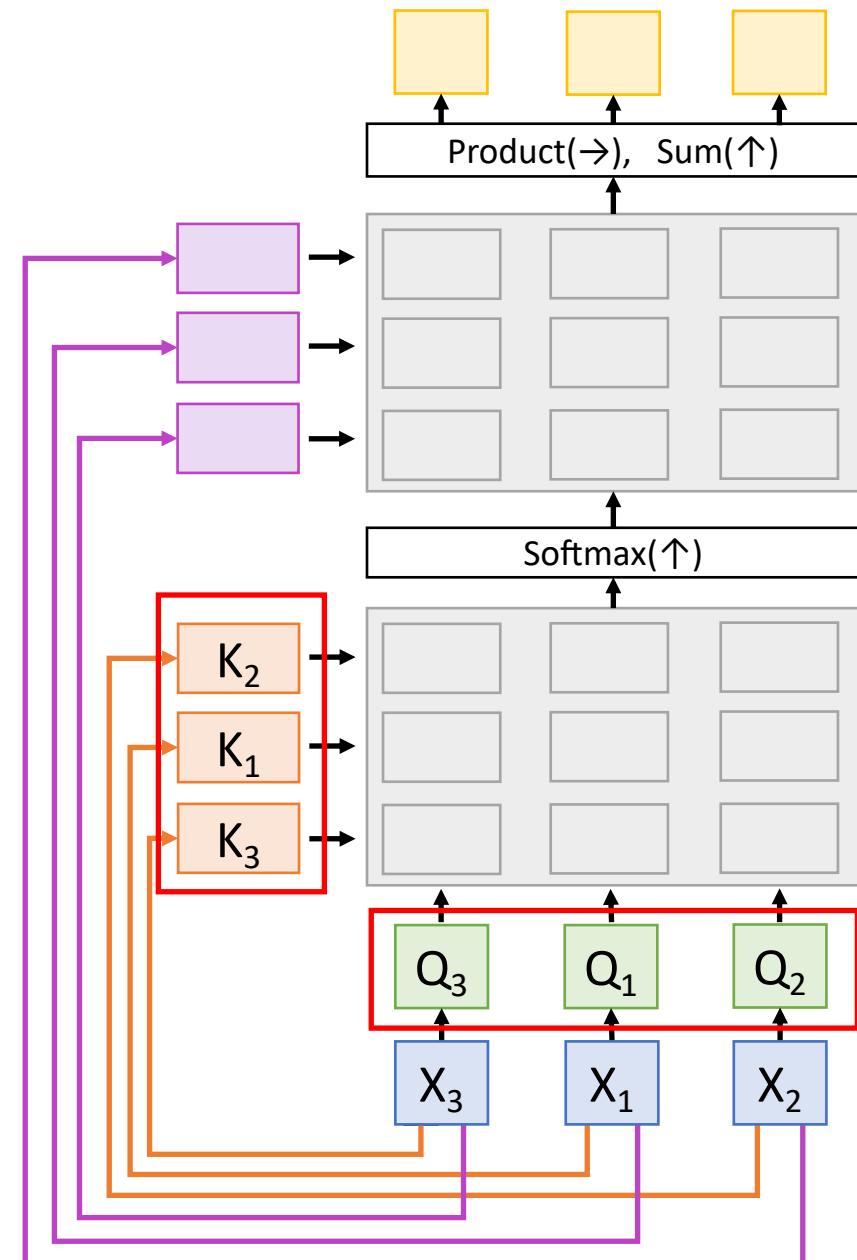
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Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Consider **permuting** the input vectors:

Queries and Keys will be the same, but permuted



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Consider **permuting** the input vectors:

Similarities will be the same, but permuted

Computation:

Query vectors: $Q = XW_Q$

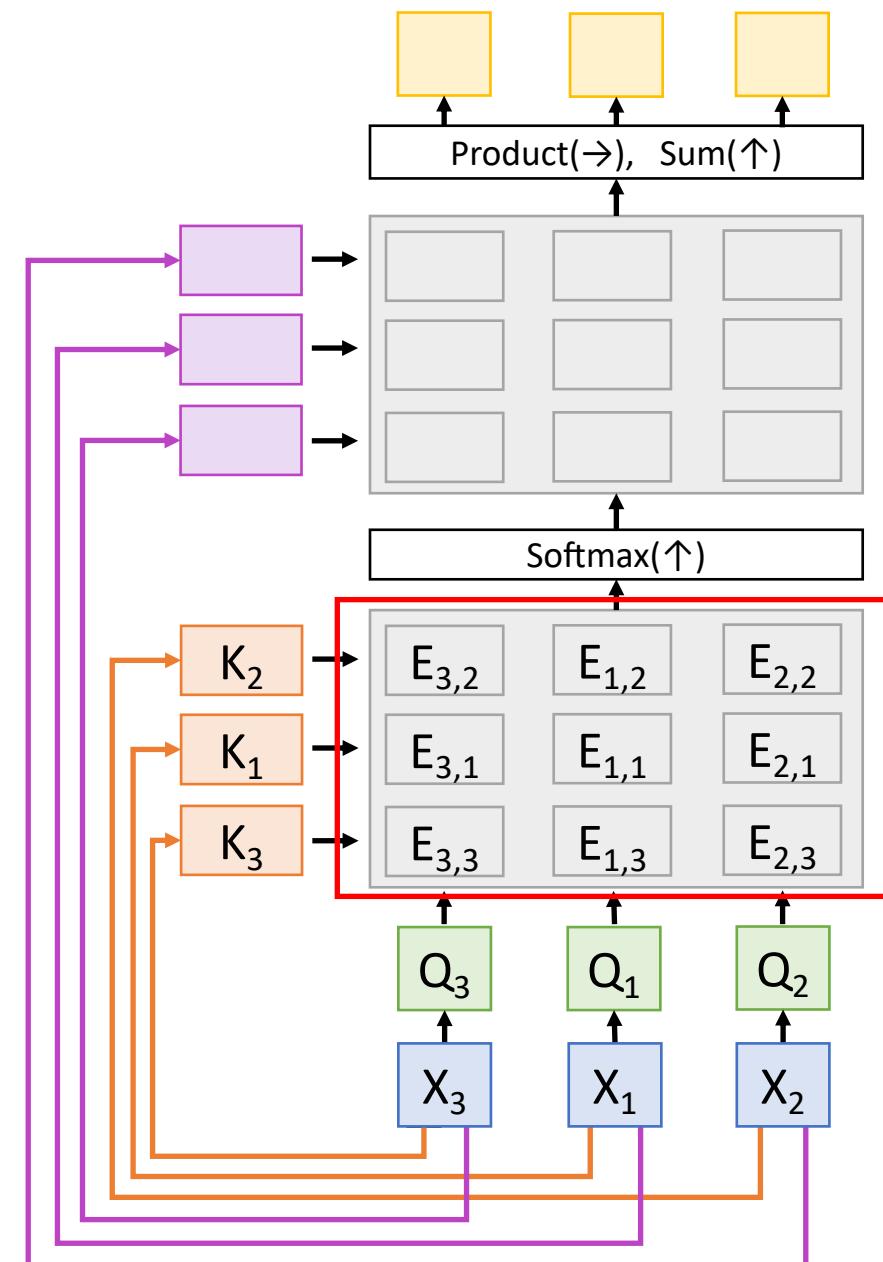
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Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

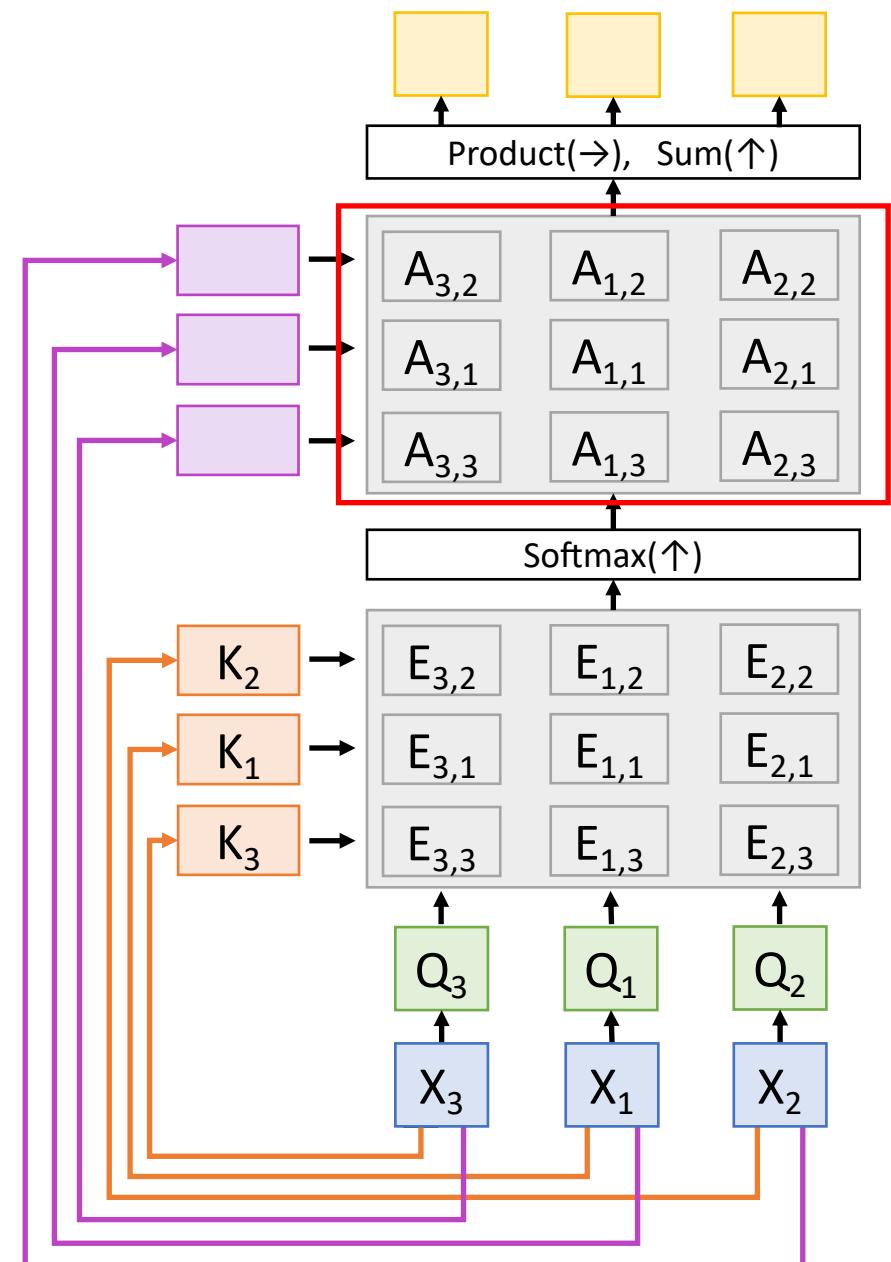
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Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Consider **permuting** the input vectors:

