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ln[1]:= Print[" The source of the data of the manuscript "];
Print[" Structure and energetics of carbon, hexagonal boron nitride, "];
Print[" and carbon/hexagonal boron nitride single-layer and bilayer nanoscrolls "];
Print[" / A.I. Siahlo, N.A. Poklonski, A.V. Lebedev, I.V.
      Lebedeva, A.M. Popov, S.A. Vyrko, A.A. Knizhnik, Yu.E. Lozovik "];
Print[" // Phys. Rev. Materials.- 2018.- V. 2, № 3.- P. 036001 (9
      pp.). [DOI: 10.1103/PhysRevMaterials.2.036001] "];
Print[" -----"];

NoL1 = 1; NoL2 = 2;
NoLp = NoL1;
Print[" I.0 The Units (nm, meV, AA)"];
"nm=10^(-9)m;";
nm = 10^(-9) m;
AA = 10^(-10) m;
JJkgms = kg m^2 / s^2;
Cl = Amper s;
"eV=JJ Electronp;";
JJ = eV/Electron;
JJms = (kg m^2) / s^2;
meV = N[eV/1000];
Print["-----"];

Print[" I.1. All Input Parameters and Constants-----"];
Print[" I.1.1. The sampling parameters"]
npRIn1 = 1000;
Print["npRIn1=", npRIn1];
Print[" I.1.2. The Input Geometry Parameters of the system"];
L14d839nm = 14.839 nm;
L12d709nm = 12.709 nm;
L129d678nm = 29.678 nm;
L1p = L129d678nm;
L1p = L14d839nm;

Print[" The carbon nanoribbon length L1=", L1p/nm, "nm"];
Lw11d8nm = 11.8 nm;
Lwp = Lw11d8nm;
Print[" The carbon nanoribbon width Lw=", Lwp/nm, "nm"];
Print[" ----- "];

Print[" Number of the layers in carbon nanoscroll NoL=", NoLp];
Print[" The length of a carbon nanoribbon L1=", L1p/nm, "nm"];
Lw1nm = 1. nm; Lwp = Lw1nm;
Print[" The carbon nanoribbon width Lw=", Lwp/nm, "nm"];
RIn1d1nm = 1.1 nm;
RIn1d2nm = 1.2 nm;
RIn1d14nm = 1.14 nm;
RIn2nm = 2.047 nm;
RIn2d1nm = 2.1 nm;
RIn2d2nm = 2.2 nm;
RIn2d3nm = 2.3 nm;
RIn2d4nm = 2.4 nm;
RIn2d5nm = 2.5 nm;
RIn2d6nm = 2.6 nm;
RIn1p = RIn2d5nm;
RIn1p = RIn2d3nm;
RIn1p = RIn2d2nm;

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RIn1p = RIn2dlnm;
RIn1p = RIn1d14nm;
Print[" The inner radius of the nanoscroll RIn1=", RIn1p/nm, "nm"];
Print[" I.1.2. The Input Energy Constants"];
Print[" epsVdW - the interlayer interaction energy per one atom of"];
Print[" the nanoscroll:"];
epsVdW35 = 35.0 meV / atom; epsVdWp = epsVdW35;
Print[" epsVdW=", epsVdWp / (eV / atom), "eV/atom"];
Print[" C - the bending elastic constant:"];
C201 = 2.01 eV AA^2 / atom;
CBN1328 = 1.328 eV AA^2 / atom;
CCp = C201;
CBNp = CBN1328;
CBNp = CCp;
Print[" CCelast=", CCp / (eV AA^2 / atom), "eV AA^2/atom"];
Print[" CCBNelast=", CBNp / (eV AA^2 / atom), "eV AA^2/atom"];
Print[" I.1.3.The Input Geometry constants-----"];
Print[" The interatomic distance aCC and the interlayer distance h"];
aCC142AA = 1.42 AA; aCCp = aCC142AA;
h335nm = 0.3354 nm; hp = h335nm;
Print["h=", hp / nm, " nm (Interlayer distance)"];
Print[" aCC=", aCCp / nm, "nm, h=", hp / nm, "nm"];
NatomsInCell2 = 2; NatomsInCellp = NatomsInCell2;
Print["NatomsInCell=", NatomsInCellp];

Print[" dPhil2 - The difference of the inner angles of the spirales"];
Print[" of the Layers"];
dPhil2eq0 = 0.0 Pi;
dPhil2eqPi = 1.0 Pi;
dPhil2p = 0.0 Pi;
dPhil2p = 1.0 Pi;
dPhil2p = 0.5 Pi;
Print[" I.5.The parameters for the visualisation"];
RIn1MinMonoScroll = hp / 5;
RIn1MinBiScroll = hp / 5;
RIn1MaxMonoScroll = 4 nm;
RIn1MaxBiScroll = 8 nm;
PlotRangeMonoScroll = {-4 eV / atom, 12 eV / atom};
PlotRangeBiScroll = {-10 eV / atom, 30 eV / atom};
ShowSpirales = True;
ShowThePlot = True;
Print[" I.6. The parameters of the output file"];
NanoscrollNamep = StringJoin["Nanoscroll", ToString[NoLp], "L", ToString[L1p / nm], "nm"];
Print[" NanoscrollName=", NanoscrollNamep];
CarbonNanoscrollEnergyVsRInFileName = StringJoin[NanoscrollNamep, ".txt"];
Print[CarbonNanoscrollEnergyVsRInFileName];
Print[" (The output of the data to a file Is Not Performed)"];
npRIn1 = 1000;
Print[" The number of the output points = ", npRIn1];
Print[" I.7. The Input Numerical Constants used in the programm"];
Print[" The Indexes used for the work with EVdW[...] function"];
iEVdW = 1; iEVdW1Un1 = 2; iEVdW1Ov1 = 3; iEVdW1Un2 = 4; iEVdW1Ov2 = 5;
iEVdW2Un1 = 6; iEVdW2Ov1 = 7;
Print[" -----End of the Input-----"];
AA = 0.1 nm; PhiIn :=  $\varphi$ In; PhiOut :=  $\varphi$ Out;
Print["-----"];

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Print[" II. The derivated parameters and the functions required"];
Print[" II.1. The derivated parameters"];
fSa[aCC_] := aCC^2 3 Sqrt[3] / 4; fSa[aCCp]; Sap = fSa[aCCp];
Print[" The cell area Sa=", fSa[aCC], "=", Sap/nm^2, "nm^2"];
Print[" II.2. The required functions-----"];
Print[" II.2.1. The function fSpiraleLen[" , NoLp, " ,  $\varphi$ In,  $\varphi$ Out, h] defines"];
Print[
  " the Length of a Spirale with the inner aple  $\varphi$ In and the outer angle  $\varphi$ Out( $\geq \varphi$ In):"];
fSpiraleLen[NoLv_, PhiInv_, PhiOutv_, hv_] :=
  (1 / (4 Pi) hv NoLv (-PhiInv Sqrt[1 + PhiInv^2] +
    PhiOutv Sqrt[1 + PhiOutv^2] - ArcSinh[PhiInv] +
    ArcSinh[PhiOutv]));
Print[" fSpiraleLen[" , NoLp,
  " ,  $\varphi$ In,  $\varphi$ Out, h]=", fSpiraleLen[NoLp, PhiIn, PhiOut, h], "."];
Print[" II.2.2. The function fElast[ $\varphi$ In, $\rho$ Out] is
  required to calculate an nanoscrollelastic energy: "];
fElast[PhiInv_,
  PhiOutv_] := (Sqrt[PhiInv^2 + 1] / PhiInv -
  Sqrt[PhiOutv^2 + 1] / PhiOutv - ArcSinh[PhiInv] + ArcSinh[PhiOutv]);
Print[" fElast[ $\varphi$ In, $\varphi$ Out] = ", fElast[PhiIn, PhiOut], "."];
Print[" II.2.3. Define the function fPhiOutvsPhiInLh[" , NoLp, " , " , PhiIn, " , L, h]."];
fPhiOutvsPhiInLh[NoLv_, PhiInv_, Lv_, hv_] :=
  Sqrt[4  $\pi$  Lv / (NoLv hv) + PhiInv^2];
Print[" The function fPhiOutvsPhiInLh[" , NoLp,
  " , " , PhiIn, " , L, h]=", fPhiOutvsPhiInLh[NoLp, PhiIn, L, h], " is a
  good approximation to obtain the value of  $\varphi$ Out for the defined  $\varphi$ In,L,h."];
fPhiInvsPhiOutLh[NoLv_, PhiOutv_, Lv_, hv_] :=
  Sqrt[PhiOutv^2 - 4  $\pi$  Lv / (NoLv hv)];
Print[" The inverse function fPhiInvsPhiOutLh[" , NoLp, " ,  $\varphi$ Out, L, h]=",
  fPhiInvsPhiOutLh[NoLp, PhiOut, L, h]];
Print[" could be used in the program applications
  if ROut (instead of RIn) is the input parameter of the system."];

