Sintaxe abstrata:

$$
\begin{array}{lll}
T & \in & \text{Types} \\
T & ::= & \text{int} \mid \text{bool} \mid T_1 \to T_2
\end{array}
$$

$$
\begin{array}{lll}
e & \in & \text{L1} \\
e & ::= & n \mid b \mid e_1 \; op \; e_2 \mid \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \\
  & \mid & x \mid e_1 \; e_2 \mid \text{fn } x{:}T \Rightarrow e \mid \text{let } x{:}T = e_1 \text{ in } e_2 \\
  & \mid & \text{let rec } f{:}T_1 \to T_2 = (\text{fn } x{:}T_1 \Rightarrow e_1) \text{ in } e_2
\end{array}
$$

$$
\begin{array}{lll}
v & \in & \text{Values} \\
v & ::= & n \mid b \mid \text{fn } x{:}T \Rightarrow e
\end{array}
$$

onde

$$
\begin{array}{llll}
n & \in & \{\ldots, -1, 0, 1, 2, \ldots\} & \textit{(numerais inteiros)} \\
b & \in & \{\text{true}, \text{false}\} & \textit{(booleanos)} \\
x & \in & \text{Ident} & \textit{(conjunto de identificadores)} \\
op & \in & \{+, -, *, >, \geq, ==, ! =, \leq, <\} & \textit{(operações aritméticas e relacionais)}
\end{array}
$$

Semântica operacional *small-step*:

$$
\frac{e_1 \longrightarrow e_1'}{e_1 \; op \; e_2 \longrightarrow e_1' \; op \; e_2} \; (\text{E-OP1})
\qquad
\frac{e_2 \longrightarrow e_2'}{v \; op \; e_2 \longrightarrow v \; op \; e_2'} \; (\text{E-OP2})
\qquad
\frac{[\![n]\!] = [\![n_1]\!] + [\![n_2]\!]}{n_1 + n_2 \longrightarrow n} \; (\text{E-OP+})
$$

$$
\frac{e_1 \longrightarrow e_1'}{e_1 \; e_2 \longrightarrow e_1' \; e_2} \; (\text{E-APP1})
\qquad
\frac{e_2 \longrightarrow e_2'}{v \; e_2 \longrightarrow v \; e_2'} \; (\text{E-APP2})
\qquad
\frac{}{(\text{fn } x{:}T \Rightarrow e) \; v \longrightarrow \{v/x\}e} \; (\text{E-}\beta)
$$

$$
\frac{e_1 \longrightarrow e_1'}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \longrightarrow \text{if } e_1' \text{ then } e_2 \text{ else } e_3} \; (\text{E-IF})
$$

$$
\frac{}{\text{if true then } e_2 \text{ else } e_3 \longrightarrow e_2} \; (\text{E-IFTRUE})
\qquad
\frac{}{\text{if false then } e_2 \text{ else } e_3 \longrightarrow e_3} \; (\text{E-IFFALSE})
$$

$$
\frac{e_1 \longrightarrow e_1'}{\text{let } x{:}T = e_1 \text{ in } e_2 \longrightarrow \text{let } x{:}T = e_1' \text{ in } e_2} \; (\text{E-LET1})
\qquad
\frac{}{\text{let } x{:}T = v \text{ in } e_2 \longrightarrow \{v/x\} \; e_2} \; (\text{E-LET2})
$$

$$
\frac{}{\text{let rec } f : T_1 \to T_2 = \text{fn } y : T_1 \Rightarrow e_1 \text{ in } e_2 \longrightarrow \{\alpha/f\} \; e_2} \; (\text{E-LETREC})
$$

onde: $\alpha \equiv \text{fn } y : T_1 \Rightarrow \text{let rec } f{:}T_1 \to T_2 = (\text{fn } y : T_1 \Rightarrow e_1) \text{ in } e_1$