Attention weights will be the same, but permuted



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Consider **permuting** the input vectors:

Values will be the same, but permuted

Computation:

Query vectors: $Q = XW_Q$

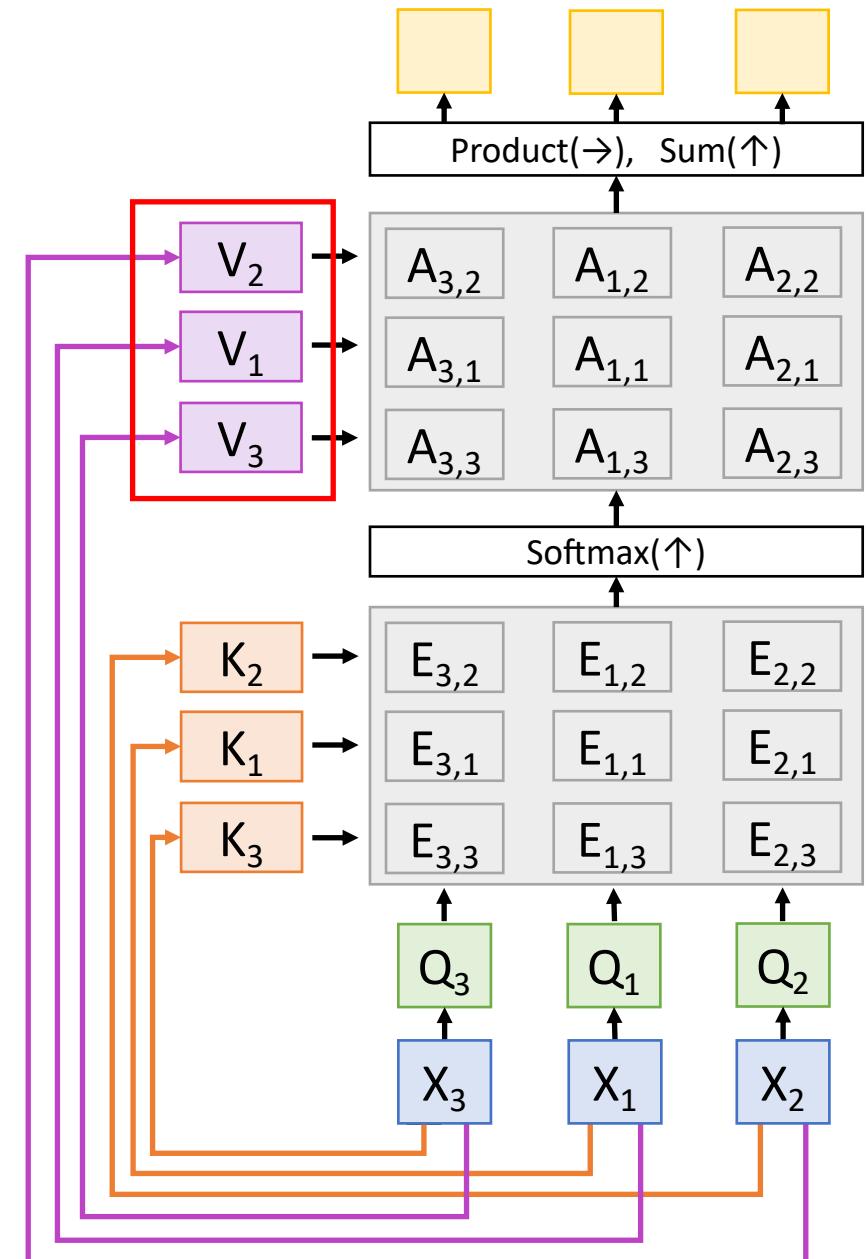
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

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Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

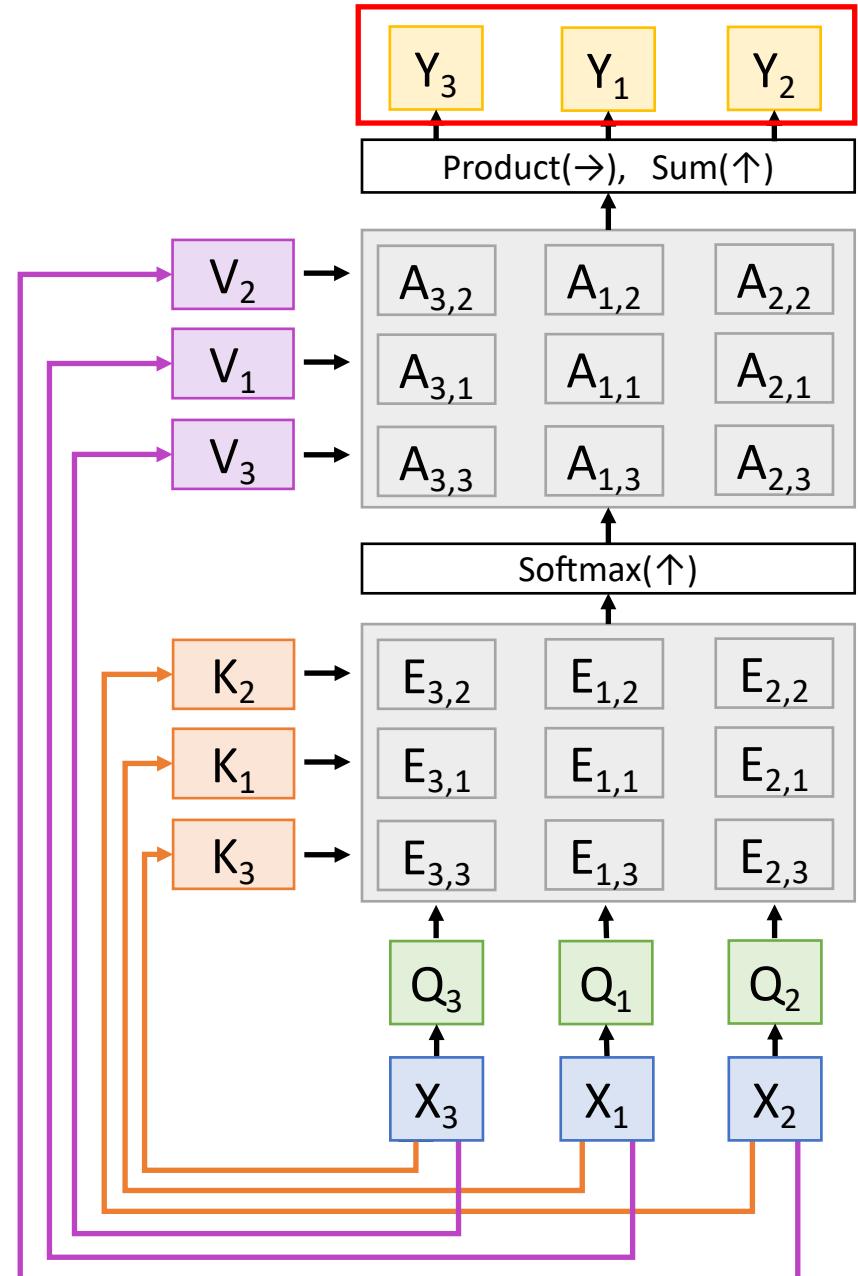
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Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Consider **permuting** the input vectors:

Outputs will be the same, but permuted



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

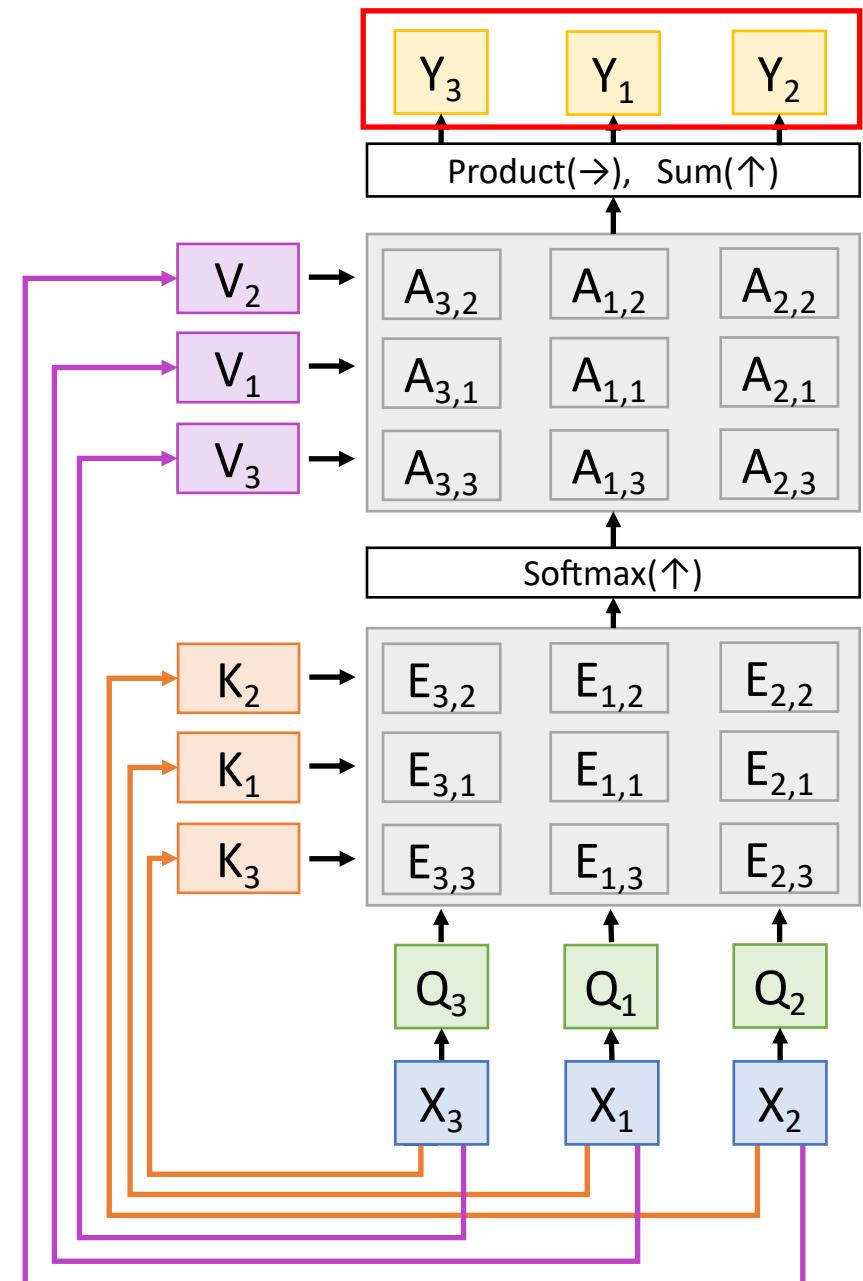
Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Consider **permuting** the input vectors:

Outputs will be the same, but permuted

Self-attention layer is **Permutation Equivariant**
 $f(s(x)) = s(f(x))$

Self-Attention layer works on **sets** of vectors



Self-Attention Layer

Self attention doesn't
“know” the order of the
vectors it is processing!