Print[" II.2.4. The functions
  fSpirale1Under(Over)Spirale1Length[NoLv,PhiInlv ,PhiOutlv,hv]" ];
fSpirale1UnderSpirale1Length[NoLv_, PhiInlv_, PhiOutlv_, hv_] :=
  fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2 Pi, hv];
fSpirale1OverSpirale1Length[NoLv_, PhiInlv_, PhiOutlv_, hv_] :=
  fSpiraleLen[NoLv, PhiInlv + 2 Pi, PhiOutlv, hv];

fSpirale1UnderSpirale2Length[NoLv_, PhiInlv_, PhiOutlv_, hv_, dPhi12v_] :=
  fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2 Pi / NoLv, hv];
fSpirale1OverSpirale2Length[NoLv_, PhiInlv_, PhiOutlv_, hv_, dPhi12v_] :=
  fSpiraleLen[NoLv, PhiInlv + 2 Pi / NoLv + dPhi12v, PhiOutlv, hv];
fSpirale2UnderSpirale1Length[NoLv_, PhiInlv_, PhiOutlv_, hv_, dPhi12v_] :=
  fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2 Pi / NoLv, hv];
fSpirale2OverSpirale1Length[NoLv_, PhiInlv_, PhiOutlv_, hv_, dPhi12v_] :=
  fSpiraleLen[NoLv, PhiInlv - dPhi12v + 2 Pi / NoLv, PhiOutlv - dPhi12v, hv];

Print[" These functiona are not required, dut could be helpful),"];

If[NoLp == 1,
  Print["fSpirale1UnderSpirale1Length[1,PhiInlv ,PhiOutlv,hv]="];
  Print[" =fSpiraleLen[NoLv,PhiInlv ,PhiOutlv-2Pi,hv]=",
    fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2 Pi, hv], ""];
  Print[" fSpirale1UnderSpirale1Length[NoLp,PhiInlp
    ,PhiOutlp,hp]=fSpirale1UnderSpirale1Length[" , NoLp, " , " ,

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    PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm, "nm] =";
Print["  =", fSpirale1UnderSpirale1Length[NoLv, PhiInlp, PhiOutlp, hp] / nm, "nm."];

Print["fSpirale1OverSpirale1Length[1,PhiInlv,PhiOutlv,hv]="];
Print["  =fSpiraleLen[NoLv,PhiInlv+2Pi,PhiOutlv,hv]=",
    fSpiraleLen[NoLv, PhiInlv + 2 Pi, PhiOutlv, hv], ";"];
Print["  fSpirale1OverSpirale1Length[NoLv,PhiInlp
    ,PhiOutlp,hp]=fSpirale1OverSpirale1Length[" , NoLv, " , " ,
    PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm, "nm] ="];
Print["  =", fSpirale1OverSpirale1Length[NoLv, PhiInlp, PhiOutlp, hp] / nm, "nm."];
];

If[NoLv == 2,
Print["  fSpirale1UnderSpirale2Length[1,PhiInlv,PhiOutlv,hv,dPhil2v]="];
Print["  fSpiraleLen[NoLv,PhiInlv,PhiOutlv -2 Pi/NoLv,hv]=",
    fSpirale1UnderSpirale2Length[1, PhiInlv, PhiOutlv, hv, dPhil2v], ";"];
Print["fSpirale1UnderSpirale2Length[NoLv,PhiInlp
    ,PhiOutlp,hp,dPhil2p]=fSpirale1UnderSpirale2Length[" ,
    NoLv, " , " , PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm,
    "nm", dPhil2p / (2 Pi), "(2Pi)] ="];
Print["  =", fSpirale1UnderSpirale2Length[NoLv, PhiInlp, PhiOutlp, hp, dPhil2p] / nm,
    "nm."];
Print["  fSpirale1OverSpirale2Length[1,PhiInlv,PhiOutlv,hv,dPhil2v]="];
Print["  =fSpiraleLen[NoLv,PhiInlv+Pi,PhiOutlv,hv]=",
    fSpiraleLen[NoLv, PhiInlv + Pi, PhiOutlv, hv], ";"];
Print["fSpirale1OverSpirale2Length[NoLv,PhiInlp
    ,PhiOutlp,hp]=fSpirale1OverSpirale1Length[" , NoLv, " , " ,
    PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm, "nm] ="];
Print["  =", fSpirale1OverSpirale2Length[NoLv, PhiInlp, PhiOutlp, hp, dPhil2p] / nm,
    "nm."];
Print[""];
Print["  fSpirale2UnderSpirale1Length[1,PhiInlv,PhiOutlv,hv]="];
Print["  fSpiraleLen[NoLv,PhiInlv,PhiOutlv -2 Pi/NoLv,hv]=",
    fSpirale1UnderSpirale2Length[1, PhiInlv, PhiOutlv, hv, dPhil2v], ";"];
Print["fSpirale2UnderSpirale1Length[NoLv,PhiInlp
    ,PhiOutlp,hp,dPhil2p]=fSpirale2UnderSpirale1Length[" ,
    NoLv, " , " , PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm,
    "nm", dPhil2p / (2 Pi), "(2Pi)] ="];
Print["  =", fSpirale1UnderSpirale2Length[NoLv, PhiInlp, PhiOutlp, hp, dPhil2p] / nm,
    "nm."];
Print["  fSpirale1OverSpirale2Length[1,PhiInlv,PhiOutlv,hv]="];
Print["  =fSpiraleLen[NoLv,PhiInlv+Pi,PhiOutlv,hv]=",
    fSpiraleLen[NoLv, PhiInlv + Pi, PhiOutlv, hv], ";"];
Print["fSpirale1OverSpirale2Length[NoLv,PhiInlp
    ,PhiOutlp,hp]=fSpirale1OverSpirale2Length[" , NoLv, " , " ,
    PhiInlp / (2 Pi), "(2Pi)", PhiOutlp / (2 Pi), "(2Pi)", hp / nm, "nm] ="];
Print["  =", fSpirale1OverSpirale2Length[NoLv, PhiInlp, PhiOutlp, hp, dPhil2p] / nm,
    "nm."];
];

Print[" II.2.4. The function fRIn1Sharp[NoLv,Llv,hv]"]
fRIn1Sharp[NoLv_, Llv_, hv_] := (Llv / (2 Pi) - (NoLv hv / 2));
Print["fRIn1Sharp[NoLv,Llv,hv]=", fRIn1Sharp[NoLv, Llv, hv]];
Print["is a good approximation to obtain
    the value of the sharp in the dependence ScrollEnergy[RIn]"];
Print["fRIn1Sharp[" , NoLv, " , " , Llp / nm, "nm", " , hp / nm, "nm] = ",
    fRIn1Sharp[NoLv, Llp, hp] / nm, "nm"];

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Print["-----"];

Print[" III. Begin of Calculation "];
If[NoLp == 1,
Print[" III.1. The inner and the outer angle of the spirale of the layer:"]];
If[NoLp == 2,
Print[" III.1. The inner and the outer angles of the spirales of the layers:"]];
Print["  $\phi_{In1}$ =", RIn1 2 Pi / (NoLp h), ",  $\phi_{Out1}$ =fPhiOutvsPhiInLh[" , NoLp, " $\phi_{In1}$ , L1, h"];"];
fPhiIn1[NoLv_, RInlv_, hv_] := RInlv 2 Pi / (NoLv hv);
PhiInlp = fPhiIn1[NoLp, RInlp, hp];
fPhiOut1[NoLv_, Llv_, RInlv_, hv_] :=
  fPhiOutvsPhiInLh[NoLv, fPhiIn1[NoLv, RInlv, hv], Llv, hv];
Print[" For RIn1=", RInlp/nm, "nm,h=", hp/nm, "nm:"];
PhiOutlp = fPhiOut1[NoLp, Llp, RInlp, hp];
ROutlp = PhiOutlp NoLp hp / (2 Pi);
Print["  $\phi_{In1}$ =", PhiInlp / (2 Pi), " (2Pi),  $\phi_{Out1}$ =", PhiOutlp / (2 Pi), " (2Pi)."];
fPhiIn2[NoLv_, RInlv_, hv_, dPhi12v_] :=
  fPhiIn1[NoLv, RInlv, hv] + dPhi12v;
PhiIn2dPhi12p =
  fPhiIn2[NoLp, RInlp, hp, 0]; (*www orig 2022.10*)
PhiIn2dPhi12p =
  fPhiIn2[NoLp, RInlp, hp, dPhi12p]; (* for dPhi12p!=0, checked 2022.10*)
PhiIn2dPhi12Pip =
  fPhiIn2[NoLp, RInlp, hp, Pi];
fPhiOut2[NoLv_, Llv_, RInlv_, hv_, dPhi12v_] :=
  fPhiOutvsPhiInLh[NoLv, fPhiIn2[NoLv, RInlv, hv, dPhi12v], Llv, hv];
PhiOut2dPhi12p =
  fPhiOut2[NoLp, Llp, RInlp, hp, dPhi12p];
If[NoLp == 2, Print["  $\phi_{In2}$ =", PhiIn2dPhi12p / (2 Pi),
  " (2Pi),  $\phi_{Out2}$ =", PhiOut2dPhi12p / (2 Pi), " (2Pi)"];
PhiOut2dPhi12Pip =
  fPhiOut2[NoLp, Llp, RInlp, hp, Pi];
Print[" for d $\phi_{12}$ =Pi:  $\phi_{In2}$ =", PhiIn2dPhi12Pip / (2 Pi),
  " (2Pi),  $\phi_{Out2}$ =", PhiOut2dPhi12Pip / (2 Pi), " (2Pi)"];];
Print["L1=", Llp/nm, "nm, RIn1=", RInlp/nm, "nm"];
If[NoLp == 1, Print[" Plot the Spirale of the layer:"]];
If[NoLp == 2, Print[" Plot Spirales of the layers:"]]; "for d $\phi_{12}$ =0";
Spirale1Plot =
  PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv, PhiInlp,
    PhiOutlp},
  PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
    1.1 ROutlp / nm}}, PlotStyle -> {Red, Thin}, Axes -> None];
If[NoLp == 1, Print[Show[Spirale1Plot]]];];
If[NoLp > 1,
Print[" Plot the Spirale of the layers:"];
Spirale2Plot = PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm,
  {Phiv, PhiIn2dPhi12p + Pi, PhiOut2dPhi12p + Pi},
  PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
    1.1 ROutlp / nm}}, PlotStyle -> {Blue, Thin}, Axes -> None];
Spirale2dPhi12PiPlot = PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm,
  {Phiv, PhiIn2dPhi12p + Pi, PhiOut2dPhi12Pip + Pi},
  PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
    1.1 ROutlp / nm}}, PlotStyle -> {Blue, Thin}, Axes -> None];
Print[Show[{Spirale1Plot, Spirale2Plot}]];
]
If[NoLp == 1,
  Spirale1OverSpirale1Plot =

