```
In[279]:= Print[" Figure3cd(Layers1L15nmRIn1d14.m, Figure3cd(Layers2L15nmRIn1d14.m"];
     Print[];
     Print[" The source of the data of the manuscript"];
     Print[" 'Structure and energetics of carbon, "];
     Print[" hexagonal boron nitride, and"];
     Print[" carbon/hexagonal boron nitride"];
     Print[" single-layer and bilayer nanoscrolls' "];
     Print[" / A.I. Siahlo, N.A. Poklonski, A.V. Lebedev,"];
     Print[" I.V. Lebedeva, A.M. Popov, S.A. Vyrko, "];
     Print[" A.A. Knizhnik, Yu.E. Lozovik "];
     Print[" // Phys. Rev. Materials.- 2018.- V. 2,"];
     Print[" № 3.- P. 036001 (9 pp.)."];
     Print[" [DOI: 10.1103/PhysRevMaterials.2.036001] "];
     Print[" -----"];
     NoL1 = 1; NoL2 = 2;
     NoLp = NoL1;
     Print[" I.O The Units (nm, meV, AA)"];
     "nm=10^(-9)m;";
     nm = 10^{(-9)} m;
     AA = 10^{(-10)} m;
     JJkgms = kg m^2/s^2;
     C1 = Amper s;
     "eV=JJ Electronp;";
     JJ = eV/Electron;
     JJms = (kg m^2)/s^2;
     meV = N[eV/1000];
     Print["----"];
     Print[" I.1. All Input Parameters and Constants-----"];
     Print[" I.1.1. The sampling parameters"]
     npRIn1 = 1000;
     Print["npRIn1=", npRIn1];
     Print[" I.1.2. The Input Geometry Parameters of the system"];
     L14d839nm = 14.839 nm;
     L12d709nm = 12.709 nm;
     L129d678nm = 29.678 nm;
     L1p = L129d678nm;
     L1p = L14d839nm;
     Print[" The carbon nanoribbon length L1=", L1p/nm, "nm"];
     Lw11d8nm = 11.8 nm;
     Lwp = Lw11d8nm;
     Print[" The carbon nanoribbon width Lw=", Lwp/nm, "nm"];
     Print["-----"];
     Print[" Number of the layers in carbon nanoscroll NoL=", NoLp];
     Print[" The length of a carbon nanoribbon L1=", L1p/nm, "nm"];
     Lw1nm = 1. nm; Lwp = Lw1nm;
     Print[" The carbon nanoribbon width Lw=", Lwp/nm, "nm"];
     RIn1d1nm = 1.1 nm;
     RIn1d2nm = 1.2 nm;
     RIn1d14nm = 1.14 nm;
     RIn2nm = 2.047 nm;
     RIn2d1nm = 2.1 nm;
     RIn2d2nm = 2.2 nm;
     RIn2d3nm = 2.3 nm;
```

```
RIn2d4nm = 2.4 nm;
RIn2d5nm = 2.5 nm:
RIn2d6nm = 2.6 nm;
RIn1p = RIn2d5nm;
RIn1p = RIn2d3nm;
RIn1p = RIn2d2nm;
RIn1p = RIn2d1nm;
RIn1p = RIn1d14nm;
Print[" The inner radius of the nanoscroll RIn1=", RIn1p/nm, "nm"];
Print[" I.1.2. The Input Energy Constants"];
Print[" epsVdW - the interlayer interaction energy per one atom of"];
Print[" the nanoscroll:"];
epsVdW35 = 35.0 meV/atom; epsVdWp = epsVdW35;
Print[" epsVdW=", epsVdWp/(eV/atom), "eV/atom"];
Print[" C - the bending elastic constant:"];
C201 = 2.01 \text{ eV AA}^2/\text{atom};
CBN1328 = 1.328 \text{ eV } AA^2/atom;
CCp = C201;
CBNp = CBN1328;
CBNp = CCp;
Print[" CCelast=", CCp/(eV AA^2/atom), "eV AA^2/atom"];
Print[" CCBNelast=", CBNp/(eV AA^2/atom), "eV AA^2/atom"];
Print[" I.1.3.The Input Geometry constants-----"];
Print[" The interatomic distance aCC and the interlayer distance h"];
aCC142AA = 1.42 AA; aCCp = aCC142AA;
h335nm = 0.3354 nm; hp = h335nm;
Print["h=", hp/nm, " nm (Interlayer distance)"];
Print[" aCC=", aCCp/nm, "nm, h=", hp/nm, "nm"];
NatomsInCell2 = 2; NatomsInCellp = NatomsInCell2;
Print["NatomsInCell=", NatomsInCellp];
Print[" dPhi12 - The difference of the inner angles of the spirales"];
Print[" of the Layers"];
dPhi12eq0 = 0.0 Pi;
dPhi12eqPi = 1.0 Pi;
dPhi12p = 0.0 Pi;
dPhi12p = 1.0 Pi;
dPhi12p = 0.5 Pi;
Print[" I.4.The parameters for the visualisation"];
RIn1MinMonoScroll = hp/5;
RIn1MinBiScroll = hp/5;
RIn1MaxMonoScroll = 4 nm;
RIn1MaxBiScroll = 8 nm;
PlotRangeMonoScroll = {-4 eV/atom, 12 eV/atom};
PlotRangeBiScroll = {-10 eV/atom, 30 eV/atom};
ShowSpirales = True;
ShowThePlot = True;
Print[" I.5. The parameters of visualization that depend on NoL=",NoLp];
PlotRangep = Switch[NoLp, 1, PlotRangeMonoScroll, 2, PlotRangeBiScroll];
RIn1Minp = Switch[NoLp, 1, RIn1MinMonoScroll, 2, RIn1MinBiScroll];
RIn1Maxp = Switch[NoLp, 1, RIn1MaxMonoScroll, 2, RIn1MaxBiScroll];
PlotRangep = Switch[NoLp, 1, PlotRangeMonoScroll, 2, PlotRangeBiScroll];
RIn1Maxp = Switch[NoLp, 1, RIn1MaxMonoScroll, 2, RIn1MaxBiScroll];
tL1 = Switch[NoLp, 1, {7. nm, 10. nm, 12.5 nm, 15. nm},
      2, {15. nm, 20. nm, 25. nm, 30. nm}];
```

```
Print[" I.6. The parameters of the output file"];
NanoscrollNamep =
   StringJoin["Nanoscroll", ToString[NoLp], "L", ToString[L1p/nm],
Print[" NanoscrollName=", NanoscrollNamep];
CarbonNanoscrollEnergyVsRInFileName =
   StringJoin[NanoscrollNamep, ".txt"];
Print[CarbonNanoscrollEnergyVsRInFileName];
Print[" (The output of the data to a file Is Not Performed)"];
npRIn1 = 1000;
Print[" The number of the output points = ", npRIn1];
Print[" I.7. The Input Numerical Constants used in the programm"];
Print[" The Indexes used for the work with EVdW[...] function"];
iEVdW = 1; iEVdW1Un1 = 2; iEVdW1Ov1 = 3; iEVdW1Un2 = 4; iEVdW1Ov2 = 5;
iEVdW2Un1 = 6; iEVdW2Ov1 = 7;
Print[" ------"];
AA = 0.1 nm; PhiIn := \varphiIn; PhiOut := \varphiOut;
Print["-----"];
Print[" II. The derivated parameters and the functions required"];
Print[" II.1. The derivated parameters"];
fSa[aCC_] := aCC^2 3 Sqrt[3]/4; fSa[aCCp]; Sap = fSa[aCCp];
Print[" The cell area Sa=", fSa[aCC], "=", Sap/nm^2, "nm^2"];
Print[" II.2. The required functions-----"];
Print[" II.2.1. The function fSpiraleLen[", NoLp,",\varphi In, \varphi Out, h] defines"];
Print[" the Length of a Spirale with the inner agle \varphiIn and the outer angle \varphiOut(>=\varphiIn):"
fSpiraleLen[NoLv_, PhiInv_, PhiOutv_, hv_] :=
     (1/(4 Pi) hv NoLv (-PhiInv Sqrt[1 + PhiInv^2] + PhiOutv Sqrt[1 + PhiOutv^2] - ArcSinh[P]
Print[" fSpiraleLen[", NoLp, ", \varphiIn, \varphiOut, h]=",
   fSpiraleLen[NoLp, PhiIn, PhiOut, h], "."];
Print[" II.2.2. The function fElast[\varphi In, \rho Out] is required to calculate an nanoscrollelasti
fElast[PhiInv_,
          PhiOutv_] := (Sqrt[PhiInv^2 + 1]/PhiInv -
             Sqrt[PhiOutv^2 + 1]/PhiOutv - ArcSinh[PhiInv] +
      ArcSinh[PhiOutv]);
Print[" fElast[\varphiIn, \varphiOut] = ",
   fElast[PhiIn, PhiOut], "."];
Print[" II.2.3. Define the function fPhiOutvsPhiInLh[", NoLp, ",", PhiIn, ",L,h]."]
fPhiOutvsPhiInLh[NoLv_, PhiInv_, Lv_, hv_] :=
      Sqrt[4 \pi Lv/(NoLv hv) + PhiInv^2];
Print[" The function fPhiOutvsPhiInLh[", NoLp, ",", PhiIn, ",L,h]=",
   fPhiOutvsPhiInLh[NoLp, PhiIn, L, h], " is a
     good approximation to obtain the value of \varphiOut for the defined \varphiIn,L,h."];
fPhiInvsPhiOutLh[NoLv_, PhiOutv_, Lv_, hv_] := Sqrt[PhiOutv^2 - 4 Pi Lv/(NoLv hv)];
Print[" The inverse function fPhiInvsPhiOutLh[", NoLp, ", \u00f3Out, L, h]]=",fPhiInvsPhiOutLh[No
Print[" could be used in the program applications if ROut (instead of RIn) is the input pa
Print[" II.2.4. The functions fSpiralelUnder(Over)SpiralelLength[NoLv,PhiIn1v ,PhiOut1v,hv
fSpirale1UnderSpirale1Length[NoLv_, PhiIn1v_ , PhiOut1v_, hv_] := fSpiraleLen[NoLv, PhiIn1
fSpirale1OverSpirale1Length[NoLv_, PhiIn1v_ , PhiOut1v_, hv_] := fSpiraleLen[NoLv, PhiIn1v
 fSpirale1UnderSpirale2Length[NoLv\_, PhiIn1v\_, PhiOut1v\_, hv\_, dPhi12v\_] := fSpiraleLen[NoLv\_, PhiOut1v\_, Ph
fSpirale1OverSpirale2Length[NoLv_, PhiIn1v_, PhiOut1v_, hv_, dPhi12v_] := fSpiraleLen[NoL
fSpirale2UnderSpirale1Length[NoLv_, PhiIn1v_ , PhiOut1v_, hv_, dPhi12v_] := fSpiraleLen[No
Print[" These functiona are not required, but could be helpful),"];
```

```
If[NoLp == 1,
  Print["fSpirale1UnderSpirale1Length[1,PhiIn1v ,PhiOut1v,hv]="];
  Print[" =fSpiraleLen[NoLv,PhiIn1v ,PhiOut1v-2Pi,hv]=",
  fSpiraleLen[NoLv, PhiIn1v , PhiOut1v - 2 Pi, hv], ";"];
  Print[" fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Le
  Print[" =",
  fSpirale1UnderSpirale1Length[NoLp, PhiIn1p, PhiOut1p, hp]/nm,
  "nm."];
  Print["fSpirale1OverSpirale1Length[1,PhiIn1v ,PhiOut1v,hv]="];