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

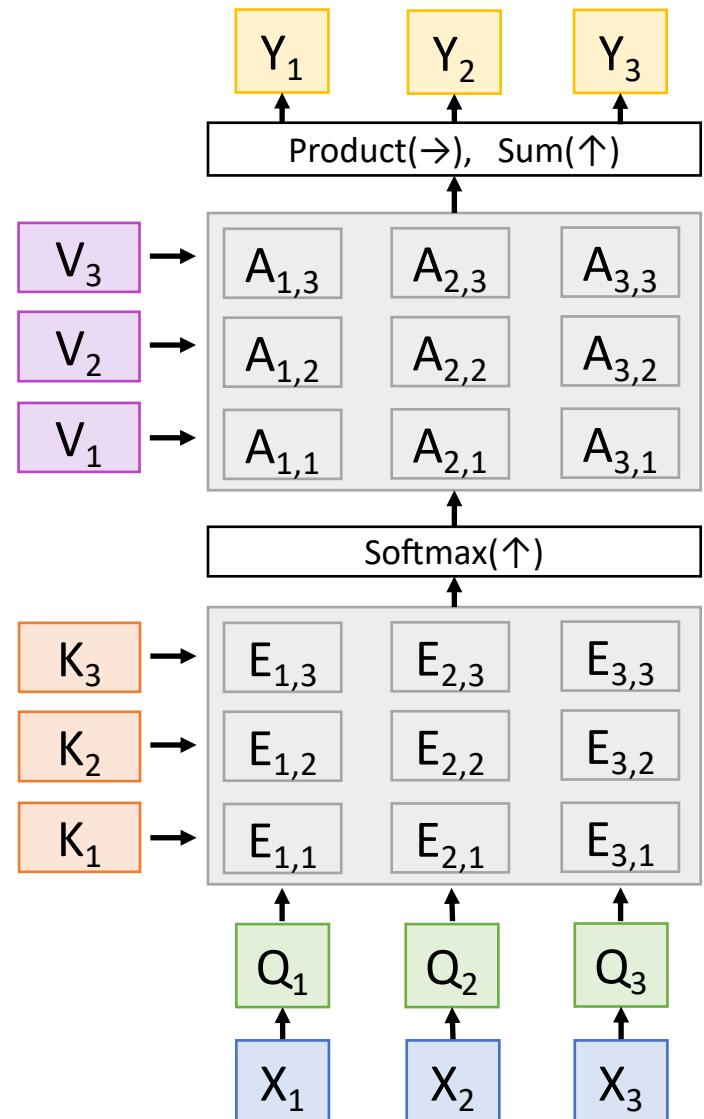
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Self attention doesn't
“know” the order of the
vectors it is processing!

In order to make
processing position-
aware, concatenate input
with **positional encoding**

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

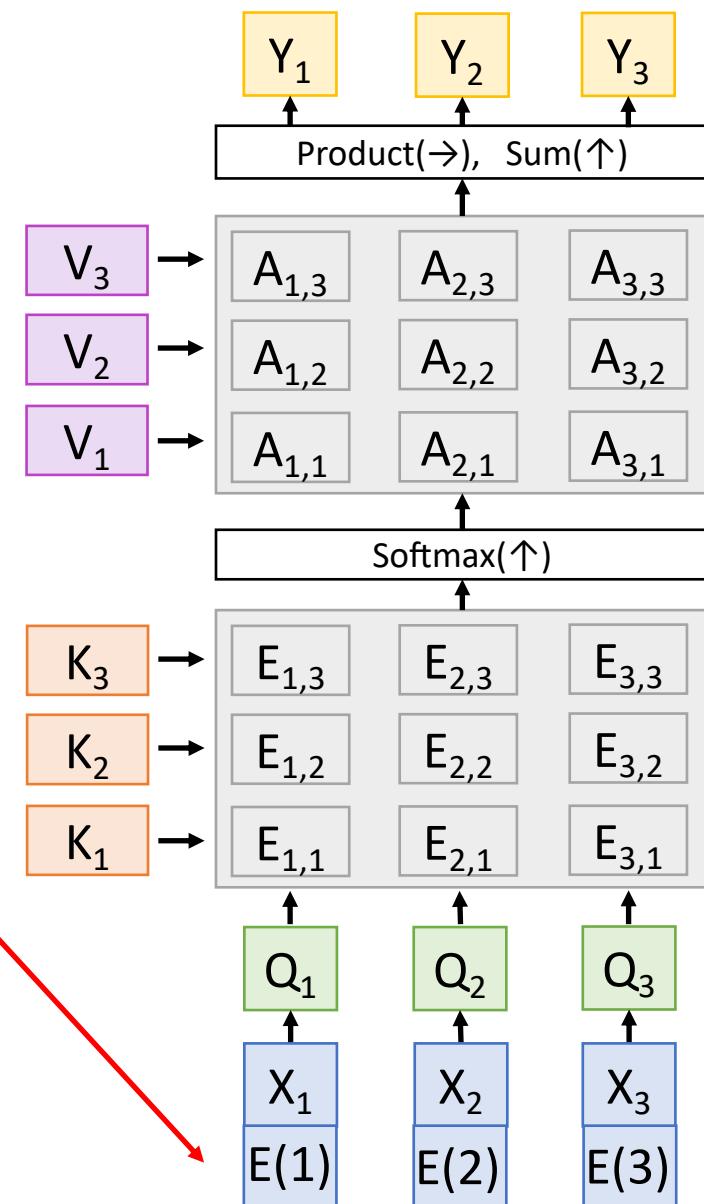
Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

E can be learned lookup
table, or fixed function



Masked Self-Attention Layer

Don't let vectors "look ahead" in the sequence

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

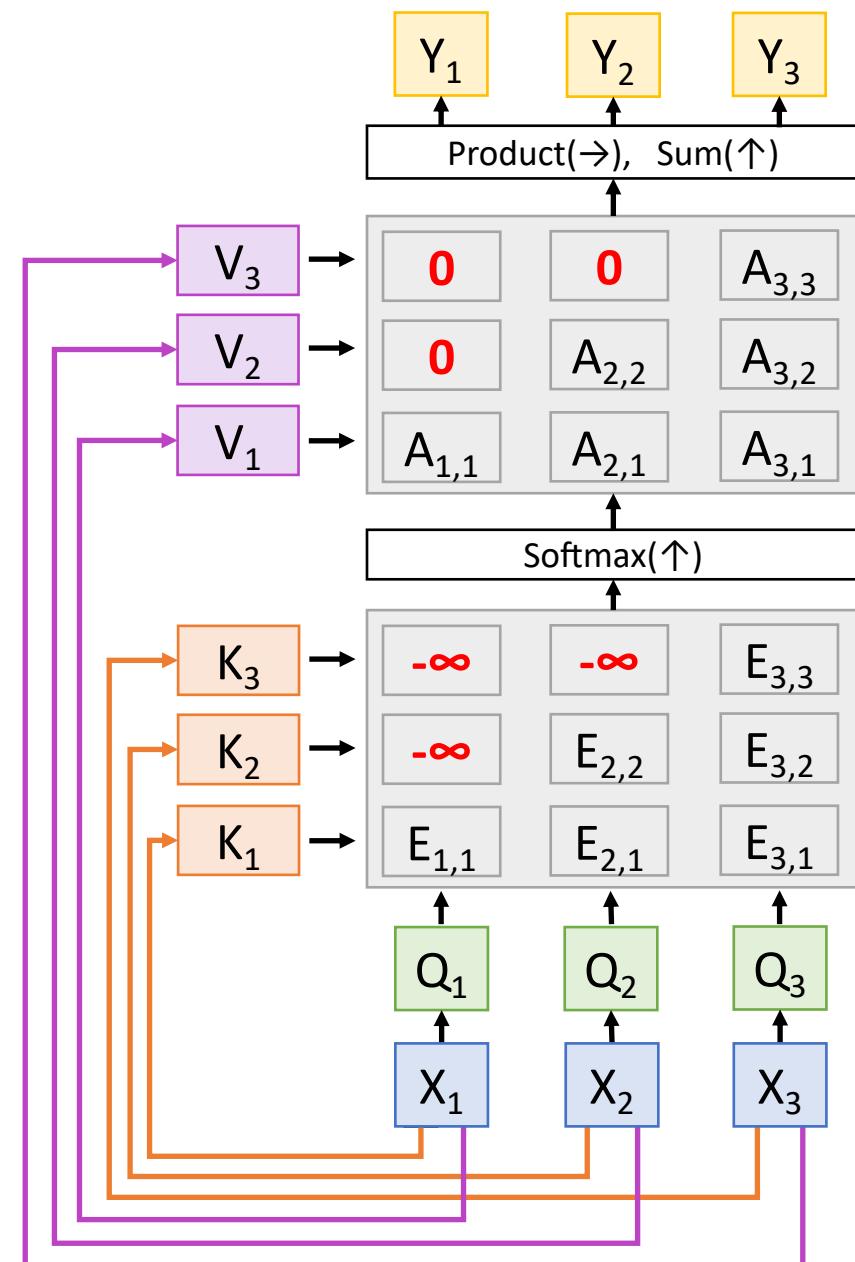
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Masked Self-Attention Layer

Don't let vectors "look ahead" in the sequence

Used for language modeling (predict next word)