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If[PhiIn1p + 2 Pi < PhiOut1p,
  PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv, PhiIn1p + 2 Pi,
    PhiOut1p},
  PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
    1.1 ROut1p / nm}}, PlotStyle -> {Red, Thick}, Axes -> None], {}];
Spirale1UnderSpirale1Plot =
If[PhiIn1p < PhiOut1p - 2 Pi,
  PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv, PhiIn1p,
    PhiOut1p - 2 Pi},
  PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
    1.1 ROut1p / nm}}, PlotStyle -> {Red, Thick}, Axes -> None], {}];
Print[" {Spirale,Spirale1UnderSpirale1},{Spirale1,Spirale1OverSpirale1}:"];
Print[Show[{Spirale1Plot, Spirale1UnderSpirale1Plot}],
  Show[{Spirale1Plot, Spirale1OverSpirale1Plot}]];
];
If[NoLp == 2,
Spirale1UnderSpirale2dPhi120Plot =
  If[PhiIn1p < PhiOut2dPhi12p - Pi,
    PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv, PhiIn1p,
      PhiOut2dPhi12p - Pi}, PlotStyle -> {Red, Thick},
    PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
      1.1 ROut1p / nm}}, {}];
Spirale1OverSpirale2dPhi120Plot =
  If[PhiIn1p + Pi < PhiOut1p,
    PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv,
      PhiIn1p + Pi +
      dPhi12p, PhiOut1p},
    PlotStyle -> {Red, Thick},
    PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
      1.1 ROut1p / nm}}, {}];
Spirale2UnderSpirale1dPhi120Plot =
  If[PhiIn2dPhi12p + Pi < PhiOut1p,
    PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm, {Phiv,
      PhiIn2dPhi12p + Pi, PhiOut1p}, PlotStyle -> {Blue, Thick},
    PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
      1.1 ROut1p / nm}}, {}];
Spirale2OverSpirale1dPhi120Plot =
  If[2 Pi + PhiIn2dPhi12p -
    dPhi12p <
    PhiOut2dPhi12p + Pi,
    PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm, {Phiv,
      2 Pi + PhiIn2dPhi12p -
      dPhi12p,
      PhiOut2dPhi12p + Pi}, PlotStyle -> {Blue, Thick},
    PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
      1.1 ROut1p / nm}}, {}];
Print[
  "Plot Spirales for dPhi12=Pi (could be NotRequired, dPhi12=0 in this program)"];
Spirale1UnderSpirale2dPhi12PiPlot =
  If[PhiIn1p < PhiOut2dPhi12p - Pi,
    PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv, PhiIn1p,
      PhiOut2dPhi12p - Pi}, PlotStyle -> {Red, Thick},
    PlotRange -> {{-1.1 ROut1p / nm, 1.1 ROut1p / nm}, {-1.1 ROut1p / nm,
      1.1 ROut1p / nm}}, {}];
Spirale1OverSpirale2dPhi12PiPlot =
  If[PhiIn1p + Pi +
    dPhi12p < PhiOut1p,

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PolarPlot[(Phiv) NoLp hp / (2 Pi) / nm, {Phiv,
  PhiInlp + Pi +
  dPhil2p, PhiOutlp},
PlotStyle -> {Red, Thick},
PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
  1.1 ROutlp / nm}}, {}];
Spirale2UnderSpirale1dPhil2PiPlot =
If[PhiIn2dPhil2p + Pi < PhiOutlp,
PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm, {Phiv,
  PhiIn2dPhil2Pip + Pi, PhiOutlp}, PlotStyle -> {Blue, Thick},
PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
  1.1 ROutlp / nm}}, {}];
Spirale2OverSpirale1dPhil2PiPlot =
If[2 Pi + PhiIn2dPhil2Pip -
  dPhil2p <
  PhiOut2dPhil2Pip + Pi,
PolarPlot[(Phiv - Pi) NoLp hp / (2 Pi) / nm, {Phiv,
  2 Pi + PhiIn2dPhil2Pip -
  dPhil2p,
  PhiOut2dPhil2Pip + Pi}, PlotStyle -> {Blue, Thick},
PlotRange -> {{-1.1 ROutlp / nm, 1.1 ROutlp / nm}, {-1.1 ROutlp / nm,
  1.1 ROutlp / nm}}, {}];

Print[" {Spirale1,Spirale2,Spirale1UnderSpirale2,Spirale2UnderSpirale1}"];
Print["      {Spirale1,Spirale2,Spirale1OverSpirale2,Spirale2OverSpirale1}"];
Print[" for dPhil2=0: ", Show[Spirale1Plot, Spirale2Plot],
Show[Spirale1Plot, Spirale2Plot, Spirale1UnderSpirale2dPhil20Plot,
Spirale2UnderSpirale1dPhil20Plot],
Show[Spirale1Plot, Spirale2Plot, Spirale1OverSpirale2dPhil20Plot,
Spirale2OverSpirale1dPhil20Plot]];
Print[" for dPhil2=Pi: ", Show[Spirale1Plot, Spirale2dPhil2PiPlot],
Show[Spirale1Plot, Spirale2dPhil2PiPlot, Spirale1UnderSpirale2dPhil2PiPlot,
Spirale2UnderSpirale1dPhil2PiPlot],
Show[Spirale1Plot, Spirale2dPhil2PiPlot, Spirale1OverSpirale2dPhil2PiPlot,
Spirale2OverSpirale1dPhil2PiPlot]];

];

Print[" III.2. The nanoscroll energy calculation"];
Print[" III.2.1. The elastic energy calculation"];
fEelastCC[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, CCv_] :=
Module[{},
Return[2 Pi CCv Lwv / (hv fSa[aCCv]) fElast[
  fPhiIn1[NoLv, RInlv, hv],
  fPhiOut1[NoLv, Llv, RInlv, hv]]]];
fEelastCBN[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, CBNv_] :=
Module[{},
Return[2 Pi CBNv Lwv / (hv fSa[aCCv]) fElast[
  fPhiIn1[NoLv, RInlv, hv],
  fPhiOut1[NoLv, Llv, RInlv, hv]]]];
EelastCCp = fEelastCC[NoLp, Lwp, Llp, RInlp, hp, aCCp, CCp];
EelastCBNp = fEelastCBN[NoLp, Lwp, Llp, RInlp, hp, aCCp, CBNp];
Print[" EelastC=", EelastCCp / (eV / atom), "eV/atom"];
If[NoLp == 2, Print[" EelastBN=", EelastCBNp / (eV / atom), "eV/atom"]];
Print[" III.2.2. The Van-der-Waals energy calculation"];
"The definition of the function ";
"'fEVdWLayer1Overlap[NoLv,Lwv,Llv, RInlv, hv, aCCv, epsVdWv]'"

```

