ALMOST PERIODIC SOLUTIONS OF LATTICE DYNAMICAL SYSTEMS WITH MONOTONE NONLINEARITY

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Let $\mathbb{R}$ (respectively, $\mathbb{Z}$) be the set of all real (respectively, integer) numbers and l_2 be the Hilbert space of all sequences $\xi = (\xi_i)_{i\in\mathbb{Z}}$ ($\xi_i \in \mathbb{R}$ for any $i \in \mathbb{Z}$) with the property $\sum\limits_{i\in\mathbb{Z}} |\xi_i|^2 < \infty$ equipped with the scalar product $\langle \xi, \eta\rangle := \sum\limits_{i\in\mathbb{Z}} \xi_i\eta_i$. Denote by $C(\mathbb{R}, l_2)$ the family of all continuous functions $\varphi : \mathbb{R} \to l_2$ equipped with the compact-open topology and $(C(\mathbb{R}, l_2), \mathbb{R}, \sigma)$ the shift dynamical system on the space $C(\mathbb{R}, l_2)$.

A subset $A \subset \mathbb{R}$ is called relatively dense if there exists a positive number l such that $[a, a + l] \bigcap \mathbb{R} \neq \emptyset$, $\forall a \in \mathbb{R}$.

Definition. A function $\varphi \in C(\mathbb{R}, l_2)$ is said to be almost periodic [1] if for any $\varepsilon > 0$ there exists a relatively dense subset $\mathcal{P}(\varepsilon)$ such that $\|\varphi(t + \tau) - \varphi(t)\|_{l_2} < \varepsilon$

In this talk we study the almost periodic solutions of the systems

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ (i \in \mathbb{Z}), \tag{1}$$

where $\lambda, \nu > 0$, $F \in C(\mathbb{R}, \mathbb{R})$ and $f \in C(\mathbb{R}, \ell_2)$ ($f(t) := (f_i(t))_{i\in\mathbb{Z}}$ for any $t \in \mathbb{R}$) is an almost periodic function.

Theorem 1. Suppose that the following conditions hold:

1. the function $f \in C(\mathbb{R}, l_2)$ is almost periodic;
2. the function F possesses the following properties:
   (a) it is locally Lipschitzian;
   (b) there exists a positive number α such that $F(s)s \leq -\alpha s^2$ for any $s \in \mathbb{R}$;
   (c) the function F is monotone [1], i.e., there exists a positive number β such that $(F(x_1)-F(x_2))(x_1-x_2) \leq -\beta|x_1-x_2|^2$ for any $x_1, x_2 \in \mathbb{R}$.

Then the equation (1) has a unique almost periodic solution.

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