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

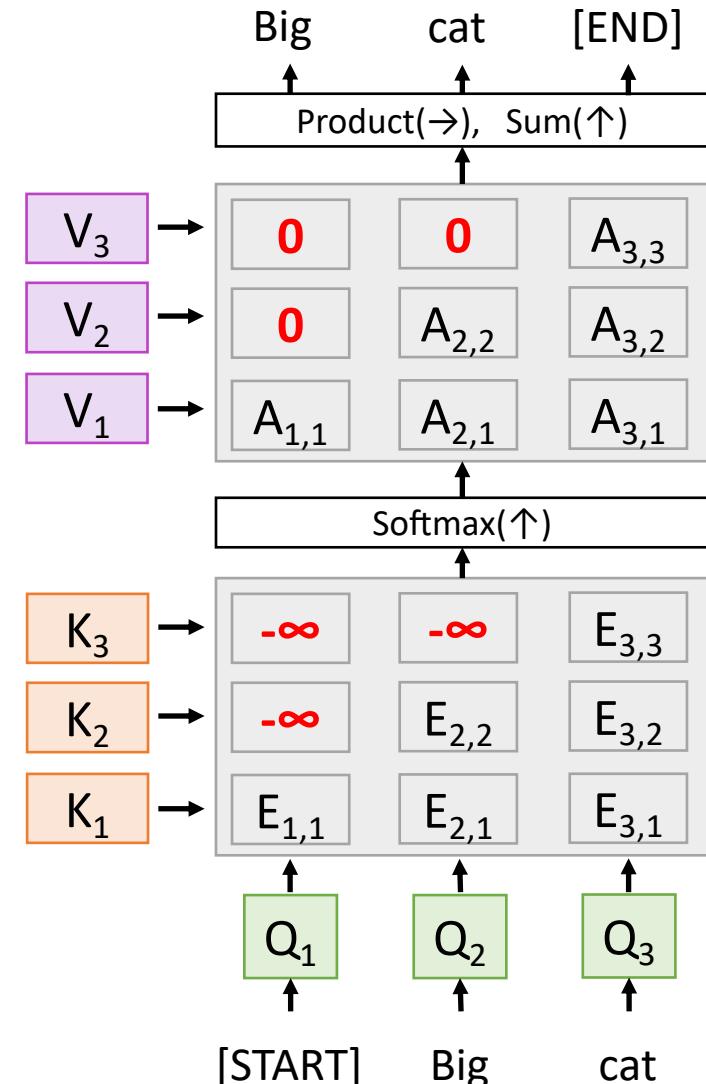
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

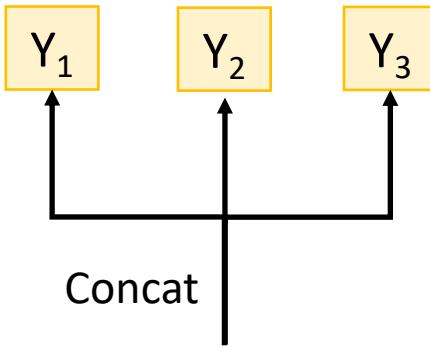
Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Multihead Self-Attention Layer

Use H independent
“Attention Heads” in parallel



Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

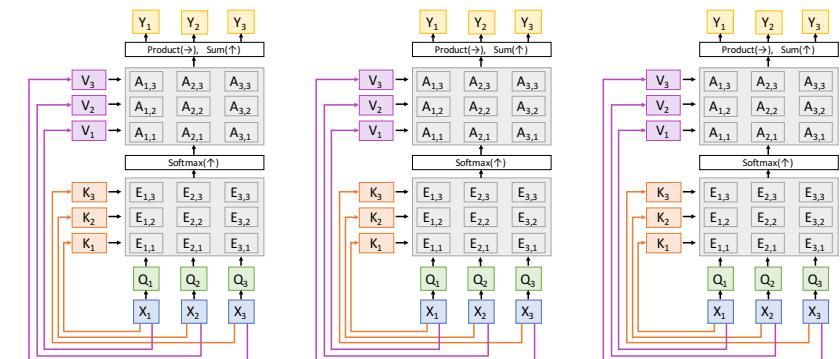
Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

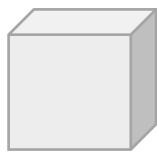
Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Hyperparameters:
Query dimension D_Q
Number of heads H



Example: CNN with Self-Attention

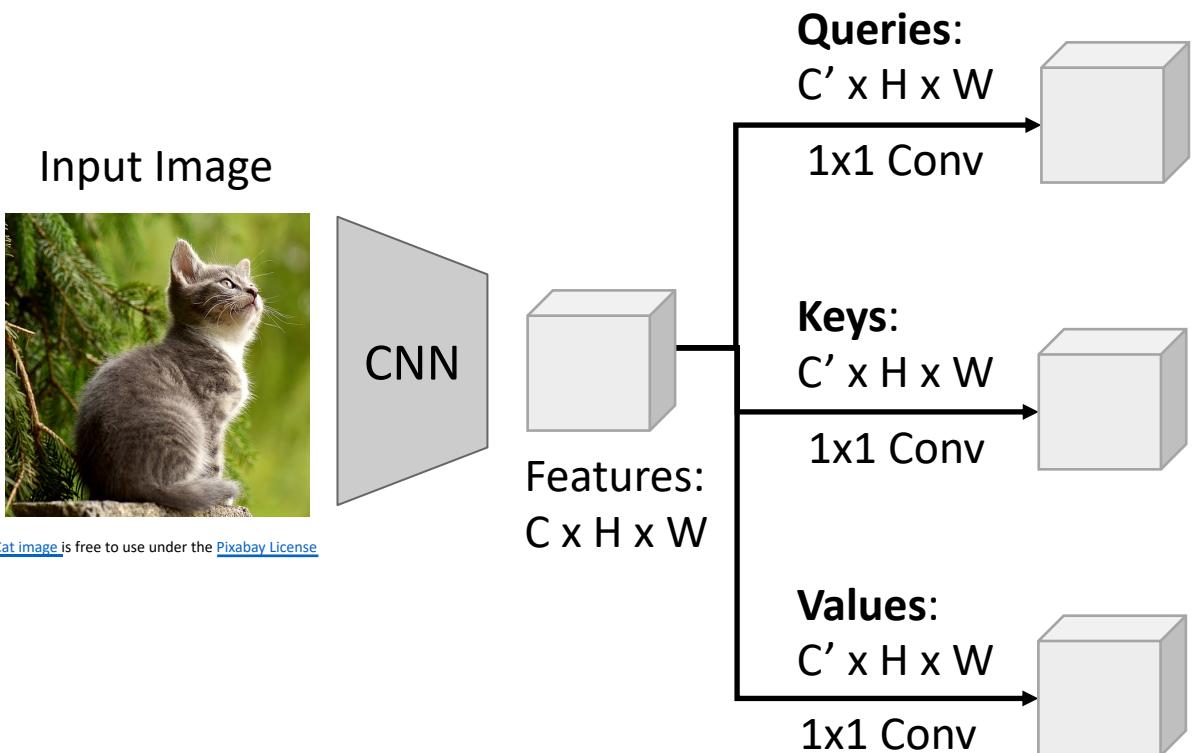
Input Image



Features:
 $C \times H \times W$

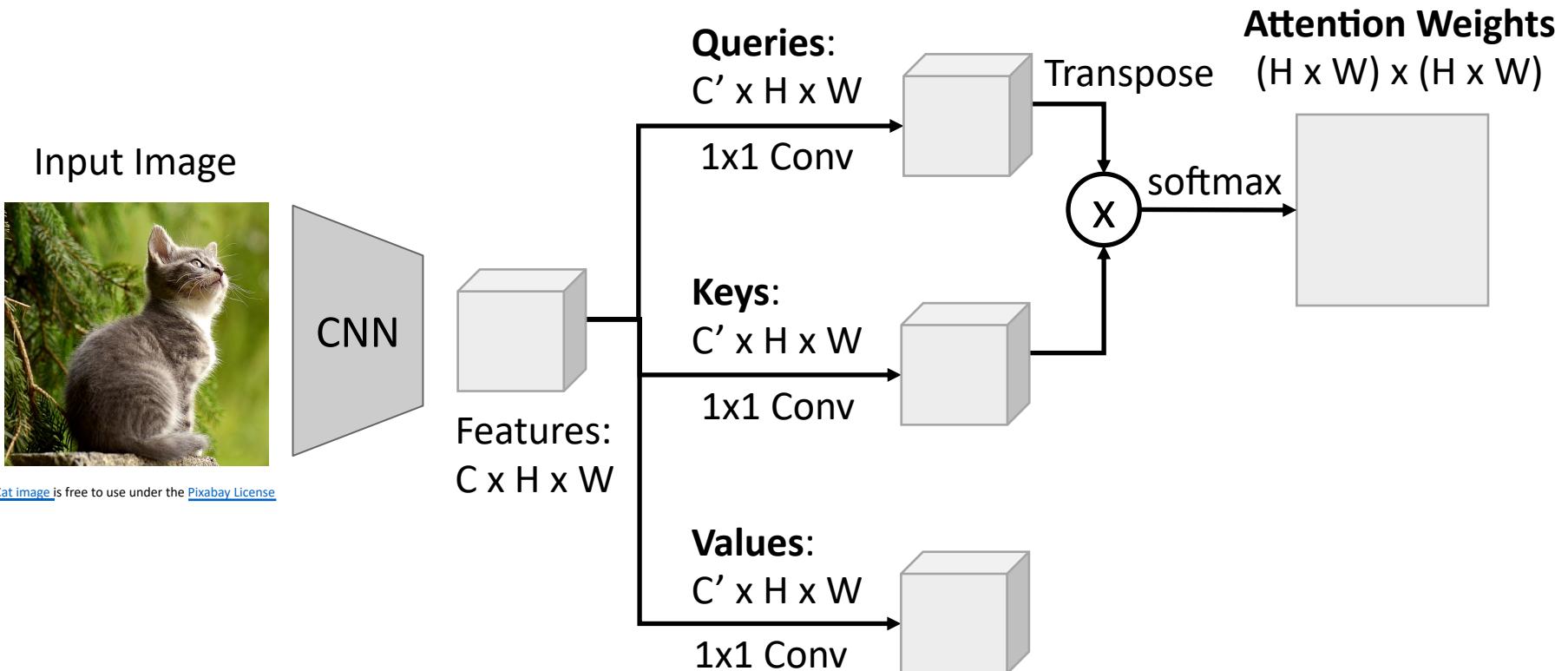
Cat image is free to use under the [Pixabay License](#)