```

"(Note: This function is omitted at calculations";
"    but could be helpful at
    calculation of VdW ebergry of monoscroll at debugging";
"    for example,";
"    fEvdWLayer1Overlap[NoL1,Lwp,15nm, 2nm, hp, aCCp, epsVdWp] ";
" and fEvdWLayersOverlap[NoL2,Lwp,Llp=15nm, 2nm, hp, aCCp, epsVdWp, 0]";
"    give the same values";
fEvdWLayer1Overlap[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, epsVdWv_] := Module[
{EvdWv,
EvdW1Unlv = 0 (eV/atom), EvdW1Ov1v = 0 (eV/atom),
Spirale1UnderSpirale1Length = 0 nm, Spirale1OverSpirale1Length = 0 nm,
PhiInlv = fPhiIn1[NoLv, RInlv, hv],
PhiOutlv = fPhiOut1[NoLv, Llv, RInlv, hv],
},
Spirale1OverSpirale1Length = fSpiraleLen[NoLv, PhiInlv + 2 Pi, PhiOutlv, hv];
Spirale1UnderSpirale1Length = fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2 Pi, hv];
"Note: Spirale1OverSpirale1Length>Spirale1UnderSpirale1Length";
EvdW1Unlv = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1UnderSpirale1Length;
EvdW1Ov1v = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1OverSpirale1Length;
EvdWv = (EvdW1Unlv + EvdW1Ov1v);
Return[{EvdWv, EvdW1Unlv, EvdW1Ov1v}];
];
"The definition of the function";
"fEvdWLayersOverlap[NoLv_,Lwv_,Llv_, RInlv_, hv_, aCCv_, epsVdWv_, dPhil2v_]";
fEvdWLayersOverlap[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, epsVdWv_, dPhil2v_] := Module[
{EvdW,
EvdW1Un1 = 0 (eV/atom), EvdW1Ov1 = 0 (eV/atom),
EvdW1Un2 = 0 (eV/atom), EvdW1Ov2 = 0 (eV/atom),
EvdW2Un1 = 0 (eV/atom), EvdW2Ov1 = 0 (eV/atom),
Spirale1UnderSpirale1Length = 0 nm, Spirale1OverSpirale1Length = 0 nm,
Spirale1UnderSpirale2Length = 0 nm, Spirale1OverSpirale2Length = 0 nm,
Spirale2UnderSpirale1Length = 0 nm, Spirale2OverSpirale1Length = 0 nm,
PhiIn1 = fPhiIn1[NoLv, RInlv, hv],
PhiIn2 = fPhiIn2[NoLv, RInlv, hv, dPhil2v],
PhiOut1 = fPhiOut1[NoLv, Llv, RInlv, hv],
PhiOut2 = fPhiOut2[NoLv, Llv, RInlv, hv, dPhil2v],
ReturnEnergiesv = {1, 2, 3, 4, 5, 6, 7}
},
If[NoLv == 1,
If[PhiIn1 < PhiOut1 - 2 Pi,
    Spirale1UnderSpirale1Length = fSpiraleLen[NoLv, PhiIn1, PhiOut1 - 2 Pi, hv];];
If[PhiIn1 + 2 Pi < PhiOut1, Spirale1OverSpirale1Length =
    fSpiraleLen[NoLv, PhiIn1 + 2 Pi, PhiOut1, hv];];
EvdW1Un1 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1UnderSpirale1Length;
EvdW1Ov1 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1OverSpirale1Length;
EvdW = (EvdW1Un1 + EvdW1Ov1);
ReturnEnergiesv = {EvdW, EvdW1Un1, EvdW1Ov1};
];
If[NoLv == 2,
    If[PhiIn1 < PhiOut2 - Pi,
        Spirale1UnderSpirale2Length = fSpiraleLen[NoLv, PhiIn1, PhiOut2 - Pi, hv];];
    If[PhiIn1 + Pi + dPhil2v < PhiOut1, Spirale1OverSpirale2Length =
        fSpiraleLen[NoLv, PhiIn1 + Pi + dPhil2v, PhiOut1, hv];];
    If[PhiIn1 + dPhil2v < PhiOut1 - Pi, Spirale2UnderSpirale1Length =
        fSpiraleLen[NoLv, PhiIn1 + dPhil2v, PhiOut1 - Pi, hv];];
    If[PhiIn1 - dPhil2v + Pi < PhiOut2 - dPhil2v, Spirale2OverSpirale1Length =
        fSpiraleLen[NoLv, PhiIn1 - dPhil2v + Pi, PhiOut2 - dPhil2v, hv];];
];

```



```

EVdW1Un2 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1UnderSpirale2Length;
EVdW1Ov2 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale1OverSpirale2Length;
EVdW2Un1 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale2UnderSpirale1Length;
EVdW2Ov1 = -epsVdWv Lwv / (2 fSa[aCCv]) Spirale2OverSpirale1Length;
EVdW = (EVdW1Un2 + EVdW1Ov2 + EVdW2Un1 + EVdW2Ov1);
ReturnEnergiesv[[iEVdW]] = EVdW;
ReturnEnergiesv[[iEVdW1Un2]] = EVdW1Un2;
ReturnEnergiesv[[iEVdW1Ov2]] = EVdW1Ov2;
ReturnEnergiesv[[iEVdW2Un1]] = EVdW2Un1;
ReturnEnergiesv[[iEVdW2Ov1]] = EVdW2Ov1;
];
Return[ReturnEnergiesv];
];

EVdWdPhil2eq0allp =
fEVdWLayersOverlap[NoL2, Lwp, Llp, RInlp, hp, aCCp, epsVdWp, dPhil2eq0];
EVdWvardPhil2allp =
fEVdWLayersOverlap[NoLp, Lwp, Llp, RInlp, hp, aCCp, epsVdWp, dPhil2p];
If[NoLp == 1,
Print[" EVdWvardPhil2allp[[iEVdW]]=",
EVdWvardPhil2allp[[iEVdW]] / (eV/atom), "eV/atom"];
Print["( EVdWvardPhil2allp[[iEVdW1Un1]]=",
EVdWvardPhil2allp[[iEVdW1Un1]] / (eV/atom), "eV/atom"];
Print[" EVdWvardPhil2allp[[iEVdW1Ov1]]=",
EVdWvardPhil2allp[[iEVdW1Ov1]] / (eV/atom), "eV/atom  "];
];
If[NoLp == 2,
Print[" for dPhil2=", dPhil2p/Pi, "Pi EVdWvardPhil2allp[[iEVdW]]=",
EVdWvardPhil2allp[[iEVdW]] / (eV/atom), "eV/atom"];
Print[" For dPhil2=", dPhil2eq0/Pi, "Pi:"];
Print[" EVdWvardPhil2allp[[iEVdW]]=",
EVdWdPhil2eq0allp[[iEVdW]] / (eV/atom), "eV/atom"];
Print[" EVdWvardPhil2allp[[iEVdW1Un2]]=",
EVdWdPhil2eq0allp[[iEVdW1Un2]] / (eV/atom), "eV/atom"];
Print[" EVdWvardPhil2allp[[iEVdW1Ov2]]=",
EVdWdPhil2eq0allp[[iEVdW1Ov2]] / (eV/atom), "eV/atom"];
Print[" EVdWvardPhil2allp[[iEVdW2Un1]]=",
EVdWdPhil2eq0allp[[iEVdW2Un1]] / (eV/atom), "eV/atom"];
Print[" EVdWvardPhil2allp[[iEVdW2Ov2]]=",
EVdWdPhil2eq0allp[[iEVdW2Ov1]] / (eV/atom), "eV/atom"];
EVdWdPhil2eqPiallp =
fEVdWLayersOverlap[NoLp, Lwp, Llp, RInlp, hp, aCCp, epsVdWp, dPhil2eqPi];
(**) Print[" For dPhil2=", dPhil2eqPi/Pi, "Pi:"];
Print[" EVdWvatdPhil2allp[[iEVdW]]=",
EVdWdPhil2eqPiallp[[iEVdW]] / (eV/atom), "eV/atom"];
Print[" EVdWvatdPhil2allp[[iEVdW1Un2]]=",
EVdWdPhil2eqPiallp[[iEVdW1Un2]] / (eV/atom), "eV/atom"];
Print[" EVdWvatdPhil2allp[[iEVdW1Ov2]]=",
EVdWdPhil2eqPiallp[[iEVdW1Ov2]] / (eV/atom), "eV/atom"];
Print[" EVdWvatdPhil2allp[[iEVdW2Un1]]=",
EVdWdPhil2eqPiallp[[iEVdW2Un1]] / (eV/atom), "eV/atom"];
Print[" EVdWvatdPhil2allp[[iEVdW2Ov2]]=",
EVdWdPhil2eqPiallp[[iEVdW2Ov1]] / (eV/atom), "eV/atom"];
EVdWEVdWdPhil2eq0p = EVdWdPhil2eq0allp[[iEVdW]];
Print[" EVdWdPhil2eq0allp=", EVdWdPhil2eq0allp / (eV/atom),
"eV/atom"];
EVdWEVdWdPhil2eqPip = EVdWvardPhil2allp[[iEVdW]];

```