  Print[" =fSpiraleLen[NoLv,PhiIn1v+2Pi ,PhiOut1v,hv]=",
  fSpiraleLen[NoLv, PhiIn1v + 2 Pi , PhiOut1v, hv], ";"];
  Print[" fSpirale1OverSpirale1Length[NoLp,PhiIn1p ,PhiOut1p,hp]=fSpirale1OverSpirale1Len
  PhiIn1p/(2 Pi), "(2Pi),", PhiOut1p/(2 Pi), "(2Pi),", hp/nm,
   "nm] ="];
  Print[" =",
  fSpiralelOverSpiralelLength[NoLp, PhiIn1p, PhiOut1p, hp]/nm,
   "nm."];
If[NoLp == 2,
  Print[" fSpirale1UnderSpirale2Length[1,PhiIn1v ,PhiOut1v,hv,dPhi12v]="];
  Print[" fSpiraleLen[NoLv,PhiIn1v, PhiOut1v -2 Pi/NoLv,hv]=",
  fSpirale1UnderSpirale2Length[1, PhiIn1v , PhiOut1v, hv, dPhi12v],
   ":"1:
  Print["fSpirale1UnderSpirale2Length[NoLp,PhiIn1p,PhiOut1p,hp,dPhi12p]=fSpirale1UnderSpir
  PhiInlp/(2 Pi), "(2Pi),", PhiOutlp/(2 Pi), "(2Pi),", hp/nm, "nm,",
   dPhi12p/(2 Pi), "(2Pi)] ="];
  Print[" =",
  fSpirale1UnderSpirale2Length[NoLp, PhiIn1p, PhiOut1p, hp,
    dPhi12p]/nm, "nm."];
  Print[" fSpirale1OverSpirale2Length[1,PhiIn1v ,PhiOut1v,hv,dPhi12v]="];
  Print[" =fSpiraleLen[NoLv,PhiIn1v+Pi ,PhiOut1v,hv]=",
   fSpiraleLen[NoLv, PhiInlv + Pi , PhiOutlv, hv], ";"];
  Print["fSpirale10verSpirale2Length[NoLp,PhiIn1p ,PhiOut1p,hp]=fSpirale10verSpirale1Lengt
  PhiIn1p/(2 Pi), "(2Pi),", PhiOut1p/(2 Pi), "(2Pi),", hp/nm,
   "nm] ="];
  Print[" =",
   fSpirale1OverSpirale2Length[NoLp, PhiIn1p , PhiOut1p, hp, dPhi12p]/
    nm, "nm."];
  Print[""];
  Print[" fSpirale2UnderSpirale1Length[1,PhiIn1v ,PhiOut1v,hv]="];
  Print[" fSpiraleLen[NoLv,PhiIn1v, PhiOut1v -2 Pi/NoLv,hv]=",
  fSpirale1UnderSpirale2Length[1, PhiIn1v , PhiOut1v, hv, dPhi12v],
  Print["fSpirale2UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp,dPhi12p]=fSpirale2UnderSpir
  PhiIn1p/(2 Pi), "(2Pi),", PhiOut1p/(2 Pi), "(2Pi),", hp/nm, "nm,",
   dPhi12p/(2 Pi), "(2Pi)] ="];
  Print[" =",
  fSpirale1UnderSpirale2Length[NoLp, PhiIn1p, PhiOut1p, hp,
     dPhi12p]/nm, "nm."];
  Print[" fSpirale10verSpirale2Length[1,PhiIn1v ,PhiOut1v,hv]="];
          =fSpiraleLen[NoLv,PhiIn1v+Pi ,PhiOut1v,hv]=",
   fSpiraleLen[NoLv, PhiIn1v + Pi , PhiOut1v, hv], ";"];
  Print["fSpirale10verSpirale2Length[NoLp,PhiIn1p ,PhiOut1p,hp]=fSpirale10verSpirale2Lengt
   PhiIn1p/(2 Pi), "(2Pi),", PhiOut1p/(2 Pi), "(2Pi),", hp/nm,
```

```
"nm] ="];
  Print[" =",
  fSpirale1OverSpirale2Length[NoLp, PhiIn1p , PhiOut1p, hp, dPhi12p]/
   nm, "nm."];
 1:
Print[" II.2.4. The function fRIn1Sharp[NoLv,L1v,hv]"]
fRIn1Sharp[NoLv_{,} L1v_{,} hv_{,}] := (L1v/(2 Pi) - (NoLv hv/2));
Print["fRIn1Sharp[NoLv,L1v,hv]=", fRIn1Sharp[NoLv, L1v, hv]];
Print["is a good approximation to obtain the value of the sharp in the dependence ScrollEn
Print["fRIn1Sharp[", NoLp, ", ", L1p/nm, "nm, ", hp/nm, "nm] = ",
  fRIn1Sharp[NoLp, L1p, hp]/nm, "nm"];
Print["----"];
Print[" III. Begin of Calculation "];
If[NoLp == 1,
 Print[" III.1. The inner and the outer angle of the spirale of the layer:"]];
If[NoLp == 2,
  Print[" III.1. The inner and the outer angles of the spirales of the layers:"]];
Print[" φIn1=", RIn1 2 Pi/(NoLp h),", φOut1=fPhiOutvsPhiInLh[", NoLp, ",φIn1,L1,h];"];
fPhiIn1[NoLv_, RIn1v_, hv_] := RIn1v 2 Pi/(NoLv hv);
PhiIn1p = fPhiIn1[NoLp, RIn1p, hp];
fPhiOut1[NoLv_, L1v_, RIn1v_, hv_] :=
    fPhiOutvsPhiInLh[NoLv, fPhiIn1[NoLv, RIn1v, hv], L1v, hv];
Print[" For RIn1=", RIn1p/nm, "nm,h=", hp/nm, "nm:"];
PhiOut1p = fPhiOut1[NoLp, L1p, RIn1p, hp];
ROut1p = PhiOut1p NoLp hp/(2 Pi);
Print[" φIn1=", PhiIn1p/(2 Pi), "(2Pi), ΨOut1=",
  PhiOut1p/(2 Pi), "(2Pi)."];
fPhiIn2[NoLv_, RIn1v_, hv_, dPhi12v_] :=
   fPhiIn1[NoLv, RIn1v, hv] + dPhi12v;
PhiIn2dPhi12p =
   fPhiIn2[NoLp, RIn1p, hp, 0];(*www orig 2022.10*)
PhiIn2dPhi12p =
   fPhiIn2[NoLp, RIn1p, hp,
  dPhi12p];(* for dPhi12p!=0, checked 2022.10*)
PhiIn2dPhi12Pip =
   fPhiIn2[NoLp, RIn1p, hp, Pi];
fPhiOut2[NoLv_, L1v_, RIn1v_, hv_, dPhi12v_] :=
    fPhiOutvsPhiInLh[NoLv, fPhiIn2[NoLv, RIn1v, hv, dPhi12v], L1v,
  hv];
PhiOut2dPhi12p =
   fPhiOut2[NoLp, L1p, RIn1p, hp, dPhi12p];
  Print[" φIn2=", PhiIn2dPhi12p/(2 Pi),"(2Pi), φOut2=", PhiOut2dPhi12p/(2 Pi), "(2Pi)"];
  PhiOut2dPhi12Pip = fPhiOut2[NoLp, L1p, RIn1p, hp, Pi];
  Print[" for dφ12=Pi: PhiIn2=", PhiIn2dPhi12Pip/(2 Pi),
   "(2Pi), φOut2=", PhiOut2dPhi12Pip/(2 Pi), "(2Pi)"];];
Print["L1=", L1p/nm, "nm, RIn1=", RIn1p/nm, "nm"];
If[NoLp == 1, Print[" Plot the Spirale of the layer:"]];
If[NoLp == 2, Print[" Plot Spirales of the layers:"]]; "for d\phi 12=0";
Spirale1Plot = PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn1p, PhiOut1p},
      PlotRange -> {{-1.1 ROut1p/nm,
      1.1 ROut1p/nm}, {-1.1 ROut1p/nm,
           1.1 ROutlp/nm}}, PlotStyle -> {Red, Thin}, Axes -> None];
If[NoLp == 1, Print[Show[Spirale1Plot]];
Print["Manipulating of Spirale1Plot for the different RIn1 and L:"];
```

```
Manipulate[PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm,
{Phiv, fPhiIn1[NoLp, RIn1nmm nm, hp],fPhiOut1[NoLp, L1nmm nm, RIn1nmm nm, hp]},
            PlotRange -> {{-1.1 ROut1p/nm,
            1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                        1.1 ROutlp/nm}}, PlotStyle -> {Red, Thin}, Axes -> None]
,{{RIn1nmm,RIn1p/nm},RIn1Minp/nm,RIn1Maxp/nm},{{L1nmm,L1p/nm},0.5 tL1[[1]]/nm,1.5 tL1[[Lenc
If[NoLp > 1,
  Print[" Plot the Spirale of the layers:"];
  Spirale2Plot =
    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn2dPhi12p + Pi,
        PhiOut2dPhi12p + Pi},
              PlotRange -> {{-1.1 ROut1p/nm,
            1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                           1.1 ROut1p/nm}}, PlotStyle -> {Blue, Thin},
      Axes -> None];
  Spirale2dPhi12PiPlot =
    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn2dPhi12p + Pi,
        PhiOut2dPhi12Pip + Pi},
              PlotRange -> {{-1.1 ROut1p/nm,
            1.1 ROut1p/nm}, {-1.1 ROut1p/nm,
                           1.1 ROut1p/nm}}, PlotStyle -> {Blue, Thin},
      Axes -> None];
  Print[Show[{Spirale1Plot, Spirale2Plot}]];
If[NoLp == 1,
        Spirale10verSpirale1Plot =
            If[PhiIn1p + 2 Pi < PhiOut1p,</pre>
                PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn1p + 2 Pi,
                        PhiOut1p},
          PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                 1.1 ROut1p/nm}}, PlotStyle -> {Red, Thick},
          Axes -> None], {}];
        Spirale1UnderSpirale1Plot =
            If[PhiIn1p < PhiOut1p - 2 Pi,</pre>
                PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn1p,
                        PhiOut1p - 2 Pi},
          \label{eq:plotRange} \mbox{->} \mbox{$\{\{-1.1$ ROutlp/nm, 1.1 ROutlp/nm}\}, $\{-1.1$ ROutlp/nm, $(-1.1)$ RO
                                 1.1 ROut1p/nm}}, PlotStyle -> {Red, Thick},
          Axes -> None], {}];
        Print[
      " {Spirale, Spirale1UnderSpirale1}, {Spirale1, Spirale1OverSpirale1}:"];
        Print[Show[{Spirale1Plot, Spirale1UnderSpirale1Plot}],
            Show[{Spirale1Plot, Spirale1OverSpirale1Plot}]];
If[NoLp == 2,
    Spirale1UnderSpirale2dPhi120Plot =
            If[PhiIn1p < PhiOut2dPhi12p - Pi,</pre>
                PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn1p,
                         PhiOut2dPhi12p - Pi}, PlotStyle -> {Red, Thick},
          PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                1.1 ROut1p/nm}}], {}];
    Spirale10verSpirale2dPhi120Plot =
            If[PhiIn1p + Pi < PhiOut1p,</pre>
```

```
PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv,
                                PhiIn1p + Pi +
                                       dPhi12p, PhiOut1p},
                          PlotStyle -> {Red, Thick},
         PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                             1.1 ROut1p/nm}}], {}];