Example: CNN with Self-Attention



Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

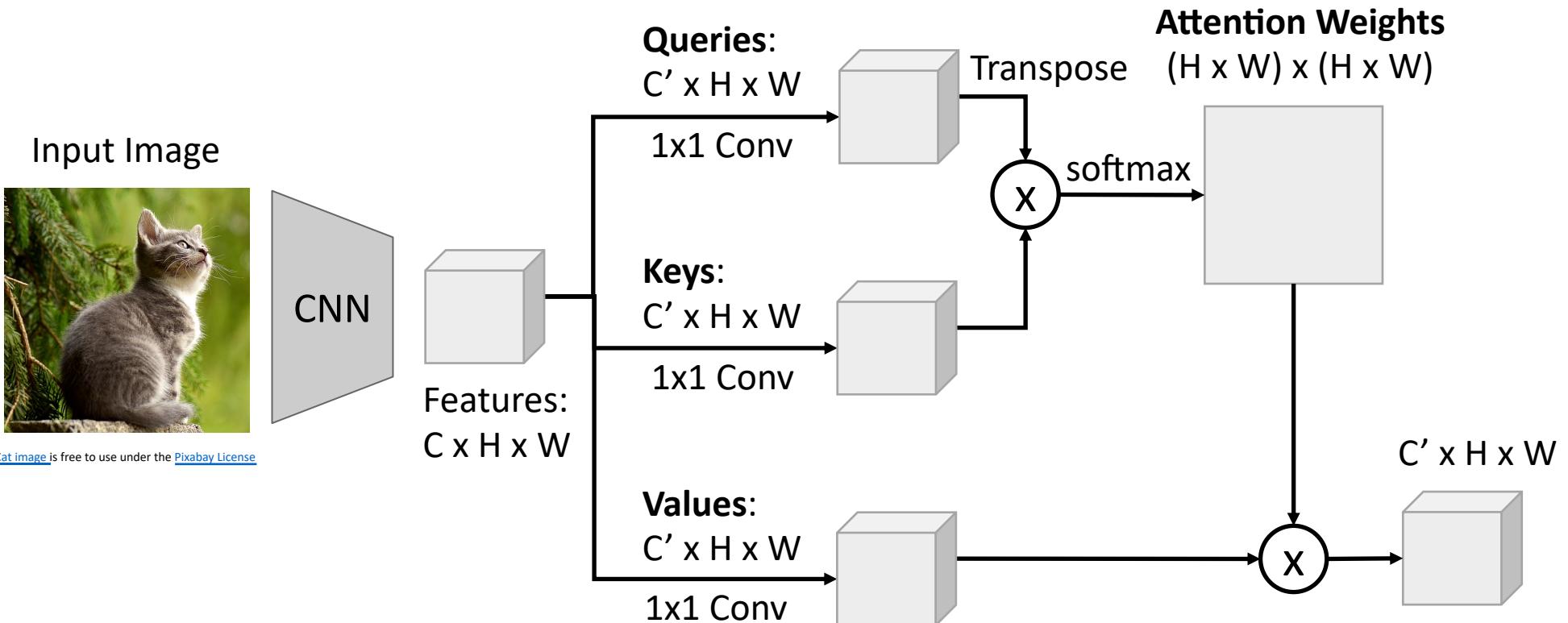
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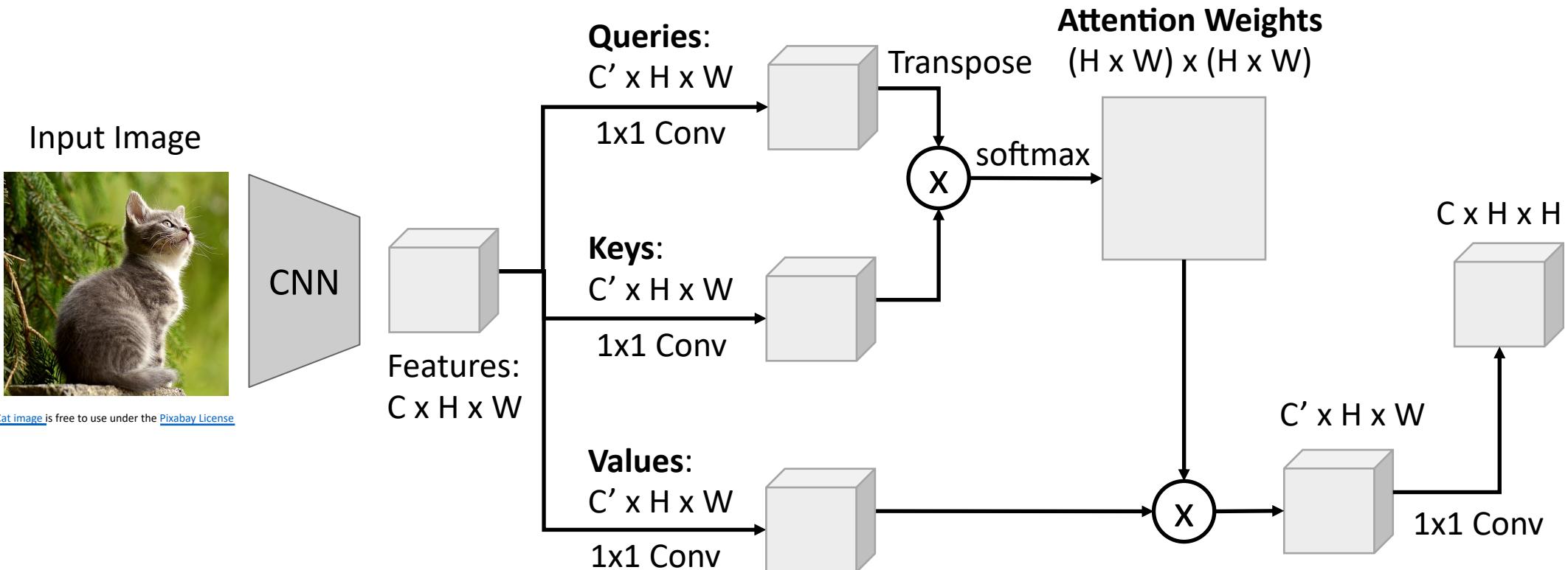
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Example: CNN with Self-Attention

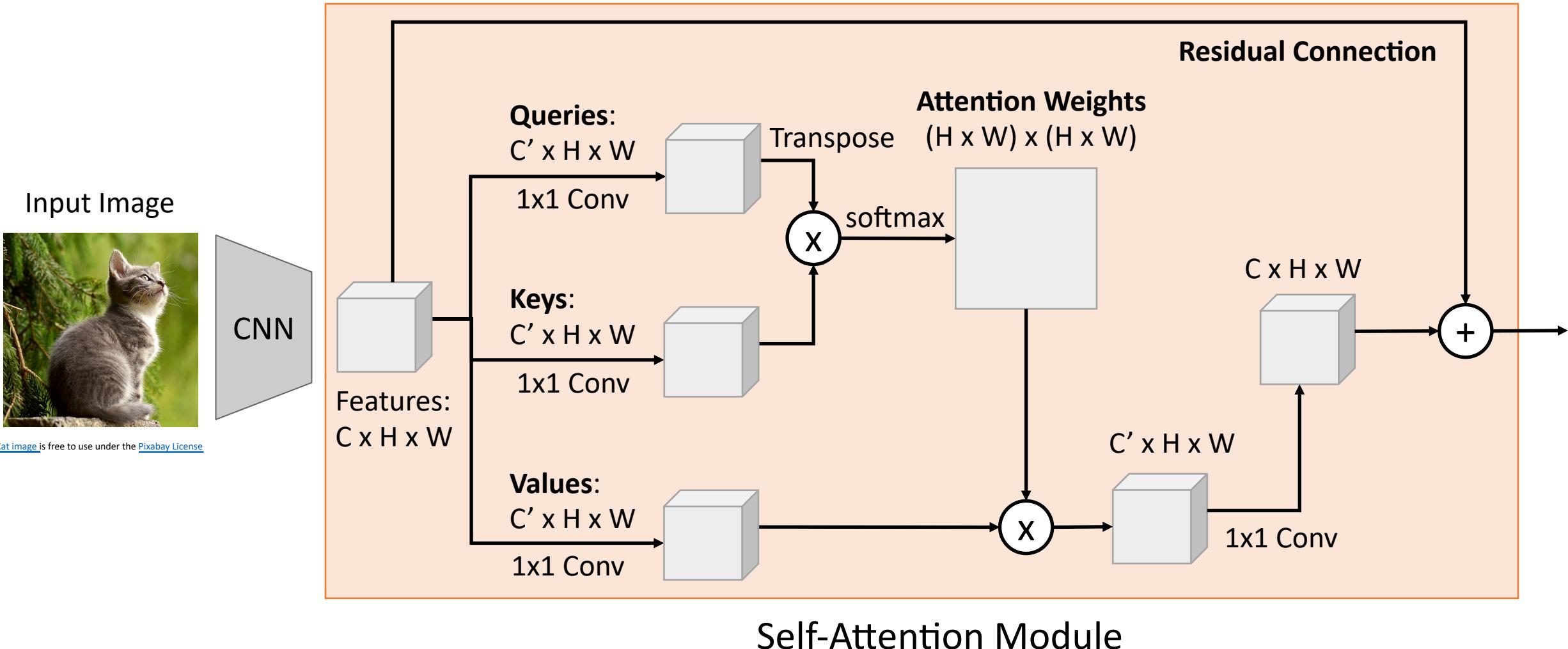


Example: CNN with Self-Attention



Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

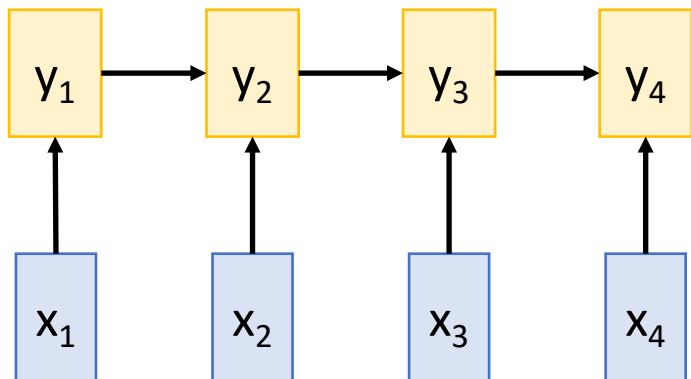
Example: CNN with Self-Attention



Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

Three Ways of Processing Sequences

Recurrent Neural Network



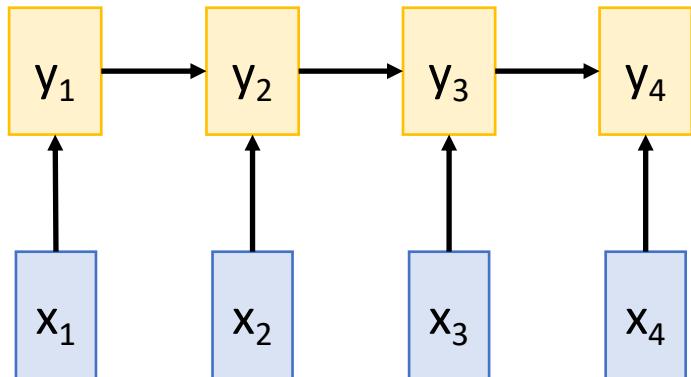
Works on **Ordered Sequences**

(+) Good at long sequences: After one RNN layer, h_T "sees" the whole sequence

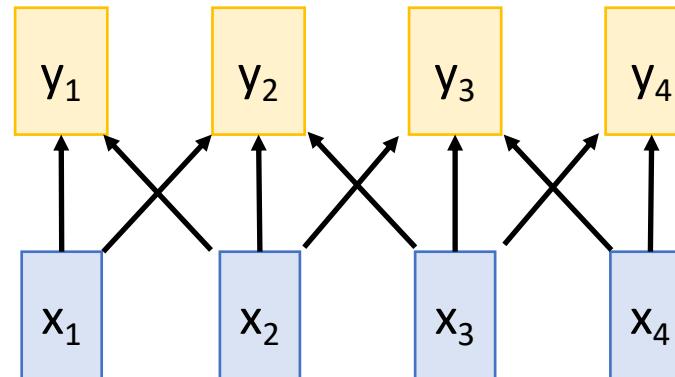
(-) Not parallelizable: need to compute hidden states sequentially