```

Print[" EVdWEVdWdPhil2eqPip=", EVdWEVdWdPhil2eqPip / (eV / atom),
      "eV/atom"];
(**)
];
If[NoLv == 2, Print[" III.3. The energy of flat planes "];];
fEnergyFlatPlanes[NoLv_, Lwv_, Llv_, aCCv_, epsVdWv_] :=
  If[NoLv == 2, -epsVdWv Lwv / fSa[aCCv] Llv, 0 eV / atom];
EnergyFlatPlanesp = fEnergyFlatPlanes[NoLv, Lwv, Llv, aCCv, epsVdWv];
If[NoLv == 2, Print[" EnergyFlatPlanes=-eps width/Sa L1(NoL-1) =",
  EnergyFlatPlanesp / (eV / atom), "eV/atom"];];
Print[" III.4. The total energy of the nanoscroll"];
fScrollEnergydPhi[NoLv_, Lwv_, Llv_,
  RInlv_, hv_, aCCv_, epsVdWv_, CCv_, CBNv_, dPhil2v_] :=
Module[{ScrollEnergyv, EVdWv, EVdWnoDimv},
  EVdWv = fEVdWLayersOverlap[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv, dPhil2v][[1]];
EVdWnoDimv = EVdWv /. {eV -> 1, atom -> 1, nm -> 1};
If[NoLv == 1,
  If[EVdWnoDimv == 0, ScrollEnergyv = fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv],
    ScrollEnergyv = EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv];];
];
If[NoLv == 2,
If[EVdWnoDimv == 0,
  ScrollEnergyv = fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
  fEelastCBN[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv],
  ScrollEnergyv = EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
  fEelastCBN[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv];];
];
Return[ScrollEnergyv];
];

fScrollEnergyVdWandElast[NoLv_, Lwv_,
  Llv_, RInlv_, hv_, aCCv_, epsVdWv_, CCv_, CBNv_] :=
Module[{ScrollEnergyVdWandElastv, EVdWv},
(*If[NoLv == 1,
  EVdWv = fEVdWLayer1Overlap[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv][[1]];
];
If[NoLv == 2,
  EVdWv = fEVdWLayersOverlap[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv][[1]];
];*)
EVdWv = fEVdWLayer1Overlap[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv][[1]];
If[NoLv == 1,
  ScrollEnergyVdWandElastv = EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv];];
If[NoLv == 2,
  ScrollEnergyVdWandElastv = EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
  fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CBNv];];

Return[ScrollEnergyVdWandElastv];
];

fScrollEnergy[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, epsVdWv_, CCv_, CBNv_] := Module[
{ScrollEnergyv = -10^20 eV / atom},
  If[RInlv / m <= fRInlSharp[NoLv, Llv, hv] / m,
  ScrollEnergyv =
    fScrollEnergyVdWandElast[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv, CCv, CBNv];
  "note: the function fScrollEnergyVdWandElast[1,...] is analytcal";
  "whereas the function fScrollEnergy[....] uses the 'If[..]'- function";
];

```

```

If[RInlv/m >= fRInlSharp[NoLv, Llv, hv] /m,
If[NoLv == 1, ScrollEnergyv = fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv];];
If[NoLv == 2, ScrollEnergyv = fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
    fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CBNv];];

];
Return[ScrollEnergyv];
];

ScrollEnergyyp = fScrollEnergy[NoLp, Lwp, Llp, RInlp, hp, aCCp, epsVdWp, CCp, CBNp];
ScrollEnergyyp = fScrollEnergy[1, Lwp, Llp, RInlp, hp, aCCp, epsVdWp, CCp, CBNp];
Print["fScrollEnergy[1,Lwp,Llp, RInlp, hp, aCCp, epsVdWp, CCp, CBNp]="];
Print["=fScrollEnergy[1, Lw=", Lwp/nm, "nm, Ll=",
    Llp/nm, "nm, RInl=", RInlp/nm, "nm, h=", hp/nm, "nm,"];
Print[" aCC=", aCCp/nm, "nm, epsVdW=", epsVdWp/(eV/atom),
    "eV/atom, CC=", CCp/(eV AA^2/atom), "(eV AA^2/atom)="];
Print["      =", ScrollEnergyyp/(eV/atom), "eV/atom"];

Print[" III.5. Determine the inner angles mismatch for the bi-layer nanoscroll
    for the high nanoribbon Length"];
Print[" For Ll=", Llp/nm, "nm,RIn=",
    RInlp/nm, "nm,h=", hp/nm, "nm and dPhil2=0:"];
Print[" ScrollEnergy=", ScrollEnergyyp/(eV/atom), "eV/atom"];
Print[" For Ll=", Llp/nm, "nm,RIn=",
    RInlp/nm, "nm,h=", hp/nm, "nm and dPhil2=Pi:"];
Print[" ScrollEnergy=", ScrollEnergyyp/(eV/atom), "eV/atom"];
Print["-----"];

Print[" IV.The potential energy of the nanoscroll"];
Print[" as a function of the inner radius RIn"];
PlotRange =
    Switch[NoLp, 1, PlotRangeMonoScroll, 2, PlotRangeBiScroll];
RInlMinp =
    Switch[NoLp, 1, RInlMinMonoScroll, 2, RInlMinBiScroll];
RInlMaxp =
    Switch[NoLp, 1, RInlMaxMonoScroll, 2, RInlMaxBiScroll];
PlotRange =
    Switch[NoLp, 1, PlotRangeMonoScroll, 2, PlotRangeBiScroll];
RInlMaxp =
    Switch[NoLp, 1, RInlMaxMonoScroll, 2, RInlMaxBiScroll];
tL1 = Switch[NoLp, 1, {7. nm, 10. nm, 12.5 nm, 15. nm},
    2, {15. nm, 20. nm, 25. nm, 30. nm}];
Print[" NoL=", NoLp];
Print[" epsVdW=", epsVdWp/(eV/atom),
    "eV/atom, C=", CCp/(eV nm^2/atom), "(eV nm^2/atom)",
    "(eV nm^2/atom),aCC=", aCCp/nm, "nm,h=", hp/nm, "nm"];
Print[" Plot ScrollEnergy[RInl/nm]/(eV/atom) for Ll=", Llp/nm,
    "nm (NoL=", NoLp, ",Lw=", Lwp/nm, "nm)"];
PlotScrollEnergyVsRInl =
    Plot[(fScrollEnergy[NoLp, Lwp, Llp, RInlnmv nm, hp, aCCp, epsVdWp,
        CCp, CBNp])/(eV/
        atom), {RInlnmv, RInlMinp/nm, RInlMaxp/nm},
        PlotRange -> PlotRange/(eV/atom)];
Print[PlotScrollEnergyVsRInl];
Print[" Plot ScrollEnergy[RInl/nm]/(eV/atom) for Ll=", tL1/nm,
    "nm (NoL=", NoLp, ",w=", Lwp/nm, "nm)"];
PlotScrollEnergyVsRInlLlth =

```

```

Plot[(fScrollEnergy[NoLp, Lwp, tL1[[1]], RIn1nmv nm, hp, aCCp, epsVdWp,
      CCp, CBNp]) / (eV /
      atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
      PlotRange -> PlotRange / (eV / atom)];
PlotScrollEnergyVsRIn1L2th =
Plot[(fScrollEnergy[NoLp, Lwp, tL1[[2]], RIn1nmv nm, hp, aCCp, epsVdWp,
      CCp, CBNp]) / (eV /
      atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
      PlotRange -> PlotRange / (eV / atom)];
PlotScrollEnergyVsRIn1L3th =
Plot[(fScrollEnergy[NoLp, Lwp, tL1[[3]], RIn1nmv nm, hp, aCCp, epsVdWp,
      CCp, CBNp]) / (eV /
      atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
      PlotRange -> PlotRange / (eV / atom)];
PlotScrollEnergyVsRIn1L4th =
Plot[(fScrollEnergy[NoLp, Lwp, tL1[[4]], RIn1nmv nm, hp, aCCp, epsVdWp,
      CCp, CBNp]) / (eV /
      atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
      PlotRange -> PlotRange / (eV / atom)];
Print[Show[{PlotScrollEnergyVsRIn1L1th, PlotScrollEnergyVsRIn1L2th,
      PlotScrollEnergyVsRIn1L3th, PlotScrollEnergyVsRIn1L4th}]];
Print["The examples of using of 'fScrollEnergy[...]' function:"]
Print["fScrollEnergy[NoLp,Lwp,tL1[[1]],RIn1p, hp, aCCp, epsVdWp,CCp, CBNp]=",
      fScrollEnergy[NoLp, Lwp, tL1[[1]], RIn1p, hp, aCCp, epsVdWp, CCp, CBNp] / (eV / atom),
      " eV/atom"];
Print["fScrollEnergy[NoLp,Lwp,tL1[[1]],1nm, hp, aCCp, epsVdWp,CCp, CBNp]=",
      fScrollEnergy[NoLp, Lwp, tL1[[1]], 1 nm, hp, aCCp, epsVdWp, CCp, CBNp] / (eV / atom),
      " eV/atom"];
Print["fScrollEnergy[NoLp,Lwp,7nm,1nm, hp, aCCp, epsVdWp,CCp, CBNp]=",
      fScrollEnergy[NoLp, Lwp, 7. nm, 1. nm, hp, aCCp, epsVdWp, CCp, CBNp] / (eV / atom),
      " eV/atom"];
Print["fEVdWLayer1Overlap[1,Lwp,7.nm,1.nm, hp, aCCp, epsVdWp][[1]]=",
      fEVdWLayer1Overlap[1, Lwp, 7. nm, 1. nm, hp, aCCp, epsVdWp][[1]] / (eV / atom),
      "eV/atom (right !=0 value, because the layer overlaps)"];
Print["fEVdWLayer1Overlap[1,Lwp,7.nm,1.5nm, hp, aCCp, epsVdWp][[1]]=",
      fEVdWLayer1Overlap[1, Lwp, 7. nm, 2.5 nm, hp, aCCp, epsVdWp][[1]] / (eV / atom),
      "eV/atom !=0, wrong value of the fEVdWLayer1Overlap[...]
      function because the layer does not not overlap"];
Print[];
Print["The analytical expressions of the fEVdWLayer1Overlap[...] function:"];
Print[];
Print["--- fEVdWLayer1Overlap[NoL1,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---"];
Print[fEVdWLayer1Overlap[NoL1, Lwv, Lpv, RInv, hv, aCCv, epsVdWv][[1]]];
Print[];
Print["--- fEVdWLayer1Overlap[NoL2,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---"];
Print[fEVdWLayer1Overlap[NoL2, Lwv, Lpv, RInv, hv, aCCv, epsVdWv][[1]]];
Print[];
Print["The Analytical expression of "];
Print["fScrollEnergyVdWandElast[",
      NoLp, ", Lwv,Llv,RInlv,hv,aCCv, epsVdWv, CCv,CBNv]:"];
Print[fScrollEnergyVdWandElast[NoLp, Lwv, Llv, RInlv, hv, aCCv, epsVdWv, CCv, CBNv]];
Print["-----"];