Spirale2UnderSpirale1dPhi120Plot =
             If[PhiIn2dPhi12p + Pi < PhiOut1p,</pre>
                    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv,
                                PhiIn2dPhi12p + Pi, PhiOut1p},
         PlotStyle -> {Blue, Thick},
         PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                             1.1 ROut1p/nm}}], {}];
Spirale2OverSpirale1dPhi120Plot =
             If[2 Pi + PhiIn2dPhi12p -
                                dPhi12p <
                          PhiOut2dPhi12p + Pi,
                    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv,
                                 2 Pi + PhiIn2dPhi12p -
                                       dPhi12p,
                                 PhiOut2dPhi12p + Pi}, PlotStyle -> {Blue, Thick},
         PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                             1.1 ROut1p/nm}}], {}];
Print["Plot Spirales for dPhi12=Pi (could be NotRequired, dPhi12=0 in this program)"];
Spirale1UnderSpirale2dPhi12PiPlot =
             If[PhiIn1p < PhiOut2dPhi12Pip - Pi,</pre>
                    PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv, PhiIn1p,
                                PhiOut2dPhi12Pip - Pi}, PlotStyle -> {Red, Thick},
         \label{eq:plotRange} \mbox{ -> } \{ \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ 1.1 ROutlp/nm} \}, \mbox{ } \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \mbox{ ROutlp/nm} \}, \mbox{ } \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \m
                                              1.1 ROut1p/nm}}], {}];
Spirale10verSpirale2dPhi12PiPlot =
             If[PhiIn1p + Pi +
                                dPhi12p < PhiOut1p,
                    PolarPlot[(Phiv) NoLp hp/(2 Pi)/nm, {Phiv,
                                PhiIn1p + Pi +
                                       dPhi12p, PhiOut1p},
                          PlotStyle -> {Red, Thick},
         \label{eq:plotRange} \mbox{ -> } \{ \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ 1.1 ROutlp/nm} \}, \mbox{ } \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \mbox{ ROutlp/nm} \}, \mbox{ } \{ -1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \mbox{ ROutlp/nm}, \mbox{ } 1.1 \m
                                             1.1 ROut1p/nm}}], {}];
Spirale2UnderSpirale1dPhi12PiPlot =
             If[PhiIn2dPhi12p + Pi < PhiOut1p,</pre>
                    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv,
                                 PhiIn2dPhi12Pip + Pi, PhiOut1p},
         PlotStyle -> {Blue, Thick},
         PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                                            1.1 ROut1p/nm}}], {}];
Spirale2OverSpirale1dPhi12PiPlot =
             If[2 Pi + PhiIn2dPhi12Pip -
                                dPhi12p <
                          PhiOut2dPhi12Pip + Pi,
                    PolarPlot[(Phiv - Pi) NoLp hp/(2 Pi)/nm, {Phiv,
```

```
2 Pi + PhiIn2dPhi12Pip -
              dPhi12p,
            PhiOut2dPhi12Pip + Pi}, PlotStyle -> {Blue, Thick},
     PlotRange -> {{-1.1 ROutlp/nm, 1.1 ROutlp/nm}, {-1.1 ROutlp/nm,
                1.1 ROut1p/nm}}], {}];
   Print[
   " {Spirale1,Spirale2,Spirale1UnderSpirale2,Spirale2UnderSpirale1},"];
   Print[
         {Spirale1,Spirale2,Spirale1OverSpirale2,Spirale2OverSpirale1}"];
   Print[" for dPhi12=0: ", Show[Spirale1Plot, Spirale2Plot],
      Show[Spirale1Plot, Spirale2Plot,
    Spirale1UnderSpirale2dPhi120Plot,
        Spirale2UnderSpirale1dPhi120Plot],
      Show[Spirale1Plot, Spirale2Plot,
    Spirale10verSpirale2dPhi120Plot,
        Spirale2OverSpirale1dPhi12OPlot]];
    Print[" for dPhi12=Pi: ",
   Show[Spirale1Plot, Spirale2dPhi12PiPlot],
      Show[Spirale1Plot, Spirale2dPhi12PiPlot,
    Spirale1UnderSpirale2dPhi12PiPlot,
        Spirale2UnderSpirale1dPhi12PiPlot],
      Show[Spirale1Plot, Spirale2dPhi12PiPlot,
    Spirale10verSpirale2dPhi12PiPlot,
        Spirale2OverSpirale1dPhi12PiPlot]];
  ];
Print[" III.2. The nanoscroll energy calculation"];
Print[" III.2.1. The elastic energy calculation"];
fEelastCC[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, CCv_] :=
    Module[{},
      Return[2 Pi CCv Lwv/(hv fSa[aCCv]) fElast[
              fPhiIn1[NoLv, RIn1v, hv],
              fPhiOut1[NoLv, Llv, RIn1v, hv]]];];
fEelastCBN[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, CBNv_] :=
    Module[{},
      Return[2 Pi CBNv Lwv/(hv fSa[aCCv]) fElast[
              fPhiIn1[NoLv, RIn1v, hv],
              fPhiOut1[NoLv, L1v, RIn1v, hv]]];];
EelastCCp = fEelastCC[NoLp, Lwp, L1p, RIn1p, hp, aCCp, CCp];
EelastCBNp = fEelastCBN[NoLp, Lwp, L1p, RIn1p, hp, aCCp, CBNp];
Print[" EelastC=", EelastCCp/(eV/atom), "eV/atom"];
If[NoLp == 2, Print[" EelastBN=", EelastCBNp/(eV/atom), "eV/atom"];];
Print[" III.2.2. The Van-der-Waals energy calculation"];
"The definition of the function ";
"'fEVdWLayer10verlap[NoLv,Lwv,L1v, RIn1v, hv, aCCv, epsVdWv]'";
"(Note: This function is omitted at calculations";
    but could be helpful at calculation of VdW ebergy of monoscroll at debugging;";
     for example,";
    fEVdWLayer1Overlap[NoL1,Lwp,15nm, 2nm, hp, aCCp, epsVdWp] ";
" and fEVdWLayersOverlap[NoL2,Lwp,L1p=15nm, 2nm, hp, aCCp, epsVdWp, 0]";
     give the same values";
fEVdWLayer10verlap[NoLv_, Lwv_, L1v_, RIn1v_, hv_, aCCv_, epsVdWv_] :=
   Module[
   {EVdWv,
     EVdW1Un1v = 0 (eV/atom), EVdW1Ov1v = 0 (eV/atom),
```

```
Spirale1UnderSpirale1Length = 0 nm,
   Spirale1OverSpirale1Length = 0 nm,
    PhiIn1v = fPhiIn1[NoLv, RIn1v, hv],
    PhiOutlv = fPhiOutl[NoLv, Llv, RInlv, hv],
  Spirale10verSpirale1Length =
   fSpiraleLen[NoLv, PhiIn1v + 2 Pi, PhiOut1v, hv];
  Spirale1UnderSpirale1Length =
   fSpiraleLen[NoLv, PhiIn1v , PhiOut1v - 2 Pi, hv];
  "Note: Spirale1OverSpirale1Length>Spirale1UnderSpirale1Length";
  EVdW1Un1v = -epsVdWv Lwv/(2 fSa[
         aCCv]) Spirale1UnderSpirale1Length;
  EVdW1Ov1v = -epsVdWv Lwv/(2 fSa[aCCv]) Spirale1OverSpirale1Length;
  EVdWv = (EVdW1Un1v + EVdW10v1v);
  Return[{EVdWv, EVdW1Un1v, EVdW1Ov1v}];
"The definition of the function";
"fEVdWLayersOverlap[NoLv_,Lwv_,Llv_, RInlv_, hv_, aCCv_, epsVdWv_, dPhi12v_]";
fEVdWLayersOverlap[NoLv_, Lwv_, L1v_, RIn1v_, hv_, aCCv_, epsVdWv_,
  dPhi12v_] := Module[
  {EVdW,
    EVdW1Un1 = 0 (eV/atom), EVdW1Ov1 = 0 (eV/atom),
    EVdW1Un2 = 0 (eV/atom), EVdW1Ov2 = 0 (eV/atom),
    EVdW2Un1 = 0 (eV/atom), EVdW2Ov1 = 0 (eV/atom),
    Spirale1UnderSpirale1Length = 0 nm,
   Spirale1OverSpirale1Length = 0 nm,
    Spirale1UnderSpirale2Length = 0 nm,
   Spirale10verSpirale2Length = 0 nm,
    Spirale2UnderSpirale1Length = 0 nm,
   Spirale2OverSpirale1Length = 0 nm,
    PhiIn1 = fPhiIn1[NoLv, RIn1v, hv],
    PhiIn2 = fPhiIn2[NoLv, RIn1v, hv, dPhi12v],
    PhiOut1 = fPhiOut1[NoLv, Llv, RInlv, hv],
    PhiOut2 = fPhiOut2[NoLv, L1v, RIn1v, hv, dPhi12v],
    ReturnEnergiesv = \{1, 2, 3, 4, 5, 6, 7\}
   },
  If[NoLv == 1,
   If[PhiIn1 < PhiOut1 - 2 Pi,</pre>
    Spirale1UnderSpirale1Length =
       fSpiraleLen[NoLv, PhiIn1, PhiOut1 - 2 Pi, hv];];
   If[PhiIn1 + 2 Pi < PhiOut1,</pre>
    Spirale1OverSpirale1Length =
      fSpiraleLen[NoLv, PhiIn1 + 2 Pi, PhiOut1, hv];];
   EVdW1Un1 = -epsVdWv Lwv/(2 fSa[
          aCCv]) Spirale1UnderSpirale1Length;
   EVdW1Ov1 = -epsVdWv Lwv/(2 fSa[aCCv]) Spirale1OverSpirale1Length;
   EVdW = (EVdW1Un1 + EVdW1Ov1);
   ReturnEnergiesv = {EVdW, EVdW1Un1, EVdW1Ov1};
   1;
  If[NoLv == 2,
       If[PhiIn1 < PhiOut2 - Pi,</pre>
    Spirale1UnderSpirale2Length =
       fSpiraleLen[NoLv, PhiIn1, PhiOut2 - Pi, hv];];
        If[PhiIn1 + Pi + dPhi12v < PhiOut1,</pre>