Three Ways of Processing Sequences

Recurrent Neural Network



1D Convolution



Works on **Ordered Sequences**

(+) **Good at long sequences:** After one RNN layer, h_T "sees" the whole sequence

(-) **Not parallelizable:** need to compute hidden states sequentially

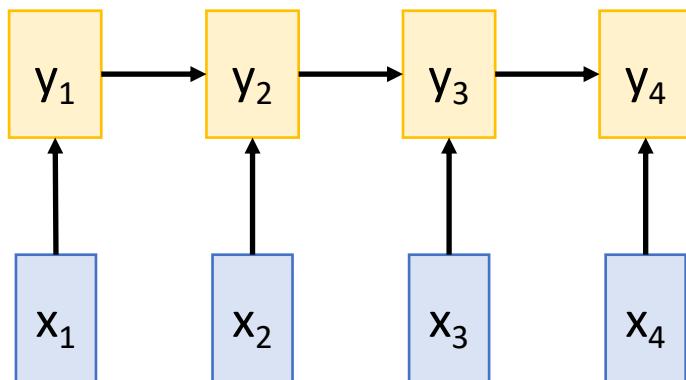
Works on **Multidimensional Grids**

(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence

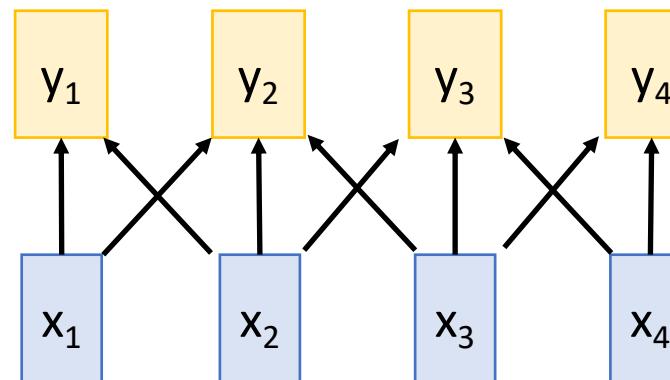
(+) **Highly parallel:** Each output can be computed in parallel

Three Ways of Processing Sequences

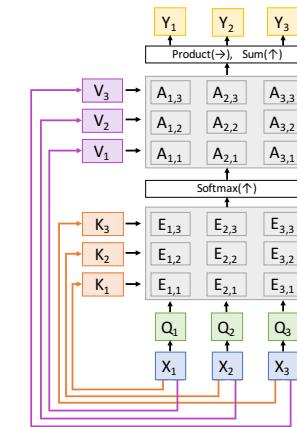
Recurrent Neural Network



1D Convolution



Self-Attention



Works on **Ordered Sequences**

- (+) Good at long sequences: After one RNN layer, h_T "sees" the whole sequence
- (-) Not parallelizable: need to compute hidden states sequentially

Works on **Multidimensional Grids**

- (-) Bad at long sequences: Need to stack many conv layers for outputs to "see" the whole sequence
- (+) Highly parallel: Each output can be computed in parallel

Works on **Sets of Vectors**

- (-) Good at long sequences: after one self-attention layer, each output "sees" all inputs!
- (+) Highly parallel: Each output can be computed in parallel
- (-) Very memory intensive

Three Ways of Processing Sequences

Recurrent Neural Network

1D Convolution

Self-Attention

Attention is all you need

Vaswani et al, NeurIPS 2017

Works on **Ordered Sequences**

(+) **Good at long sequences:** After one RNN layer, h_T "sees" the whole sequence

(-) **Not parallelizable:** need to compute hidden states sequentially

Works on **Multidimensional Grids**

(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence

(+) **Highly parallel:** Each output can be computed in parallel

Works on **Sets of Vectors**

(-) **Good at long sequences:** after one self-attention layer, each output "sees" all inputs!

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(-) **Very memory intensive**

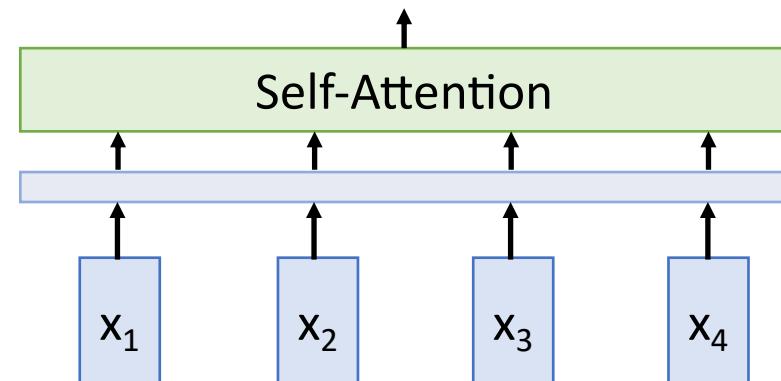
The Transformer



Vaswani et al, "Attention is all you need", NeurIPS 2017

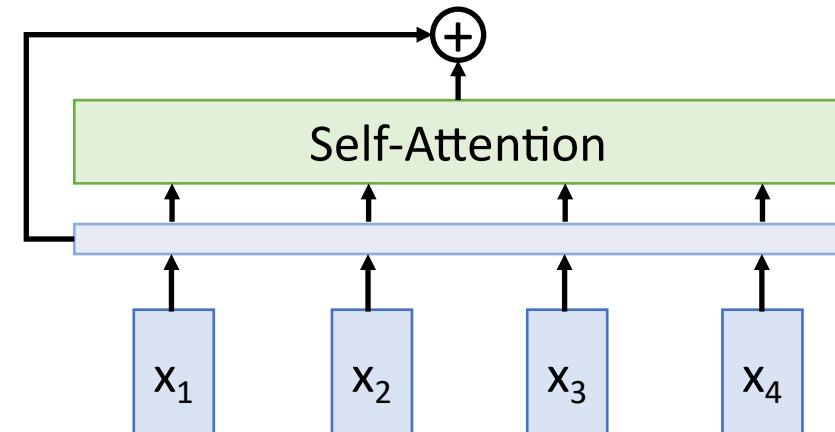
The Transformer

All vectors interact
with each other



The Transformer

Residual connection
All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

Given h_1, \dots, h_N (Shape: D)

scale: γ (Shape: D)

shift: β (Shape: D)

$\mu_i = (1/D) \sum_j h_{i,j}$ (scalar)

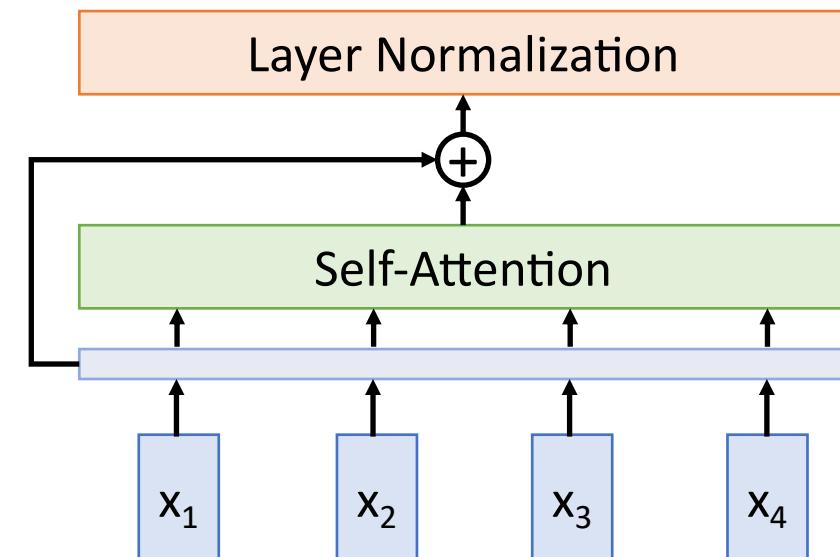
$\sigma_i = (\sum_j (h_{i,j} - \mu_i)^2)^{1/2}$ (scalar)

$z_i = (h_i - \mu_i) / \sigma_i$

$y_i = \gamma * z_i + \beta$

Ba et al, 2016

Residual connection
All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

Given h_1, \dots, h_N (Shape: D)

scale: γ (Shape: D)

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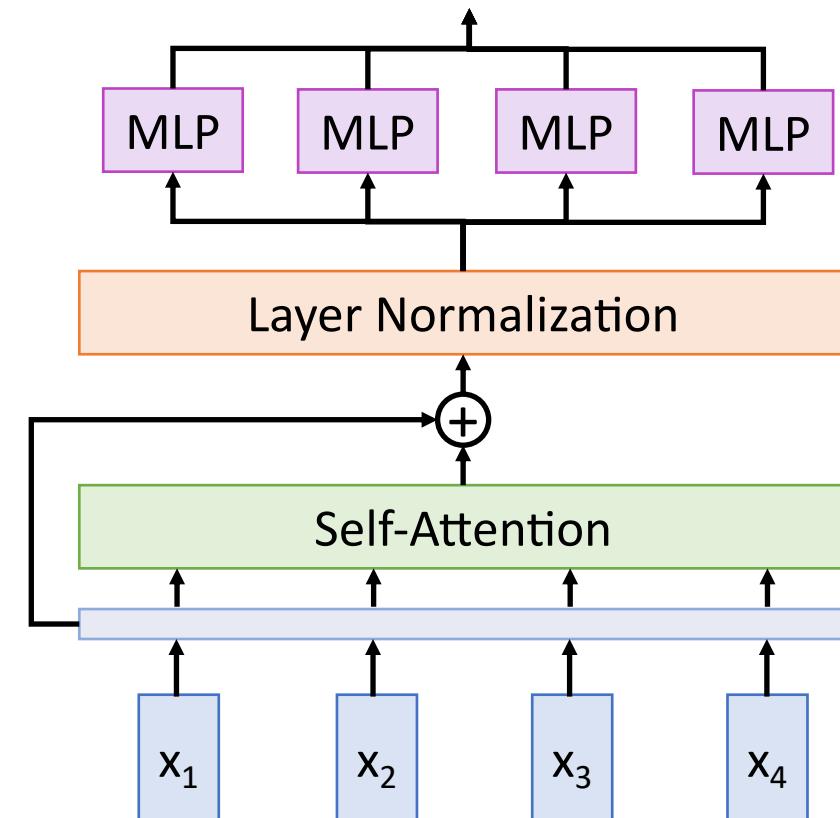
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$y_i = \gamma * z_i + \beta$