Print["V. Export the data of the plots of the nanoscroll energy"];
Print[" as a function of the inner radius"];
Print["The parameters of the output file"];
Print["The number of the output points = ", npRIn1];

```

```

Print["Export the plot data to the files:"];
tRInlnmRegular =
  Table[(RInlMinp + (RInlMaxp - RInlMinp) iiRin / (npRInl)) / nm, {iiRin, 1, npRInl}];
tScrollEnergy = tRInlnmRegular;
tScrollEnergyeVatom = tRInlnmRegular;
tPlotEvsRin = Table[{}, {ii, 1, Length[tL1]}];
AllPlotsEVsRin = {};

For[iiL1 = 1, iiL1 <= Length[tL1], iiL1++,
  Llpi = tL1[[iiL1]];
  NanoscrollNameep =
    StringJoin["Nanoscroll", ToString[NoLp], "L", ToString[Llpi / nm], "nm"];
  Print["NanoscrollName=", NanoscrollNameep];
  ScrollEnergyFileName = StringJoin["EvsRInl", NanoscrollNameep, ".dat"];
  Print["ScrollEnergyFileName=", ScrollEnergyFileName];

  For[iiRInl = 1, iiRInl <= npRInl, iiRInl++,
    RInlpi = tRInlnmRegular[[iiRInl]] nm;
    tScrollEnergy[[iiRInl]] = fScrollEnergy[NoLp, Lwp, Llpi, RInlpi, hp, aCCp, epsVdWp,
      CCp, CBNp];

    tScrollEnergyeVatom[[iiRInl]] = (tScrollEnergy[[iiRInl]]) / (eV / atom);
    tPlotEvsRin[[iiL1]] = ListPlot[Transpose[{tRInlnmRegular, tScrollEnergyeVatom}],
      PlotRange -> PlotRange / (eV / atom)];
    Print[tPlotEvsRin[[iiL1]]];
    AllPlotsEVsRin = Join[{AllPlotsEVsRin, tPlotEvsRin[[iiL1]]}];
    CarbonNanoscrollEnergyVsRinFileName = StringJoin[NanoscrollNameep, "dat"];
    Export[ToFileName[NotebookDirectory[], ScrollEnergyFileName],
      Transpose[{Insert[tRInlnmRegular, "RInl[nm]", 1],
        Insert[tScrollEnergyeVatom, "E[eV/atom]", 1]}]];
  ];
  Print["Plot ScrollEnergy[RInl/nm]/(eV/atom) for L1=", tL1 / nm,
    "nm (NoL=", NoLp, ", Lw=", Lwp / nm, "nm)"];
  Print[Show[AllPlotsEVsRin]];

```

The source of the data of the manuscript

Structure and energetics of carbon, hexagonal boron nitride,

and carbon/hexagonal boron nitride single-layer and bilayer nanoscrolls

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I.0 The Units (nm, meV, ÅÅ)

I.1. All Input Parameters and Constants-----

I.1.1. The sampling parameters

npRInl=1000

I.1.2. The Input Geometry Parameters of the system

The carbon nanoribbon length L1=14.839nm

The carbon nanoribbon width Lw=11.8nm

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Number of the layers in carbon nanoscroll NoL=1
The length of a carbon nanoribbon L1=14.839nm
The carbon nanoribbon width Lw=1.nm
The inner radius of the nanoscroll RIn1=1.14nm
I.1.2. The Input Energy Constants
epsVdW - the interlayer interaction energy per one atom of
the nanoscroll:
epsVdW=0.035eV/atom
C - the bending elastic constant:
CCelast=2.01eV AA^2/atom
CCBNelast=2.01eV AA^2/atom
I.1.3.The Input Geometry constants-----
The interatomic distance aCC and the interlayer distance h
h=0.3354 nm (Interlayer distance)
aCC=0.142nm, h=0.3354nm
NatomsInCell=2
dPhil2 - The difference of the inner angles of the spirales
of the Layers
I.5.The parameters for the visualisation
I.6. The parameters of the output file
Nanoscroll1Name=Nanoscroll1L14.839nm
Nanoscroll1L14.839nm.txt
(The output of the data to a file Is Not Performed)
The number of the output points = 1000
I.7. The Input Numerical Constants used in the programm
The Indexes used for the work with EVdW[...] function
-----End of the Input-----
-----
II. The derivated parameters and the functions required
II.1. The derivated parameters
The cell area  $Sa = \frac{3\sqrt{3} aCC^2}{4} = 0.0261938nm^2$ 
II.2. The required functions-----
II.2.1. The function fSpiraleLen[1,φIn, φOut, h] defines
the Length of a Spirale with the inner angle φIn and the outer angle φOut(>=φIn):

```

$$fSpiraleLen[1, \varphi In, \varphi Out, h] = \frac{h \left(-\varphi In \sqrt{1 + \varphi In^2} + \varphi Out \sqrt{1 + \varphi Out^2} - \text{ArcSinh}[\varphi In] + \text{ArcSinh}[\varphi Out] \right)}{4 \pi}.$$

II.2.2. The function $fElast[\varphi In, \rho Out]$ is required to calculate an nanoscrolllelastic energy:

$$fElast[\varphi In, \varphi Out] = \frac{\sqrt{1 + \varphi In^2}}{\varphi In} - \frac{\sqrt{1 + \varphi Out^2}}{\varphi Out} - \text{ArcSinh}[\varphi In] + \text{ArcSinh}[\varphi Out].$$

II.2.3. Define the function $fPhiOutvsPhiInLh[1, \varphi In, L, h]$.

The function $fPhiOutvsPhiInLh[1, \varphi In, L, h] = \sqrt{\frac{4 L \pi}{h} + \varphi In^2}$ is a good approximation to obtain the value of φOut for the defined $\varphi In, L, h$.

The inverse function $fPhiInvsPhiOutLh[1, \varphi Out, L, h] = \sqrt{-\frac{4 L \pi}{h} + \varphi Out^2}$

could be used in the program applications

if $Rout$ (instead of RIn) is the input parameter of the system.

II.2.4. The functions $fSpirale1Under(Over)Spirale1Length[NoLv, PhiInlv, PhiOutlv, hv]$

These functiona are not required, dut could be helpful),

$$\begin{aligned} fSpirale1UnderSpirale1Length[1, PhiInlv, PhiOutlv, hv] = \\ = fSpiraleLen[NoLv, PhiInlv, PhiOutlv - 2\pi, hv] = \frac{1}{4 \pi} \\ hv NoLv \left(-\Phi Inlv \sqrt{1 + \Phi Inlv^2} + \sqrt{1 + (\Phi Outlv - 2 \pi)^2} (\Phi Outlv - 2 \pi) - \right. \\ \left. \text{ArcSinh}[\Phi Inlv] + \text{ArcSinh}[\Phi Outlv - 2 \pi] \right); \\ fSpirale1UnderSpirale1Length[NoLv, PhiInlp, PhiOutlp, hp] = fSpirale1UnderSpirale1Length[\\ 1, \frac{\Phi Inlp}{2 \pi} (2\pi), \frac{\Phi Outlp}{2 \pi} (2\pi), 0.3354 \text{nm}] = \\ = 0.0266903 \left(-\Phi Inlp \sqrt{1 + \Phi Inlp^2} + \right. \\ \left. \sqrt{1 + (\Phi Outlp - 2 \pi)^2} (\Phi Outlp - 2 \pi) - \text{ArcSinh}[\Phi Inlp] + \text{ArcSinh}[\Phi Outlp - 2 \pi] \right) \text{nm}. \end{aligned}$$

$$\begin{aligned} fSpirale1OverSpirale1Length[1, PhiInlv, PhiOutlv, hv] = \\ = fSpiraleLen[NoLv, PhiInlv + 2\pi, PhiOutlv, hv] = \frac{1}{4 \pi} \\ hv NoLv \left(\Phi Outlv \sqrt{1 + \Phi Outlv^2} + (-\Phi Inlv - 2 \pi) \sqrt{1 + (\Phi Inlv + 2 \pi)^2} + \right. \\ \left. \text{ArcSinh}[\Phi Outlv] - \text{ArcSinh}[\Phi Inlv + 2 \pi] \right); \\ fSpirale1OverSpirale1Length[NoLv, PhiInlp, PhiOutlp, hp] = fSpirale1OverSpirale1Length[\\ 1, \frac{\Phi Inlp}{2 \pi} (2\pi), \frac{\Phi Outlp}{2 \pi} (2\pi), 0.3354 \text{nm}] = \\ = 0.0266903 \left(\Phi Outlp \sqrt{1 + \Phi Outlp^2} + \right. \\ \left. (-\Phi Inlp - 2 \pi) \sqrt{1 + (\Phi Inlp + 2 \pi)^2} + \text{ArcSinh}[\Phi Outlp] - \text{ArcSinh}[\Phi Inlp + 2 \pi] \right) \text{nm}. \end{aligned}$$