    Spirale10verSpirale2Length =
       fSpiraleLen[NoLv, PhiIn1 + Pi + dPhi12v, PhiOut1, hv];];
        If[PhiIn1 + dPhi12v < PhiOut1 - Pi,</pre>
```

```
Spirale2UnderSpirale1Length =
       fSpiraleLen[NoLv, PhiIn1 + dPhi12v, PhiOut1 - Pi, hv];];
        If[PhiIn1 - dPhi12v + Pi < PhiOut2 - dPhi12v,</pre>
     Spirale2OverSpirale1Length =
       fSpiraleLen[NoLv, PhiIn1 - dPhi12v + Pi, PhiOut2 - dPhi12v,
        hv1:1:
    EVdW1Un2 = -epsVdWv Lwv/(2 fSa[
          aCCv]) Spirale1UnderSpirale2Length;
         EVdW10v2 = -epsVdWv Lwv/(2 fSa[
          aCCv]) Spirale1OverSpirale2Length;
         EVdW2Un1 = -epsVdWv Lwv/(2 fSa[
          aCCv]) Spirale2UnderSpirale1Length;
         EVdW2Ov1 = -epsVdWv Lwv/(2 fSa[
          aCCv]) Spirale2OverSpirale1Length;
         EVdW = (EVdW1Un2 + EVdW1Ov2 + EVdW2Un1 + EVdW2Ov1);
         ReturnEnergiesv[[iEVdW]] = EVdW;
         ReturnEnergiesv[[iEVdW1Un2]] = EVdW1Un2;
         ReturnEnergiesv[[iEVdW1Ov2]] = EVdW1Ov2;
         ReturnEnergiesv[[iEVdW2Un1]] = EVdW2Un1;
         ReturnEnergiesv[[iEVdW2Ov1]] = EVdW2Ov1;
   Return[ReturnEnergiesv];
EVdWdPhi12eq0allp =
    fEVdWLayersOverlap[NoL2, Lwp, L1p, RIn1p, hp, aCCp, epsVdWp,
   dPhi12eq0];
EVdWvardPhi12allp =
    fEVdWLayersOverlap[NoLp, Lwp, L1p, RIn1p, hp, aCCp, epsVdWp,
   dPhi12p];
If[NoLp == 1,
  Print[" EVdWvardPhi12allp[[iEVdW]]=",
   EVdWvardPhi12allp[[iEVdW]]/(eV/atom), "eV/atom"];
  Print["( EVdWvardPhi12allp[[iEVdW1Un1]]=",
   EVdWvardPhi12allp[[iEVdW1Un1]]/(eV/atom), "eV/atom"];
    Print[" EVdWvardPhi12allp[[iEVdW10v1]]=",
   EVdWvardPhi12allp[[iEVdW1Ov1]]/(eV/atom), "eV/atom )"];
 ];
If[NoLp == 2,
  Print[" for dPhi12=", dPhi12p/Pi,
   "Pi EVdWvardPhi12allp[[iEVdW]]=",
     EVdWvardPhi12allp[[iEVdW]]/(eV/atom), "eV/atom"];
    Print[" For dPhi12=", dPhi12eq0/Pi, "Pi:"];
    Print[" EVdWvardPhi12allp[[iEVdW]]=",
      EVdWdPhi12eq0allp[[iEVdW]]/(eV/atom), "eV/atom"];
    Print[" EVdWvardPhi12allp[[iEVdW1Un2]]=",
      EVdWdPhi12eq0allp[[iEVdW1Un2]]/(eV/atom), "eV/atom"];
    Print[" EVdWvardPhi12allp[[iEVdW1Ov2]]=",
      EVdWdPhi12eq0allp[[iEVdW1Ov2]]/(eV/atom), "eV/atom"];
    Print[" EVdWvardPhi12allp[[iEVdW2Un1]]=",
      EVdWdPhi12eq0allp[[iEVdW2Un1]]/(eV/atom), "eV/atom"];
    Print[" EVdWvardPhi12allp[[iEVdW2Ov2]]=",
      EVdWdPhi12eq0allp[[iEVdW2Ov1]]/(eV/atom), "eV/atom"];
    EVdWdPhi12eqPiallp :
      fEVdWLayersOverlap[NoLp, Lwp, L1p, RIn1p, hp, aCCp, epsVdWp,
    dPhi12eqPi];
  (**) Print[" For dPhi12=", dPhi12eqPi/Pi, "Pi:"];
```

```
Print[" EVdWvatdPhi12allp[[iEVdW]]=",
      EVdWdPhi12eqPiallp[[iEVdW]]/(eV/atom), "eV/atom"];
    Print[" EVdWvatdPhi12allp[[iEVdW1Un2]]=",
      EVdWdPhi12eqPiallp[[iEVdW1Un2]]/(eV/atom), "eV/atom"];
    Print[" EVdWvatdPhi12allp[[iEVdW10v2]]=",
      EVdWdPhi12eqPiallp[[iEVdW1Ov2]]/(eV/atom), "eV/atom"];
    Print[" EVdWvatdPhi12allp[[iEVdW2Un1]]=",
      EVdWdPhi12eqPiallp[[iEVdW2Un1]]/(eV/atom), "eV/atom"];
    Print[" EVdWvatdPhi12allp[[iEVdW2Ov2]]=",
      EVdWdPhi12eqPiallp[[iEVdW2Ov1]]/(eV/atom), "eV/atom"];
    EVdWEVdWdPhi12eq0p = EVdWdPhi12eq0allp[[iEVdW]];
    Print[" EVdWdPhi12eq0allp=", EVdWdPhi12eq0allp/(eV/atom),
      "eV/atom"];
    EVdWEVdWdPhi12eqPip = EVdWvardPhi12allp[[iEVdW]];
    Print[" EVdWEVdWdPhi12eqPip=", EVdWEVdWdPhi12eqPip/(eV/atom),
      "eV/atom"];
  (**)
 1:
If[NoLp == 2, Print[" III.3. The energy of flat planes "];];
fEnergyFlatPlanes[NoLv_, Lwv_, L1v_, aCCv_, epsVdWv_] :=
  If[NoLv == 2, -epsVdWv Lwv/fSa[aCCv] L1v, 0 eV/atom];
EnergyFlatPlanesp = fEnergyFlatPlanes[NoLp, Lwp, L1p, aCCp, epsVdWp];
If[NoLp == 2, Print[" EnergyFlatPlanes=-eps width/Sa L1(NoL-1) =",
      EnergyFlatPlanesp/(eV/atom), "eV/atom"];];
Print[" III.4. The total energy of the nanoscroll"];
fScrollEnergydPhi[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, epsVdWv_,
   CCv_, CBNv_, dPhi12v_] :=
   Module[{ScrollEnergyv, EVdWv, EVdWnoDimv},
     EVdWv =
    fEVdWLayersOverlap[NoLv, Lwv, Llv, RInlv, hv, aCCv, epsVdWv,
      dPhi12v][[1]];
   EVdWnoDimv = EVdWv /. \{eV \rightarrow 1, atom \rightarrow 1, nm \rightarrow 1\};
     If[NoLv == 1,
    If(EVdWnoDimv == 0,
      ScrollEnergyv =
       fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv],
      ScrollEnergyv =
        (EVdWv/(eV/atom) + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv]/(eV/atom))(eV/at
        1;
     If[NoLv == 2,
    If[EVdWnoDimv == 0,
      ScrollEnergyv =
       fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
                fEelastCBN[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv],
      ScrollEnergyv =
        (EVdWv + fEelastCC[NoLv, Lwv, L1v, RIn1v, hv, aCCv, CCv]/(eV/atom) +
                 fEelastCBN[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv]/(eV/atom))(eV/atom);];
     Return[ScrollEnergyv];
   1;
fScrollEnergyVdWandElast[NoLv_, Lwv_, Llv_, RIn1v_, hv_, aCCv_,
   epsVdWv_, CCv_, CBNv_] :=
   Module[{ ScrollEnergyVdWandElastv, EVdWv},
   (*If[NoLv == 1,
```

```
EVdWv=fEVdWLayer1Overlap[NoLv,Lwv,L1v,RIn1v,hv,aCCv,
   epsVdWv][[1]];
   ];
   If[NoLv == 2,
   EVdWv=fEVdWLayersOverlap[NoLv,Lwv,Llv,RIn1v,hv,aCCv,
   epsVdWv][[1]];
   1:*)
   EVdWv =
   fEVdWLayer1Overlap[NoLv, Lwv, L1v, RIn1v, hv, aCCv,
      epsVdWv][[1]];
   If[NoLv == 1,
      ScrollEnergyVdWandElastv =
      EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv];];
   If[NoLv == 2,
      ScrollEnergyVdWandElastv =
      EVdWv + fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
       fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CBNv];];
   Return[ScrollEnergyVdWandElastv];
fScrollEnergy[NoLv_, Lwv_, Llv_, RInlv_, hv_, aCCv_, epsVdWv_, CCv_,
   CBNv_] := Module[
   {ScrollEnergyv = -10^20 eV/atom},
   If[RIn1v/m <= fRIn1Sharp[NoLv, L1v, hv]/m,</pre>
    ScrollEnergyv =
     fScrollEnergyVdWandElast[NoLv, Lwv, Llv, RInlv, hv, aCCv,
      epsVdWv, CCv, CBNv];
    "note: the function fScrollEnergyVdWandElast[1,...] is analytycal";
    "whereas the function fScrollEnergy[....] uses the 'If[..]'- function";
    If[RIn1v/m >= fRIn1Sharp[NoLv, L1v, hv]/m,
    If[NoLv == 1,
     ScrollEnergyv =
       fEelastCC[NoLv, Lwv, L1v, RIn1v, hv, aCCv, CCv];];
    If[NoLv == 2,
     ScrollEnergyv =
       fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CCv] +
        fEelastCC[NoLv, Lwv, Llv, RInlv, hv, aCCv, CBNv];];
   1;
   Return[ScrollEnergyv];
   ];
ScrollEnergyp =
  fScrollEnergy[NoLp, Lwp, L1p, RIn1p, hp, aCCp, epsVdWp, CCp,
   CBNp];
ScrollEnergyp =
  fScrollEnergy[1, Lwp, L1p, RIn1p, hp, aCCp, epsVdWp, CCp, CBNp];
Print["fScrollEnergy[1,Lwp,L1p, RIn1p,hp, aCCp, epsVdWp,CCp,CBNp]="];
Print["=fScrollEnergy[1, Lw=", Lwp/nm, "nm, L1=", L1p/nm, "nm, RIn1=",
   RIn1p/nm, "nm, h=", hp/nm, "nm,"];
Print[" aCC=", aCCp/nm, "nm, epsVdW=", epsVdWp/(eV/atom),
  "eV/atom, CC=", CCp/(eV AA^2/atom), "(eV AA^2/atom)="];
Print["
                  =", ScrollEnergyp/(eV/atom), "eV/atom"];
Print[" III.5. Determine the inner angles mismatch for the bi-layer nanoscroll
```

```
for the high nanoribbon Length"];
Print[" For L1=", L1p/nm, "nm,RIn=", RIn1p/nm, "nm,h=", hp/nm,
  "nm and dPhi12=0:"];
Print[" ScrollEnergy=", ScrollEnergyp/(eV/atom), "eV/atom"];
Print[" For L1=", L1p/nm, "nm,RIn=", RIn1p/nm, "nm,h=", hp/nm,
 "nm and dPhi12=Pi:"];
Print[" ScrollEnergy=", ScrollEnergyp/(eV/atom), "eV/atom"];
Print["-----
Print[" IV.The potential energy of the nanoscroll"];
Print[" as a function of the inner radius RIn"];
Print[" NoL=", NoLp];
Print[" epsVdW=", epsVdWp/(eV/atom), "eV/atom, C=",
  CCp/(eV nm^2/atom), "(eV nm^2/atom)",
    "(eV nm^2/atom),aCC=", aCCp/nm, "nm,h=", hp/nm, "nm"];
Print[" Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1=", L1p/nm,