Ba et al, 2016

MLP independently
on each vector

Residual connection
All vectors interact
with each other



The Transformer

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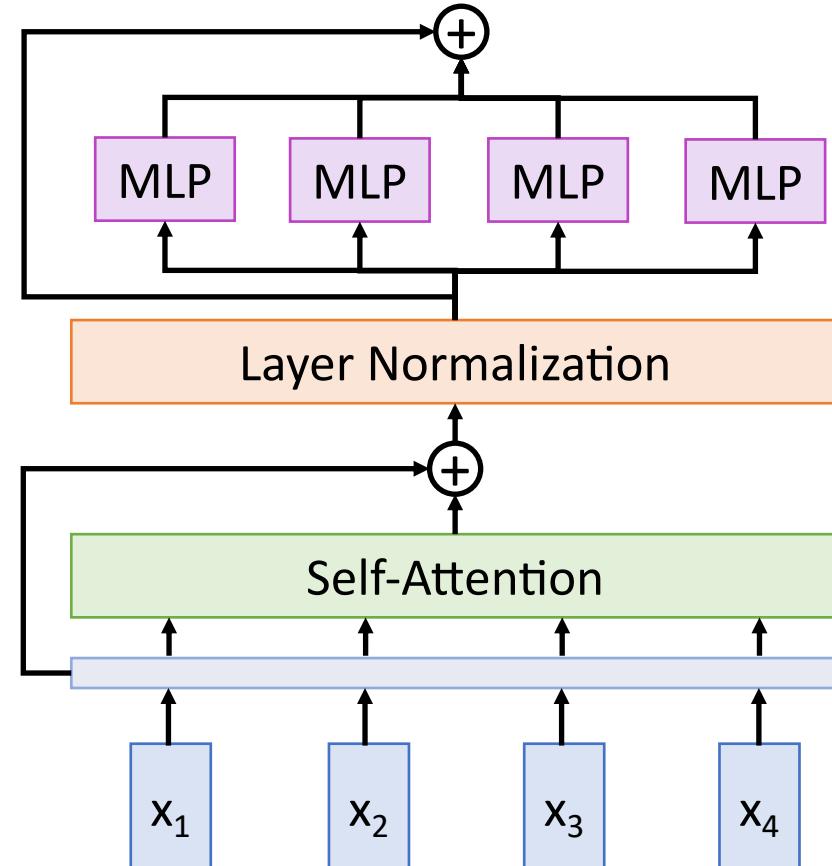
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

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scale: γ (Shape: D)

shift: β (Shape: D)

$\mu_i = (1/D) \sum_j h_{i,j}$ (scalar)

$\sigma_i = (\sum_j (h_{i,j} - \mu_i)^2)^{1/2}$ (scalar)

$z_i = (h_i - \mu_i) / \sigma_i$

$y_i = \gamma * z_i + \beta$

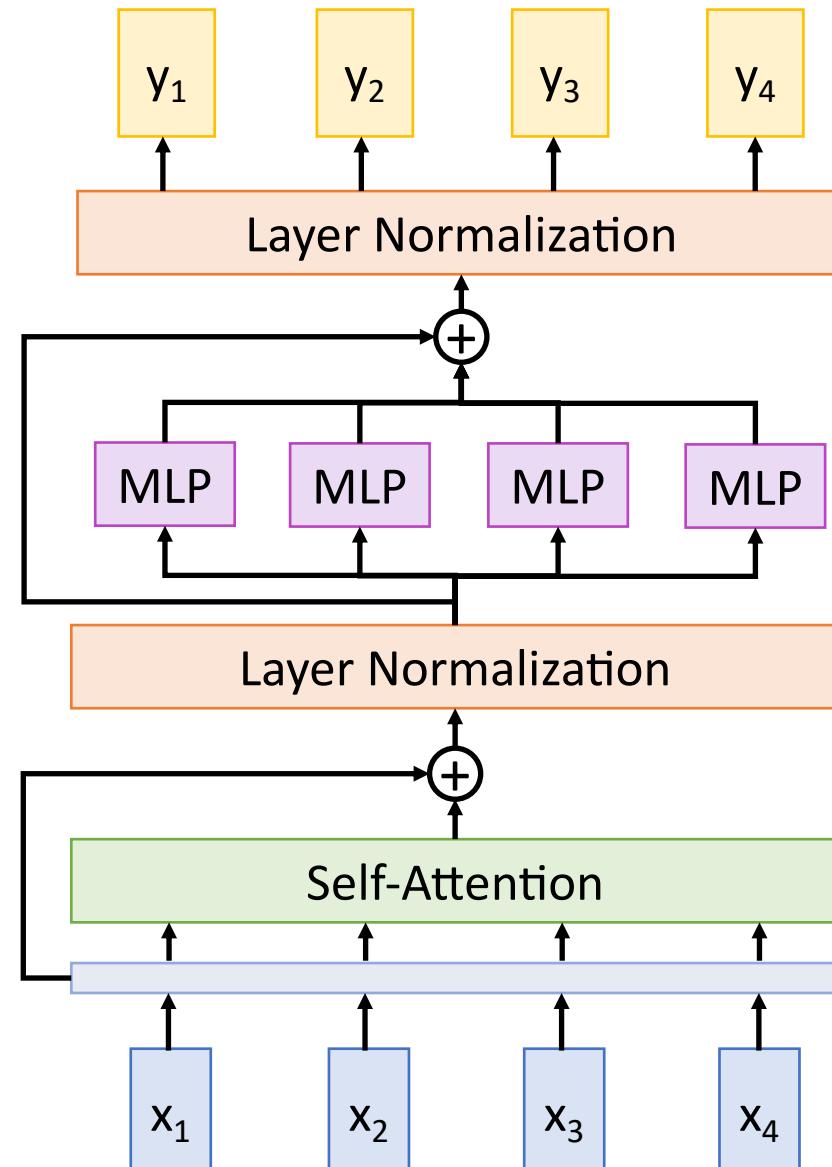
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
with each other



The Transformer

Transformer Block:

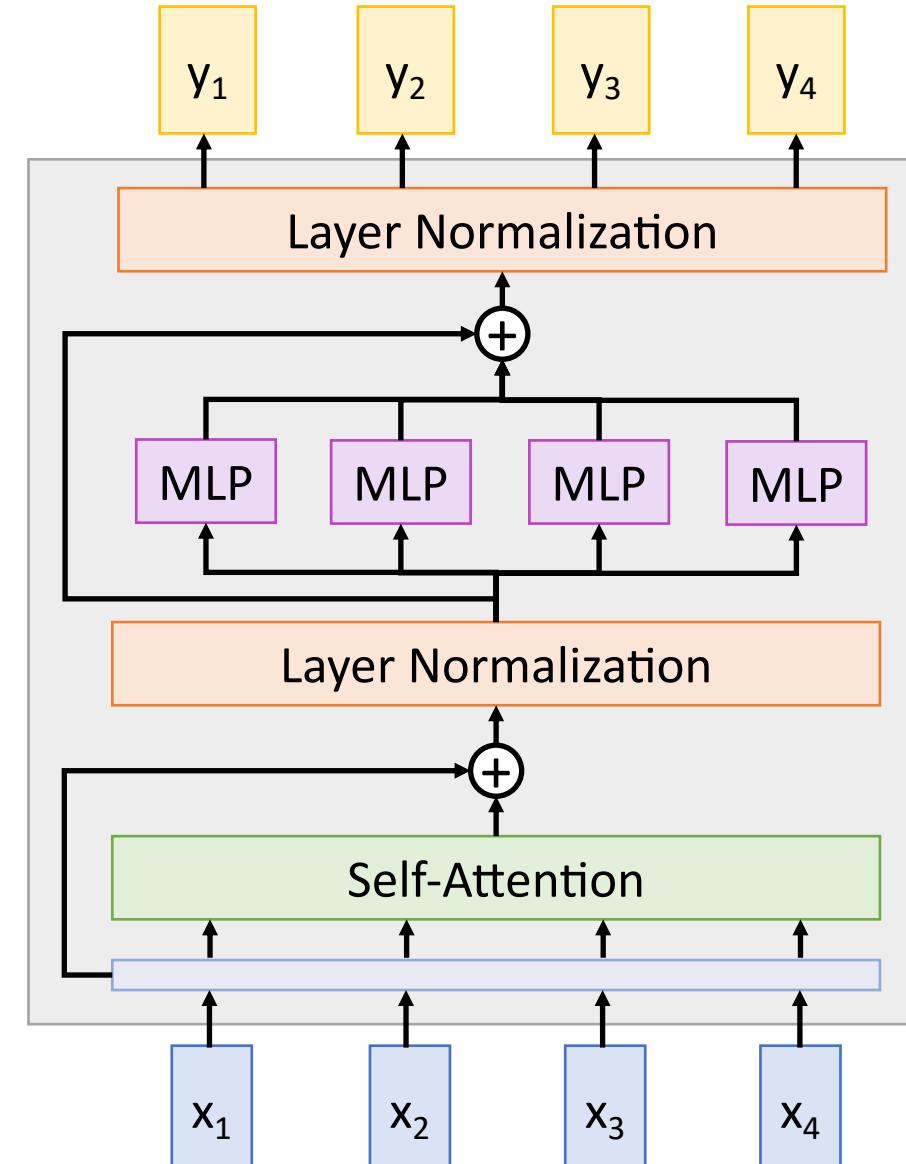
Input: Set of vectors x

Output: Set of vectors y

Self-attention is the only
interaction between vectors!

