

Worksheet 3

Practical Lab Numerical Computing

Andrii Lischishin

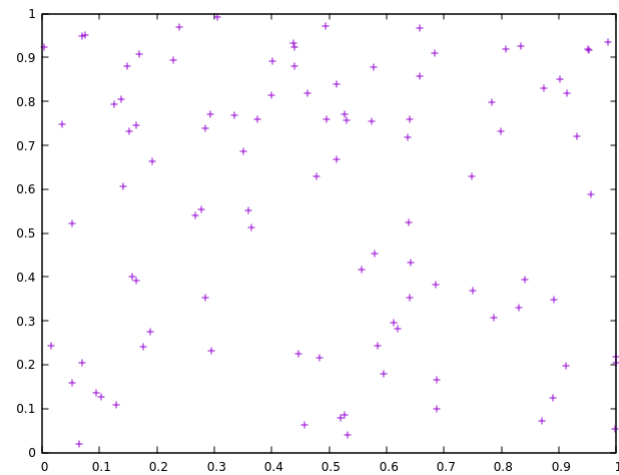
Lars Schleithoff

Hendrik Kleikamp

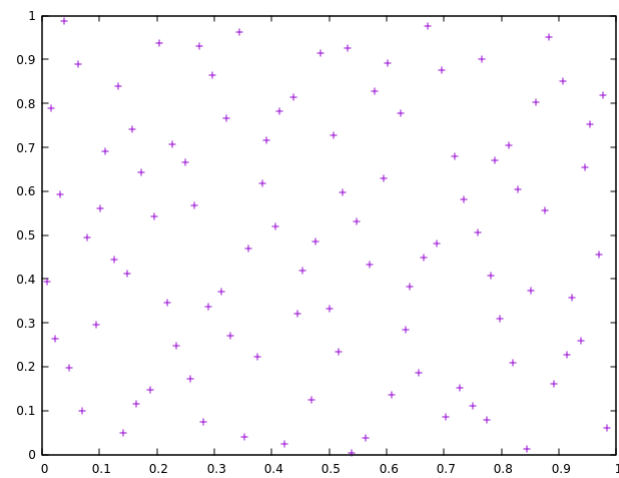
June 16, 2017

Task 7

Uniform random numbers in $(0, 1)^2$:



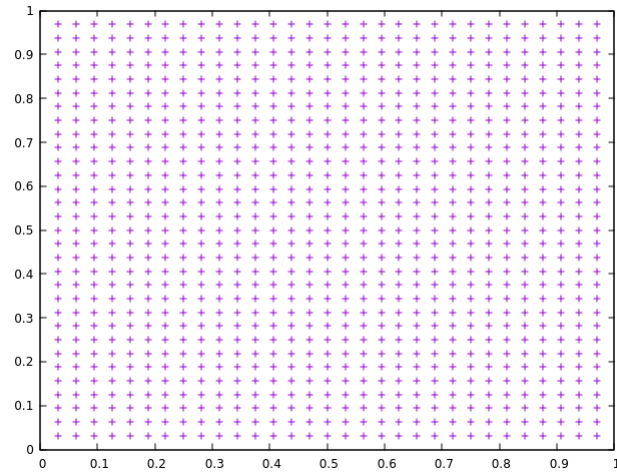
Halton sequence:



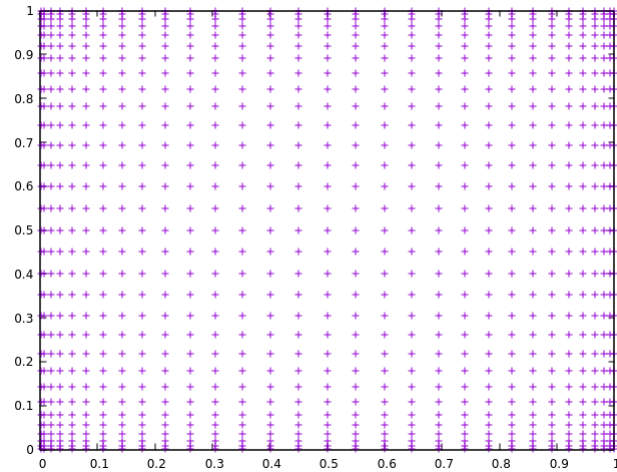
One can see that there are some regions in $(0,1)^2$ where no uniform random numbers are set. This is not the case in the point set calculated by the Halton sequence. The Halton sequence gives a very uniform spread set without any holes.