    "nm (NoL=", NoLp, ",Lw=", Lwp/nm, "nm"];
PlotScrollEnergyVsRIn1 =
  Plot[(fScrollEnergy[NoLp, Lwp, Llp, RInlnmv nm, hp, aCCp, epsVdWp,
              CCp, CBNp])/(eV/
              atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
   PlotRange -> PlotRangep/(eV/atom)];
Print[PlotScrollEnergyVsRIn1];
Print[" Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1=", tL1/nm,
    "nm (NoL=", NoLp, ",w=", Lwp/nm, "nm)"];
PlotScrollEnergyVsRIn1L1th =
    Plot[(fScrollEnergy[NoLp, Lwp, tL1[[1]], RIn1nmv nm, hp, aCCp,
      epsVdWp,
             CCp, CBNp] )/(eV/
              atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
   PlotRange -> PlotRangep/(eV/atom)];
PlotScrollEnergyVsRIn1L2th =
   Plot[(fScrollEnergy[NoLp, Lwp, tL1[[2]], RIn1nmv nm, hp, aCCp,
      epsVdWp,
              CCp, CBNp] )/(eV/
              atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
   PlotRange -> PlotRangep/(eV/atom)];
PlotScrollEnergyVsRIn1L3th =
    Plot[(fScrollEnergy[NoLp, Lwp, tL1[[3]], RIn1nmv nm, hp, aCCp,
      epsVdWp,
              CCp, CBNp] )/(eV/
              atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
   PlotRange -> PlotRangep/(eV/atom)];
PlotScrollEnergyVsRIn1L4th =
   Plot[(fScrollEnergy[NoLp, Lwp, tL1[[4]], RIn1nmv nm, hp, aCCp,
      epsVdWp,
              CCp, CBNp])/(eV/
              atom), {RIn1nmv, RIn1Minp/nm, RIn1Maxp/nm},
   PlotRange -> PlotRangep/(eV/atom)];
Print[Show[{PlotScrollEnergyVsRIn1L1th, PlotScrollEnergyVsRIn1L2th,
        PlotScrollEnergyVsRIn1L3th, PlotScrollEnergyVsRIn1L4th}]];
Print["The examples of using of 'fScrollEnergy[..]' function:"]
Print["fScrollEnergy[NoLp,Lwp,tL1[[1]],RIn1p,hp, aCCp, epsVdWp,CCp,CBNp]=", fScrollEnergy[
    CCp, CBNp]/(eV/atom), " eV/atom"];
Print["fScrollEnergy[NoLp,Lwp,tL1[[1]],1nm,hp, aCCp, epsVdWp,CCp, CBNp]=", fScrollEnergy[N
    CCp, CBNp]/(eV/atom), " eV/atom"];
Print["fScrollEnergy[NoLp,Lwp,7nm,1nm,hp, aCCp, epsVdWp,CCp, CBNp]=",
```

```
fScrollEnergy[NoLp, Lwp, 7. nm, 1. nm, hp, aCCp, epsVdWp, CCp,
    CBNp]/(eV/atom), " eV/atom"];
Print["fEVdWLayer10verlap[1,Lwp,7.nm,1.nm,hp,aCCp,epsVdWp][[1]]=",
  fEVdWLayer10verlap[1, Lwp, 7. nm, 1. nm, hp, aCCp,
     epsVdWp][[1]]/(eV/atom),
  "eV/atom (right !=0 value, because the layer overlaps"];
Print["fEVdWLayer1Overlap[1,Lwp,7.nm,1.5nm,hp,aCCp,epsVdWp][[1]]=",
  fEVdWLayer10verlap[1, Lwp, 7. nm, 2.5 nm, hp, aCCp,
     epsVdWp][[1]]/(eV/atom),
  "eV/atom !=0, wrong value of the fEVdWLayer1Overlap[..] function because the layer does
Print[];
{\tt Print["The\ analytical\ expressions\ of\ the\ fEVdWLayer1Overlap[...]\ function:"];}
Print[];
Print["--- fEVdWLayer10verlap[NoL1,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---"];
Print[fEVdWLayer10verlap[NoL1, Lwv, Lpv, RInv, hv, aCCv,
    epsVdWv][[1]];
Print[]:
Print["--- fEVdWLayer10verlap[NoL2,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---"];
Print[fEVdWLayer10verlap[NoL2, Lwv, Lpv, RInv, hv, aCCv,
    epsVdWv][[1]];
Print[];
Print["The Analytical expression of "];
Print["fScrollEnergyVdWandElast[", NoLp,
  ",Lwv,Llv,RIn1v,hv,aCCv, epsVdWv, CCv,CBNv]:"];
Print[fScrollEnergyVdWandElast[NoLp, Lwv, Llv, RIn1v, hv, aCCv,
   epsVdWv, CCv, CBNv]];
Print["----"];
Print["V. Export the data of the plots of the nanoscroll energy"];
Print[" as a function of the inner radius"];
Print["The parameters of the output file"];
Print["The number of the output points = ", npRIn1];
Print["Export the plot data to the files:"];
tRIn1nmRegular =
  Table[(RIn1Minp + (RIn1Maxp - RIn1Minp) iiRin/(npRIn1))/nm, {iiRin,
    1, npRIn1}];
tScrollEnergy = tRIn1nmRegular;
tScrollEnergyeVatom = tRIn1nmRegular;
tPlotEvsRin = Table[{}, {ii, 1, Length[tL1]}];
AllPlotsEVsRin = {};
For[iiL1 = 1, iiL1 <= Length[tL1], iiL1++,</pre>
 L1pi = tL1[[iiL1]];
 NanoscrollNamep =
  StringJoin["Nanoscroll", ToString[NoLp], "L", ToString[Llpi/nm],
  Print["NanoscrollName=", NanoscrollNamep];
  ScrollEnergyFileName =
  StringJoin["EvsRIn1", NanoscrollNamep, ".dat"];
  Print["ScrollEnergyFileName=", ScrollEnergyFileName];
  For[iiRIn1 = 1, iiRIn1 <= npRIn1, iiRIn1++,</pre>
  RIn1pi = tRIn1nmRegular[[iiRIn1]] nm;
   tScrollEnergy[[iiRIn1]] =
    fScrollEnergy[NoLp, Lwp, L1pi, RIn1pi, hp, aCCp, epsVdWp,
            CCp, CBNp];
```

```
tScrollEnergyeVatom[[iiRIn1]] = (tScrollEnergy[[iiRIn1]])/(eV/atom);];
       tPlotEvsRin[[iiL1]] =
        ListPlot[Transpose[{tRIn1nmRegular, tScrollEnergyeVatom}],
         PlotRange -> PlotRangep/(eV/atom)];
       Print[tPlotEvsRin[[iiL1]]];
       AllPlotsEVsRin = Join[{AllPlotsEVsRin, tPlotEvsRin[[iiL1]]}];
       CarbonNanoscrollEnergyVsRinFileName =
        StringJoin[NanoscrollNamep, "dat"];
       Export[ToFileName[NotebookDirectory[], ScrollEnergyFileName],
        Transpose[{Insert[tRIn1nmRegular, "RIn1[nm]", 1],
          Insert[tScrollEnergyeVatom, "E[eV/atom]", 1]}]]
       1;
     Print["Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1=", tL1/nm,
         "nm (NoL=", NoLp, ",Lw=", Lwp/nm, "nm)"];
     Print[Show[AllPlotsEVsRin]];
     Print[];
     Print["Manipulating of the plot of the nanoscroll energy as the function"];
     Print[" of the nanoscroll inner radius:"];
     Print[" "];
     Print["(Manipulate[Plot[fScrollEnergy[...,L1nmm,RIn1,hp,...]]],"];
     Print["
                                   where L1nmm (is L1 in nanometers) "];
     Print["
                                   is the manipulated value"];
     Print["
                                  )"];
     Manipulate[
      Plot[(fScrollEnergy[NoLp, Lwp, L1nmm nm, RIn1nmv nm, hp, aCCp,
          epsVdWp,
                        CCp, CBNp])/(eV/atom), {RIn1nmv, RIn1Minp/nm,
        RIn1Maxp/nm], PlotRange -> PlotRangep/(eV/atom)], {{L1nmm, L1p/nm},
        0.5 tL1[[1]]/nm, 1.5 tL1[[Length[tL1]]]/nm}]
 Figure3cd(Layers1L15nmRIn1d14.m, Figure3cd(Layers2L15nmRIn1d14.m
 The source of the data of the manuscript
  'Structure and energetics of carbon,
 hexagonal boron nitride, and
 carbon/hexagonal boron nitride
  single-layer and bilayer nanoscrolls'
/ A.I. Siahlo, N.A. Poklonski, A.V. Lebedev,
I.V. Lebedeva, A.M. Popov, S.A. Vyrko,
A.A. Knizhnik, Yu.E. Lozovik
// Phys. Rev. Materials. - 2018. - V. 2,
№ 3.- P. 036001 (9 pp.).
[DOI: 10.1103/PhysRevMaterials.2.036001]
I.O The Units (nm, meV, AA)
```

```
I.1. All Input Parameters and Constants-----
I.1.1. The sampling parameters
npRIn1=1000
I.1.2. The Input Geometry Parameters of the system
The carbon nanoribbon length L1=14.839nm
The carbon nanoribbon width Lw=11.8nm
Number of the layers in carbon nanoscroll NoL=1
The length of a carbon nanoribbon L1=14.839nm
The carbon nanoribbon width Lw=1.nm
The inner radius of the nanoscroll RIn1=1.14nm
I.1.2. The Input Energy Constants
epsVdW - the interlayer interaction energy per one atom of
the nanoscroll:
epsVdW=0.035eV/atom
C - the bending elastic constant:
CCelast=2.01eV AA^2/atom
CCBNelast=2.01eV AA^2/atom
I.1.3. The Input Geometry constants-----
The interatomic distance aCC and the interlayer distance \boldsymbol{h}
h=0.3354 nm (Interlayer distance)
aCC=0.142nm, h=0.3354nm
NatomsInCell=2
dPhi12 - The difference of the inner angles of the spirales
  of the Layers
I.4. The parameters for the visualisation
I.5. The parameters of visualization that depend on NoL=1
I.6. The parameters of the output file
NanoscrollName=Nanoscroll1L14.839nm
Nanoscroll1L14.839nm.txt
 (The output of the data to a file Is Not Performed)
The number of the output points = 1000
I.7. The Input Numerical Constants used in the programm
The Indexes used for the work with EVdW[...] function
 -----End of the Input-----
  _____
```