Substituição

$\{\_/\_\}\_ : \text{L1} \times \text{Ident} \times \text{L1} \to \text{L1}$

$$
\begin{array}{lcl}
\{v/x\} \; n & = & n \\
\{v/x\} \; b & = & b \\
\{v/x\} \; (e_1 \text{ op } e_2) & = & \{v/x\}e_1 \text{ op } \{v/x\}e_2 \\
\{v/x\} \; (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) & = & \text{if } \{v/x\}e_1 \text{ then } \{v/x\}e_2 \text{ else } \{v/x\}e_3 \\
\{v/x\} \; (e_1 \; e_2) & = & \{v/x\}e_1 \; \{v/x\}e_2 \\
\{v/x\} \; x & = & v \\
\{v/x\} \; y & = & y \;\; (se \; x \neq y) \\
\{v/x\} \; (\text{fn } x : T \Rightarrow e) & = & \text{fn } x : T \Rightarrow e \\
\{v/x\} \; (\text{fn } y : T \Rightarrow e) & = & \text{fn } y : T \Rightarrow \{v/x\}e \;\; (\text{se } x \neq y) \\
\{v/x\}(\text{let } x{:}T = e_1 \text{ in } e_2) & = & \text{let } x{:}T = \{v/x\}e_1 \text{ in } e_2 \\
\{v/x\}(\text{let } y{:}T = e_1 \text{ in } e_2) & = & \text{let } y{:}T = \{v/x\}e_1 \text{ in } \{v/x\}e_2 \;\; (\text{se } x \neq y) \\
\{v/f\}(\text{let rec } f : T_1 \to T_2 = (\text{fn } y{:}T_1 \Rightarrow e_1) & = & \text{let rec } f : T_1 \to T_2 = (\text{fn } y{:}T_1 \Rightarrow e_1) \\
\quad \text{in } e_2) & & \quad \text{in } e_2 \\
\{v/x\}(\text{let rec } f : T_1 \to T_2 = (\text{fn } y{:}T_1 \Rightarrow e_1) & = & \text{let rec } f : T_1 \to T_2 = \{v/x\}(\text{fn } y{:}T_1 \Rightarrow e_1) \\
\quad \text{in } e_2) & & \quad \text{in } \{v/x\} \; e_2 \\
& & \quad \text{se } x \neq f
\end{array}
$$

Sistema de Tipos:

$$\frac{}{\Gamma \vdash n : \mathsf{int}} \quad (\text{T\textsc{int}}) \qquad \frac{}{\Gamma \vdash b : \mathsf{bool}} \quad (\text{T\textsc{bool}}) \qquad \frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad (\text{T\textsc{var}})$$

$$\frac{\Gamma \vdash e_1 : \mathsf{int} \qquad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1 + e_2 : \mathsf{int}} \, (\text{T\textsc{op}+}) \qquad \frac{\Gamma, x{:}T \vdash e : T'}{\Gamma \vdash (\mathsf{fn}\ x{:}T \Rightarrow e) : T{\to}T'} \, (\text{T\textsc{fn}}) \qquad \frac{\Gamma \vdash e_1 : T{\to}T' \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1\ e_2 : T'} \, (\text{T\textsc{app}})$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : T \qquad \Gamma \vdash e_3 : T}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 : T} \, (\text{T-\textsc{if}}) \qquad \frac{\Gamma \vdash e_1 : T \qquad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \mathsf{let}\ x{:}T = e_1\ \mathsf{in}\ e_2 : T'} \, (\text{T-\textsc{let}})$$

$$\frac{\Gamma, f : T_1{\to}T_2,\ x{:}T_1 \vdash e_1 : T_2 \qquad \Gamma, f : T_1{\to}T_2 \vdash e_2 : T}{\Gamma \vdash \mathsf{let\ rec}\ f{:}T_1 \to T_2 = (\mathsf{fn}\ x{:}T_1 \Rightarrow e_1)\ \mathsf{in}\ e_2 : T} \qquad (\text{T-\textsc{letrec}})$$