Task 9

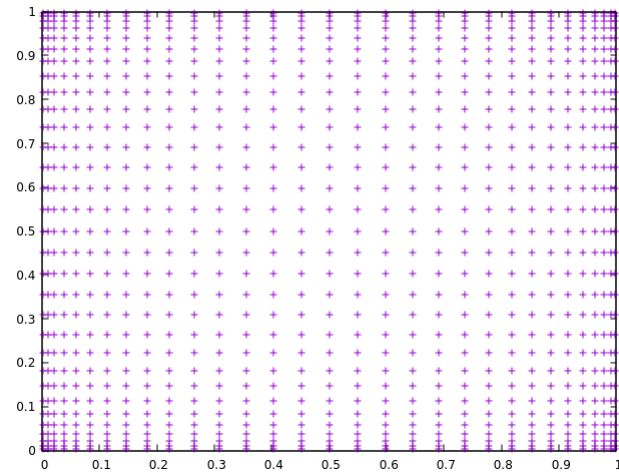
Quadrature nodes of the two-dimensional product rule for trapezoidal rule:



Quadrature nodes of the two-dimensional product rule for Gauss-Legendre:



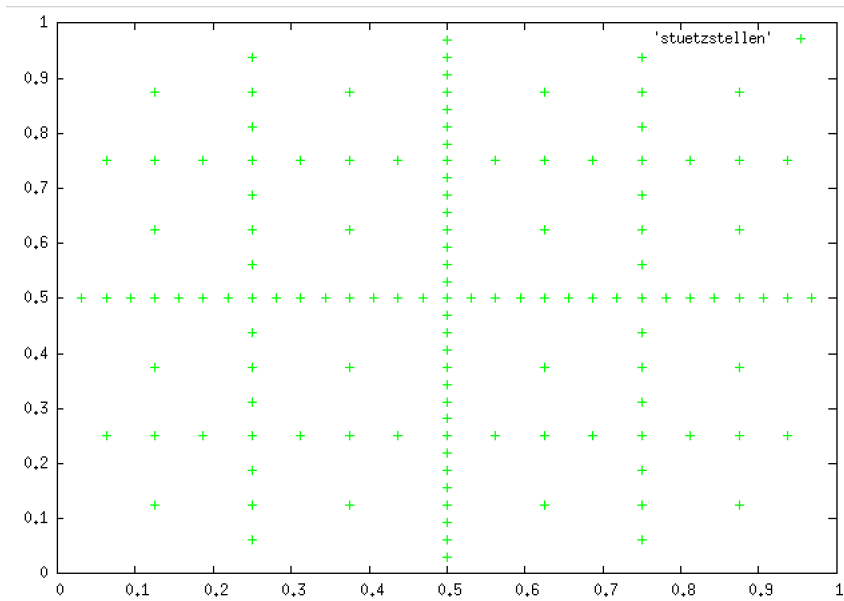
Quadrature nodes of the two-dimensional product rule for Clenshaw Curtis:



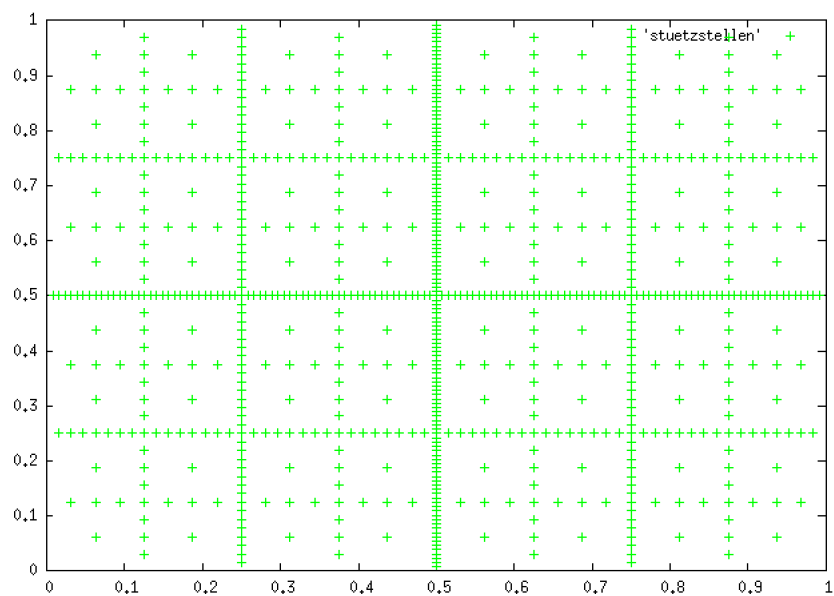
Both Gauß-Quadrature and Clenshaw-Curtis-Quadrature have an increased amount of nodes at the edges of the integration region. The trapeziodal rule distributes the nodes equidistantly over the integration region.

Task 11

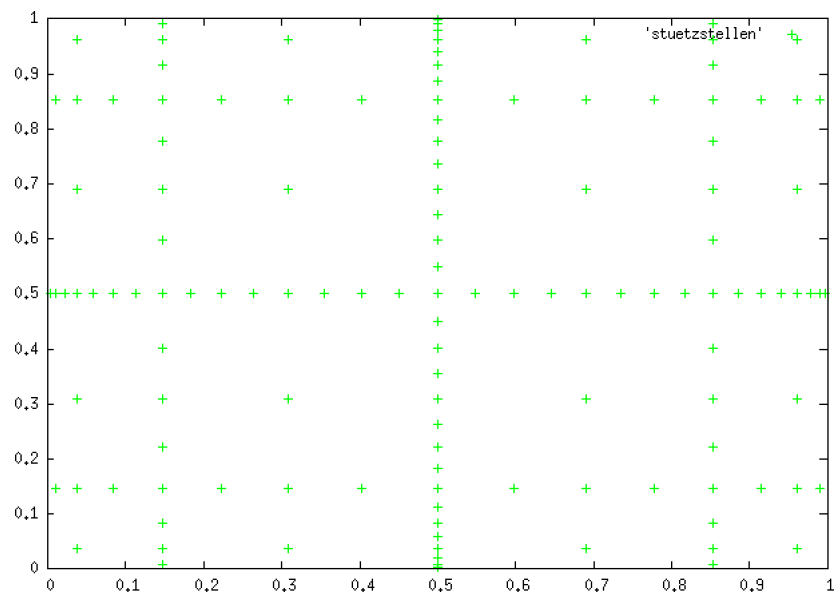
Two-dimensional Sparse Grid using Trapeziodal Rule for $l = 5$:



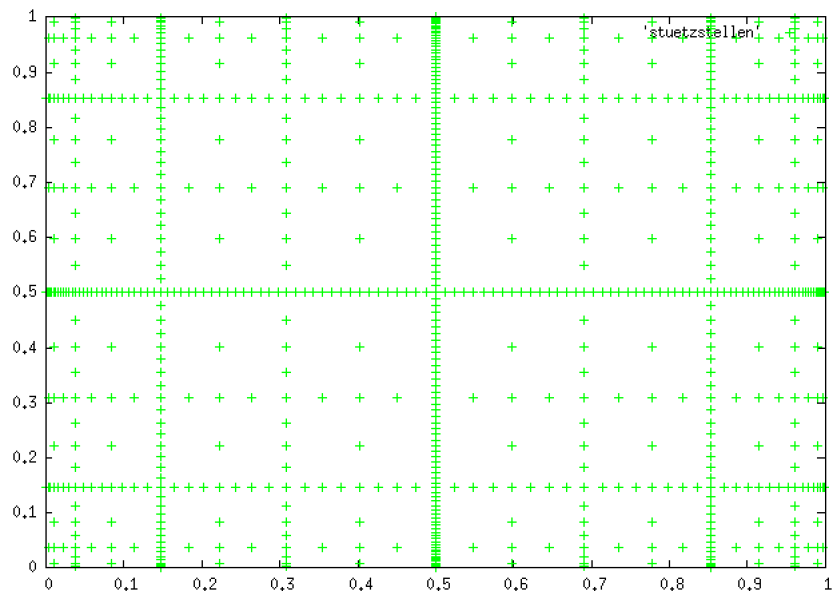
Two-dimensional Sparse Grid using Trapezoidal Rule for $l = 7$:



Two-dimensional Sparse Grid using Clenshaw-Curtis Quadrature Rule for $l = 5$:



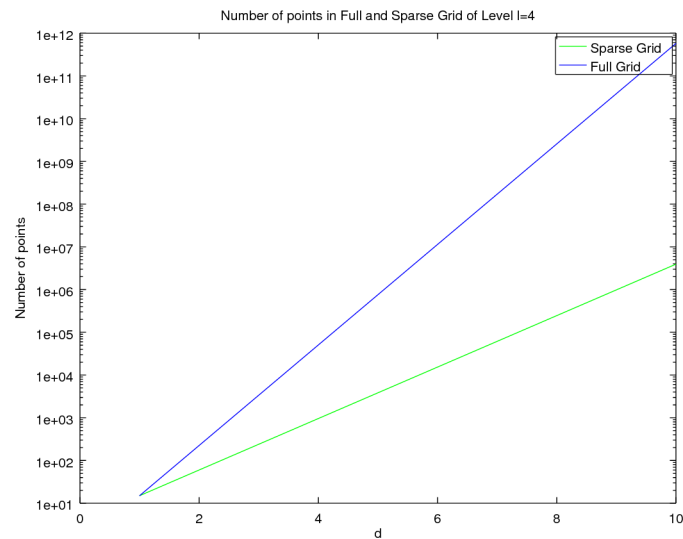
Two-dimensional Sparse Grid using Clenshaw-Curtis Quadrature Rule for $l = 7$:



For Clenshaw-Curtis Quadrature, with increasing level, more nodes are situated at the edges of the integration region. For Trapezoidal Rule, this is not the case.

Task 12

Number of points in Sparse Grid and Full Grid in Level $l = 4$:



One can see that the number of points increases much faster in the Full Grid, then in the Sparse Grid.

Task 18

The difference between MC and QMC is, that MC uses actual pseudo random numbers, while QMC uses deterministic sequences. For this reason, QMC Quadrature Nodes are more evenly distributed over the integration region, which enhances the convergence rate of QMC (which is $\mathcal{O}(N)$) compared to the convergence of $\mathcal{O}(N^{1/2})$ of Monte Carlo Method.

Both, QMC and MC, can integrate a wide range of functions, regardless of how smooth these functions are.

Sparse Grids and Full Grids make use of nodes and weights which result from quadrature methods like Clenshaw-Curtis or Trapezoidal rule. These methods can only be applied to somewhat smoother functions, but they have much better convergence rates. Full Grids are heavily affected by the dimension, with the number of nodes increasing exponentially with increasing dimension. For sparse grids, dependence on dimension is considerably lower.

Therefore, if it is known that the integrand will be relatively smooth, using Sparse Grid methods is the best way to compute integrals in high dimensions. However, if the integrand is unknown or if the integrand is a function that is not differentiable in many points, MC and QMC integration is the preferred method, especially in high dimensions.