II. The derivated parameters and the functions required

II.1. The derivated parameters

The cell area
$$Sa = \frac{3\sqrt{3} \ aCC^2}{4} = 0.0261938 nm^2$$

II.2. The required functions-----

II.2.1. The function $fSpiraleLen[1, \varphi In, \varphi Out, h]$ defines

the Length of a Spirale with the inner agle φ In and the outer angle φ Out(>= φ In):

$$\text{fSpiraleLen[1, φIn, φOut, h] = } \frac{\text{h}\left(-\varphi\text{In}\,\sqrt{1+\varphi\text{In}^2}\right. + \varphi\text{Out}\,\sqrt{1+\varphi\text{Out}^2}\right. - \text{ArcSinh}[\varphi\text{In}] + \text{ArcSinh}[\varphi\text{Out}]\right)}{4\,\pi}$$

II.2.2. The function $fElast[\varphi In, \rho Out]$ is required to calculate an nanoscrollelastic energy:

$$\texttt{fElast}[\varphi \texttt{In}, \varphi \texttt{Out}] \ = \ \frac{\sqrt{1 + \varphi \texttt{In}^2}}{\varphi \texttt{In}} - \frac{\sqrt{1 + \varphi \texttt{Out}^2}}{\varphi \texttt{Out}} - \texttt{ArcSinh}[\varphi \texttt{In}] + \texttt{ArcSinh}[\varphi \texttt{Out}] \,.$$

II.2.3. Define the function $fPhiOutvsPhiInLh[1, \varphi In, L, h]$.

The function fPhiOutvsPhiInLh[1, φ In,L,h] = $\sqrt{\frac{4 \text{ L} \pi}{\text{L}} + \varphi$ In² is a good approximation to obtain the value of φ Out for the defined φ In,L,h.

The inverse function fPhiInvsPhiOutLh[1, φ Out, L, h]] = $\sqrt{-\frac{4 \text{ L} \pi}{\text{L}} + \varphi \text{Out}^2}$

could be used in the program applications

if ROut (instead of RIn) is the input parameter of the system.