II.2.4. The function $fRIn1Sharp[NoLv, Llv, hv]$

$$fRIn1Sharp[NoLv, Llv, hv] = -\frac{hv NoLv}{2} + \frac{Llv}{2 \pi}$$

is a good approximation to obtain the value of the sharp in the dependence ScrollEnergy[RIn]
 $fRIn1Sharp[1, 14.839nm, 0.3354nm] = 2.194nm$

 III. Begin of Calculation

III.1. The inner and the outer angle of the spirale of the layer:

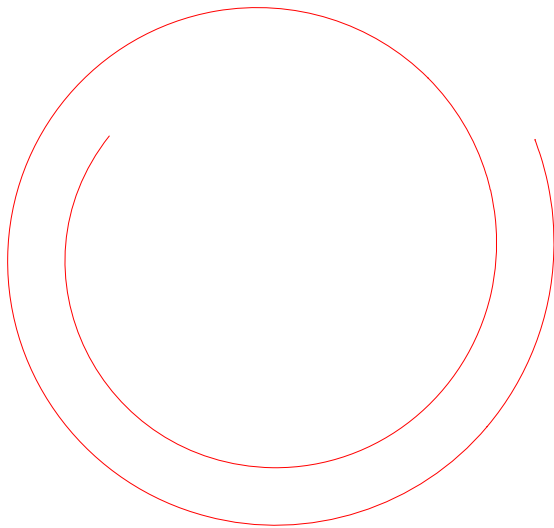
$$\varphi_{In1} = \frac{2\pi R_{In1}}{h}, \quad \varphi_{Out1} = f\Phi_{OutvsPhiInLh}[1, \varphi_{In1}, L1, h];$$

For $R_{In1}=1.14nm, h=0.3354nm$:

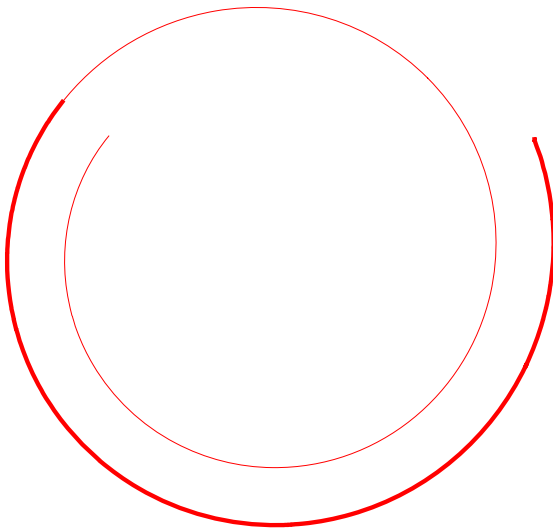
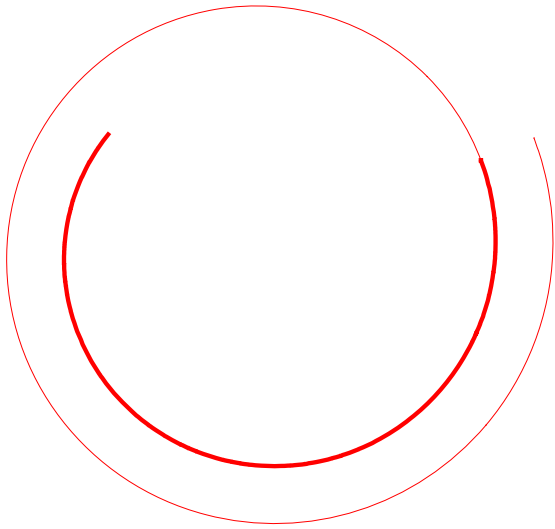
$$\varphi_{In1}=3.39893(2\pi), \quad \varphi_{Out1}=5.06316(2\pi).$$

$L1=14.839nm, R_{In1}=1.14nm$

Plot the Spirale of the layer:



$\{Spirale, Spirale1UnderSpirale1\}, \{Spirale1, Spirale1OverSpirale1\}:$



III.2. The nanoscroll energy calculation

III.2.1. The elastic energy calculation

EelastC=5.7333eV/atom

III.2.2. The Van-der-Waals energy calculation

EVdWwardPhi12allp[[iEVdW]]=-7.91941eV/atom

(EVdWwardPhi12allp[[iEVdWlUn1]]=-3.49244eV/atom

EVdWwardPhi12allp[[iEVdWlOv1]]=-4.42696eV/atom)

III.4. The total energy of the nanoscroll

fScrollEnergy[1,Lwp,Llp, RInlp,hp, aCCp, epsVdWp,CCp,CBNp]=

=fScrollEnergy[1, Lw=1.nm, L1=14.839nm, RIn1=1.14nm, h=0.3354nm,

```
aCC=0.142nm, epsVdW=0.035eV/atom, CC=2.01(eV AA^2/atom)=
=-2.18611eV/atom
```

III.5. Determine the inner angles mismatch for the bi-layer nanoscroll
for the high nanoribbon Length

```
For L1=14.839nm,RIn=1.14nm,h=0.3354nm and dPhi12=0:
```

```
ScrollEnergy=-2.18611eV/atom
```

```
For L1=14.839nm,RIn=1.14nm,h=0.3354nm and dPhi12=Pi:
```

```
ScrollEnergy=-2.18611eV/atom
```

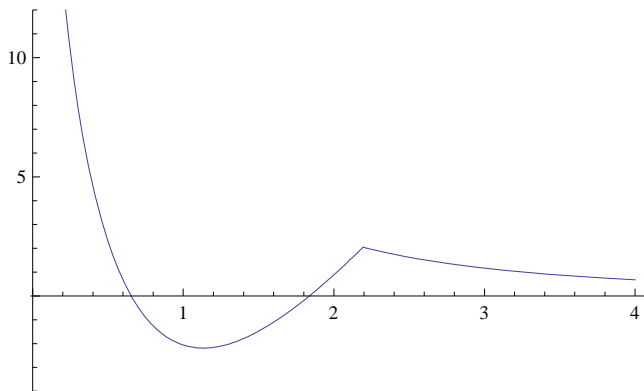
IV.The potential energy of the nanoscroll

as a function of the inner radius RIn

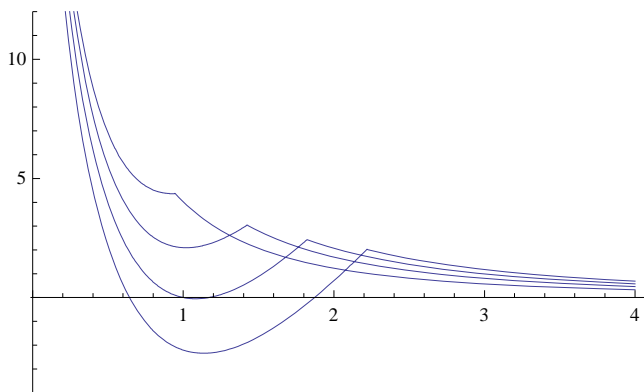
```
NoL=1
```

```
epsVdW=0.035eV/atom, C=0.0201(eV nm^2/atom)(eV nm^2/atom),aCC=0.142nm,h=0.3354nm
```

```
Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1=14.839nm (NoL=1,Lw=1.nm
```



```
Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1={7., 10., 12.5, 15.}nm (NoL=1,w=1.nm)
```



The examples of using of 'fScrollEnergy[...]' function:

```
fScrollEnergy[NoLp,Lwp,tL1[[1]],RInlp,hp, aCCp, epsVdWp,CCp, CBNp]=3.26809 eV/atom
```

```
fScrollEnergy[NoLp,Lwp,tL1[[1]],1nm,hp, aCCp, epsVdWp,CCp, CBNp]=4.01569 eV/atom
```

```
fScrollEnergy[NoLp,Lwp,7nm,1nm,hp, aCCp, epsVdWp,CCp, CBNp]=4.01569 eV/atom
```

```
fEVdWLayer1Overlap[1,Lwp,7.nm,1.nm,hp,aCCp,epsVdWp][[1]]=
0.39374eV/atom (right !=0 value, because the layer overlaps
```

```
fEVdWLayer1Overlap[1,Lwp,7.nm,1.5nm,hp,aCCp,epsVdWp][[1]]=
12.2479eV/atom !=0, wrong value of the
fEVdWLayer1Overlap[..] function because the layer does not overlap
```