Layer norm and MLP work
independently per vector

Highly scalable, highly
parallelizable



The Transformer

Transformer Block:

Input: Set of vectors x

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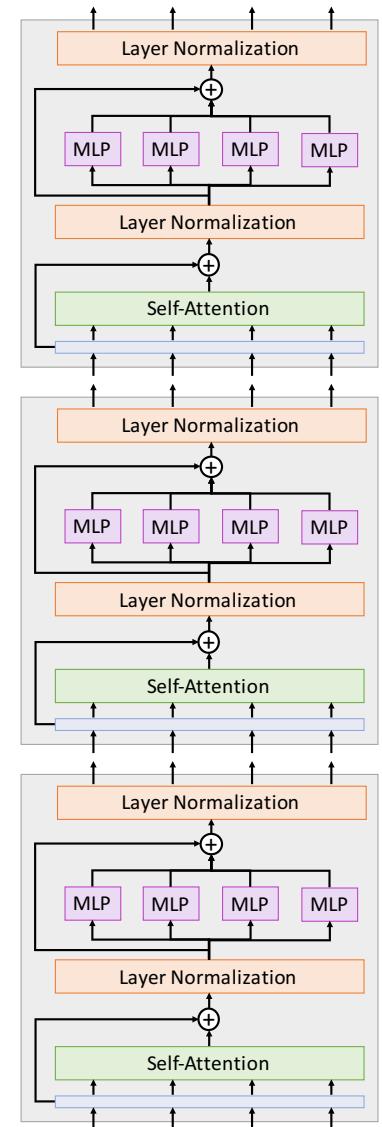
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A **Transformer** is a sequence of transformer blocks

Vaswani et al:
12 blocks, $D_Q=512$, 6 heads



The Transformer: Transfer Learning

“ImageNet Moment for Natural Language Processing”

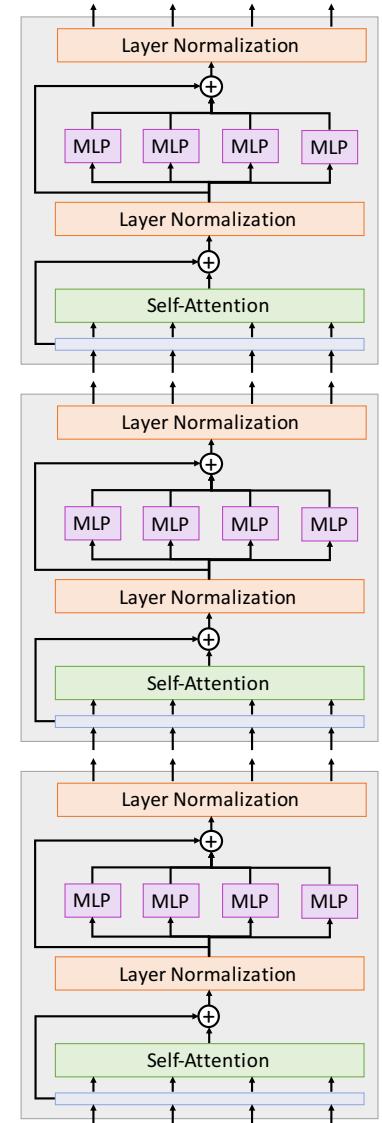
Pretraining:

Download a lot of text from the internet

Train a giant Transformer model for language modeling

Finetuning:

Fine-tune the Transformer on your own NLP task



Scaling up Transformers

Model	Layers	Width	Heads	Params	Data	Training
Transformer-Base	12	512	8	65M		8x P100 (12 hours)
Transformer-Large	12	1024	16	213M		8x P100 (3.5 days)

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Devlin et al, "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding", EMNLP 2018

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Yang et al, "XLNet: Generalized Autoregressive Pretraining for Language Understanding", 2019
Liu et al, "RoBERTa: A Robustly Optimized BERT Pretraining Approach", 2019

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Radford et al, "Language models are unsupervised multitask learners", 2019

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Shoeybi et al, "Megatron-LM: Training Multi-Billion Parameter Language Models using Model Parallelism", 2019

Scaling up Transformers

~\$430,000 on Amazon AWS!

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PROMPT (Human-written): *In a shocking finding, scientist discovered a herd of unicorns living in a remote, previously unexplored valley, in the Andes Mountains. Even more surprising to the researchers was the fact that the unicorns spoke perfect English.*

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COMPLETION (Transformer-written): The scientist named the population, after their distinctive horn, Ovid's Unicorn. These four-horned, silver-white unicorns were previously unknown to science.

Now, after almost two centuries, the mystery of what sparked this odd phenomenon is finally solved.

Dr. Jorge Pérez, an evolutionary biologist from the University of La Paz, and several companions, were exploring the Andes Mountains when they found a small valley, with no other animals or humans. Pérez noticed that the valley had what appeared to be a natural fountain, surrounded by two peaks of rock and silver snow.

Pérez and the others then ventured further into the valley. "By the time we reached the top of one peak, the water looked blue, with some crystals on top," said Pérez.

Pérez and his friends were astonished to see the unicorn herd. These creatures could be seen from the air without having to move too much to see them – they were so close they could touch their horns.

While examining these bizarre creatures the scientists discovered that the creatures also spoke some fairly regular English. Pérez stated, "We can see, for example, that they have a common 'language,' something like a dialect or dialectic."

Dr. Pérez believes that the unicorns may have originated in Argentina, where the animals were believed to be descendants of a lost race of people who lived there before the arrival of humans in those parts of South America.

While their origins are still unclear, some believe that perhaps the creatures were created when a human and a unicorn met each other in a time before human civilization. According to Pérez, "In South America, such incidents seem to be quite common."

However, Pérez also pointed out that it is likely that the only way of knowing for sure if unicorns are indeed the descendants of a lost alien race is through DNA. "But they seem to be able to communicate in English quite well, which I believe is a sign of evolution, or at least a change in social organization," said the scientist.

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Try it yourself:

<https://talktotransformer.com>

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OpenAI, "Better Language Models and their Implications", 2019, <https://openai.com/blog/better-language-models/>

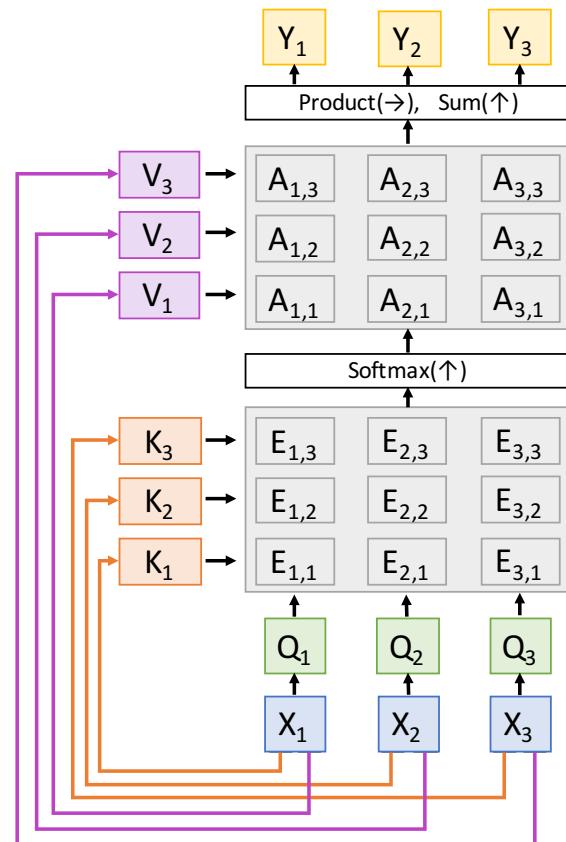
Summary

Adding **Attention** to RNN models lets them look at different parts of the input at each timestep

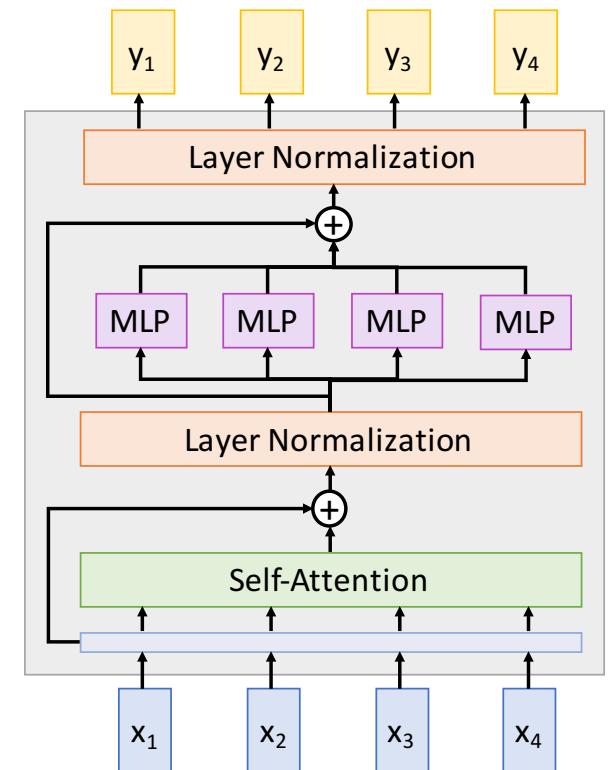


A dog is standing on a hardwood floor.

Generalized **Self-Attention** is new, powerful neural network primitive



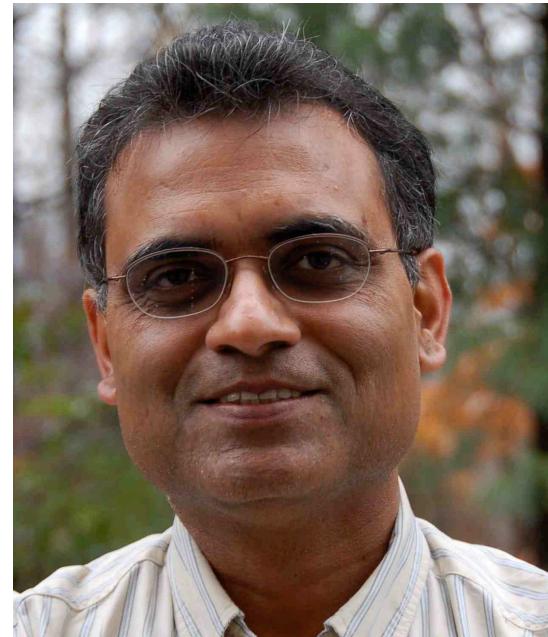
Transformers are a new neural network model that only uses attention



Next Week: Guest Lectures



Monday 10/28
Luowei Zhou
Vision and Language



Wednesday 10/30
Prof. Atul Prakash
Adversarial Machine Learning