II.2.4. The functions fSpiralelUnder(Over)SpiralelLength[NoLv,PhiIn1v ,PhiOut1v,hv]

These functiona are not required, but could be helpful),

fSpirale1UnderSpirale1Length[1,PhiIn1v ,PhiOut1v,hv] =

=fSpiraleLen[NoLv,PhiIn1v ,PhiOut1v-2Pi,hv] =
$$\frac{1}{4\pi}$$

$$\texttt{hv}\,\,\texttt{NoLv}\,\left(-\,\texttt{PhiIn1v}\,\sqrt{\,1\,+\,\texttt{PhiIn1v}^2}\,\,+\,\sqrt{\,1\,+\,\left(\,\texttt{PhiOut1v}\,-\,2\,\,\pi\right)^{\,2}}\,\,\left(\,\texttt{PhiOut1v}\,-\,2\,\,\pi\right)\,\,-\,3\,\,\text{PhiIn1v}\,\sqrt{\,1\,+\,\texttt{PhiIn1v}^2}\,\,+\,\sqrt{\,1\,+\,\left(\,\texttt{PhiOut1v}\,-\,2\,\,\pi\right)^{\,2}}\,\,\left(\,\texttt{PhiOut1v}\,-\,2\,\,\pi\right)^{\,2}}$$

$$ArcSinh[PhiIn1v] + ArcSinh[PhiOut1v - 2\pi]$$
;

 $\verb|fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiIn1p,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,PhiOut1p,hp]=fSpirale1UnderSpirale1Length[NoLp,hp]=fSpirale1UnderSpirale1Length[NoLp,hp]=fSpirale1UnderSpirale1Length[NoLp,hp]=fSpirale1UnderSpirale1Length[NoLp,hp]=fSpirale1UnderSpirale1Length[NoLp,hp]=fSpirale1UnderSpirale1U$ 1,3.39893(2Pi),5.06316(2Pi),0.3354nm =

=5.22745nm.

fSpiralelOverSpiralelLength[1,PhiInlv ,PhiOutlv,hv] =

=fSpiraleLen[NoLv,PhiIn1v+2Pi ,PhiOut1v,hv] =
$$\frac{1}{4\pi}$$

hv NoLv
$$\left(\text{PhiOutlv} \sqrt{1 + \text{PhiOutlv}^2} + (-\text{PhiIn1v} - 2\pi) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} \right) + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right) \sqrt{1 + (\text{PhiIn1v} + 2\pi)^2} + \left(-\text{PhiIn1v} - 2\pi \right)$$

$$ArcSinh[PhiOutlv] - ArcSinh[PhiInlv + 2 \pi]$$
;

fSpirale1OverSpirale1Length[NoLp,PhiIn1p ,PhiOut1p,hp]=fSpirale1OverSpirale1Length[1,3.39893(2Pi),5.06316(2Pi),0.3354nm =

=6.62623nm.

II.2.4. The function fRIn1Sharp[NoLv,L1v,hv]

fRIn1Sharp[NoLv,L1v,hv] =
$$-\frac{hv NoLv}{2} + \frac{L1v}{2 \pi}$$

is a good approximation to obtain the value of the sharp in the dependence ScrollEnergy[RIn] fRIn1Sharp[1, 14.839nm, 0.3354nm] = 2.194nm

III. Begin of Calculation

III.1. The inner and the outer angle of the spirale of the layer:

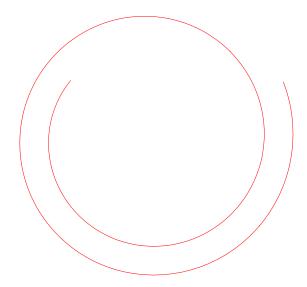
$$\varphi In1 = \frac{2 \pi RIn1}{h}, \quad \varphi Out1 = fPhiOutvsPhiInLh[1, \varphi In1, L1, h];$$

For RIn1=1.14nm, h=0.3354nm:

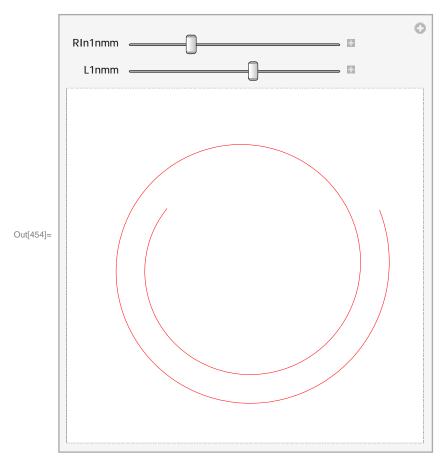
 φ In1=3.39893(2Pi), Φ Out1=5.06316(2Pi).

L1=14.839nm, RIn1=1.14nm

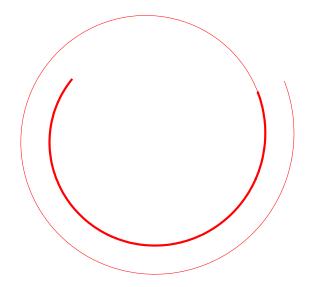
Plot the Spirale of the layer:

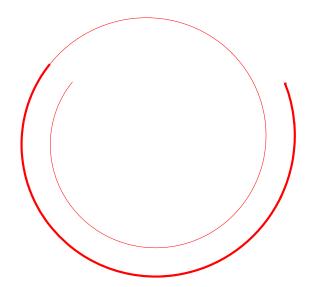


Manipulating of Spirale1Plot for the different RIn1 and L:



 $\{ \verb|Spirale| a, \verb|Spirale| 1| \}, \{ \verb|Spirale| a, \verb|Spirale| 1| \};$





III.2. The nanoscroll energy calculation III.2.1. The elastic energy calculation EelastC=5.7333eV/atom III.2.2. The Van-der-Waals energy calculation EVdWvardPhi12allp[[iEVdW]]=-7.91941eV/atom (EVdWvardPhi12allp[[iEVdW1Un1]]=-3.49244eV/atom EVdWvardPhi12allp[[iEVdW1Ov1]]=-4.42696eV/atom) III.4. The total energy of the nanoscroll fScrollEnergy[1,Lwp,L1p, RIn1p,hp, aCCp, epsVdWp,CCp,CBNp]= =fScrollEnergy[1, Lw=1.nm, L1=14.839nm, RIn1=1.14nm, h=0.3354nm,

```
aCC=0.142nm, epsVdW=0.035eV/atom, CC=2.01(eV AA^2/atom)=
          =-2.18611eV/atom
```

III.5. Determine the inner angles mismatch for the bi-layer nanoscroll for the high nanoribbon Length

For L1=14.839nm,RIn=1.14nm,h=0.3354nm and dPhi12=0:

ScrollEnergy=-2.18611eV/atom

For L1=14.839nm, RIn=1.14nm, h=0.3354nm and dPhi12=Pi:

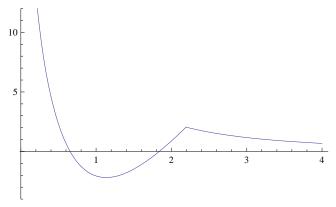
ScrollEnergy=-2.18611eV/atom

IV. The potential energy of the nanoscroll

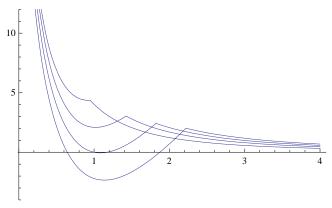
as a function of the inner radius RIn

NoL=1

 ${\tt epsVdW=0.035eV/atom,\ C=0.0201(eV\ nm^2/atom)(eV\ nm^2/atom),aCC=0.142nm,h=0.3354nm,m=0.33540nm,m=0.3354nm,m=0.33540nm,m=0.33540nm,m=0.33540nm,m=0.33540nm,m=0.33540nm,m=0.33540nm,m=0$ Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1=14.839nm (NoL=1,Lw=1.nm



Plot ScrollEnergy[RIn1/nm]/(eV/atom) for L1={7., 10., 12.5, 15.}nm (NoL=1,w=1.nm)



The examples of using of 'fScrollEnergy[..]' function:

fScrollEnergy[NoLp,Lwp,tL1[[1]],RIn1p,hp, aCCp, epsVdWp,CCp,CBNp]=3.26809 eV/atom fScrollEnergy[NoLp,Lwp,tL1[[1]],1nm,hp, aCCp, epsVdWp,CCp, CBNp]=4.01569 eV/atom fScrollEnergy[NoLp,Lwp,7nm,1nm,hp, aCCp, epsVdWp,CCp, CBNp]=4.01569 eV/atom

fEVdWLayer10verlap[1, Lwp, 7.nm, 1.nm, hp, aCCp, epsVdWp][[1]]= 0.39374eV/atom (right != 0 value, because the layer overlaps fEVdWLayer10verlap[1,Lwp,7.nm,1.5nm,hp,aCCp,epsVdWp][[1]]= 12.2479eV/atom !=0, wrong value of the $\verb|fevolute| \verb| fevolute| \verb| function| because the layer does not not overlap| \\$

The analytical expressions of the fEVdWLayer1Overlap[..] function:

--- fEVdWLayer1Overlap[NoL1, Lwv, Lpv, RInv, hv, aCCv, epsVdWv][[1]]: ---

$$-\frac{1}{6\sqrt{3}} \text{epsVdWv hv Lwv}$$

$$\left(\sqrt{\frac{4 \text{ Lpv } \pi}{h \text{v}} + \frac{4 \pi^2 \text{ RInv}^2}{h \text{v}^2}} \sqrt{1 + \frac{4 \text{ Lpv } \pi}{h \text{v}} + \frac{4 \pi^2 \text{ RInv}^2}{h \text{v}^2}} + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right) \sqrt{1 + \left(2 \pi + \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} \right) + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right) \sqrt{1 + \left(2 \pi + \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} \sqrt{1 + \left(-2 \pi - \frac{2 \pi \text{ RInv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \pi^2 \text{ RInv}^2} - \frac{1}{2 \pi \pi^2$$

$$\operatorname{ArcSinh}\left[2\,\pi + \frac{2\,\pi\,\operatorname{RInv}}{\operatorname{hv}}\right] + \operatorname{ArcSinh}\left[\sqrt{\frac{4\,\operatorname{Lpv}\,\pi}{\operatorname{hv}} + \frac{4\,\pi^2\,\operatorname{RInv}^2}{\operatorname{hv}^2}}\,\right] - \frac{1}{6\,\sqrt{3}\,\operatorname{aCCv}^2\,\pi}\operatorname{epsVdWv}\operatorname{hv}\operatorname{Lwv}\left[\sqrt{\frac{4\,\operatorname{Lpv}\,\pi}{\operatorname{hv}^2} + \frac{4\,\pi^2\,\operatorname{RInv}^2}{\operatorname{hv}^2}}\,\right]$$

$$\left(-\frac{2 \pi \text{RInv} \sqrt{1 + \frac{4 \pi^2 \text{RInv}^2}{h v^2}}}{h v} + \left(-2 \pi + \sqrt{\frac{4 \text{Lpv} \pi}{h v} + \frac{4 \pi^2 \text{RInv}^2}{h v^2}} \right) \sqrt{1 + \left(-2 \pi + \sqrt{\frac{4 \text{Lpv} \pi}{h v} + \frac{4 \pi^2 \text{RInv}^2}{h v}} \right)^2} - \frac{1}{h v} \right)^2 } \right)$$

$$\operatorname{ArcSinh}\left[\frac{2\,\pi\,\operatorname{RInv}}{\operatorname{hv}}\right] - \operatorname{ArcSinh}\left[2\,\pi - \sqrt{\frac{4\,\operatorname{Lpv}\,\pi}{\operatorname{hv}} + \frac{4\,\pi^2\,\operatorname{RInv}^2}{\operatorname{hv}^2}}\,\right]$$