Semântica operacional *big step* com substituição:

$$\frac{}{n \Downarrow n} \quad (\textsc{bs-num}) \qquad \frac{}{\mathsf{fn}\ x \Rightarrow e \Downarrow \mathsf{fn}\ x \Rightarrow e} \quad (\textsc{bs-fn}) \qquad \frac{}{b \Downarrow b} \quad (\textsc{bs-bool})$$

$$\frac{e_1 \Downarrow \mathsf{true} \qquad e_2 \Downarrow v}{\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \, (\textsc{bs-ift}) \qquad \frac{e_1 \Downarrow \mathsf{false} \qquad e_3 \Downarrow v}{\rho \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \, (\textsc{bs-iff}) \qquad \frac{e_1 \Downarrow v' \qquad \{v'/x\}\ e_2 \Downarrow v}{\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Downarrow v} \, (\textsc{bs-let})$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2 \qquad [\![n]\!] = [\![n_1]\!] + [\![n_2]\!]}{e_1 + e_2 \Downarrow n} \, (\textsc{bs-op}+) \qquad \frac{e_1 \Downarrow \mathsf{fn}\ x \Rightarrow e \qquad e_2 \Downarrow v_2 \qquad \{v_2/x\}\ e \Downarrow v}{e_1\ e_2 \Downarrow v} \, (\textsc{bs-app})$$

$$\frac{\{\alpha/f\}\ e_2 \Downarrow v}{\mathsf{let\ rec}\ f = \mathsf{fn}\ x \Rightarrow e_1\ \mathsf{in}\ e_2 \Downarrow v} \qquad (\textsc{bs-letrec})$$

onde: $\alpha \equiv \mathsf{fn}\ y : T_1 \Rightarrow \mathsf{let\ rec}\ f{:}T_1{\to}T_2 = (\mathsf{fn}\ y : T_1 \Rightarrow e_1)\ \mathsf{in}\ e_1$

Semântica operacional *big step* com ambientes:

$$\begin{array}{llll} v & \in & \mathsf{Values} & \\ v & ::= & n \mid b \mid \langle x, e, \rho \rangle \mid \langle f, x, e, \rho \rangle & \end{array} \qquad \begin{array}{llll} \rho & \in & \mathsf{Env} \\ \rho & ::= & [\,] \mid \rho, x \mapsto v \end{array}$$

$$\rho \vdash n \Downarrow n \qquad (\textsc{bs-num}) \qquad\qquad \frac{\rho \vdash e_1 \Downarrow \mathsf{false} \qquad \rho \vdash e_3 \Downarrow v}{\rho \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \quad (\textsc{bs-iffls})$$

$$\rho \vdash b \Downarrow b \qquad (\textsc{bs-bool}) \qquad\qquad \rho \vdash (\mathsf{fn}\ x \Rightarrow e) \Downarrow \langle x, e, \rho \rangle \qquad (\textsc{bs-fn})$$

$$\frac{\rho(x) = v}{\rho \vdash x \Downarrow v} \qquad (\textsc{bs-id}) \qquad\qquad \frac{\rho \vdash e_1 \Downarrow v' \qquad \rho, x \mapsto v' \vdash e_2 \Downarrow v}{\rho \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Downarrow v} \quad (\textsc{bs-let})$$

$$\frac{\rho \vdash e_1 \Downarrow \mathsf{true} \qquad \rho \vdash e_2 \Downarrow v}{\rho \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \quad (\textsc{bs-iftr}) \qquad \frac{\rho, f \mapsto \langle f, x, e_1, \rho \rangle \vdash e_2 \Downarrow v}{\rho \vdash \mathsf{let\ rec}\ f = \mathsf{fn}\ x \Rightarrow e_1\ \mathsf{in}\ e_2 \Downarrow v} (\textsc{bs-letrec})$$

$$\frac{\rho \vdash e_1 \Downarrow \langle x, e, \rho' \rangle \qquad \rho \vdash e_2 \Downarrow v' \qquad \rho', x \mapsto v' \vdash e \Downarrow v}{\rho \vdash e_1\ e_2 \Downarrow v} \qquad (\textsc{bs-app})$$

$$\frac{\rho \vdash e_1 \Downarrow \langle f, x, e, \rho' \rangle \qquad \rho \vdash e_2 \Downarrow v' \qquad \rho', x \mapsto v', \mapsto \langle f, x, e, \rho' \rangle \vdash e \Downarrow v}{\rho \vdash e_1\ e_2 \Downarrow v} \qquad (\textsc{bs-apprec})$$