The analytical expressions of the fEVdWLayer1Overlap[..] function:

```
--- fEVdWLayer1Overlap[NoL1,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---
```

$$\begin{aligned}
& - \frac{1}{6\sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv} \text{hv} \text{Lwv} \\
& \left(\sqrt{\frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} \sqrt{1 + \frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} + \left(-2\pi - \frac{2 \pi \text{RInv}}{\text{hv}} \right) \sqrt{1 + \left(2\pi + \frac{2 \pi \text{RInv}}{\text{hv}} \right)^2} - \right. \\
& \quad \left. \text{ArcSinh}\left[2\pi + \frac{2 \pi \text{RInv}}{\text{hv}} \right] + \text{ArcSinh}\left[\sqrt{\frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} \right] \right) - \frac{1}{6\sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv} \text{hv} \text{Lwv} \\
& \left(- \frac{2 \pi \text{RInv} \sqrt{1 + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}}}{\text{hv}} + \left(-2\pi + \sqrt{\frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} \right) \sqrt{1 + \left(-2\pi + \sqrt{\frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} \right)^2} - \right. \\
& \quad \left. \text{ArcSinh}\left[\frac{2 \pi \text{RInv}}{\text{hv}} \right] - \text{ArcSinh}\left[2\pi - \sqrt{\frac{4 \text{Lpv} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RInv}^2}{\text{hv}^2}} \right] \right)
\end{aligned}$$

```
--- fEVdWLayer1Overlap[NoL2,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---
```

$$\begin{aligned}
& - \frac{1}{3\sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv} \text{hv} \text{Lwv} \\
& \left(\sqrt{\frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} \sqrt{1 + \frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} + \left(-2\pi - \frac{\pi \text{RInv}}{\text{hv}} \right) \sqrt{1 + \left(2\pi + \frac{\pi \text{RInv}}{\text{hv}} \right)^2} - \right. \\
& \quad \left. \text{ArcSinh}\left[2\pi + \frac{\pi \text{RInv}}{\text{hv}} \right] + \text{ArcSinh}\left[\sqrt{\frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} \right] \right) - \frac{1}{3\sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv} \text{hv} \text{Lwv} \\
& \left(- \frac{\pi \text{RInv} \sqrt{1 + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}}}{\text{hv}} + \left(-2\pi + \sqrt{\frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} \right) \sqrt{1 + \left(-2\pi + \sqrt{\frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} \right)^2} - \right. \\
& \quad \left. \text{ArcSinh}\left[\frac{\pi \text{RInv}}{\text{hv}} \right] - \text{ArcSinh}\left[2\pi - \sqrt{\frac{2 \text{Lpv} \pi}{\text{hv}} + \frac{\pi^2 \text{RInv}^2}{\text{hv}^2}} \right] \right)
\end{aligned}$$

The Analytical expression of

fScrollEnergyVdWandElast[1,Lwv,Llv,RInlv,hv,aCCv, epsVdWv, CCv,CBNv]:

$$\begin{aligned}
 & \frac{8 \text{CCv Lwv } \pi \left(\frac{h\nu \sqrt{1 + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}}{2 \pi \text{RInlv}} - \frac{\sqrt{1 + \frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}}{\sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}} - \text{ArcSinh}\left[\frac{2 \pi \text{RInlv}}{h\nu}\right] + \text{ArcSinh}\left[\sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}\right] \right)}{3 \sqrt{3} \text{aCCv}^2 h\nu} - \\
 & \frac{1}{6 \sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv } h\nu \text{Lwv} \\
 & \left(\sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}} \sqrt{1 + \frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}} + \left(-2 \pi - \frac{2 \pi \text{RInlv}}{h\nu}\right) \sqrt{1 + \left(2 \pi + \frac{2 \pi \text{RInlv}}{h\nu}\right)^2} - \right. \\
 & \left. \text{ArcSinh}\left[2 \pi + \frac{2 \pi \text{RInlv}}{h\nu}\right] + \text{ArcSinh}\left[\sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}\right] \right) - \frac{1}{6 \sqrt{3} \text{aCCv}^2 \pi} \text{epsVdWv } h\nu \text{Lwv} \\
 & \left(-\frac{2 \pi \text{RInlv}}{h\nu} \sqrt{1 + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}} + \left(-2 \pi + \sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}\right) \sqrt{1 + \left(-2 \pi + \sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}\right)^2} - \right. \\
 & \left. \text{ArcSinh}\left[\frac{2 \pi \text{RInlv}}{h\nu}\right] - \text{ArcSinh}\left[2 \pi - \sqrt{\frac{4 \text{Llv } \pi}{h\nu} + \frac{4 \pi^2 \text{RInlv}^2}{h\nu^2}}\right] \right)
 \end{aligned}$$

V. Export the data of the plots of the nanoscroll energy

as a function of the inner radius

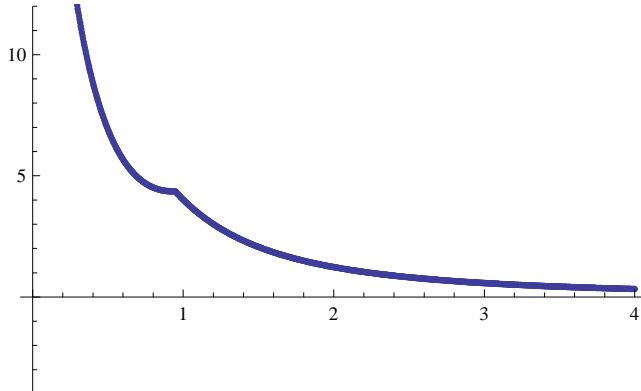
The parameters of the output file

The number of the output points = 1000

Export the plot data to the files:

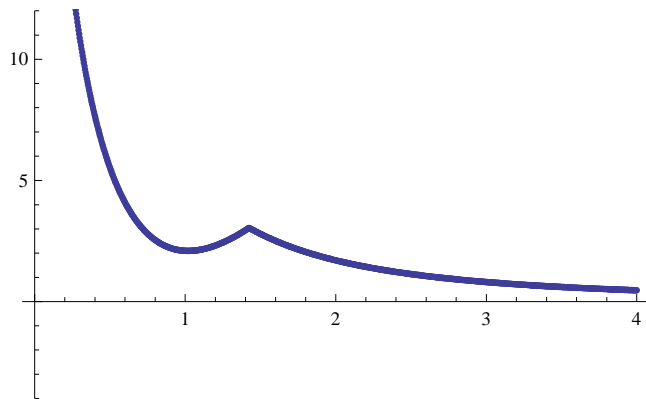
NanoscrollName=Nanoscroll1L7.nm

ScrollEnergyFileName=EvsRIn1Nanoscroll1L7.nm.dat



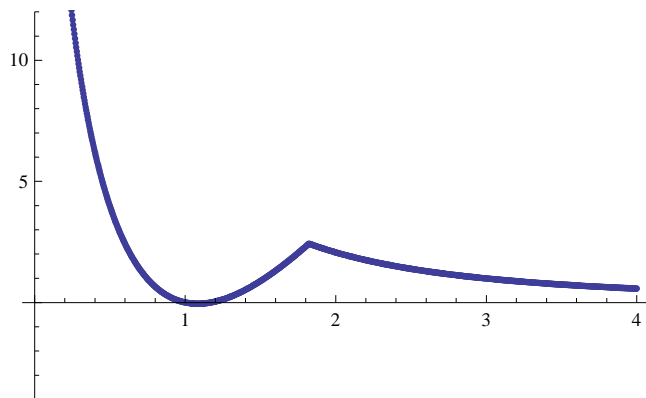
NanoscrollName=Nanoscroll11L10.nm

ScrollEnergyFileName=EvsRIn1Nanoscroll11L10.nm.dat



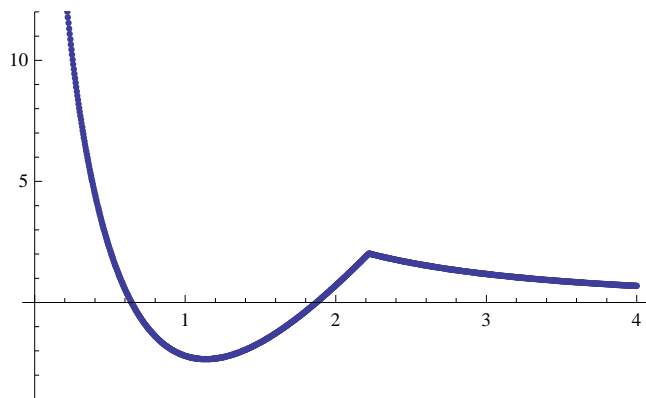
NanoscrollName=Nanoscroll11L12.5nm

ScrollEnergyFileName=EvsRIn1Nanoscroll11L12.5nm.dat



NanoscrollName=Nanoscroll11L15.nm

ScrollEnergyFileName=EvsRIn1Nanoscroll11L15.nm.dat



Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1={7., 10., 12.5, 15.}nm (NoL=1,Lw=1.nm)