--- fEVdWLayer10verlap[NoL2,Lwv,Lpv,RInv,hv,aCCv,epsVdWv][[1]]: ---

$$-\frac{1}{3\sqrt{3}}$$
 accv² π

$$\texttt{epsVdWv hv Lwv} \left(\sqrt{\frac{2 \texttt{Lpv}\,\pi}{\texttt{hv}} + \frac{\pi^2 \, \texttt{RInv}^2}{\texttt{hv}^2}} \, \sqrt{1 + \frac{2 \, \texttt{Lpv}\,\pi}{\texttt{hv}} + \frac{\pi^2 \, \texttt{RInv}^2}{\texttt{hv}^2}} \right. \\ + \left(-2 \, \pi - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}^2} \right) \\ = \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \left(-2 \, \pi - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right) \sqrt{1 + \left(2 \, \pi + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} + \frac{\pi \, \texttt{RInv}}{\texttt{hv}} \right)^2} - \frac{\pi \, \texttt{RInv}}{\texttt{hv}} + \frac{\pi$$

$$\operatorname{ArcSinh} \left[2\,\pi + \frac{\pi\,\text{RInv}}{\text{hv}} \right] + \operatorname{ArcSinh} \left[\sqrt{\frac{2\,\text{Lpv}\,\pi}{\text{hv}} + \frac{\pi^2\,\text{RInv}^2}{\text{hv}^2}} \,\right] - \frac{1}{3\,\sqrt{3}\,\operatorname{aCCv}^2\pi} \operatorname{epsVdWv} \operatorname{hv} \operatorname{Lwv} \right] = 0$$

$$\left(-\frac{\pi \operatorname{RInv} \sqrt{1 + \frac{\pi^2 \operatorname{RInv}^2}{hv^2}}}{hv} + \left(-2 \pi + \sqrt{\frac{2 \operatorname{Lpv} \pi}{hv} + \frac{\pi^2 \operatorname{RInv}^2}{hv^2}} \right) \sqrt{1 + \left(-2 \pi + \sqrt{\frac{2 \operatorname{Lpv} \pi}{hv} + \frac{\pi^2 \operatorname{RInv}^2}{hv^2}} \right)^2} - \right)$$

$$\operatorname{ArcSinh}\left[\frac{\pi\operatorname{RInv}}{\operatorname{hv}}\right] - \operatorname{ArcSinh}\left[2\pi - \sqrt{\frac{2\operatorname{Lpv}\pi}{\operatorname{hv}} + \frac{\pi^2\operatorname{RInv}^2}{\operatorname{hv}^2}}\right]$$

The Analytical expression of

fScrollEnergyVdWandElast[1,Lwv,L1v,RIn1v,hv,aCCv, epsVdWv, CCv,CBNv]:

$$8 \; \text{CCv Lwv} \; \pi \left(\frac{\text{hv} \sqrt{1 + \frac{4 \, \pi^2 \, \text{Rinlv}^2}{\text{hv}^2}}}{2 \, \pi \, \text{RInlv}} - \frac{\sqrt{1 + \frac{4 \, \text{Llv} \, \pi}{\text{hv}} + \frac{4 \, \pi^2 \, \text{RInlv}^2}{\text{hv}^2}}}{\sqrt{\frac{4 \, \text{Llv} \, \pi}{\text{hv}} + \frac{4 \, \pi^2 \, \text{RInlv}^2}{\text{hv}^2}}} \right) - \text{Arcsinh} \left[\frac{2 \, \pi \, \text{Rinlv}}{\text{hv}} \right] + \text{Arcsinh} \left[\sqrt{\frac{4 \, \text{Llv} \, \pi}{\text{hv}} + \frac{4 \, \pi^2 \, \text{Rinlv}^2}{\text{hv}^2}} \right] \right]$$

$$3\sqrt{3}$$
 accv² hv

$$\frac{1}{6\sqrt{3}} \frac{1}{\text{aCCv}^2 \pi} \text{epsVdWv hv Lwv}$$

$$\left(\sqrt{\frac{4 \text{ Llv } \pi}{h \text{v}} + \frac{4 \pi^2 \text{ RInlv}^2}{h \text{v}^2}} \sqrt{1 + \frac{4 \text{ Llv } \pi}{h \text{v}} + \frac{4 \pi^2 \text{ RInlv}^2}{h \text{v}^2}} + \left(-2 \pi - \frac{2 \pi \text{ RInlv}}{h \text{v}}\right) \sqrt{1 + \left(2 \pi + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} \right) + \left(-2 \pi - \frac{2 \pi \text{ RInlv}}{h \text{v}}\right) \sqrt{1 + \left(2 \pi + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}\right)^2} - \frac{1}{2 \pi \text{ RInlv}} + \frac{2 \pi \text{ RInlv}}{h \text{v}}$$

$$\operatorname{ArcSinh}\left[2\,\pi + \frac{2\,\pi\,\mathrm{RIn1v}}{\mathrm{hv}}\right] + \operatorname{ArcSinh}\left[\sqrt{\frac{4\,\operatorname{L1v}\,\pi}{\mathrm{hv}} + \frac{4\,\pi^2\,\mathrm{RIn1v}^2}{\mathrm{hv}^2}}\,\right] - \frac{1}{6\,\sqrt{3}\,\mathrm{aCCv}^2\,\pi}\mathrm{epsVdWv\,hv\,Lwv}$$

$$\left(-\frac{2 \pi \text{RIn1v} \sqrt{1 + \frac{4 \pi^2 \text{RIn1v}^2}{\text{hv}^2}}}{\text{hv}} + \left(-2 \pi + \sqrt{\frac{4 \text{L1v} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RIn1v}^2}{\text{hv}^2}} \right) \sqrt{1 + \left(-2 \pi + \sqrt{\frac{4 \text{L1v} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RIn1v}^2}{\text{hv}^2}} \right)^2} - \frac{1}{1 + \left(-2 \pi + \sqrt{\frac{4 \text{L1v} \pi}{\text{hv}} + \frac{4 \pi^2 \text{RIn1v}^2}{\text{hv}^2}} \right)^2} \right) } \right)$$

$$\operatorname{ArcSinh}\left[\frac{2\,\pi\,\mathrm{RIn1v}}{\mathrm{hv}}\right]-\operatorname{ArcSinh}\left[2\,\pi-\sqrt{\frac{4\,\mathrm{L1v}\,\pi}{\mathrm{hv}}+\frac{4\,\pi^2\,\mathrm{RIn1v}^2}{\mathrm{hv}^2}}\,\right]$$

V. Export the data of the plots of the nanoscroll energy as a function of the inner radius

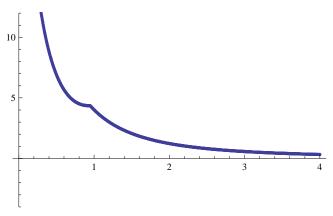
The parameters of the output file

The number of the output points = 1000

Export the plot data to the files:

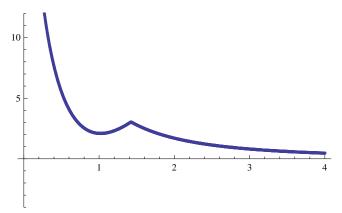
NanoscrollName=Nanoscroll1L7.nm

ScrollEnergyFileName=EvsRIn1Nanoscroll1L7.nm.dat



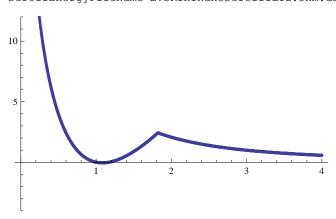
NanoscrollName=Nanoscroll1L10.nm

 ${\tt ScrollEnergyFileName=EvsRIn1Nanoscroll1L10.nm.dat}$



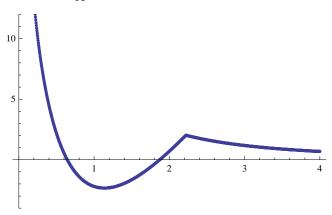
NanoscrollName=Nanoscroll1L12.5nm

ScrollEnergyFileName=EvsRIn1Nanoscroll1L12.5nm.dat

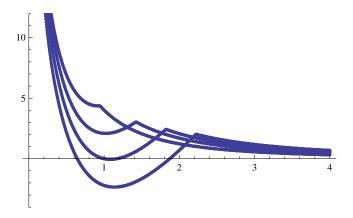


NanoscrollName=Nanoscroll1L15.nm

 ${\tt ScrollEnergyFileName=EvsRIn1Nanoscroll1L15.nm.dat}$



 $\label{eq:polynomial} \texttt{Plot ScrollEnergy[RIn1/nm]/(eV/atom)} \ \ \texttt{for L1=\{7., 10., 12.5, 15.\}nm} \ \ (\texttt{NoL=1,Lw=1.nm})$



Manipulating of the plot of the nanoscroll energy as the function of the nanoscroll inner radius:

```
(Manipulate[Plot[fScrollEnergy[...,L1nmm,RIn1,hp,...]]],
                       where L1nmm (is L1 in nanometers)
                      is the manipulated value
                      )
```

